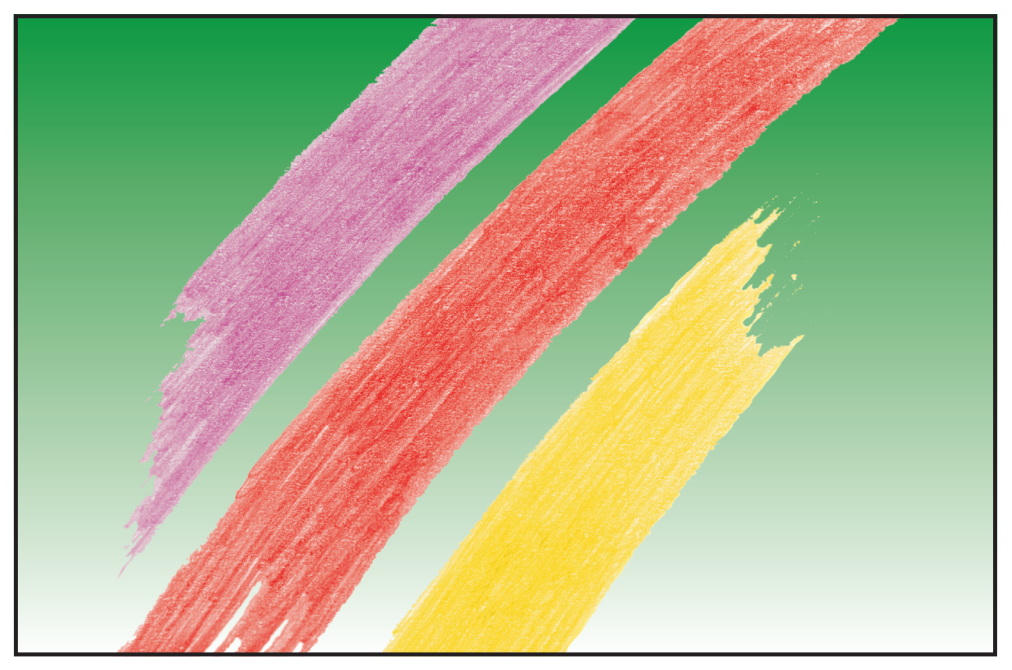
QUANTITATIVE METHODS IN ECONOMICS

Multiple Criteria Decision Making XXII



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CONTENT

IMPACT OF MIXED FREQUENCY DESTINATION ALLOCATION STRATEGIES ON THROUGHPUT OF AMRS-BASED SORTING SYSTEMS: A SIMULATION STUDY

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Abstract

This study investigates destination allocation strategies in Autonomous Mobile Robots -based sorting systems using simulation. Through simulation experiments, we analyze the impact of a promising strategies on system throughput and AMR performance, in a complex layout. Results show that strategy effectiveness varies based on destination distribution of a set of items to be sorted. This research offers insights into designing efficient sorting systems in dynamic logistics environments.

Keywords: *autonomous mobile robots, simulation, sorting systems, allocation*

JEL Classification: L920, C630.
AMS Classification: 37M05, 90B06.

1 INTRODUCTION

Intensive flows of goods with short delivery times and expanding assortments have necessitated the use of sorting as an activity that, although an additional operation required on goods, results in efficient product distribution to end-users. In warehouse systems, sorting enables the implementation of zone picking, which involves less movement of pickers between picking locations and faster item selection. In the express delivery industry, the hub-and-spoke network configuration, i.e., leveraging economies of scale in transportation to and from the hub, would not be justified without an efficient sorting system. Until recently, conveyor-based sorting systems were the only option, involving the application of various combinations of roller and belt conveyors, slides, and auxiliary unloading equipment from the system. Such systems are characterized by high speeds of movement but also limited flexibility and significant space required for implementation. Therefore, any changes to the configuration of an existing system, or to the capacity require substantial resources, funding, and downtime.

On the other hand, intensive technological development in the past 15 years (Siegwart et al., 2011) in hardware (sensors, batteries (McNulty et al., 2022), ...) and software, primarily the application of artificial intelligence in various functional segments (Fragapane et al., 2021), has led to the emergence and increasing use of Autonomous Mobile Robots (AMRs). Due to their adaptability, AMRs have found application in various material handling activities.

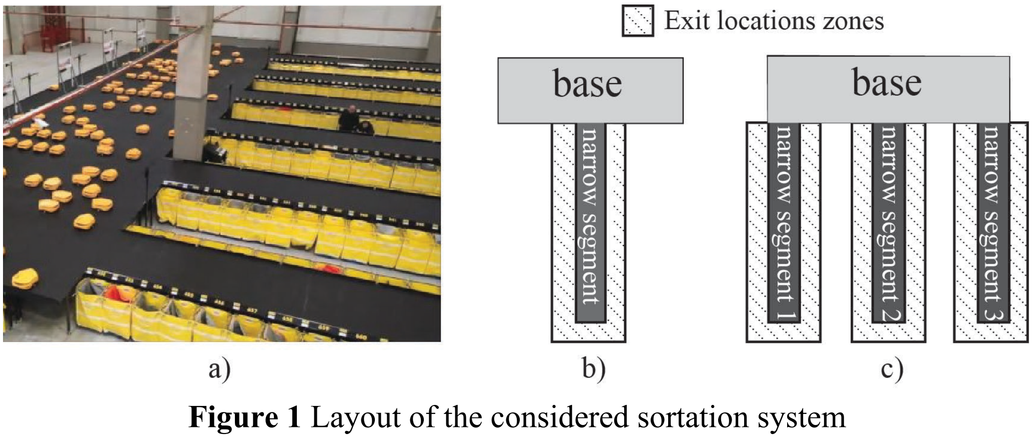
Although they represent a new option for handling tasks, the operation of such systems entails making decisions that are inherently similar to those characterizing dominating conveyor-based solutions. Namely, questions such as how many resources are needed, what loading capacity can be achieved, how to allocate goods over the space, which routing algorithms to use, where are idle points, how to zone space, etc., are integral parts of designing and managing systems based on AMRs. Of course, in providing answers to these questions, i.e., in solving relevant problems, it is necessary to consider the specificities of the observed system. In this regard, this paper is an extension of the paper (Bjelić and Golubović, 2023) in which authors considered the problem of allocating the set of destination locations to sorting system exit locations for the case of AMR-based sortation system with frequently used layout. In this research we go deeper into the simulation analysis of the allocation strategy that provided the highest throughput.

The paper is organized into 5 chapters. The second chapter describes the observed problem with a brief review of relevant literature. The third chapter presents the simulation model. All details of the simulation experiment, including the most important results, are provided in Chapter four. Finally, concluding considerations are presented at the end, within the fifth chapter.

2 PROBLEM DESCRIPTION

The sorting process in systems with AMRs involves the independent operation of each robot. In general, each of the AMRs goes through the following stages in operation: picking up the item at one of the loading locations, transporting the item to the exit location, unloading the item at the exit location (into a container, bag, box, etc.), and finally returning to the loading location to load a new item. Naturally, designing and managing such systems is extremely complex because it involves a whole range of decisions made at various hierarchical levels, which have different levels of impact on sorting efficiency. Some of these decisions include: loading method (manual or using auxiliary robots), defining the number and locations of loading locations, managing battery recharges (number of recharge locations, locations, recharge technology...), defining robot paths, regulating vehicle deadlocks, allocating destinations to exit locations...

We considered a widely used layout of the system, implemented for example in one of the largest Turkish parcel distribution companies in 2023. (Figure 1a), as well as in one of the leading logistics provider in Serbia in 2024. This layout is characterized by the ability to position a large number of exit locations but with reduced maneuvering space for robots in certain areas. Specifically, the observed type of layout is known as the T-layout due to its resemblance to the letter "T" (Figure 1b). The operational space consists of a wider segment (base), within which the loading and battery charging locations are positioned; and narrower segments, positioned orthogonal to the base, along which the exit locations are positioned, and along which the robots move to unload goods into the appropriate exit location. It should be mentioned that it is also possible to locate exit locations alongside edges of the base segment as well. However, that solution is less preferred compared to the alternative in which additional parallel narrow segments are added to the base, accompanied by the increase of the base's surface so that all segments are connected (Figure 1c). Accordingly, in this research we considered only the settings of the system in which exit locations are aligned alongside the narrow segments. The minimum width of the narrower segment is two widths of AMRs to allow for their passing. However, in order to minimize deadlocks of robots we assumed the widths of narrower segments of three AMRS' widths.



a) b) c)

Figure 1 Layout of the considered sortation system

Taking into account the reduced maneuverability of AMRs in narrower segments, it is clear that the allocation of destination points to exit locations, i.e., destination allocation, can have a significant impact on obstructing and potentially locking AMRs. It should be emphasized that obstructing, in this context, refers to an increase in the travel time of AMRs to the exit location, which occurs due to the presence of other AMRs. With an increase in the number of AMRs, the intensity of obstructing increases, as well as the likelihood of AMRs reaching a deadlock state, which refers to a situation where two or more AMRs cannot determine a path to the exit location due to being surrounded by other AMRs in subsequent time periods. In this sense, it is clear that destination allocation to exit locations can impact the level of obstruction and deadlock. More precisely, if a group of destinations with a higher exiting frequency is located in one narrower segment, there is a greater chance of obstruction and deadlock of robots. Additionally, if exits for more frequent destinations are located closer to the base of the narrower segment, and for less frequent destinations farther from the base, obstructing of AMRs carrying items to less frequent destinations will also have a high probability of obstructing and deadlocking. In (Bjelić and Golubović, 2023) simulation study showed that allocation of destinations to exits in which frequent destinations are allocated as far as possible of the base segment (referred as the Furthest strategy) gave the lowest levels of obstructing and deadlocks. However, since locating frequent destinations in groups, as in mentioned research, may influence levels of deadlocks and obstructions in this research we intended to check effects of mixed allocation strategy in which groups of most and least frequent destinations are allocated next to each. We experimented with group sizes of 1, 2, 4, 6, 8 and 10 destinations (referred to as Mix1, Mix2, Mix4, Mix6, Mix8 and Mix10 strategies in the rest of the paper), and compared obtained results with allocation strategies from mentioned research. Examples of allocating groups of 1, 2, and 4 destinations on parts of different narrow segments are depicted on Figure 2, respectively.

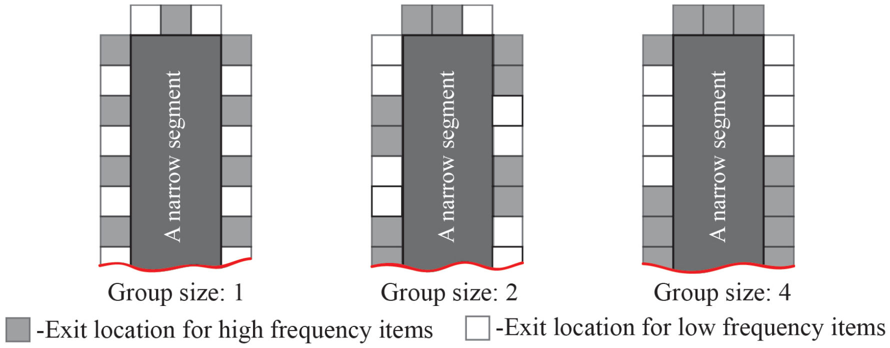


Figure 2 An illustrative examples of Mix1, Mix2 and Mix4 allocation strategies within narrow

As already mentioned, the frequency of occurrence of a destination is one of the parameters that can influence the destination allocation problem. It may be very useful to consider this parameter because it is known in a sufficiently long period before the sorting process begins. The impact of this parameter is respected, as in (Bjelić and Golubović, 2023) by analyzing three sets of results obtained in three practical situations based on the Pareto principle. The first case implies even distribution of items to exits, i.e., 80% of items are unloaded at 80% of exit locations. In the second case, 80% of unloading operations is realized at 50% of exit locations (destinations). The third case implies domination of several destinations, so we assumed that 80% of unloads are realized at 20% of exit locations. In accordance with the percentage of destinations to which 80% of items are unloaded, cases are referenced by the labels "80," "50," and "20."

The use of AMRs in sorting systems, as well as in other handling operations, is a relatively new concept, so it is entirely expected that the number of models used for managing systems with AMRs is extremely small. Besides mentioned work of Bjelić and Golubović, we would like to mention paper (Fragapane et al., 2021). In it the authors provide an overview of management issues for systems based on AMRs, including issues related to fleet size dimensioning, routing, time engagement, zoning, dispatching, etc. On the other hand, the specificity of each observed system requires an individualized approach to managing such systems. For these reasons, it is clear that simulation is widely used in managing such systems. In the mentioned review paper, the authors state that in problems defining the level of decentralization in systems with AMRs, defining the number and types of robots, zoning, and resource management, simulation is used in 32 out of 56 reviewed papers.

3 SIMULATION MODEL OF THE SORTING SYSTEM

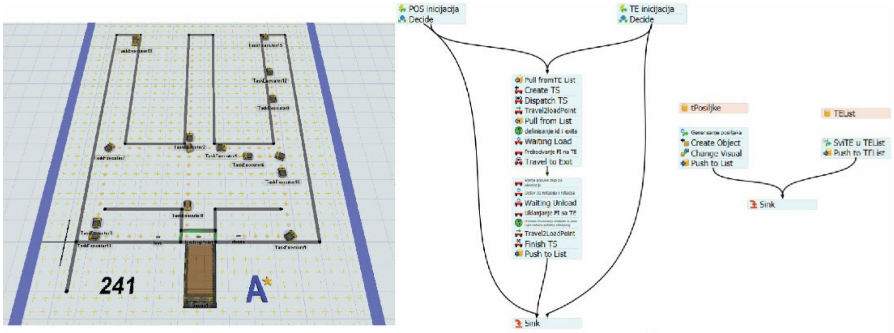
The number of items transferred from loading(input) to exit (unloading) locations during an hour of operation is an ultimate parameter which encompasses the influence of all respected characteristics of the sorting system. For this reason, (Fragapane et al., 2021) cites this parameter as the most commonly used in studies employing simulation as a solving approach. Accordingly, in this research, achieved capacity is regarded as the primary indicator of system efficiency in various configurations of input settings.

By its nature, the operation of sorting systems falls into the category of processes that cannot continue indefinitely. In other words, they are characterized by work that always starts from the same initial conditions and ends with the sorting of the last item. In line with the above and as stated in (Marklund and Laguna, 2018; Kelton et al., 2015; Banks et al., 2014), it is clear that analyzing the behavior of such a system is only possible with data resulting from multiple repetitions of the simulation experiment with different (random) sets of items, yet following the appropriate destination allocation strategy. However, as each experiment realization needs to be entirely independent of the others, it is necessary to respect the existence of a warm-up period in each experiment realization, after which the AMRs will be positioned at random locations within the layout.

The simulation model was developed using the FlexSim 2021 software package, employing a combined implementation of 3D models and ProcessFlow tools (Figure 3). In practice, this modeling combination has proven to be very efficient because it allows for the straightforward implementation of model logic through the ProcessFlow tool, while, on the other hand, enabling easier model validation by visualizing the 3D model (Figure 3a). FlowItems were used to represent items in the 3D model, and their generation was controlled by applying the appropriate list and push and pull activities in ProcessFlow. Upon creating a FlowItem in the 3D object of the Queue type, located near the input location, i.e., in the middle of the base part of the "T" layout on the opposite side of the narrower segments, the mechanism for assigning cargo to AMRs is activated. AMRs are modeled by using TaskExecuters, whereby the required number of AMRs is generated upon loading the model and placed alternately at locations to the left and right of the input location. To avoid scenarios in which there are no items for sorting at the loading location, i.e. situations in which the number of sorted items is not caused by the system’s allocation settings but by the shortage of items for sorting, we set the model in such way that there are always items for sorting at the input point.

After assigning the task of transferring the item to the appropriate exit location, the AMR moves from the left/right location to the loading location, where it remains for the specified time interval for operations such as loading cargo onto the robot, scanning documents, etc. After loading the item, the robot continues moving towards the exit location. Loading area is set in such way that it can contain only one AMR, so that the next AMR for loading enters the loading area only when currently loading AMR leaves it.

The implementation of the desired layout and the characteristics of AMR movement in the narrow segment (presence of three paths) were realized by incorporating Flexsim's 3D A* tool, where setting barriers defines the parts of the surface along which AMRs can move. Additionally, the grid on which robots move is divided into segments that can only contain one AMR at a time. The selection of a sequence of segments for robot movement from the input to the output location, i.e., defining the path, was achieved by applying the widely implemented A* routing algorithm (De Ryck et al., 2020; Mahulea et al., 2019; Siegwart et al., 2011), which is an integral part of Flexsim's tool and after which the tool was named. AMRs move along fields with orthogonal movement with 90-degree rotations at each change of direction. The robot turning time is a parameter specified in the implemented A* tool.



a) b)

Figure 3 3D and ProcessFlow views of the simulation model

Upon reaching the desired exit location, depending on the relationship between the required and current orientation of the robot, the robot is turned in the desired direction, and the cargo unit is unloaded, i.e., the AMR is deleyed at the location for the time specified by the technical features of the system. After unloading and incrementing the sorting counter, the AMR is directed to the location to the left or right of the input location. Upon arrival at that location, AMRs become available for new loading. The barrier system around the input location ensures that only one AMR from each side is available for loading at all times because until it is engaged in loading, by standing on the segment where the left/right location is, it locks others from approaching that location and completing their active task.

The behavior of AMRs, in terms of the sequence of operation realization, is implemented by applying the concept of Task Sequences, as shown in Figure 3b. It should be emphasized that all input parameters regarding the system settings and all its elements are defined by using appropriate tables.

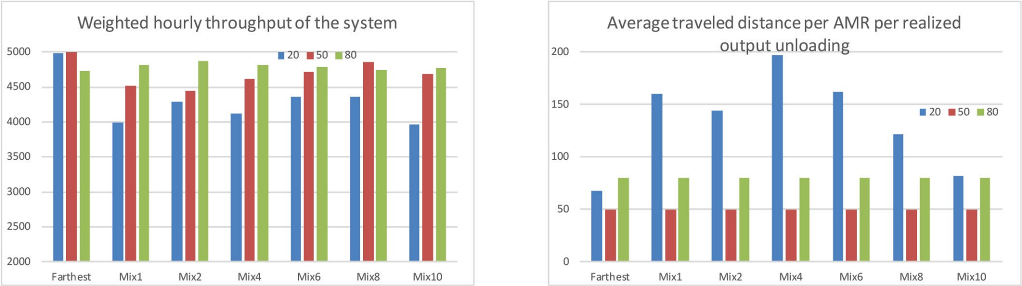
4 NUMERICAL EXPERIMENTS

The simulation model concept, described in the previous section, was applied to a specific example of a sorting system with the characteristics exactly the same as in the case of the research done by (Bjelić and Golubović, 2023) with the only change related to the size of AMRs fleet. Namely, in this study we considered fleet size of 30 AMRs because in the beforementioned study that number turned out to provide the highest system throughput.

To ensure high reliability of the obtained results 100 experiment replications were conducted. In addition to data related to the number of completed unloads we collected data that gave us the possibility to respect occurrences of deadlocks and therefore calculate weighted hourly throughput (*WHT*) of the system. By weighted throughput we consider a throughput which would have been achieved during one hour if deadlock had not occurred and the system had worked whole hour. If *CU* denotes the number of actually completed unloads and t_d the moment of dead lock, i.e. the moment of last completed unloading task prior to the deadlock, than WHT can be calculated as:

$$WHT = \frac{3600}{t_d} CU \tag{1}$$

However, it should be mentioned that weighted throughput is respected only for experiments in which moment of the deadlock happened after the threshold level of 70s after the end of the warm-up period. Otherwise, our conclusions would have been made on outliers in samples.



a) b)

Figure 4 Results of the numerical experiments

Based on the numerical experiments' results from Figure 4 it can be seen that the structure of destinations distribution has a significant impact on the allocation strategy to be implement in the considered sorting system. More precisely, it can be seen that the expected throughput is the highest for the Furthest strategy in the case of "20" and "50" distribution scenarios, but in the case of "80" scenario "Mix2" allocation strategy provided highest throughput. On the other hand, performance of the AMR related to their expected traveled distance during the simulated period showed the absolute resilience on the implemented allocation strategy in the case of "50" and "80" scenarios. In the case of "20" scenario Furthest allocation strategy showed the best performance, but just slightly lower then Mix10 allocation strategy which is the most resembling strategy to the Furthest strategy.

5 CONCLUSIONS

Sorting systems based on the implementation of AMRs, due to advantages such as ease of robot integration/deployment, project implementation speed, and reduced spatial requirements, represent a highly promising alternative to currently dominant conveyor-based sorting systems. In this research, we focused on analyzing the problem of destination allocation in AMRs-based sorting systems and on understanding the impact of different allocation strategies on the efficiency of the sorting system, especially in complex layouts such as the T-layout. The tool we used for reaching the goal was simulation modelling realized by Flexsim 2021 simulation package.

Through simulation experiments, we investigated how different allocation strategies affect the overall system throughput, as well as the performance of AMRs in terms of traveled distance and potential deadlocks. Our results showed that the optimal allocation strategy depends on the distribution of destination locations, i.e., that allocation strategy should be changed in accordance to the structure of items' destinations.

References

[1] Banks, J., Carson, J., Nelson, B. and Nicol, D. (2014) Discrete-Event System Simulation, 5th Ed., Pearson Education Limited.
[2] Bjelić, N. and Golubović, N. (2023) 'Analiza uticaja strategija alokacije odredišta i veličine flote autonomnih mobilnih sortirnih robota na proizvodnost primenom simulacionog modela u Flexsim-u', XLI Symposium of New Technologies in Postal Transportation and Telecomunication Traffic – POSTEL 2023, University of Belgrade - Faculty of Transport and Traffic Engineering, p. 21.-30.
[3] De Ryck, M., Versteyhe, M. and Debrouwere, F. (2020) 'Automated guided vehicle systems, state-of-the-art control algorithms and techniques', Journal of Manufacturing Systems, vol. 54, pp. 152–173.
[4] Fragapane, G., de Koster, R., Sgarbossa, F. and Strandhagen, J. O. (2021) 'Planning and control of autonomous mobile robots for intralogistics: Literature review and research agenda', European Journal of Operational Research, vol. 294, no. 2, pp. 405–426.
[5] Kelton, D., Sadowski, R. and Zupick, N. (2015) Simulation with Arena, 6th Ed., NY, USA, McGraw-Hill Education.
[6] Mahulea, C., Kloetzer, M. and Gonzalez, R. (2019) Path Planning of Cooperative Mobile Robots Using Discrete Event Models, 1st edition., Hoboken, Wiley-IEEE Press.
[7] Marklund, M. and Laguna, J. (2018) Business Process Modeling, Simulation and Design, 3rd Ed., New York, Chapman and Hall/CRC.
[8] McNulty, D., Hennessy, A., Li, M., Armstrong, E. and Ryan, K. M. (2022) 'A review of Li-ion batteries for autonomous mobile robots: Perspectives and outlook for the future', Journal of Power Sources, vol. 545, p. 231943.
[9] Siegwart, R., Reza Nourbakhsh, I. and Scaramuzza, D. (2011) Introduction to Autonomous Mobile Robots, The MIT Press

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HOW TO ADAPT THE FRAME PROCEDURE FOR MULTI-CRITERIA DECISION-MAKING TO MEASURE ARTIFICIAL INTELLIGENCE SUPPORT TO HUMAN RESOURCE MANAGEMENT

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Abstract

This paper aims to adapt the frame procedure for multi-criteria decision-making based on assigning weights to measure the level of artificial intelligence (AI) support to human resource management (HRM). Based on an extensive survey on AI in Slovenian enterprises, the methodology used includes factor analysis and an analytic hierarchy process. The results of factor analysis were used to structure the hierarchical multi-criteria model. Pairwise comparisons were used to elicit the individual experts' judgments on the criteria's importance, and group weights were obtained by geometric mean from individual weights and normalization. Increasing linear value functions were used to measure the local alternatives' values based on their means. This paper also aims to illustrate the developed procedure in selected industries. The results show that among the industries under consideration, the highest level of AI support to HRM was found in financial and insurance activities, which should serve as a benchmark concerning 'AI-supported appropriate training and development of employees' and 'reducing the workload of employees with AI.' According to 'AI-supported acquiring and retaining talented employees', professional, scientific, and technical activities stand out.

Keywords: *Analytic hierarchy process, Artificial intelligence, Factor analysis, Geometric mean, Human resource management, Multiple criteria*

JEL Classification: C44, C83, M50
AMS Classification: 62H25, 90B50, 91B06

1 INTRODUCTION

Artificial intelligence (AI) in human resource management (HRM) significantly transforms talent acquisition, training practices, and workload management, enabling companies to optimize their processes and enhance efficiency (Jatobá et al., 2023). With advanced analytical tools, HR departments can more effectively identify and attract talented candidates, providing personalized training and development programs tailored to individual needs and goals (Kambur and Akar, 2022). Moreover, AI automates routine administrative tasks, reduces workload, and allows employees to focus on more complex and value-added activities (Qiu et al., 2022). This improves productivity and employee satisfaction and contributes to a more dynamic and competitive working environment (Agarwal, 2023).

To take and consequently implement appropriate measures to improve AI support for HRM, it is, therefore, necessary to first measure its values according to individual criteria at various levels and according to all criteria, and then compare them for individual alternatives and determine the ones that could serve as benchmarks. For this purpose, the frame procedure for multi-criteria decision-making (MCDM) based on assigning weights (Čančer, 2012) can be followed. This paper therefore aims to adapt the frame procedure for MCDM based on assigning weights to measure the level of AI support to HRM. It introduces solutions on how to integrate statistical analysis results into multi-criteria measurement. It also aims to illustrate the developed procedure in selected industries in Slovenia.

This paper aims to answer the following research questions:

- What methods are appropriate to be included in the frame procedure for MCDM so that it can be used to measure AI support for HRM?
- Which service sector industry has the highest level of AI support for HRM?
- Which service sector industries should serve as a benchmark according to each construct describing AI support to HRM?

The structure of the rest of the paper is as follows. The next section introduces the methodology used, followed by the presentation of the results. The paper concludes by answering the research questions, discussing the main findings, and pointing out limitations and further research possibilities.

2 METHODOLOGY

The mixed methodology including the questionnaire-based survey, descriptive and factor analysis, and analytic hierarchy process together with benchmarking was employed.

2.1 Data gathering and analysis

This research is part of the extensive survey on AI in Slovenian enterprises. A closed-type questionnaire with the statements describing constructs ‘AI-supported acquiring and retaining talented employees’ (AR), ‘AI-supported appropriate training and development of employees’ (T), and ‘reducing the workload of employees with AI’ (RW) was completed in 473 Slovenian enterprises. The respondents indicated their agreement with the statements based on a 5-point Likert-type scale from 1 – strongly disagree to 5 – strongly agree. We adopted items for construct AR from Kambur and Akar (2022), construct T from Pillai and Sivathanu (2020), and construct RW from Qiu et al. (2022).

Factor analysis was used to determine the structure of each construct describing AI support to HRM. To justify its use, the Kaiser-Meyer-Olkin measure of sampling adequacy (KMO > 0.5) (Kaiser, 1974) and the results of Bartlett’s test of sphericity were considered. The internal consistency was measured by Cronbach’s alpha coefficient (Costello and Osborne, 2005). The values of communalities were checked if it was necessary to exclude variables with communalities lower than 0.4 (Costello and Osborne, 2005), and thus the structure of each factor, i.e., criterion (in case of one factor) or each sub-criterion (in case of two or more factors) was determined. In the case of more than one factor, the varimax rotation method was used to support the description of factors. Factor weights determined the strength of the relationship between each factor and each variable. Among descriptive statistics, means by each lowest level criterion were calculated and employed to measure the local alternative’s value, i.e., the alternative’s value concerning the lowest level criterion.

2.2 Frame procedure for multi-criteria measurement of AI support to HRM

The frame procedure for MCDM based on assigning weights (Čančer, 2012) – from problem definition and structuring, through criteria weighting, measuring local and aggregate values of alternatives, to sensitivity analysis – has been well verified for the selection of the most appropriate components. This paper adapts this procedure for multi-criteria measurement of AI support to HRM, emphasizing problem structuring, criteria weighting and measuring the local values of alternatives, and using performance analysis for benchmarking.

In problem structuring, factor analysis results were used to determine the problem hierarchy. When one factor was formed from the statements, i.e., variables of each construct, it represented the first-level criterion, and variables with a sufficiently large communality represented the second-level criteria. When two or more factors were formed, they represented the second-level criteria, and variables with a sufficiently large communality represented the third-level criteria in the criteria hierarchy. Groups of sample elements represented alternatives.

Individual weights of the criteria are derived using Saaty's method (Saaty and Sodenkamp, 2010) and then aggregated by geometric mean:

$$g_k = \left(\prod_{i=1}^{n} u_{k,i}\right)^{\frac{1}{n}}, \tag{1}$$

where g_k is the geometric mean for the kth criterion, $u_{k,i}$ is the individual weight of the kth criterion based on judgments of the ith participant, and n is the number of participants making the judgments on the criteria's importance, and normalized:

$$w_k = \frac{g_k}{\sum_{k=1}^{m} g_k}, \tag{2}$$

where w_k is the group weight of the kth criterion, and m is the number of criteria concerning the upper criterion. The calculation of weights for all levels of sub-criteria is carried out according to the same principle.

Increasing linear value functions were used to measure local alternatives‘ values based on the means by lowest-level criteria for each alternative. To make a clear distinction between alternatives, the lower bound was equal to the lowest mean, and the upper bound to the highest mean within each lowest-level criterion. In sensitivity analysis, emphasis was given to performance sensitivity analysis to determine the main advantages and disadvantages in specific industries by criteria and sub-criteria of AI support to HRM and thus enabled benchmarking of AI support to HRM.

3 RESULTS

This section illustrates the adapted frame procedure for MCDM based on assigning weights to measure the level of AI support to HRM in selected industries in Slovenia.

3.1 Problem structuring

The values of the KMO measure of sampling adequacy presented in Tables 1 and 2 (KMO > 0.5) and the results of Bartlett's test of sphericity (p < 0.001) justify the use of factor analysis. High values of Cronbach alpha indicate exemplary internal consistency of each construct. As all communalities were higher than 0.4, no variable has been eliminated.

Table 1 Factor analysis results for the first construct

Construct	Statement	Commu-nality	Factor loading
AR. AI-supported acquiring and retaining talented employees. Cronbach alpha: 0.901	AR1. AI helps in conducting primary interviews of bulk candidates using chatbots.	0.774	0.880
	AR2. AI helps in a better quality of decisions for recruiting and selecting candidates.	0.832	0.912
	AR3. AI technology saves the monotony of the job done during the process of finding candidates.	0.711	0.843
	AR4. AI technology reduces the time spent in finding candidates.	0.773	0.879
Kaiser-Meyer-Olkin measure of sampling adequacy: 0.818, total variance explained – cumulative %: 77.235			

Factor analysis for the first construct AR resulted in one factor (Table 1). Table 2 shows that factor analysis for the second construct T resulted in two factors. Considering the statements with higher absolute values of factor loadings, they were described as ‘contributions of AI technology to in-company training courses’ (T1) and ‘contributions of the AI-supported in-company training courses' (T2). Similarly, the factors of RW were described as 'reducing the burden on administrative staff with AI' (RW1) and 'capabilities of AI to reduce the workload of employees' (RW2).

Table 2 Factor analysis results for the second and the third constructs

Construct	Sub-construct	Statement	Commu-nality	Factor loadings	
T. AI-supported appropriate training and development of employees. Cronbach alpha: 0.819	T1. Contributions of AI technology to in-company training courses. Cronbach alpha: 0.867	T1.1. AI technology reduces the time spent on in-company training courses.	0.676	-0.004	0.822
		T1.2. AI technology reduces the attention deficit experienced by employees in classical in-company training courses.	0.844	-0.065	0.917
		T1.3. AI technology increases accessibility to in-company training courses.	0.839	-0.026	0.915
	T2. Contributions of the AI-supported in-company training courses. Cronbach alpha: 0.823	T2.1. In-company training courses with artificial intelligence technology lead to a successful training program.	0.444	0.662	0.079
		T2.2. Employee professional knowledge will be updated with in-company training courses through artificial intelligence technology.	0.465	0.679	0.059
		T2.3. When the in-company training courses take place with artificial intelligence technology, the restrictions regarding to place where the training will be given will be removed.	0.804	0.869	-0.220
		T2.4. Employees are provided with the required training to deal with AI applications.	0.787	0.880	-0.114
Kaiser-Meyer-Olkin measure of sampling adequacy: 0.706, total variance explained – cumulative %: 69.417					
RW. Reducing the workload of employees with AI. Cronbach alpha:0.807	RW1. Reducing the burden on administrative staff with AI.	RW1. With AI we reduce the burden on administrative staff in the company.	0.969	0.047	0.983
	RW2. Capabilities of AI to reduce the workload of employees. Cronbach alpha:0.834	RW2.1. The AI technology applied in our company can take orders and complete tasks which reduces the workload of employees.	0.608	0.748	0.220
		RW2.2. The AI technology applied in our company can communicate with users/customers which reduces the workload of employees.	0.711	0.837	0.100
		RW2.3. The AI technology applied in our company can search and analyze information which reduces the workload of employees.	0.729	0.853	-0.034
		RW2.4. Artificial intelligence can help in getting the job done which saves employees work time.	0.676	0.820	-0.062
Kaiser-Meyer-Olkin measure of sampling adequacy: 0.750, total variance explained – cumulative %: 73.888					

The criteria hierarchy is presented in Figure 1. To structure the problem hierarchy, alternatives should be connected to the lowest level criteria of each criterion. The following industries are defined as alternatives in our research: Wholesale and Retail Trade; Repair of Motor Vehicles and Motorcycles (G), Information and Communication Activities (J), Financial and Insurance Activities (K), Professional, Scientific, and Technical Activities (M).

3.2 Criteria weighting

Experts from three main areas of this research, namely AI, HRM, and decision-making, individually expressed their judgments on the criteria's importance hierarchically, i.e., concerning the upper criterion by pairwise comparisons, using the AHP scale of the strengths of importance (Saaty and Sodenkamp, 2010), thus enabling the calculation of individual weights based on the eigenvalues method using computer program Expert Choice. They are presented in Figure 1.

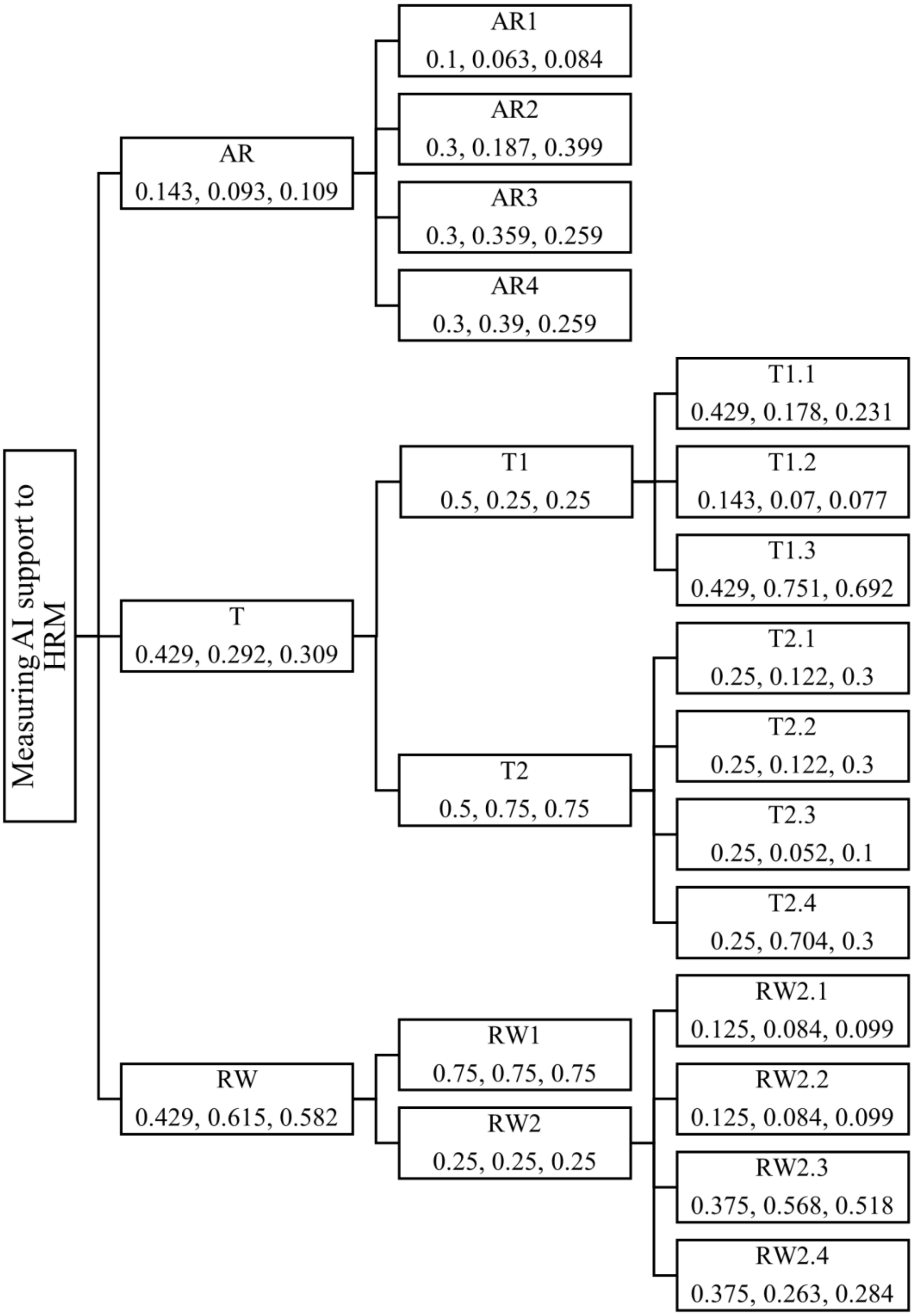

Figure 1 Criteria hierarchy with individual weights

Notes: The symbols of criteria are explained in Tables 1 and 2. In the second row of each node, the individual weights of the criteria are written, derived by Expert Choice, based on the expert judgments in the following order: Human Resource Management, Artificial Intelligence, and Decision-making.

Table 3 presents the geometric mean values, obtained by (1), together with the group weights, obtained by (2), which were written directly in the group model as the local weights.

Table 3 Group criteria weights

Level	Criterion	Geometric mean	Group weight
1	AR	0.113	0.115
2	AR1	0.081	0.083
2	AR2	0.282	0.299
2	AR3	0.303	0.310
2	AR4	0.312	0.319
1	T	0.338	0.343
2	T1	0.315	0.325
3	T1.1	0.260	0.271
3	T1.2	0.092	0.096
3	T1.3	0.606	0.633
2	T2	0.655	0.675
3	T2.1	0.209	0.232
3	T2.2	0.209	0.232
3	T2.3	0.109	0.121
3	T2.4	0.375	0.416
1	RW	0.535	0.543
2	RW1	0.750	0.750
2	RW2	0.250	0.250
3	RW2.1	0.101	0.102
3	RW2.2	0.101	0.102
3	RW2.3	0.480	0.487
3	RW2.4	0.304	0.308

Notes: The symbols of criteria are explained in Tables 1 and 2.

Table 3 shows that RW is the most important first-level criterion, followed by T and AR. Concerning RW, RW1 is three times more important than RW2, and among RW2 sub-criteria, the most important is RW2.3, followed by RW2.4 having the weight more than three times higher than RW2.2 and RW2.1. Concerning T, T2 is more than twice as important as T1. Among T1 sub-criteria, the most important is T1.3, and among T2 sub-criteria, T2.4 has the highest weight. Concerning AR, the importance of AR4, AR3, and AR2 does not significantly differ, while the importance of AR1 is significantly lower.

3.3 Measuring local and global alternatives' values and sensitivity analysis

Table 4 presents the alternatives' values concerning the lowest-level criteria obtained with increasing value functions as described in the methodology section. Table 4 shows that acquiring and retaining talented employees is best AI supported in M and worst AI supported in K, concerning each of the second-level criteria of AR. Moreover, the capabilities of AI to reduce the workload of employees with AI are the lowest in J concerning each of the third-level criteria of RW, and reducing the burden on administrative staff with AI is the lowest in G.

Table 5 shows that concerning all criteria, HRM is best supported with AI in K, closely followed by M, and worst supported in G. Acquiring and retaining talented employees is best AI supported in M, followed by G, J, and K. Appropriate training and development of employees is best supported with AI in K, followed by J, M, and G. Concerning the sub-criteria of T, contributions of AI technology to in-company training courses are the highest in K, followed by J, G and M, and contributions of the AI-supported in-company training courses are the highest in K, followed by M, J, and G. Reducing the workload of employees with AI is the best in K, followed by M, J, and G. Concerning the sub-criteria of RW, K is the best in reducing the burden on administrative staff, as well, followed by M, J and G, and M has the highest capabilities of AI to reduce the workload of employees, followed by K, G and J.

Table 4 Local values of alternatives

Lowest level criterion		Alternative			
Level	Symbol	G	J	K	M
2	AR1	0.223	0.231	0	0.546
2	AR2	0.356	0.173	0	0.471
2	AR3	0.365	0.108	0	0.527
2	AR4	0.361	0.226	0	0.414
3	T1.1	0	0.391	0.426	0.183
3	T1.2	0	0.411	0.589	0
3	T1.3	0.271	0.350	0.379	0
3	T2.1	0.266	0.188	0.547	0
3	T2.2	0	0.143	0.457	0.400
3	T2.3	0	0.167	0.472	0.361
3	T2.4	0	0.156	0.467	0.378
2	RW1	0	0.200	0.433	0.367
3	RW2.1	0.042	0	0.583	0.375
3	RW2.2	0.051	0	0.423	0.526
3	RW2.3	0.158	0	0.408	0.434
3	RW2.4	0.158	0	0.389	0.453

Notes: G – Wholesale and Retail Trade; Repair of Motor Vehicles and Motorcycles, J – Information and Communication Activities, K – Financial and Insurance Activities, M – Professional, Scientific, and Technical Activities. The symbols of criteria are explained in Tables 1 and 2.

Table 5 Aggregate values of alternatives

Aggregate value concerning	Alternative			
	G	J	K	M
AR	0.349	0.174	0	0.476
T	0.097	0.228	0.460	0.214
T1	0.172	0.367	0.412	0.050
T2	0.062	0.161	0.484	0.293
RW	0.034	0.150	0.430	0.386
RW1	0	0.200	0.433	0.367
RW2	0.135	0	0.422	0.443
All criteria	0.092	0.180	0.391	0.337

Notes: Values concerning RW1 are not the aggregate ones as RW1 is the lowest level criterion. They are presented for a clearer comparison. The symbols of criteria are explained in Tables 1 and 2, and the symbols of alternatives under Table 4.

The results of the performance sensitivity analysis are further discussed in the last section.

4 DISCUSSION AND CONCLUSIONS

The presentation and illustration of the adapted frame procedure for MCDM let us answer the first research question. Factor analysis proved useful in structuring the multi-criteria problem of measuring the AI support to HRM. One of the main advantages of the analytic hierarchy process, i.e., pairwise comparisons using linguistic and numerical ways to express judgments on criteria's importance came into the forefront, and geometric mean was used to obtain group weights from the individual ones as experts from different fields participated in criteria weighting. On the other hand, pairwise comparisons were not selected to measure local alternatives' values, as they were based on the means of alternatives according to individual criteria. When we have a homogeneous space of criteria, i.e., when they are measured in equal units, it is appropriate to unify the procedure for measuring the values of alternatives concerning individual criteria. In such cases, each participating expert does not need to carry out the entire multicriteria decision-making process as is necessary for some computer programs (where individual aggregate values are combined to group aggregate values) to support group decision-making, but we can only elicit their individual judgments on the criteria's importance and then combine them into group weights. Performance sensitivity analysis helped determine AI support's main strengths and weaknesses to HRM in specific industries.

The results obtained let us answer the second and the third research questions set in the introduction. Among the service sector industries under consideration, the highest level of AI support to HRM was found in financial and insurance activities. According to 'AI supported acquiring and retaining talented employees,' professional, scientific, and technical activities should serve as a benchmark. According to 'AI supported appropriate training and development of employees,' financial and insurance activities should serve as a benchmark; the same conclusion can be drawn for its second-level criteria, namely 'contributions of AI technology to in-company training courses' and 'contributions of the AI-supported in-company training courses.' According to 'reducing the workload of employees with AI,' financial and insurance activities should serve as a benchmark, which is not the case for its second-level criterion 'capabilities of AI to reduce the workload of employees,' according to which professional, scientific, and technical activities stand out. Financial and insurance activities often lead to AI integration into HRM processes, due to their high degree of digitalization and the need to process large volumes of data (Hidayat and Defitri, 2024). Han et al. (2023) emphasize that financial and insurance activities serve as a benchmark for the use of AI in HRM, which is expected due to their natural inclination towards digitalization and automation. These industries have extensive experience with digital technologies, which facilitates the integration of AI into various processes, including HRM (Kaur et al., 2024). Additionally, the high regulatory demands of these sectors encourage the search for innovative solutions to manage complexity, where AI offers significant advantages (Han et al., 2023). According to Kaur et al. (2024), machine learning is rapidly becoming a significant force of innovation within the financial services industry. The adoption of machine learning in the financial sector underscores its potential to drive progress and innovation, marking a pivotal shift in how financial services are delivered and managed.

The survey was limited to selected industries in Slovenia. The presented frame procedure can be used to measure AI support to HRM in other industries and consider other classifications of alternatives, e.g., according to company size, and to compare results between different countries when conducting an international survey.

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References

[1] Agarwal, A. (2023). AI adoption by human resource management: a study of its antecedents and impact on HR system effectiveness. *Foresight*, 25(1), pp. 67–81.

[2] Čančer, V. (2012). How to Determine and Consider the Weights of Interacting Criteria in MCDM. In: International Scientific Conference Quantitative Methods in Economics

(Multiple Criteria Decision Making XVI). [online] Bratislava: EKONÓM, pp. 28–32. Available at: http://www.fhi.sk/files/katedry/kove/ssov/proceedings/Zbornik2012.pdf [Accessed 16 Feb. 2024].

[3] Costello, A. B. and Osborne, J. (2005). Best Practices in Exploratory Factor Analysis: Four Recommendations for Getting the Most from Your Analysis. *Practical Assessment, Research, and Evaluation*, 10(7), pp. 1–9.

[4] Han, Y., Chen, J., Dou, M., Wang, J. and Feng, K. (2023). The Impact of Artificial Intelligence on the Financial Services Industry. *Academic Journal of Management and Social Sciences*, 2(3), pp. 83–85.

[5] Hidayat, M. and Defitri, S. J. (2024). The Impact of Artificial Intelligence (AI) on Financial Management. *Management Studies and Business Journal*, 1(1), pp. 123–129.

[6] Jatobá, M. N., Ferreira, J. J., Fernandes, P. O. and Teixeira, J. P. (2023). Intelligent human resources for the adoption of artificial intelligence: a systematic literature review. *Journal of Organizational Change Management*, 36(7), pp. 1099–1124.

[7] Kaiser, H. F. (1974). An Index of Factorial Simplicity. *Psychometrika*, 39(1), pp. 31–36.

[8] Kambur, E. and Akar, C. (2022). Human resource developments with the touch of artificial intelligence: a scale development study. *International Journal of Manpower*, 43(1), pp. 168–205.

[9] Kaur, K., Kumar, Y. and Kaur, S. (2024). Artificial Intelligence and Machine Learning in Financial Services to Improve the Business System. In: Kautish, S., Chatterjee, P., Pamucar, D., Pradeep, N. and Singh, D. (Eds). Computational Intelligence for Modern Business Systems. Singapore: Springer, pp. 3–30.

[10] Pillai, R. and Sivathanu, B. (2020). Adoption of artificial intelligence (AI) for talent acquisition in IT/ITeS organizations. *Benchmarking An International Journal*, 27(9), pp. 1–22.

[11] Qiu, H., Li, M., Bai, B., Wang, N. and Li, Y. (2022). The impact of AI-enabled service attributes on service hospitableness: the role of employee physical and psychological workload. *International Journal of Contemporary Hospitality Management*, 34(4), pp. 1–17.

[12] Saaty, T. L. and Sodenkamp, M. (2010). The Analytic Hierarchy and Analytic Network Measurement Processes: The Measurement of Intangibles. In: Zopounidis, C. and Pardalos, P. M. (Eds.). *Handbook of Multicriteria Analysis*. Berlin Heidelberg: Springer, pp. 91–166.

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COOPERATION IN THE TRANSPORT SYSTEM WITH INTERMEDIATE WAREHOUSES

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Abstract

The vehicle routing problem is a very discussed problem and many variants of it can be found in the literature. In this paper, we will focus on some possibilities of cooperation in a specific vehicle routing problem. We assume a transport system with one central warehouse and several intermediate warehouses, which are served from the central warehouse by large vehicles (trucks). Each of the intermediate warehouses has its own set of customers, which it serves with vehicles with a smaller capacity. The first considered possibility of cooperation with the aim of minimizing transport costs will be the creation of a coalition of intermediate warehouses. The second option will be the creation of a service system for all customers from a central warehouse with vehicles of greater or smaller capacity.

Keywords: *Vehicle Routing Problem, Cooperation, Savings*

JEL Classification: C610, C71
AMS Classification: 91A12, 90B06

1 INTRODUCTION

Logistics costs, and thus transport costs in particular, often represent a large part of companies' operating costs. One of the approaches to reduce these costs is mutual cooperation between logistics companies (Zibaei et al., 2016). Several researchers have adopted Cooperative Game Theory as an approach to the cooperation of logistics firms, e.g. Lozano et al. (2013) presented a mathematical model for quantifying the benefits resulting from merging the transport requirements of different companies. He proved that there is a reduction in common transport costs precisely because of the use of larger or more vehicles and the increased number of connected roads. Several authors have proven that cooperation between independent owners of logistics network companies can bring an effect. For example, McCain (2008) focused on analyzing cooperative games between organizations in order to increase their profit. The cooperation between companies could lead to the reduction of transport costs and thus can lead to an increase in performance and overall profit.

In this paper, we will explore the possibilities of cooperation in a transport system with one central warehouse and intermediate warehouses (which are served from the central warehouse), where each of the intermediate warehouses has its own customers. We will focus on quantifying the savings in the case that intermediate warehouses form a coalition in such a way that it is possible to serve the customer from any intermediate warehouse. Finally, we will consider the possibility of creating one central service system from a central warehouse.

2 MATHEMATICAL MODEL OF SELECTED ROUTING PROBLEMS

We will define mathematical models of routing problems on a complete, continuous, edge-valued graph $G = (V_0, H)$, where $V_0 = V \cup \{0\}$ represents a non-empty n + 1-element set of nodes of the graph, while $\{0\}$ represents central warenhouse and $V = \{1,2,...n\}$ represents sets of customers. Each of the customers has a request (known in advance) and we will assume that these are implemented only in a whole. Then $H \subset V_0 \times V_0$ represents the set of edges from the node *i* to the node *j*, where each oriented edge h_{ij} is assigned by a real number d_{ij} called the cost of the edge (usually the shortest distance).

The basic problem is a vehicle routing problem (Pekár et al., 2012). Each of the customers located in the *i*-th node, $i \in V$, requests from the center $\{0\}$ the import of a predetermined number of units of goods, which we denote as q_i, $i \in V$. Service is carried out using a vehicle (vehicles) with limited capacity (g). The goal is to determine such vehicle journeys in which the requirements of all customers will be satisfied, while the route traveled by the vehicles will be minimal.

The mathematical model of vehicle routing problem can be written as follows:
Sets and parameters:
n – number of nodes
$V = \{1,2,\ldots n\}$ – sets of customers (visited nodes except central warenhouse)
$V_0 = V \cup \{0\}$ – set of all nodes including central warenhouse
d_{ij}, $i, j \in V_0$, $i \neq j$ – shortest distances between all nodes
q_i, $q_i \leq g$, $i \in V$ – customer requirements
g – maximal vehicle capacity
Variables:
$x_{ij} \in \{0,\ 1\}$, $i, j \in V_0$, $i \neq j$ – binary variables represent, whether the edge (i,j) is the part of final route or not
$u_i \in \langle 0, n \rangle$, $i \in V$ – continuous bounded variables representing the order of nodes in the resulting path
$u_0 = 0$ – variable assigned to the starting node

Mathematical model:

$$\min f(\mathbf{X},\mathbf{u}) = \sum_{i\in V_0} \sum_{\substack{j\in V_0 \\ i\neq j}} d_{ij} x_{ij} \tag{1}$$

$$\sum_{i\in V_0} x_{ij} = 1,\ j \in V,\ i \neq j \tag{2}$$

$$\sum_{j\in V_0} x_{ij} = 1,\ i \in V,\ i \neq j \tag{3}$$

$$u_i + q_j - g(1 - x_{ij}) \leq u_j,\ i \in V_0,\ j \in V,\ i \neq j \tag{4}$$

$$q_i \leq u_i \leq g,\ j \in V \tag{5}$$

Now we will suppose that the vehicles are located in $k+1$ intermediate depots, which serve m customers. Let $V_1 = \{0,1,\ldots k\}$ be a set of nodes represents depots and the set $V_2 = \{k+1, k+2, \ldots m\}$ represents the set of nodes of customers, the set $V = V_1 \cup V_2$ is than the set of all nodes of the graph.

The mathematical model of the vehicle routing problem with several depots can then be written as follows (Pekár et al., 2012):

Sets and parameters:
k – number of depots
m– number of customers
$V_1 = \{0,1,...k\}$ - set of depots
$V_2 = \{k+1, k+2,...m\}$ – set of customers
$V = V_1 \cup V_2$ - set of all nodes
d_{ij}, $i, j \in V$, $i \neq j$ – shortest distances between all nodes
q_i, $q_i \leq g$, $i \in V_2$,– customer requirements
g – maximal vehicle capacity

Variables:
$x_{ij} \in \{0, 1\}$, $i, j \in V$, $i \neq j$ – binary variables represent, whether the edge (i,j) is the part of final route or not
$u_i \in \langle q_i, g \rangle$, $i \in V_2$– continuous bounded variables representing the order of nodes in the resulting path
$u_i = 0$, $i \in V_1$– variable assigned to the starting node

Mathematical model:

$$\min f(\mathbf{X}, \mathbf{u}) = \sum_{i \in V} \sum_{\substack{j \in V \\ i \neq j}} d_{ij} x_{ij} \tag{6}$$

$$\sum_{i \in V} x_{ij} = 1,\ j \in V_2,\ i \neq j \tag{7}$$

$$\sum_{j \in V} x_{ij} = 1,\ i \in V_2,\ i \neq j \tag{8}$$

$$\sum_{j \in V_2} x_{ij} \leq 1,\ i \in V_1 \tag{9}$$

$$u_i + q_j - g(1 - x_{ij}) \leq u_j,\ i \in V,\ j \in V_2,\ i \neq j \tag{10}$$

$$q_i \leq u_i \leq g,\ i \in V_2 \tag{11}$$

$$\sum_{j \in V} x_{ij} = \sum_{j \in V} x_{ji},\ i \in V \tag{12}$$

Finally, we will present a model with a heterogeneous transport fleet, which allows modeling customer service using vehicles with different capacities. We assume the set of vehicles available to serve customers $H = \{1, 2, ...r\}$ where r denotes the maximum number of vehicles, while each h-th vehicle $h \in H$, has its own capacity g_h. When servicing customers, we will assume that the vehicle can be used, but it can also remain unused at the center.

Mathematical model of distribution with a heterogeneous transport fleet can be written as follows (Pekár et al., 2012):
Sets and parameters:
n – number of nodes
$V = \{1,2, \dots n\}$ – sets of customers (visited nodes except central warehouse)
$V_0 = V \cup \{0\}$ – set of all nodes including central warehouse

$d_{ij},\ i,j \in V_0,\ i \neq j$ – shortest distances between all nodes
$q_i,\ q_i \leq g,\ i \in V$ – customer requirements
r – number of vehicles
$H = \{1,\ 2,\ ...r\}$ – set of vehicles
$g_h, h \in H$ – maximal capacity of the *h*-th vehicle
$c_h, h \in H$ – cost per unit of distance for *h*-th vehicle

Variables:
$x_{ijh} \in \{0,\ 1\},\ i,j \in V_0,\ i \neq j,\ h \in H$ – binary variables represent, whether the edge (*i,j*) is the part of final route of the *h*-th vehicle or not
$u_{ih} \in \langle q_i, g_h \rangle,\ i \in V, h \in H$ – continuous bounded variables representing the order of nodes in the route of the *h*-th vehicle
$u_{0h} = 0, h \in H$ – variable assigned to the starting node for the *h*-th vehicle

Mathematical model:

$$\min f(\mathbf{X}, \mathbf{u}) = \sum_{i \in V_0} \sum_{\substack{j \in V_0 \\ j \neq i}} \sum_{h \in H} c_h d_{ij} x_{ijh} \tag{13}$$

$$\sum_{i \in V_0} \sum_{h \in H} x_{ijh} = 1,\ j \in V,\ i \neq j \tag{14}$$

$$\sum_{j \in V_0} \sum_{h \in H} x_{ijh} = 1,\ i \in V,\ i \neq j \tag{15}$$

$$u_{ih} + q_j - g_h\left(1 - x_{ijh}\right) \leq u_{jh},\ i \in V_0,\ j \in V,\ h \in H, i \neq j \tag{16}$$

$$q_i \leq u_{ih} \leq g_h,\ i \in V,\ h \in H \tag{17}$$

Presented models will be used in the next part.

3 ILLUSTRATIVE EXAMPLE

Let's assume a transport system consisting of one central warehouse and three intermediate warehouses (players). Two intermediate warehouses represent separate entities, one intermediate warehouse location is identical to the location of the central warehouse. From the central warehouse, deliveries are made to intermediate warehouses, and intermediate warehouses carry out deliveries to customers themselves, while each of the intermediate warehouses has its own set of customers. The shortest distances between all nodes are given in Table 1.

In the central warehouse, vehicles with a capacity of 24 (trucks) and also smaller vehicles with a capacity of 10 are available. Only vehicles with a smaller capacity of 10 are placed in the intermediate warehouses. We assume the following vehicle operating costs: 0,22 per unit distance for a vehicle with a capacity of 24 and 0,15 per unit distance for a vehicle with a capacity of 10. The central warehouse is located in node 0, intermediate warehouses are located in nodes 0, 1 and 2. Each of the intermediate warehouses has a strictly defined set of customers, but at the same time it also represents a point of consumption. To the intermediate warehouse in node 0 is assigned the set of nodes {3, 5, 12, 15, 17}, to the intermediate warehouse in node

1 is assigned the set {7, 8, 9, 13, 16} and the intermediate warehouse in node 2 is assigned with the set {4, 6, 10, 11, 14}.

Table1: Shortest distances between all nodes

Node	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0	0	208	213	41	269	76	268	119	155	155	113	195	55	125	259	142	166	89
1	208	0	391	249	464	263	446	88	103	82	321	403	239	104	437	127	47	200
2	213	391	0	172	73	147	55	302	327	346	92	35	169	316	46	303	349	230
3	41	249	172	0	229	53	227	160	196	195	78	155	77	165	218	182	207	129
4	269	464	73	229	0	211	26	375	391	411	156	74	233	367	49	367	422	294
5	76	263	147	53	211	0	217	196	182	202	55	137	24	159	188	158	236	86
6	268	446	55	227	26	217	0	357	397	401	162	80	239	371	23	373	404	300
7	119	88	302	160	375	196	357	0	71	50	232	315	174	33	348	85	46	140
8	155	103	327	196	391	182	397	71	0	20	235	317	158	50	368	24	54	97
9	155	82	346	195	411	202	401	50	20	0	255	337	178	30	392	44	34	117
10	113	321	92	78	156	55	162	232	235	255	0	83	77	211	134	211	279	138
11	195	403	35	155	74	137	80	315	317	337	83	0	159	294	71	293	361	221
12	55	239	169	77	233	24	239	174	158	178	77	159	0	134	210	134	212	62
13	125	104	316	165	367	159	371	33	50	30	211	294	134	0	362	52	57	107
14	259	437	46	218	49	188	23	348	368	392	134	71	210	362	0	344	395	272
15	142	127	303	182	367	158	373	85	24	44	211	293	134	52	344	0	78	73
16	166	47	349	207	422	236	404	46	54	34	279	361	212	57	395	78	0	151
17	89	200	230	129	294	86	300	140	97	117	138	221	62	107	272	73	151	0

Source: The authors

Customer requirements are given in the Table 2:

Table2: Requirements of the customers

Node	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Demand		3	4	5	2	6	2	5	2	3	5	6	1	2	4	6	2	4

Source: The authors

Using the model (1)-(5), we will determine the distribution structure and the individual transport costs of the players.
Central warehouse in 0: Delivery by large vehicle to intermediate warehouses: 0-1-0-2-0. Delivered quantity: 40. Route length: 842. Route costs: 185,24.

Intermediate warehouse in 0 (*Player 0*): Delivery by small vehicle: 0-3-0-5-12-0-17-15-0. Delivered quantity: 22. Route length: 541. Route costs: 81,15.
Intermediate warehouse in 1 (*Player 1*): Delivery by small vehicle: 1-7-13-1-9-8-16-1. Delivered quantity: 14. Route length: 428. Route costs: 64,2.
Intermediate warehouse in 2 (*Player 2*): Delivery by small vehicle: 2-4-6-14-2-10-2-11-2. Delivered quantity: 19. Route length: 422. Route costs: 63,3.
The total cost of the system: 393,89.

Suppose that players serving intermediate warehouses have the possibility to form a coalition, that is, a player can serve a customer in any node. Now we use the model (6)-(12) and the results are:

Central warehouse in 0: Delivery by large vehicle to intermediate warehouses: 0-1-0-2-0. Delivered quantity: 41. Route length: 842. Route costs: 185,24.

Intermediate warehouse in 0 (*Player 0*)*:* Delivery by small vehicle: 0-17-0-3-10-0-5-12-0. Quantity delivered: 21. Route length: 565. Route costs: 84,75.
Intermediate warehouse in 1 (*Player 1*): Delivery by small vehicle: 1-9-13-7-1-15-8-16-1. Quantity delivered: 20. Route length: 485. Route costs: 72,75.
Intermediate warehouse in 2 (*Player 2*): Delivery by small vehicle: 2-11-2-14-6-4-2. Delivered quantity: 14. Route length: 238. Route costs: 35,7.
The total cost of the system is: 378,44.

The results are summarized in the Table 3:

Table3: Cost of players

	Cost before cooperation	Cost after cooperation	Savings
Player 0	81,15	84,75	-3,6
Player 1	64,2	72,75	-8,55
Player 2	63,3	35,7	27,6
Sum	208,65	193,2	15,45

Source: The authors

It is clear that the savings from cooperation is approximately 7,4%. Player 0 and player 1 have increased costs; costs of player 2 have decreased. This kind of cooperation would only be possible in the case of transferable winnings, which means that the players will redistribute the costs among themselves. Several approaches can be used to redistribute winnings. In (Čičková, Figurová, 2019) one can find the egalitarian solution, the Nash solution, the utilitarian solution, and the Shapley value applied to cost distribution in the case of a distribution system. The total costs of the system are obtained by adding delivery by large vehicle from the central warehouse to intermediate warehouses, i.e. they are 393,89 in the initial case and 378,44 after cooperation.

Let's now examine the possibility of canceling intermediate storage. In this case, all customers will be served from the center using a large or small vehicle. Using the model (13)-(17), we get the route:
Large vehicle: 0-11-4-6-14-2-10-12-0-13-15-8-9-16-1-7-0 Small vehicle: 0-3-0-5-17-0. Total cost of the route: 291,29, which represents a saving of 102,6 (compared to 393,89), or 26,05%.

4 CONCLUSIONS

In the paper, we focused on the possibilities of cooperation in a specific transport system, which included a central warehouse and intermediate warehouses, where vehicles with different capacities are used for service. We investigated two possibilities of cooperation. The first option was the mutual sharing of customers between intermediate warehouses. In this way, it would be possible to save 7,4% of costs. The second option is the creation of one central system (cancellation of intermediate warehouses). This method proved to be more effective, as we achieved savings of up to 26,05%. There are many ways to extend this approach. For example, dynamically examine the location of intermediate warehouses. This remains an open question for further research.

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References

1. Pekár, J., Brezina, I., Čičková, Z. and Reiff, M., (2012). *Modelovanie rozmiestňovania recyklačných centier*. Bratislava: Vydavateľstvo EKONÓM.
2. Lozano, S., Moreno, P., Adenso-Díaz, B. and Algaba, E., (2013). Cooperative game theory approach to allocating benefits of horizontal cooperation. *European Journal of Operational Research*, 229(2), pp.444-452.
3. Čičková, A. and Figurová, D., (2019). Benefits From Cooperation Behaviour in Vehicle Routing Problem In: *LOGIC*, Belgrade: University of Belgrade, Faculty of Transport and Traffic Engineeiring, pp. 19-25.
4. McCain, R.A., (2008). Cooperative games and cooperative organizations. *The Journal of Socio-Economics*, *37*(6), pp.2155-2167.
5. Zibaei, S., Hafezalkotob, A. and Ghashami, S.S., (2016). Cooperative vehicle routing problem: an opportunity for cost saving. *Journal of Industrial Engineering International*, *12*, pp.271-286.

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MODELING OF SUSTAINABLE SUPPLY CHAINS

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Abstract

A supply chain is a network system of interconnected members from supplier to customer. In this paper is proposed modeling combined approach for sustainable supply chains. The sustainability of supply chains can be measured by multiple criteria, such as economic, social, environmental, economic, and others. Multiple criteria analysis is useful for assessing sustainability of supply chains. De Novo approach focus on designing optimal systems. Finding an equilibrium between the interests of members of a sustainable supply chain is a very important problem. Game theory has become a useful instrument in the analysis of supply chains with multiple members. Biform games combine the advantages of both cooperative and non-cooperative games into new dynamic, which can be used to not only generate more profits but also to change nature of the business environment in benefit of members.

Keywords: *Supply chain, Sustainability, Multiple criteria, Game theory*

JEL Classification: L22, C70
AMS Classification: 93A30, 91A40

1 INTRODUCTION

A supply chain is a complex and dynamic supply and demand network of agents, activities, resources, technology and information involved in moving a product or service from supplier to customer. Supply chain structure and behavior is changing dynamically. The sustainability of supply chains can be measured by multiple criteria, such as environmental, social, economic, and others.

The objective of supply chain sustainability is to create, protect and grow long-term environmental, social and economic value for all stakeholders involved in bringing products and services to market. The main objective of the paper is to create a comprehensive model and solution methods for sustainable supply chains. Sustainable supply chains are modeled as network systems with multiple agents, which are evaluated according to multiple criteria. It is usually impossible to optimize all criteria together in a given system. Searching for a better portfolio of resources leads to reshaping of given system boundaries and better criteria values. Supply chain sustainability is also given by creating equilibrium relationships between agents. Biform games are used for searching an equilibrium in sustainable supply chains. Information sharing affects reducing inefficiencies and reducing material flows, leading to reduced environmental pollution.

The rest of the paper is organized as follows. Section 2 presents basics of sustainable supply chains. Section 3 is devoted to the design of a simple multi-criteria supply chain model using standard and De Novo approaches. Game theory approqaches for searching equilibrium in sustainable supply chains are proposed in section 4. Section 5 presents conclusions.

2 SUSTAINABLE SUPPLAY CHAINS

Sustainability in supply chain management has become a highly relevant topic for researchers and practitioners. Carter and Rogers (2008) perform a large-scale literature review and use conceptual theory building to introduce the concept of sustainability and demonstrate the relationships among environmental, social, and economic performance within a supply chain management context. Seuring (2013) analyzes more than 300 papers on the topic of sustainable (forward) supply chains. Looking at the research methodologies employed, only 36 papers apply quantitative models. This is in contrast to the field of reverse supply chains where several reviews on respective quantitative models have already been provided. The paper summarizes research on quantitative models for forward supply chains. There are three dominant approaches: equilibrium models, multi-criteria decision making and analytical hierarchy process. The paper of Brandenburg et al. (2014) provides a content analysis of 134 carefully identified papers on quantitative models that address sustainability aspects in the forward supply chain. It was found that most were analytically based with a focus on multiple criteria decision making. The tools most often used comprise the analytical hierarchy process or the analytical network process, as well as life cycle analysis.

The supply chain is defined as a network system of clusters with: suppliers, manufacturers, distributors, retailers, customers, where material, financial, information, decision flows connect participants in both directions (see Fiala 2005).

The paper proposes a procedure for tmodeling and analyses of sustainable supply chains based on multiple criteria analysis and game theory approaches. The proposed procedure combines both approaches and uses specific De Novo multiple criteria optimization procedures and biform games for the solution. The authors propose to divide the biform games into so-called sequential and simultaneous shapes. Other sustainability aspects and the impact of technology development may be included in the model.

3 MULTIPLE CRITERIA APPROACHES

A theory and solution methods were proposed to consider multiple evaluation criteria (see for example Steuer, 1986, Ehrgott, 2005)). The multi-criteria approach was modified to relax the constraints and search for optimal systems on the De Novo approach (Zeleny, 2010).

3.1 Standard multiple critera approaches

Standard multiple criteria approaches focus on valuation of already given systems. Multi-objective linear programming (MOLP) is a model of optimizing a given system by multiple objectives:

$$\begin{aligned} &\text{“Min”} \quad z = Cx \\ &\text{s.t.} \quad Ax \geq b,\ x \geq 0 \end{aligned} \tag{1}$$

where C is a (k, n) – matrix of objective coefficients, A is an (m, n) – matrix of structural coefficients, b is an m-vector of known resource restrictions, x is an n-vector of decision variables. For multi-objective programming problems, the concept of efficient solutions is used. A compromise solution is selected from the set of efficient solutions. Many methods are proposed for solving the problem. Most of the methods are based on trade-offs. The interactive method STEM was used for solving the case study.

Multiple criteria supply chain model

In the next part, a multiple criteria supply chain dproblem is formulated (Fiala, Majovská, 2020). The mathematical program determines the ideal locations for each facility and allocates the activity at each facility such that the multiple objectives are considered and the constraints of meeting the customer demand and the facility capacity are satisfied. The presented model of a supply chain consists of 4 layers with m suppliers, S_1, S_2, ... S_m, n potential producers, P_1, P_2, ... P_n, p potential distributors, D_1, D_2, ... D_p, and r customers, C_1, C_2, ... C_n.

The following notation is used:
a_i = annual supply capacity of supplier i, b_j = annual potential capacity of producer j,
w_k = annual potential capacity of distributor k, d_l = annual demand of customer l,
f_j^P = fixed cost of potential producer j, f_k^D = fixed cost of potential distributor k,
c_{ij}^S = unit transportation cost from S_i to P_j, c_{jk}^P = unit transportation cost from P_j to D_k,
c_{kl}^D = unit transportation cost from D_k to C_l, e_{ij}^S = unit pollution from S_i to P_j,
e_{jk}^P = unit pollution from P_j to D_k, e_{kl}^D = unit environmental pollution from D_k to C_l,
x_{ij}^S = number of units transported from S_i to P_j, x_{jk}^P = number of units transported from P_j to D_k, x_{kl}^D = number of units transported from D_k to C_l,
y_j^P = bivalent variable for build-up of the fixed capacity of producer j,
y_k^D = bivalent variable for build-up of the fixed capacity of distributor k.

Using the above notations the problem can be formulated as follows:

The model has two objectives. The first one expresses minimizing total costs. The second one expresses minimizing total environmental pollution.

Minimize 2 objectives:

$$z_1 = \sum_{j=1}^{n} f_j^P y_j^P + \sum_{k=1}^{p} f_k^D y_k^D + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^S x_{ij}^S + \sum_{j=1}^{n}\sum_{k=1}^{p} c_{jk}^P x_{jk}^P + \sum_{k=1}^{p}\sum_{l=1}^{r} c_{kl}^D x_{kl}^D$$

$$z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} e_{ij}^S x_{ij}^S + \sum_{j=1}^{n}\sum_{k=1}^{p} e_{jk}^P x_{jk}^P + \sum_{k=1}^{p}\sum_{l=1}^{r} e_{kl}^D x_{kl}^D$$

Subject to the following constraints:

the amount sent from the supplier to producers cannot exceed the capacity

$$\sum_{j=1}^{n} x_{ij} \le a_{i,} \quad i = 1, 2, ..., m,$$

the amount produced by the producer cannot exceed the producer capacity

$$\sum_{k=1}^{p} x_{jk} \le b_j y_j, \quad j = 1, 2, ..., n,$$

the amount shipped from the distributor should not exceed the distributor capacity

$$\sum_{l=1}^{r} x_{kl} \le w_k y_k, \quad k = 1, 2, ..., p,$$

the amount shipped to the customer must equal the customer demand

$$\sum_{k=1}^{p} x_{kl} = d_l, \quad l = 1, 2, ..., r,$$

the amount shipped out of producers cannot exceed units received from suppliers

$$\sum_{i=1}^{m} x_{ij} - \sum_{k=1}^{p} x_{jk} \ge 0, \quad j = 1, 2, ..., n,$$

the amount shipped out of distributors cannot exceed quantity received from producers

$$\sum_{j=1}^{n} x_{jk} - \sum_{l=1}^{r} x_{kl} \ge 0, \quad k = 1, 2, ..., p,$$

binary and non-negativity constraints

$$y_j, y_k \in \{0,1\},$$

$$x_{ij}, x_{jk}, x_{kl} \ge 0, \quad i = 1, 2, ..., m, \; j = 1, 2, ..., n, \; k = 1, 2, ..., p, \; l = 1, 2, ..., r.$$

The formulated model is a multi-objective linear programming problem (MOLP). The problem can be solved by some MOLP methods.

3.2 De Novo approaches

De Novo approach focus on designing optimal systems. The approach is based on reformulation of the problem by given prices of resources and the given budget. Searching for a better portfolio of resources leads to a continuous reconfiguration and reshaping of systems boundaries.

By given prices of resources and the given budget the MOLP problem (1) is reformulated in the MODNLP problem (2)

$$\text{“Min” } z = Cx$$
$$\text{s.t. } Ax - b \geq 0,\ pb \leq B,\ x \geq 0 \tag{2}$$

where b is an m-vector of unknown resource restrictions, p is an m-vector of resource prices, and B is the given total available budget.

The De Novo approach was adapted for supply chain design.

Multi-objective De Novo supply chain model

The De Novo approach can be useful in the modeling of the multi criteria supply chain. Only a partial relaxation of constraints is adopted. Producer and distributor capacities are relaxed. Unit costs for capacity build-up are computed:

$p_j^P = \frac{f_j^P}{b_j}$ = cost of the unit capacity of potential producer j,

$p_k^D = \frac{f_k^D}{w_k}$ = cost of the unit capacity of potential distributor k.

Variables for build-up capacities are introduced:

u_j^P = variable for the flexible capacity of producer j,

u_k^D = variable for the flexible capacity of producer k.

The constraints for non-exceeding producer and distributor fixed capacities are replaced by the flexible capacity constraints and the budget constraint:

$$\sum_{k=1}^{p} x_{jk} - u_j^P \leq 0,\quad j = 1,\ 2,\ \ldots,\ n,$$

$$\sum_{l=1}^{r} x_{kl} - u_k^D \leq 0,\quad k = 1,\ 2,\ \ldots,\ p,$$

$$\sum_{j=1}^{n} p_j^P u_j^P + \sum_{k=1}^{p} p_k^D u_k^D \leq B.$$

3.3 Case study

The De Novo approach was tested on a case study. A supply chain is proposed with 3 potential suppliers, 3 potential manufacturers, 3 potential distributors, 3 customers. The chain is evaluated according to 2 criteria, the first criterion is aimed at minimizing total costs and the second one at minimizing overall environmental pollution.
Inputs for the model are as follows:

Capacities a_i = 100, i = 1, 2, 3; b_j = 100, j = 1, 2, 3;
w_k = 100, k = 1, 2, 3; d_l = 50, l = 1, 2, 3.
Fixed costs f_1^P = 110, f_2^P = 100, f_3^P = 120, f_1^D = 120,
f_2^D = 110, f_3^D = 150.

Unit transportation costs and unit pollution are shown in Table 1 and Table 2.

c_{ij}^S	1	2	3	c_{jk}^P	1	2	3	c_{kl}^D	1	2	3
1	5	10	6	1	7	5	9	1	8	3	10
2	8	9	7	2	6	8	4	2	6	5	4
3	3	6	8	3	5	7	9	3	7	3	5

Table 1 Unit transportation costs

e_{ij}^S	1	2	3	e_{jk}^P	1	2	3	e_{kl}^D	1	2	3
1	4	3	8	1	8	7	9	1	8	6	2
2	8	9	2	2	6	8	4	2	8	9	8
3	7	6	8	3	4	7	9	3	5	3	5

Table 2 Unit pollution

This model was solved by different approaches. The first two approaches minimize each criterion separately. The compromise solution is calculated by the traditional STEM interactive approach for multi-criteria problems and the De Novo approach was used. The following are non-zero values of the variables that express the number of units of product shipped between each supply chain layer.

These values are given for each problem-solving approach:

Min z_1: $x_{13}^S = 50$, $x_{31}^S = 100$, $x_{12}^P = 100$, $x_{31}^P = 50$, $x_{12}^D = 50$, $x_{21}^D = 50$, $x_{23}^D = 50$.
Min z_2: $x_{12}^S = 100$, $x_{23}^S = 50$, $x_{23}^P = 100$, $x_{31}^P = 50$, $x_{13}^D = 50$, $x_{31}^D = 50$, $x_{32}^D = 50$.
STEM: $x_{11}^S = 58.13$, $x_{23}^S = 91.87$, $x_{12}^P = 58.13$, $x_{31}^P = 91.87$, $x_{12}^D = 46.87$, $x_{13}^D = 45$, $x_{21}^D = 50$, $x_{22}^D = 3.12$, $x_{23}^D = 50$.
De Novo: $x_{23}^S = 62.86$, $x_{32}^S = 87.14$, $x_{21}^P = 10$, $x_{23}^P = 77.14$, $x_{31}^P = 62.86$, $x_{12}^D = 50$, $x_{13}^D = 22.86$, $x_{31}^D = 50$, $x_{33}^D = 27.14$.

The criteria values z_1 a z_2 and budget B are compared according to these solutions. De Novo solution is better in all values than the STEM solution. De Novo approach provides better solutions on both criteria and also with a lower budget due to flexible capacity constraints. The capacities of supply chain members have been optimized for flows in the supply chain and budget. The comparison of results is shown in Table 3.

	Min z_1	Min z_2	STEM	De Novo
z_1	2460	3490	3070	3000
z_2	3100	1800	2030	2000
B	460	490	460	365.71

Table 3 Comparison of solution results

4 GAME THEORY APPROACHES

The proposed sustainable supply chain model can be extended to consider the interrelationships between the members of the chain. Games are used for behavior modeling of sustainable supply chains and focus on allocation of resources, capacities, costs, revenues and profits (Myerson, 1997). The second component of the proposed procedure is searching for equilibrium. Most supply chains are composed of independent agents with individual interests and preferences. There are numerous opportunities to create hybrid models that combine competitive and cooperative behavior. Biform games are used for searching an equilibrium in sustainable supply chains. A biform game is a combination of non-cooperative and cooperative games for searching an equilibrium (Brandenburger, Stuart, 2007). We propose to divide biform games into sequential and simultaneous shapes (Fiala, 2022).

4.1 Sequential biform games

The sequential biform game is a two-stage game: in the first stage, players choose their strategies in a non-cooperative way, thus forming the second stage of the game, in which the players cooperate. First, suppliers make initial proposals and take decisions. This stage is analyzed using a non-cooperative game theory approach. The players search for Nash equilibrium by solving the problem. Nash equilibrium is a set of decisions from which no player can improve the value of his pay-off function by unilaterally deviating from it.

The cooperative game theory looks at the set of possible outcomes, studies what the players can achieve, what coalitions will form, how the coalitions that do form divide the outcome, and whether the outcomes are stable and robust. Allocation mechanisms are based on different approaches such as Shapley values, contracts, auctions, negotiations, etc.

Confidence indices $0 \leq \alpha^i \leq 1,$ for all $i = 1, 2, \ldots, n$, are introduced. The indices show players' anticipation of the pay-off they will receive in the cooperative stage, i.e. the proportion of the difference between the maximum and minimum core allocation achievable for players. Confidence indices of the players provide the link between the non-cooperative and cooperative stages of the biform game.

4.2 Simultaneous biform games

The simultaneous biform game is a one-stage model where combinations of concepts for cooperative and non-cooperative games are applied. The combinations will be changed according situations in problems. At this stage, multi-round negotiations take place. The first problem is a classification of situations. The situations are affected by:

- which players can cooperate,
- to what scope they can cooperate.

If all players can cooperate fully, then a standard cooperative model can be used with subsequent distribution of the result according to the Shapley values. If no one can cooperate even in a partial content, a standard non-cooperative model is used.

The general simultaneous biform games are based on negotiation process with multiple criteria (see Fiala, 1999). The pressure negotiation concept is based on the assumption that each negotiating subject decides under pressure of objective context, subject to a variety of internal and external pressures. The scope of cooperation is determined by the various constraints that result from the fact that players are under internal and external pressures. The scope of cooperation is dynamic and changes over time. The effects of pressures will be reflected in restrictive conditions.

4.3 Negotiation Model

Suppose n negotiation participants. Denote X as decision space for the negotiating process. Elements of this space are decisions $\mathbf{x} \in X$, which are vectors whose components represent the parameters of the decision. A consensus decision $\mathbf{x}^*$ should be chosen from the decision space X. The traditional game concepts assume a fixed structure and fixed sets of strategies. Sets of strategies will be taken as dynamic $X_i(t)$, for players $i = 1, 2, \ldots, n$, depending on the discrete time periods $t = 1, 2, \ldots, T$. Dynamic evaluations of strategies will be also considered.

Each participant evaluates decisions by multiple criteria and compares the decisions with the target values. Multiple criteria analysis from the first component of proposed procedure is applied. The criteria are in the form of criteria functions, that all participants want to optimize their values. Each participant in negotiations may have a different number of criteria. Denote $\mathbf{f}^1(\mathbf{x})$, $\mathbf{f}^2(\mathbf{x})$, ..., $\mathbf{f}^n(\mathbf{x})$ vector criteria functions that transform decision **x** into the vectors of target values $\mathbf{y}^1$, $\mathbf{y}^2$, ..., $\mathbf{y}^n$ of the target spaces of the participants Y^1, Y^2, ..., Y^n. These achievements, however, the participant tries to not reveal his interests and his strategy to all players. Own negotiations and exchanges of information between participants are happening in the decision space.

The negotiation process is dynamic, and suppose that there is at discrete time points $t = 1, 2, ..., T$. At the time T the process is completed by finding a trajectory to time horizon T. Negotiation process over time can be modeled as a gradual change of the negotiation space, which is a subset of the decision space containing acceptable decisions of participants in the negotiation time until a single element negotiation space is reached.

For each participant, a set of acceptable decisions is formulated, which is a set of decisions that are permissible and acceptable in terms of the required aspiration levels of criteria functions. The aspiration levels $\mathbf{b}^i(t)$, $i = 1, 2, ..., n$, $t = 1, 2, ..., T$, of criteria functions represent opportunities for added values. At the beginning of the negotiations it has the form

$$X_i(0) = \{\mathbf{x};\ \mathbf{x} \in X,\ \mathbf{f}^i(\mathbf{x}) \le \mathbf{b}^i(0)\},\ i = 1, 2, ..., n. \tag{3}$$

Then the negotiation space is defined at the beginning of the negotiations as an intersection of sets of acceptable decisions of all participants in negotiations

$$X_0(0) = \bigcap_{i=1}^{r} X_i(0) \tag{4}$$

If the negotiation space $X_0(0)$ is a single element set, then the negotiation problem is trivial. This element is the consensus. Negotiation problem becomes interesting when the negotiation space is empty or contains more than one element. In the first case, participants have to reduce some or all of the aspiration levels of criteria functions but participants are involved in the reduction of certain criteria more and other less. In the latter case, each element of the negotiation space is acceptable to all participants but different elements are evaluated differently, because they meet the criteria of the participants on different levels. Further negotiations are conducted in time points $t = 1, 2, ..., T$,

$$X_i(t) = \{\mathrm{x};\ \mathrm{x} \in \mathrm{X},\ \mathbf{f}^{\mathbf{i}}(\mathbf{x}) \le \mathbf{b}^{\mathbf{i}}(t)\},\ i = 1, 2, ..., n, \tag{5}$$

and should lead to a consensus decision, to achieve single-element negotiation space $X_0(t)$.

5 CONCLUSION

This paper proposes and discusses the procedure for designing sustainable supply chains. The procedure takes into account multiple agents in the system and multiple evaluation criteria to solve the design problem. The procedure is flexible enough. It is generally open to other types of criteria and other types of agents. De Novo approach was applied for multiple criteria supply chain design problem and provides better solution than traditional approaches applied on fixed constraints. The approach is not oriented towards the optimization of some criteria but seeks for a trade-off free solution by reformulating resource constraints only limited by the budget. The resources are saved by drawing only in the amount necessary to reach a balanced solution. The multi-criteria approach is applied in seeking equilibrium for interested agents using biform game procedures. Biform games combine cooperative and non-cooperative approaches of game theory. The paper proposes to divide biform games into sequential and simultaneous shapes, The procedure is open to complement other concepts and approaches, for example, allocation mechanisms can be based on different approaches such as Shapley values, contracts, auctions, and negotiations. A combination of these concepts and approaches can be a powerful instrument for designing supply chains.

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References

[1] Brandenburg, M., Govindan, K., Sarkis, J., & Seuring, S. (2014). Quantitative models for sustainable supply chain management: Developments and directions. *European journal of operational research*, 233(2), pp. 299-312.

[2] Brandenburger, A. and Stuart, H. (2007). Biform games. *Management Science,* 53(4), pp. 537-549.

[3] Carter, C. R., & Rogers, D. S. (2008). A framework of sustainable supply chain management: moving toward new theory. International Journal of Physical Distribution & Logistics Management, 38(5), pp. 360-387.

[4] Ehrgott, M. (2005). *Multicriteria Optimization*. Berlin: Springer Science & Business Media.

[5] Fiala, P. (1999). Modeling of negotiation process with multiple evaluation criteria. *Politická ekonomie*, 47(2), pp. 253-268.

[6] Fiala, P. (2005). Information sharing in supply chains. *Omega: The International Journal of Management Science,* 33(4), pp. 419-423.

[7] Fiala, P. (2022). Modelling and analysis of co-opetition in network industries by biform games. *Central European Journal of Operations Research*, 30(2), pp. 647-665.

[8] Fiala, P. and Majovská, R. (2020). Modeling the Design Phase of Sustainable Supply Chains. In: Khan, S. A. R. (ed) Global Perspectives on Green Business Administration and Sustainable Supply Chain Management. Hershey PA: IGI Global.

[9] Myerson, R. B. (1997*). Game Theory: Analysis of Conflict*. Cambridge: Harvard University Press.

[10] Seuring, S. (2013). A review of modeling approaches for sustainable supply chain management. *Decision Support Systems*, 54(4), pp. 1513-1520.

[11] Steuer, R. E. (1986). *Multiple Criteria Optimization: Theory, Computation and Application*. New York: Wiley.

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OPTIMIZING STORE PLACEMENT IN URBAN ENVIRONMENTS

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Abstract

Decision-making on establishment placement involves various rules and methods, often influenced by real city possibilities and historical contexts. Despite theories aiming for even spatial distribution, practical outcomes often differ. Data about population density can be a good base for looking at the ideal placement through optimization models.

A case study illustrates the integration of theoretical frameworks with practical considerations. While urban plans and historical factors primarily influence placement, optimization methodologies based on population density and geographical proximity can provide insights for maximizing location utility. By integrating theoretical models and empirical data, optimal placement solutions can be derived, leading to the efficient utilization of available space. This approach ensures that establishments are strategically positioned, balancing market demand and accessibility.

Keywords: *Hotelling's theorem, Store Location Optimization, Python*

JEL Classification: C02, C61, C70
AMS Classification: 91B42, 91C05

1 INTRODUCTION

A number of rules, principles or methods can be used to decide on the location of establishments. In practice, however, it is often based on the real possibilities of the city, or space, and possibly historical facts. Very common is the use of Hotelling's Law, which is characterized by the location of identical or similar shops in one place or one location. There are also several theories and location models that deal with evenly spaced stores in a space so that the store is equally accessible to customers. These models are used in practice to locate distribution centers or service points that consider a certain catchment area, and their choice of location is not subject to market principles.

1.1 Methods

The main methods have already been mentioned, the main view of location is based on the location of individual competitors' establishments, i.e. already existing establishments. This approach is the Hotelling’s law, or the issue is known as Hotelling's game or Hotelling's rule.

This approach is one of the first to emerge in the field of spatial competition and the model consists of two or more firms seeking the most advantageous market position in a particular space. The model is the basis of several theories of product differentiation and location. It is based on the fact that the number of customers served is the same, allowing for equilibrium market strategies of all firms involved. On this basis, we can judge that, given the same set of conditions, firms tend to choose locations close to each other.

The second approach used is based on optimization problems, namely the problem of locating an establishment in Euclidean space. Thus, the location of the operation is based on an optimization criterion that represents the total air distance from the potential location of the operation to each area where customers are located. Using the above criterion, the location of the location of the operation can be located based on the air distances.

In this way, the point (store) with the overall shortest distance from the initial vertex x^0 to all n other vertices x^i can be found. The optimally located operation (point) x^0 is located at the center of the space with coordinates $[x_1^0, x_2^0]$. The distance $d(x^i, x^0)$ is computed according to the L2 metric, i.e., by forming a "square" with coordinates $[x_1^i, x_2^i]$ in order to minimize the sum of all distances $d(\mathbf{x^i}, \mathbf{x^0})$ ($i = 1, 2,\ldots n$). The problem thus described can be formulated as a free extremum finding problem (Brezina 2020):

$$f(x_1^0, x_2^0) = \sum_{i=1}^{n} \sqrt{(x_1^i - x_1^0)^2 + (x_2^i - x_2^0)^2} \rightarrow \min \tag{1}$$

respectively with assigned weights w_i ($i = 1, 2,\ldots n$) it can be formulated:

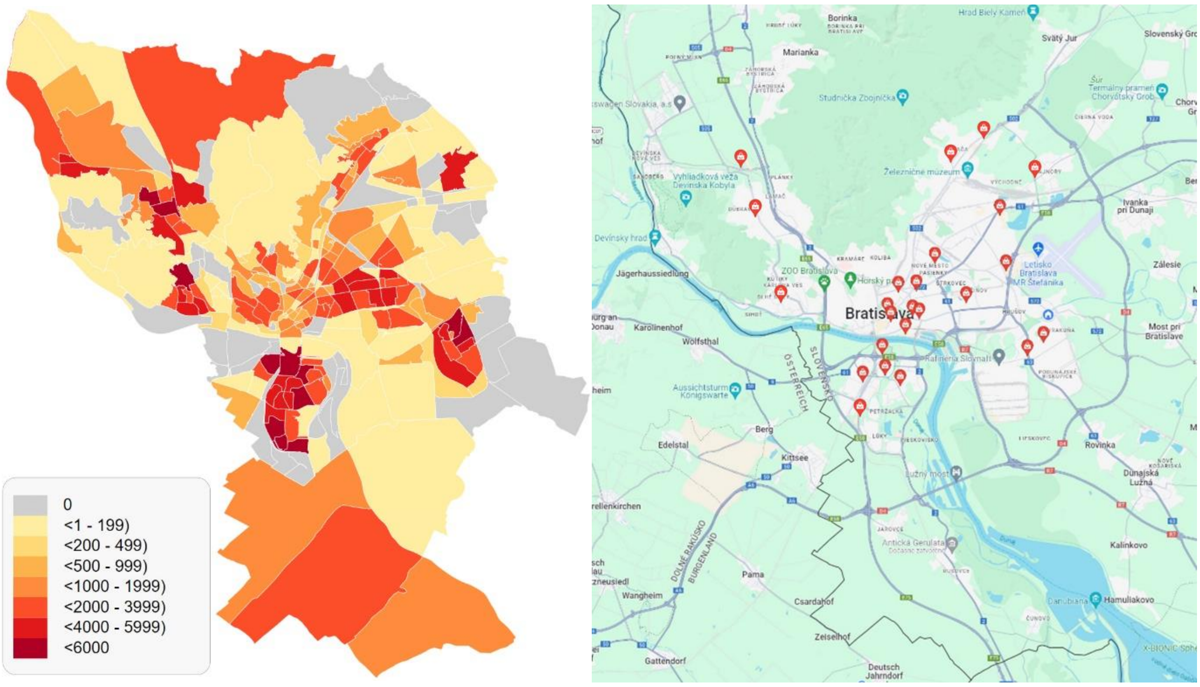
$$f(x_1^0, x_2^0) = \sum_{i=1}^{n} w_i \sqrt{(x_1^i - x_1^0)^2 + (x_2^i - x_2^0)^2} \rightarrow \min \tag{2}$$

1.2 Data

Because Bratislava has recently joined the "open metropolises" and that the website opendata.bratislava.sk provides a lot of datasets on various indicators, it is not a problem to obtain data for optimal decision-making.

This page shows the basic breakdown of Bratislava and the population of each urban district. Regarding the issue under discussion, it is advisable to use the population information from the data mentioned above, since the aim is to adapt the location of the plant to the customers. For this reason, it is advisable to respect the population density when locating establishments.

Fig. 1: Map of Bratislava by population and location of selected retail outlets in Bratislava



Source: http://www.sodbtn.sk/obce/bratislava.php
Source: author/googlemaps

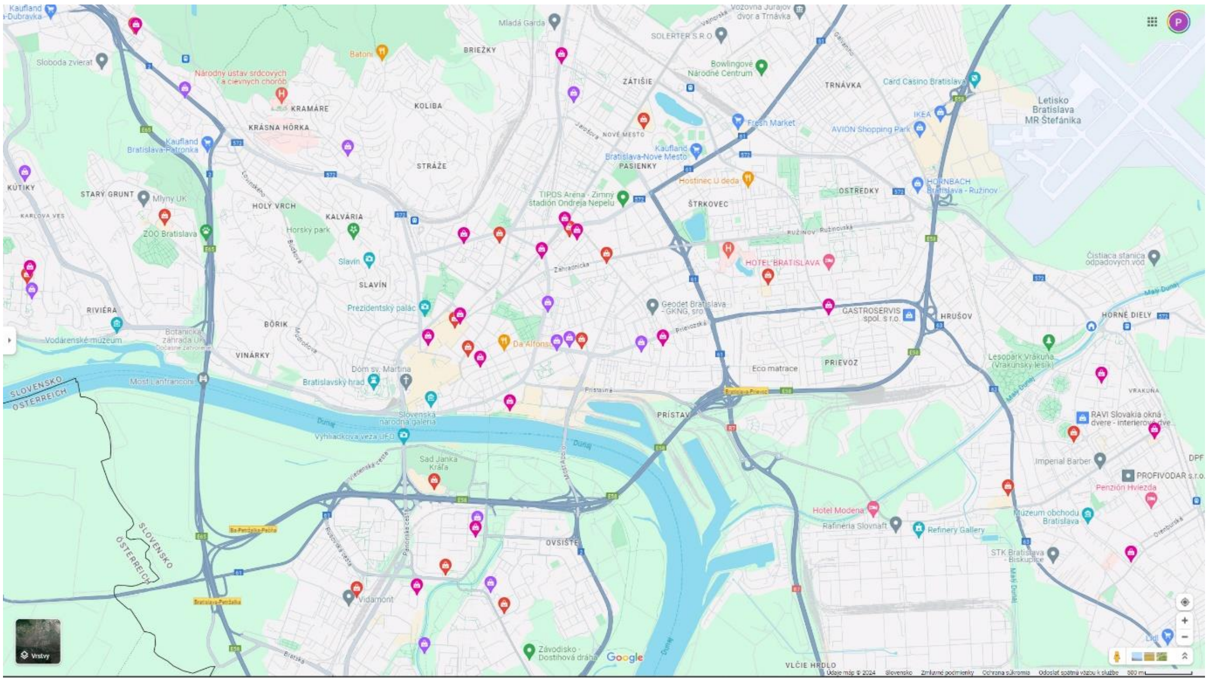
Bratislava, the capital city of the Slovak Republic, is administratively divided into 5 urban districts, 17 urban districts and 264 basic residential units (BRU), which are shown in the population density figure. For optimization purposes, the center of the BRU would need to be appropriately selected and its coordinates (latitude and longitude) determined.

Subsequently, through the traffic location model in the Euclidean plane, it is possible to determine the approximate location where the traffic serving the selected BRUs should be established. These centers of each BRU would represent individual vertices according to graph theory (hereafter referred to as x^i). The solution computed by the optimization then gives the point with the overall shortest distance from the designated traffic $\mathbf{x^0}$ to the individual vertices $\mathbf{x^i}$ with assigned weights w_i ($i = 1, 2, \ldots n$) representing the population in the BRU.
The optimization of the location of the traffic depends essentially on the distance of its shortest link as the crow flies from the other vertices in the considered planar (planar) network (i.e., the BRU centers). This distance represents the length of the line segment between the location point and the point of a given vertex.

2 CASE STUDIES

The case study looks at the distribution of outlets of the three largest chains in the selected segment according to the number of outlets in the study area. The location of the outlets is monitored in the capital of the Slovak Republic, Bratislava, because of the familiar environment and also because of the number of the mentioned outlets, which is high in relation to the size and population.

Fig. 2: Map of Bratislava and location of three comparable retail outlets, which are the most frequented in the given segment



Source: author

Three examples are described:

- The first one, where the location of the operations respects both approaches, i.e. the operations are in the center of the space under study and at the same time they are all located close to each other.
- The second examines an entire urban area that can be divided into two halves, while one half does not contain any of the retail outlets mentioned above.
- The third example shows a situation where the outlets are located in the middle and on the edges of the parts with higher population density.

As mentioned above, the examples below refer to three retail outlets within the same segment providing the same product range. All three provide a variety of global and Slovakian brands and, indeed, all have private labels as well and hence their outlets are well suited to this type of research. The locations of the three retailers are marked in the map of the capital city of Slovakia - Bratislava.

2.1 A placement respecting both approaches

The example below highlights a situation where the two theories above come together in practice. Both theories can be applied to the location of operations on a transnational scale (selecting a site/location that would be appropriate in terms of accessibility to selected states), on a national scale (selecting a city that would be appropriate with respect to other cities and towns in the state), and smaller tasks (selecting a site/location in a city or in a housing estate).

It is evident from the City of Bratislava map image and the location of the three comparable retail stores that are the most popular in the segment that their locations were based primarily on the urban plan options and historical assumptions. A closer look leads to the conclusion that Hotelling's Law does not best represent the location of establishments in the Karlova Ves district. Here it is possible to observe a situation where three supermarkets with the same or similar assortment have outlets located within a range of less than 300 m.

The above mathematical approach would be based on the BRU data. The table below shows the selected BRUs for the Karlova Ves district. 7 BRUs were selected to represent urban development.

Other BRUs that are sparsely populated and have 0-500 inhabitants were abstracted. The BRUs were also selected due to the accessibility and coherence of the selected area (the unselected areas are separated from the selected BRUs by natural barriers, e.g. a hill, and there is no direct transport communication between them, or they are served by other public transport connections). At the same time, the number of inhabitants in the selected BRUs represents 96% of the population of the Karlova Ves urban district (i.e. out of 32648 inhabitants).

Fig. 3: Source code and output in Python programming language to solve the problem of traffic location in Euclidean space

```
import numpy as np
from scipy.optimize import fmin
def f(x):
    function=0
    for i in V:
        function += w[i]*(((latitude[i]-x[0])**2+(longitude[i]-x[1])**2)**0.5)
    return function
BRU=["Kútiky","Dlhé diely - západ","Dlhé diely - východ","Dlhé diely - stred","Dlhé diely -
sever","Riviera","Poliklinika KV"]
latitude=[48.161631,48.152700,48.149449,48.154457,48.154866,48.151448,48.156672]
longitude=[17.048760,17.041795,17.050348,17.047605,17.051972,17.058720,17.056266]
w=[7147,3793,5396,4725,2633,4019,3665]
n, V = len(BRU), set(range(len(BRU)))
xopt, fopt, iter, funcalls, warnflag = fmin(f,np.array([0,0]),full_output=True)
print(xopt, fopt)
```

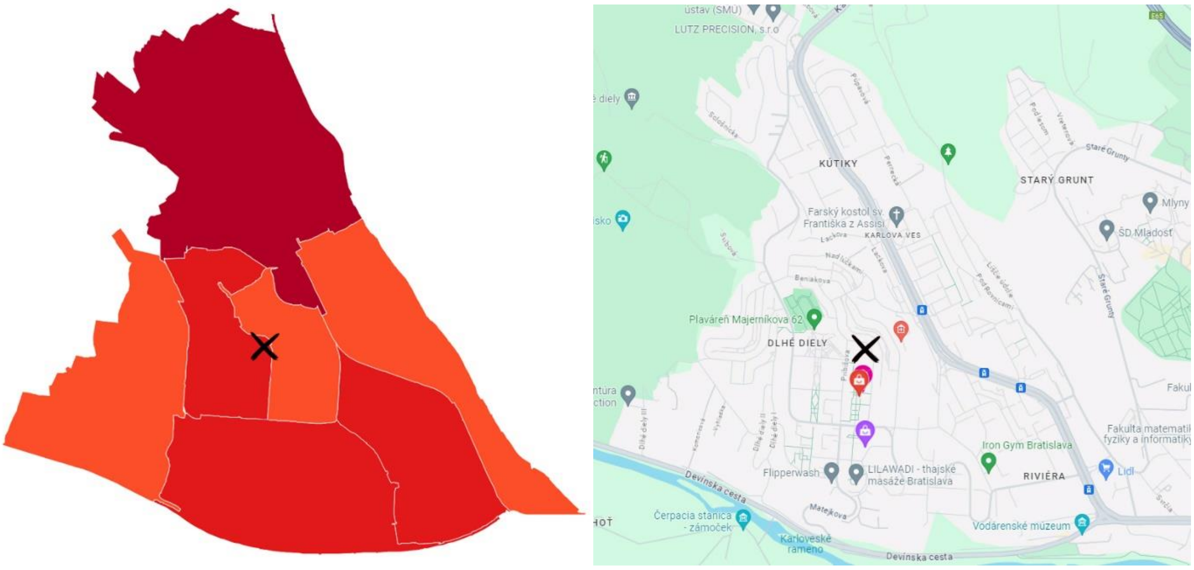
```
Optimization terminated successfully.
         Current function value: 188.726478
         Iterations: 100
         Function evaluations: 184
[48.15475332 17.05028888] 188.72647779603793
```

Source: author

Based on the calculation, the point that determines the optimal location of the plant, given the coordinates of the centers of the individual BRUs and their populations, would have the following coordinates: 48.15475332 17.05028888 (shown as a black cross in the figures).

The given point is plotted in both maps of the urban area with the selected BRUs, both according to the population and according to the location of the establishments of the three retailers mentioned above. The suitability of the location of these shops is demonstrated by the fact that the nearest of the aforementioned establishments is only approximately 150 m away from the calculated point. This fact points to a situation where it is possible to combine the two theoretical approaches mentioned above, which should result in the maximum possible use of the site.

Fig. 4: Map of selected BRUs of the Karlova Ves district, location of selected retail outlets and location of the optimal point of space for the given outlets



Source: author

2.2 Disproportionate placement and optimal placement

The second example deals with a situation where all the outlets of the selected retailers are located in only one part of the selected urban area. Namely, in the Old Town, which can be imaginatively divided into two parts by one, the busiest road. Consequently, it can be observed that in one of these parts there are no establishments even though the population of this part represents 45% of the population of the Old Town urban area. In this part there is also the centre of the whole urban district, where it would be optimal to locate the establishments, but the existing establishments are far from this point. Thus, a more theoretical model according to Hotelling's theory can be observed in the district, although not as visibly as in the previous example. Applying the above-mentioned optimization approach only to the part where there are not yet any establishments would look as follows. The source code and output in the Python programming language would look like the previous example, only the input data would be changed.

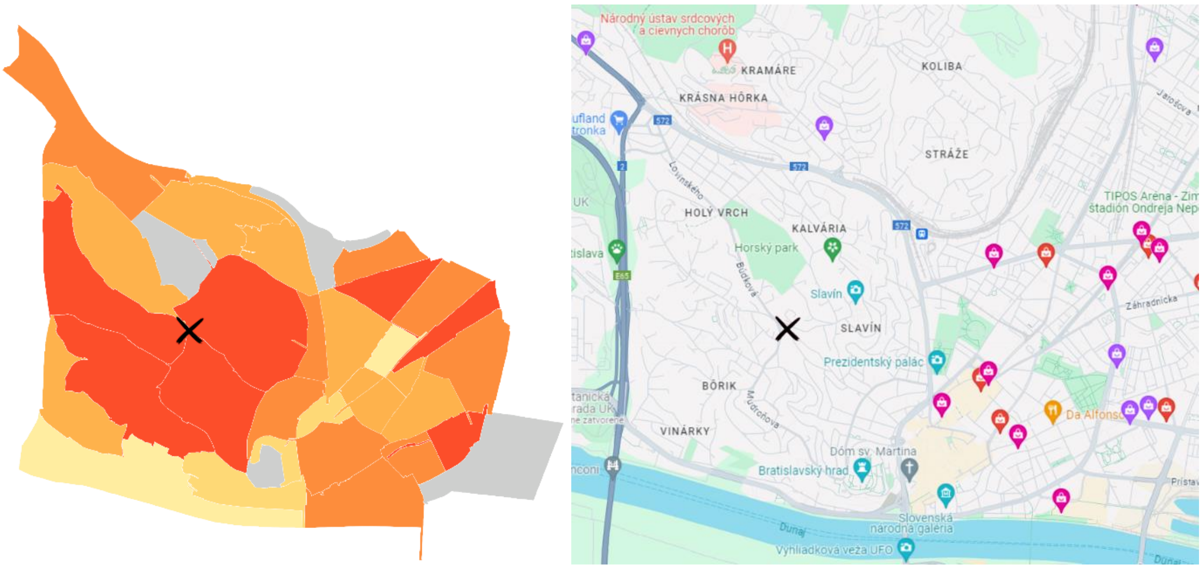
Fig. 5: Input data for the urban district of Staré mesto

```
BRU=["Dubová ulica","Sokolovňa","Kalvária","Holý vrch","Na Hrebienku","Slavín","Štefánka",
"Lýceum","Nemocnica - Partizánska","Bórik","Vinárky","Park kultúry","Podhradie","Hradný svah"]
latitude=[48.163958,48.158748,48.160122,48.157575,48.154597,48.153681,48.152078,48.147267,48.147668,
48.146809,48.149415,48.143631,48.140968,48.143946]
longitude=[17.080875,17.100874,17.094608,17.084781,17.081648,17.099071,17.105423,17.102676,17.095638,
17.085639,17.079245,17.083493,17.097355,17.101775]
w=[1631,843,689,605,2690,2139,1195,1898,2167,2190,811,44,349,208]
```

Source: author

The result would be the coordinates of the optimal location 48.15140034 17.09345585. This location represents the proposed location for a possible new operation. That location is marked in the figure. This new location would provide great potential since, as mentioned, it is 45%, or 17459 of the 38655 population.

Fig. 6: Map of the whole of the Old Town, the location of the selected retail outlets and the location of the optimum point in the area for the proposed location



Source: author

2.3 Specific placement

The third example is about the situation in the urban district of Ružinov, which has 68574 inhabitants, i.e. twice as many inhabitants as the previous urban districts. The location of the three retail outlets in this district is specific in the fact that the outlets are in shopping centers. Another specificity is that they are located in BRUs with a small population or on the outskirts of the urban area.

In this example, there is no visible use of the Hotteling Rule, and the establishments are not located in the middle of the urban district either. Given that this is again a large urban district, but one that also contains large BRUs with small populations, the example only addresses searches in selected BRUs with populations over 1000. These selected BRUs together have 95% of the total population of the urban district.

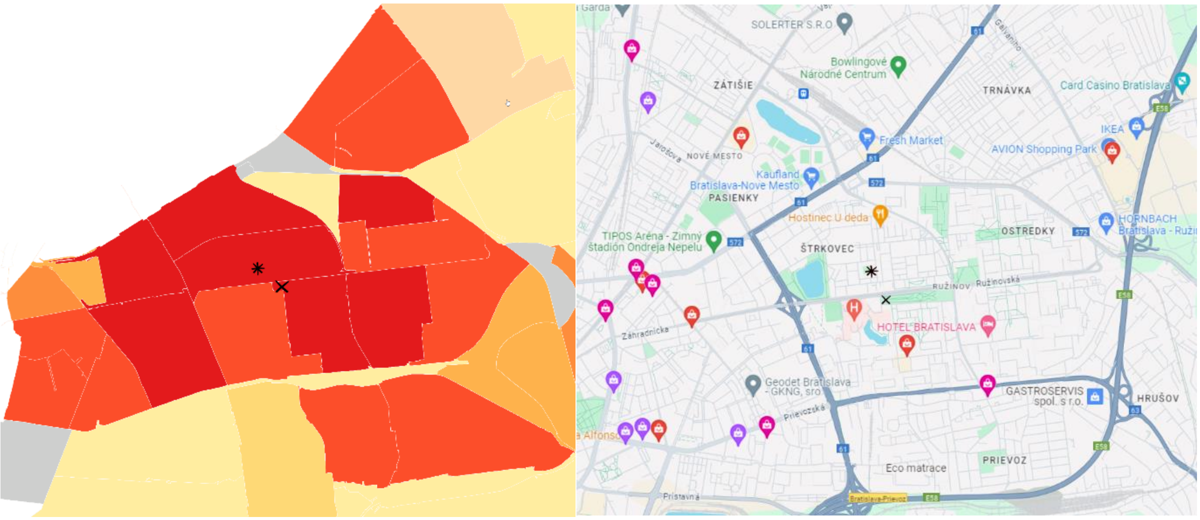
Fig. 7: Input data for the urban district of Ružinov

```
BRU=["Jelačíčova ulica","Koceľova ulica","Sídlisko 500 bytov","Pošen - západ","Trnávka - juh",
"Prievoz - východ","Prievoz - západ","Trnávka - sever","Ostredky - juh","Zvolenská ulica",
"Trávniky - západ","Štrkovec - sever","Ostredky - sever","Štrkovec - juh","Pošen - východ",
"Starý Ružinov","Ružová dolina","Trávniky - východ"]
latitude=[48.155988, 48.152930, 48.148645, 48.154642, 48.168246, 48.146224, 48.146310, 48.171566, 48.159939,
48.148669, 48.153080, 48.161840, 48.163288, 48.159206, 48.154156, 48.158262, 48.152151, 48.153452]
longitude=[17.128682, 17.131064, 17.130198, 17.176728, 17.170168, 17.163510, 17.175784, 17.163988, 17.171536,
17.136946, 17.152737, 17.150677, 17.167931, 17.154325, 17.168489, 17.139691, 17.142525, 17.160033]
w=[1992, 2089, 2248, 2647, 3001, 3025, 3219, 3288, 3541, 3613, 3703, 4113, 4223, 4518, 4567, 4827, 5121, 5362]
```

Source: author

The result of the search for the optimal location is the coordinates: 48.15653363 17.15597795, i.e., the location in the center of the urban area (**x** in the figure). There are only two establishments relatively close to that location (600 meters and 1300 meters, respectively).
When focusing on BRUs with a population of more than 4000 (in the figure in deep red) and neither of them contains establishments of the segment under study, the coordinates are 48.15913505 17.15431974, marked ***** in the figure. This location is even further away from the two establishments that are located in the urban area than the previous calculated location.

Fig. 8: Map of selected BRUs of the Ružinov district, location of selected retail outlets and location of the optimal point in the area for the proposed location



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3 CONCLUSIONS

Deciding where to locate establishments is a complex process that takes into account a number of factors, including urban structure, population density and historical factors. Mainstream methods and theoretical models, such as Hotelling's Law and optimization problems, provide a framework for decision making, but their application requires a situation-specific approach. The case studies analyze in detail the different approaches to the location of retail outlets in different districts of Bratislava. The simple application of theoretical models often has to be subordinated to the specific characteristics of each location and its inhabitants.
In all three examples, optimizing the location of outlets is a complex process that requires the consideration of multiple factors. In the examples, it can also be seen that the best result is in the section that is built last of the three mentioned.
Success in the field of plant location depends not only on theoretical models, but also on the ability to adapt to the specific conditions and possibilities of the chosen location. Given the availability of data and technological tools, it is possible to use modern methods and techniques to optimize the location of emerging plants.

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References

[1] Brezina, I., Pekár, J. and Gežík, P. (2020). Metódy logistiky prepravy, rozmiestňovania a rozvrhovania (Aplikácie matematických modelov v jazyku Python) [Methods of logistics of transportation, placement and scheduling (Applications of mathematic models in Python language)]. Letra Edu.

[2] Čičková, Z. , Reiff, M. and Holzerová P. (2022). *Applied Multi-criteria Model of Game Theory on Spatial Allocation Problem with the Influence of the Regulator*. In: Polish Journal of Management Studies. - Czestochowa : Faculty of Management, Czestochowa University of Technology. - ISSN 2081-7452. - Vol. 26, no. 2 (2022), pp. 112-128.

[3] Holzerová P. and Čičková, Z. (2022). *Duopoly Pricing Strategy in Spatial Competition Using Constant-sum Games*. In: Tem Journal: Technology, Education, Management, Informatics : Journal of the Association for Information Communication Technologies, Education and Science. - Novi Pazar : UIKTEN. - ISSN 2217-8309. - Vol. 11, no. 4 (2022), pp. 1439-1450.

[4] Stránka venovaná vybraným charakteristikám obcí na slovensku - Sčítanie 2021 - doplnené vybrané štatistiky. [Page dedicated to selected characteristics of municipalities in Slovakia - Census 2021 - supplemented with selected statistics.] [online] Available at: http://www.sodbtn.sk/obce/bratislava.php [Accessed 1 April 2024].

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HEATING WITH THE SOLID FUEL: A SPATIAL ECONOMETRIC APPROACH

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Abstract

This paper deals with the environmental challenge in Slovakia regarding the issue of heating. The results of the 2021 Population and Housing Census showed that up to 21.32 % of the Slovak dwellings was heated with solid fuel. The paper analyses this Census data for all the 79 Slovak districts. In the first step, the spatial data analysis proved the impact of location on the ratio of dwellings heated with the solid fuel, i.e. existence of the global positive spatial autocorrelation. While the statistically significant coldspots were located in the western part of Slovakia, the hotspots were mainly located in the middle part of Slovakia. In the second part of the paper, the estimation of the spatial autoregressive model (SAR) enabled to capture not only the link of the selected explanatory variables to the dependent variable in the analysed regions, but also to calculate the spillovers to neighbouring regions.

Keywords: *2021 Population and Housing Census, Spatial analysis, Getis-Ord statistics, SAR model*

JEL Classification: C21, R11
AMS Classification: 62P20, 91B72

1 INTRODUCTION

The environment and improvement of its quality was one of the basic priorities of the strategic document Europe 2020 (European Commission, 2012). The main focus of this priority was the achievement of sustainable growth, the support of more efficient use of resources with the vision of achieving greener and more competitive economies. The green aspect is also part of the current EU cohesion policy framework for the period 2021-2027, which is focused on five investment priorities: a smarter Europe, a low-carbon and greener Europe, a more connected Europe, a more social Europe and a Europe closer to citizens (Euractiv, 2020). Regarding the paper framework, we will focus on the priority of low-carbon and greener Europe.

In order to make Europe the first climate-neutral continent in 2050, the so-called European Green Deal was adopted. This new growth strategy aims to transform the EU into a fair and prosperous society with a modern and competitive economy that uses resources efficiently, where net greenhouse gas emissions will be zero by 2050 and where economic growth does not depend on the use of resources (European Commission, 2019). A significant contribution to improving the quality of air, water or the environment in general is the increase in energy efficiency and the support of renewable energy sources (Euractiv, 2020).

The Slovak Republic, like other EU countries, faces several environmental challenges, and in this context the new Envirostrategy 2030, approved in 2019, plays an important role. Prioritized will be issues of waste management, air quality and protection of habitats and species (Ministry of the Environment of the Slovak Republic, 2019). Both, at the national and international level, ambitious goals aimed at reducing CO2 emissions and the transition from the use of fossil fuels to the use of renewable energy sources have been set.

An important factor in the analysis of emissions is their origin. As pointed out by e.g. Chocholatá (2023b), the emissions from households (specifically from heating) play a crucial role. Therefore, decarbonisation and the associated reduction of greenhouse gas emissions would be the great challenge for heating.

The main aim of the paper is to investigate the data from the 2021 Population and Housing Census in the Slovak Republic. Regarding the energy sources used for heating, the Census data showed that up to 66.22 % of Slovak dwellings are heated with gas, followed by heating with solid fuel (21.32 % of dwellings). Furthermore, we assume that the closer the regions are in terms of geographical distance, the more likely it is that they will influence each other also in the area of environmental quality.

Therefore, we hypothesize that the ratio of dwellings heated with the solid fuel (in %) across the 79 Slovak districts (regions) is influenced by other characteristics of the region (the average nominal monthly salary of an employee, population density and the existence of gas connection). To consider the relevance of the geographical location of the region in space, as well as to calculate the spatial spillovers, the spatial econometric analysis is provided.

The rest of the paper is organized as follows: section 2 deals with methodology, section 3 presents the data and empirical results and section 4 closes with concluding remarks.

2 METHODOLOGY

This section will provide the brief information about the main methodological issues/instruments used in analysis. Even in regional data, the issue of the region's geographical location has not been automatically incorporated. With the advent of the new millennium, models allowing the inclusion of spatial issues – spatial autocorrelation and spatial heterogeneity has become more and more popular, various spatial econometric models and estimation techniques have been developed (Chasco, García and Vicéns, 2007). Since the analysed data has the spatial character, the issues of spatial autocorrelation and spatial spillovers will be presented. The second issue, the spatial heterogeneity, will not be considered (see e.g. Chocholatá, 2023a).

The issue of spatial autocorrelation is identified in case when the value taken by a given variable is related to the value of the same variable located nearby. To capture the character of spatial links (spatial proximity) among regions, the appropriately constructed spatial weights matrix **W** is used. The measurement of the spatial autocorrelation can be done by global and local spatial autocorrelation statistics such as Moran´s I statistic, Geary´s c statistic or Getis-Ord statistics $G_i(d)$.

The aim of this paper is to assess the role of region location in the regional modelling of the ratio of dwellings using the solid fuel heating. To verify the hypothesis formulated in section 1, the preliminary spatial analysis will be followed by estimation of the spatial autoregressive model (SAR) assuming the spatial spillover effects within the spatial lag of dependent variable (global spillover). The spatial spillover effect means that for a particular region, the data generating process is influenced by the nature of the dependent variables related to the nearby regions. SAR model[1], in matrix notation, can be formulated as follows:

---

[1] Actually, we present the spatial extension of the standard regression model (combination of the spatial autoregressive structure with a standard regression model) labelled sometimes also as mixed-regressive, spatial-regressive model (see LeSage and Pace, 2009).

$$\mathbf{y} = \rho\mathbf{W}\mathbf{y} + \boldsymbol{l_N}\alpha + \mathbf{X}\boldsymbol{\beta} + \mathbf{u} \tag{1}$$

where $\mathbf{y}$ is the N dimensional vector of the dependent variable for all N locations, $\mathbf{X}$ denotes the $N \times k$ matrix of exogenous explanatory variables (k represents the number of explanatory variables), $\boldsymbol{l_N}$ represents the N dimensional vector of ones associated with the intercept α, $\boldsymbol{\beta}$ is the k dimensional vector of unknown parameters to be estimated, $\mathbf{u} \sim N(\mathbf{0}, \sigma_u^2 \mathbf{I}_\mathrm{N})$ is the N dimensional vector of random errors, σ_u^2 is random error variance, $\mathbf{W}$ is the N dimensional spatial weights matrix and $\mathbf{Wy}$ denotes the N dimensional vector of the spatially lagged dependent variable. Spatial autoregressive parameter ρ indicates the direction and the strength of spatial dependence. In case that the parameter ρ takes the value of zero, there is no spatial dependence in $\mathbf{y}$**,** the model (1) reduces to the standard (least-squares) regression model.

As pointed out by LeSage and Pace (2009), the expected value of the dependent variable in region i is no longer influenced only by the exogenous characteristics of the region i, but also by the exogenous characteristics of all other regions through a spatial multiplier $(\mathbf{I}_\mathrm{N} - \rho\mathbf{W})^{-1}$. The data generating process associated to model (1) can be thus written as:

$$\mathbf{y} = (\mathbf{I}_\mathrm{N} - \rho\mathbf{W})^{-1}\mathbf{X}\boldsymbol{\beta} + (\mathbf{I}_\mathrm{N} - \rho\mathbf{W})^{-1}\mathbf{u} \tag{2}$$

A change in a single region related to any explanatory variable will have *a direct impact* on the region itself as well as potential *indirect impact* on all the other regions. Furthermore, LeSage and Pace (2009) presented the summary impact measures (summary average measures of marginal effects) as well as their computational forms. The average total impact can be separated into two parts, the first part is related to the direct effects – average direct impact and the second part measures the indirect effects – average indirect impact (Furková, 2018).

3 DATA AND EMPIRICAL RESULTS

This section presents the data for the spatial analysis of the 79 Slovak districts (regions). As a dependent variable, the ratio of dwellings heated with the solid fuel (in %) is considered. As explanatory variables, three variables were used: the average nominal monthly salary of an employee (in Euro), population density per km^2 and houses with the gas connection (in %). The data were retrieved from two sources – the 2021 Population and Housing Census[2] (ratio of dwellings heated with the solid fuel and ratio of houses with gas connection) and from the DATAcube database of the Statistical Office of the Slovak Republic[3] (the average nominal monthly salary of an employee and population density per km^2). In the further text of this paper, the data will be marked as follows: dependent variable – "solid_fuel", natural logarithm of average nominal monthly salary of an employee – "wage", natural logarithm of population density per km^2 – „dens" and the ratio of houses with the gas connection – "gas". The analyses were provided in free downloadable softwares GeoDa and R.

Figure 1 graphically illustrates the values of the dependent variable in individual regions in the form of the box-map (the extended version of the quartile map with the separate identification of the outliers in the first and fourth quartile, respectively). To account for spatial interactions between regions, the binary spatial weights matrix $\mathbf{W}$ of the "queen" type was used. Although based on the box-map, the spatial clusters could be identified, to provide information about the

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[2] https://www.scitanie.sk/en/dwellings/basic-results/structure-of-dwellings-by-energy-sources-used-for-heating/SR/SK0/OK

[3] http://datacube.statistics.sk/

statistical significance of the clusters, the local version of the Getis-Ord statistics was calculated. Upon the graphical assessment, the statistically significant clusters, so-called hotspot and coldspot regions are clearly specified. It seems to be evident that the regions with the low ratios of dwellings heated by solid fuel are mainly located in the western part of Slovakia, whereas the hotspots are significantly present in the central part of Slovakia. Furthermore, the calculated Moran's I global statistics of 0.476 showed the existence of a strong positive spatial autocorrelation.

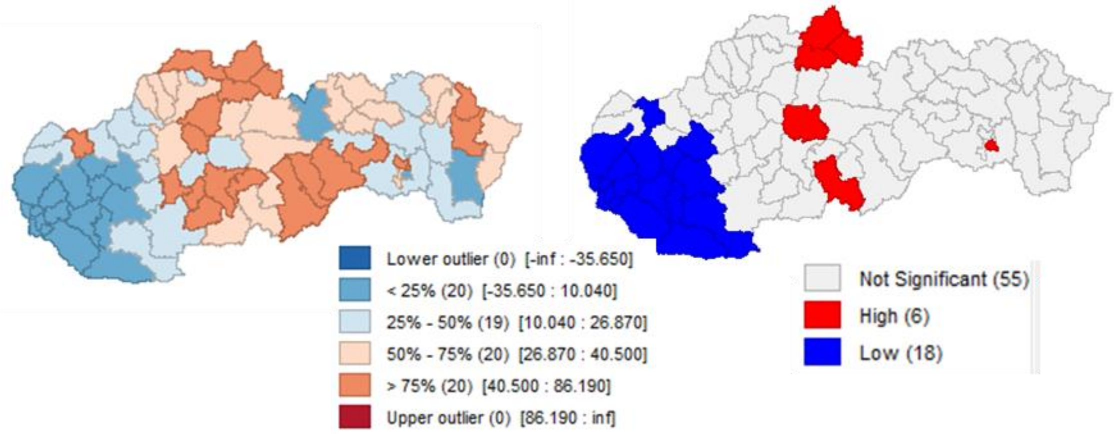

Figure 1 Box-map and Getis-Ord cluster map for the dependent variable (solid_fuel)

Source: constructed by the author in GeoDa

The previous confirmation of the significant spatial pattern of the dependent variable will be taken into account by the following spatial econometric modelling. Firstly, the standard (least-squares) regression model was estimated, i.e. model (1) without the term $\rho\mathbf{Wy}$. Estimation results as well as diagnostic results are gathered in Table 1 (column: Linear model). All the estimated parameters were statistically significant, the expected negative relationship between the ratio of dwellings heated with the solid fuel and the variables average nominal monthly salary of an employee (*wage*) and the ratio of houses with the gas connection (*gas*), respectively was confirmed. This indicates that increase of wages as well as increase in gasification of regions will lead to the decline of dwellings heated by the solid fuel. On the other hand, in case of the variable population density per km^2 (*dens*) the positive impact was identified, indicating that the rise of population density in the region will lead to the increasing ratio of dwellings heated by solid fuel. Based on values of the robust LM statistics (since the robust LM (lag) statistic is statistically significant while the robust LM (error) statistic is not), we decided for estimation of the SAR model (1). As already mentioned above, the inclusion of the spatial component thus seems to be inevitable in order to overcome the problem of possibly biased results and hence misleading conclusions from estimation. Estimation results of the SAR model are collected in Table 1 (column: SAR model).

Due to the confirmed spatial autocorrelation, we will not pay further attention to the estimation results of the standard linear regression model. The ML estimation of the SAR model (1) produced statistically significant estimates of all parameters with expected signs. The estimation of the spatial autoregressive parameter ρ is statistically significant as well, so it confirms that the explicit incorporation of the spatially lagged dependent variable was adequate. In addition, this was also confirmed by LR test results (see Table 1).

Table 1 Estimation Results of Linear Regression Model and SAR model

	Linear model	**SAR model**
Estimation	OLS	ML
α	513.412***	413.772***
β_1 ($wage_i$)	-67.239***	-55.787***
β_2 ($dens_i$)	7.673***	7.364***
β_3 (gas_i)	-0.663***	-0.526***
ρ	-	0.392***
R^2	0.518	-
Log likelihood	-314.339	-308.753
Diagnostic tests		
Moran's *I* (error)	2.664***	-
LM (lag)	11.896***	-
Robust LM (lag)	8.519***	-
LM (error)	4.309**	-
Robust LM (error)	0.933	-
LR test	-	11.171***

Note: Symbols *** and ** indicate the statistical significance at 1% and 5% level of significance, respectively. Explanation of abbreviations: OLS – Ordinary Least Squares, ML – Maximum Likelihood, LM – Lagrange Multiplier, LR – Likelihood Ratio.
Source: Author's calculations in Geoda.

Following LeSage and Pace (2009), we have done the calculations and statistical verification of summary impact measures (average total impact, average direct impact and average indirect impact) – see Table 2.

Table 2 Summary measures of direct, indirect and total impacts

	wage	***dens***	***gas***
Parameter estimate (β_1, β_2, β_3)	-55.787	7.364	-0.526
Average direct impact (*ADI*)	-57.967	7.651	-0.546
Difference *ADI* and parameter estimate	-2.180	0.287	-0.020
Average indirect impact (*AII*)	-33.783	4.459	-0.318
Average total impact (*AT*I)	-91.750	12.111	-0.865
ADI/ATI	0.632	0.632	0.632
AII/ATI	0.368	0.368	0.368

Source: authors' calculations in R

Let us consider the SAR model estimates for individual explanatory variables. The estimate for the first explanatory variable, *wage*, yields -55.787, the corresponding average direct impact is different and equals -57.967. The difference between ADI and parameter estimate of 2.180 indicates the feedback effects among regions. The average total impact of -91.750 mirrors that 1 % rise of wages will lead to average decline of dwellings heated by the solid fuel of 0.918 percentage points.

Now, let us take a closer look at the impacts associated with the *dens* variable. Again, we can notice that, the average direct impact (7.651) does not match the estimate of the parameter (7.364). The difference of 0.287 is perceived as the amount of feedback that arises from the effects passing through the neighbouring regions, and is reversed by the region itself. The 1 %

increase in the region's density will contribute to the average increase of dwellings heated by the solid fuel of 0.077 percentage points.

As for the third explanatory variable, *gas*, the difference of 0.020 between the ADI and parameter estimate corresponds to the feedback effect, as well. The average total impact of -0.865 shows that the 1 percentage point increase in the gasification of the region will cause an average decline of 0.865 percentage points in the ratio of dwelling heated by the solid fuel.

The percentage shares of the average direct and average indirect impacts respectively, on the average total impacts are always the same for all explanatory variables in the SAR model, i.e. 63.2 % and 36.8 %, respectively – see Table 2.

4 CONCLUSION

This paper aimed at the analysis of the ratio of dwellings heated with the solid fuel across the Slovak regions. The relevance of the geographical location of the region was proved by the spatial analysis, the clusters of hotspots and coldspots were identified. In the second step of analysis, the econometric analysis with concentration on the ML estimation of the SAR model confirmed that the analysed variable is positively linked with its spatial lagged values and by the other variables - the average nominal monthly salary of an employee, population density and the existence of gas connection. Calculation of the summary measures of direct, indirect and total impacts indicated a higher portion of total impact attributable to the direct impacts (63.2 %) compared to the 36.8 % assignable to indirect impacts.

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References

[1] Chasco, C., García, I. and Vicéns, J. (2007). Modeling spatial variations in household disposable income with Geographically Weighted Regression. [online] Available at: https://mpra.ub.uni-muenchen.de/1682/ [Accessed 5 Feb. 2020].

[2] Chocholatá, M. (2023a). Heating with Solid Fuel in Slovak Dwellings: A GWR approach. [online] In: *Proceedings of the 17th Symposium on Operational Research in Slovenia*. Available at: https://drustvo-informatika.si/uploads/documents/6a1c2595-7d3f-4dd2-ab6c-9ed9b168c19d//SOR23Proceedings.pdf [Accessed 30 Apr. 2023].

[3] Chocholatá, M. (2023b). Emisie oxidu uhoľnatého v Slovenskej republike pre rôzne úrovne regionálneho členenia. [online] In: *Medzinárodný seminár mladých vedeckých pracovníkov Katedry ekonometrie FIS VŠE v Praze a Katedry operačného výskumu a ekonometrie FHI EU v Bratislave*. Available at: https://sekarl.euba.sk/arl-eu/sk/detail/?&idx=eu_un_cat*0297387 [Accessed 20 Apr. 2024].

[4] Energieportal. (2021). Pomôžu plynové kotly znížiť emisie CO2? Vodík môže byť výhodné riešenie pre vykurovanie. [online] Available at: https://www.energie-portal.sk/Dokument/pomozu-plynove-kotly-znizit-emisie-co2-vodik-moze-byt-vyhodne-riesenie-pre-vykurovanie-107032.aspx [Accessed 30 Apr. 2023].

[5] Euractiv. (2020). Kohézna politika 2021 až 2027. [online] Available at: https://euractiv.sk/section/ekonomika-a-euro/video/%F0%9F%94%B4-nazivo-kohezna-politika-2021-az-2027/ [Accessed 30 Apr. 2023].

[6] European Commission. (2012). Consolidated versions of the Treaty on European Union and the Treaty on the Functioning of the European Union. *Official Journal of the European Union*, 55, pp. 1-390. [online] Available at: http://eur-

lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:C:2012:326:FULL:EN:PDF [Accessed 15 Feb. 2015].
[7] European Commission. (2019). Európsky ekologický dohovor. [online] Available at: https://eur-lex.europa.eu/resource.html?uri=cellar:b828d165-1c22-11ea-8c1f-01aa75ed71a1.0018.02/DOC_1&format=PDF [Accessed 30 Mar. 2024].
[8] Furková, A. (2018). R&D Spatial Spillovers across the EU regions: Quantification and sensitivity analysis based on the spatial econometric approach. In*: AIESA Building of Society based on Knowledge (18th International Scientific conference)*, pp. 439-446. Available at: https://fhi.euba.sk/www_write/files/veda-vyskum/konferencie/aiesa/AIESA_2018/Zbornik_AIESA2018.pdf [Accessed 30 Mar. 2024].
[9] LeSage, J. and Pace, K. (2009). *Introduction to Spatial Econometrics*. Boca Raton: Chapman and Hall/CRC.
[10] Ministry of the Environment of the Slovak Republic. (2019). Zelenšie Slovensko. Stratégia environmentálnej politiky Slovenskej republiky do roku 2030. [online] Available at: https://www.minzp.sk/files/iep/03_vlastny_material_envirostrategia2030_def.pdf [Accessed 2 Nov. 2023].
[11] Population and Housing Census. (2021). Number of dwellings by energy sources used for heating in the Slovak Republic at 1. 1. 2021. [online] Available at: https://www.scitanie.sk/en/dwellings/basic-results/structure-of-dwellings-by-energy-sources-used-for-heating/SR/SK0/OK [Accessed 30 Apr. 2023].
[12] Statistical Office of the Slovak Republic. (2023). DATAcube. [online] Available at: http://datacube.statistics.sk/ [Accessed 30 Apr. 2023].

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SCIENTOMETRIC ANALYSIS OF OPERATIONS RESEARCH & MANAGEMENT SCIENCE JOURNALS: A DEA APPROACH

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Abstract

The paper deals with the scientometric analysis of journals indexed in the Web of Science database included in the category Operations Research & Management Science (currently there are 86 journals). The study considers five main indicators that describe the performance of the journals - impact factor, article influence score, immediacy index, journal citation index, and 5-yaer impact factor. Rankings defined by the indicators are aggregated using a discrete optimization model that minimizes the weighted sum of deviations of the final ranking from the partial ones. This result is compared with the ranking obtained by a DEA model without explicit inputs. Relationships between all pairs of indicators is discussed on a real dataset for the year 2022. The paper compares the classification of journals into categories Q1 to Q4 according to the traditional methodology and according to the results of the proposed methodology.

Keywords: *Impact factor, Article influence score, Data envelopment analysis, Optimization, Ranking*

JEL Classification: C44
AMS Classification: 90C08

1 INTRODUCTION

Scientific journals are ranked and evaluated according to the various scientometric measures. The position of the journal within the set of journals of the similar scientific field is an important indicator for evaluation of scientific outputs of researchers and allocation of funds among individual researchers and/or departments, faculties etc. The aim of this paper is to analyze the set of 86 journals included in the Web of Science category Operations Research and Management Science (ORMS) using several the most important scientometric measures and by alternative approaches. Many researchers dealt with the similar topic, namely with the evaluation of ORMS journals using alternative approaches as data envelopment analysis (DEA) models, multiple criteria decision making (MCDM) techniques, and other modelling tools.

One of the first papers which uses DEA models for evaluation of journals is (Halkos and Tzemeres, 2011). The authors evaluate 229 economic journals indexed in the Web of Science. They used just one input and two outputs where all of them are constructed as composite indicators from several other partial indicators. For application of DEA models, it is not an ideal approach. Lozano and Salmerón (2011) investigated rather the administrative performance of ORMS journals by traditional DEA models – they considered the variables that describe the length of the reviewing process, the time from acceptance to publication of articles and others. Xu et al. (2011) construct a new scientometric indicator for ORMS journals based on PageRank method that allows a differentiation of citations by quality. Their approach takes into consideration not only the quantity of citations but also their quality which is similar to the definition of article influence score (AIS). Liao et al. (2019) published a study about highly cited papers in the ORMS field. The aim of this study is to identify the most influential papers, authors, institutions and countries. Chen et al. (2021) developed and applied integer DEA models for evaluation of ORMS journals. The variables used in their study are taken from main citation metrics as AIS, impact factor (IF), h-index, and others.

In this paper, we will evaluate the citation impact of 85 ORMS journals (one journal was removed because not all variables are available) included in the Web of Science database. The next section presents the applied methodology. Section 3 contains the results of the study and the last section points out the main findings and discusses the future research.

2 METHODOLOGY

Application of DEA or MCDM models requires definition of variables – inputs/outputs when using the terminology of DEA or criteria when MCDM models are applied. They are used for evaluation of decision making units (DMUs) in DEA or alternatives in MCDM. In out study, the DMUs are the journals being evaluated. Their impact or importance is described by various indicators that are published every year in the Clarivate Journal Citation Report. We will further consider the following indicators:

- Impact factor (IF) in year *t* is a total number of citations in this year of articles published in the preceding two years divided by the number of citable items in these two years.
- 5-year impact factor (5Y IF) is defined in the same way as the IF but this indicator considers 5-year period.
- Immediacy index (IMI) is the number of citations to a journal in year *t* divided by the number of articles published in this year.
- Article influence score (AIS) is a relatively new indicator that reflects the importance of journals that cite the articles published in the considered journal. Its calculation is much more complex than calculation of the other ones.
- Journal citation index (JCI) is a new measure introduced by Clarivate company few years ago. It is quite strongly correlated with the journal IF and its added value is limited.

In our study, we are trying to apply alternative approaches for ranking of journals based on the above-mentioned measures. First of them is DEA based approach. DEA models were introduced in (Charnes et al., 1978). DEA itself is originally not a tool for ranking of the set of units but there are extensions that allow to offer this information – e.g. Andersen and Petersen (1993) model. We will use DEA models without explicit inputs and the five measures (variables) are considered as outputs of the model. Mathematical formulation of a super-efficiency DEA model without explicit inputs is as follows:

$$\begin{aligned} &\text{Maximize} \quad && \text{eff(DMU}_q) = \sum_{k=1}^{r} u_k y_{qk}, \\ &\text{subject to} && \\ & && \sum_{k=1}^{r} u_k y_{ik} \le 1, \quad i = 1,\dots,n, i \ne q, \\ & && u_k \ge \varepsilon, \ k = 1,\dots,r, \end{aligned} \tag{1}$$

where n is the number of the units in the set, r is the number of outputs, q is the index of the unit under evaluation, and y_{ik} is the value of the k-th output for the i-th unit. The units can be ranked in descending order according to the objective function values – the value greater or equal to 1 indicates efficiency, the values less than 1 inefficiency of the unit under evaluation. Lower values mean lower position in the final ranking.

The second approach that is applied for the ranking of ORMS journals is based on the aggregation of several (five in our case) partial rankings. This approach was used for ranking of nations at Summer Olympic Games in (Jablonsky, 2018). The partial rankings are aggregated by solving the following integer linear optimization problem:

$$\begin{aligned} &\text{Minimize} \quad \sum_{i=1}^{n}\sum_{j=1}^{s} w_j (d_{ij}^{-} + d_{ij}^{+}) \\ &\text{subject to} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2) \\ &r_{ij} + d_{ij}^{-} - d_{ij}^{+} = x_i, \qquad i = 1,\dots,n,\ j = 1,\dots,s, \\ &\sum_{j=1}^{s} y_{ij} = 1, \qquad i = 1,\dots,n, \\ &\sum_{i=1}^{n} y_{ij} = 1, \qquad j = 1,\dots,s, \\ &x_i = \sum_{j=1}^{s} j.y_{ij}, \qquad i = 1,\dots,n, \\ &y_{ij} - binary, \end{aligned}$$

where s is the number of partial rankings to be aggregated into one final ranking, r_{ij}, i = 1, …, n, j = 1, …, s is the position of the i-th unit in the j-th partial ranking, x_i, i = 1, …, n is the position of the i-th unit in the final ranking, w_j, j = 1, …, s expresses the importance of the j-th partial ranking, d_{ij}^{-}, d_{ij}^{+} are negative and positive deviations of the final ranking and the partial rankings, and y_{ij}, , i = 1, …, n, j = 1, …, s are artificial binary variables that help to ensure the uniqueness of the final ranking.

3 RESULTS

In this section, the results of both approaches will be illustrated on a dataset of 85 ORMS journals indexed in the Clarivate Web of Science database. In addition, they will be compared with the most influential scientometric measures – IF and AIS in the last year edition.

Table 1 – *Correlation between variables (2022)*

	IF	JCI	5-Y IF	Immediacy	AIS
IF	1.0000				
JCI	0.8903	1.0000			
5-Y IF	0.9658	0.9095	1.0000		
Immediacy	0.8200	0.7279	0.7561	1.0000	
AIS	0.6038	0.7195	0.7433	0.3742	1.0000

Correlations between all pairs of variables are presented in Table 1. It is not surprising that the highest correlation level is between the IF and 5-Y IF because they are based on the same principle, differing only in the time horizon. Journal IF is strongly correlated with the JCI and the immediacy index, the level of correlation of this variable with the AIS is significantly lower. AIS is a measure that is relatively new. It is not based on the number of citations as the other metrics but on the importance of citing journals. That is why the level of correlation of the AIS measure with all other metrics is lower. In our numerical experiments. We have worked with two models as described in the previous section. The DEA model without explicit inputs does not consider any weight restrictions. The aggregation procedure – model (2) – considers weights of variables. They have been set as follows: IF – 0.3, JCI, 5-Y IF, Immediacy index – 0.1, and AIS – 0.4. We did not analyze the sensitivity of the results on the setting of the weights. This is to avoid overemphasizing the weight of the impact factor that is strongly correlated with the other three measures.

The results are presented in Table 2. This table contains (for comparison purposes) the journal IF 2022 and the AIS 2022 of all 85 journals including their assignment to the Q1 to Q4 categories which is important for the evaluation of the publication outputs at universities ad other research institutions. Except this, Table 2 presents efficiency and super-efficiency scores computed by model (1) and results of the aggregation procedure – model (2) including the assignment into Q1 to Q4 categories. The last 3 rows of Table 2 contain minimum, median and maximum value of the measures.

Table 2 - *Results*

	Impact 2022		*AIS 2022*		*DEA*		*Aggregation*	
Journal	*Value*	*Q*	*Value*	*Q*	*CCR*	*Q*	*Rank*	*Q*
4OR-Q J OPER RES	2.00	Q3	0.591	Q3	0.214	Q3	55	Q3
ANN OPER RES	4.80	Q1	0.798	Q2	0.420	Q2	21	Q1
APPL STOCH MOD BUS	1.40	Q4	0.396	Q4	0.150	Q4	67	Q4
ASIA PAC J OPER RES	1.40	Q4	0.221	Q4	0.143	Q4	80	Q4
CENT EUR J OPER RES	1.70	Q4	0.389	Q4	0.167	Q4	69	Q4
COMPUT OPER RES	4.60	Q2	1.157	Q2	0.456	Q2	22	Q2
COMPUT OPTIM APPL	2.20	Q3	1.229	Q1	0.356	Q2	32	Q2
DECIS SUPPORT SYST	7.50	Q1	1.539	Q1	0.693	Q1	10	Q1
DISCRETE EVENT DYN S	2.00	Q3	0.844	Q2	0.296	Q2	54	Q3
DISCRETE OPTIM	1.10	Q4	0.669	Q2	0.243	Q3	65	Q4
ENG ECON	1.20	Q4	0.216	Q4	0.117	Q4	82	Q4
ENG OPTIMIZ	2.80	Q2	0.517	Q3	0.241	Q3	57	Q3
EUR J IND ENG	1.00	Q4	0.219	Q4	0.371	Q2	81	Q4
EUR J OPER RES	6.40	Q1	1.395	Q1	0.612	Q1	14	Q1
EXPERT SYST APPL	8.50	Q1	1.276	Q1	0.692	Q1	8	Q1
FLEX SERV MANUF J	2.70	Q2	0.563	Q3	0.243	Q3	49	Q3
FUZZY OPTIM DECIS MA	4.70	Q2	0.818	Q2	0.394	Q2	24	Q2
IEEE SYST J	4.40	Q2	0.885	Q2	0.395	Q2	26	Q2
IISE TRANS	2.60	Q2	0.780	Q2	0.283	Q3	37	Q2
IMA J MANAG MATH	1.70	Q4	0.282	Q4	0.209	Q3	68	Q4
INFOR	1.30	Q4	0.337	Q4	0.130	Q4	74	Q4
INFORMS J APPL ANAL	1.40	Q4	0.393	Q4	0.144	Q4	71	Q4
INFORMS J COMPUT	2.10	Q3	1.029	Q2	0.297	Q2	43	Q3
INT J COMPUT INTEG M	4.10	Q2	0.629	Q3	0.342	Q2	29	Q2
INT J IND ENG COMP	3.30	Q2	0.548	Q3	0.272	Q3	47	Q3
INT J INF TECH DECIS	4.90	Q1	0.442	Q4	0.392	Q2	28	Q2
INT J PROD ECON	12.00	Q1	1.731	Q1	0.960	Q1	5	Q1
INT J PROD RES	9.20	Q1	1.317	Q1	0.736	Q1	11	Q1
INT J SYST SCI	4.30	Q2	0.720	Q2	0.355	Q2	30	Q2
INT J TECHNOL MANAGE	2.80	Q2	0.351	Q4	0.224	Q3	58	Q3
INT T OPER RES	3.10	Q2	0.606	Q3	0.277	Q3	31	Q2
J GLOBAL OPTIM	1.80	Q3	0.721	Q2	0.231	Q3	52	Q3
J IND MANAG OPTIM	1.30	Q4	0.261	Q4	0.144	Q4	77	Q4
J MANUF SYST	12.10	Q1	1.750	Q1	1.055	Q1	2	Q1

J OPER MANAG	7.80	Q1	2.906	Q1	1.042	Q1	3	Q1
J OPER RES SOC	3.60	Q2	0.660	Q3	0.308	Q2	39	Q2
J OPTIMIZ THEORY APP	1.90	Q3	0.776	Q2	0.246	Q3	59	Q3
J QUAL TECHNOL	2.50	Q2	0.935	Q2	0.311	Q2	33	Q2
J SCHEDULING	2.00	Q3	0.570	Q3	0.207	Q3	56	Q3
J SIMUL	2.50	Q2	0.409	Q4	0.205	Q3	53	Q3
J SYST ENG ELECTRON	2.10	Q3	0.328	Q4	0.169	Q4	73	Q4
J SYST SCI SYST ENG	1.20	Q4	0.268	Q4	0.171	Q4	76	Q4
M&SOM-MAN SERV OP	6.30	Q1	2.801	Q1	0.855	Q1	9	Q1
MANAGE SCI	5.40	Q1	3.989	Q1	1.250	Q1	15	Q1
MATH METH OPER RES	1.20	Q4	0.509	Q3	0.157	Q4	72	Q4
MATH OPER RES	1.70	Q4	1.641	Q1	0.425	Q2	45	Q3
MATH PROGRAM	2.80	Q2	2.296	Q1	0.633	Q1	23	Q2
MATH PROG COMPUT	6.30	Q1	3.498	Q1	1.353	Q1	4	Q1
MEMET COMPUT	4.70	Q2	0.888	Q2	0.445	Q2	27	Q2
MIL OPER RES	0.70	Q4	0.208	Q4	0.074	Q4	84	Q4
NAV RES LOG	2.20	Q3	0.596	Q3	0.223	Q3	50	Q3
NETW SPAT ECON	2.40	Q3	0.686	Q2	0.258	Q3	41	Q2
NETWORKS	2.10	Q3	1.372	Q1	0.419	Q2	25	Q2
OMEGA-INT J MANAG S	6.90	Q1	1.501	Q1	0.691	Q1	13	Q1
OPER RES	2.70	Q2	2.140	Q1	0.537	Q1	34	Q2
OPER RES LETT	1.10	Q4	0.520	Q3	0.151	Q4	79	Q4
OPER RES PERSPECT	2.50	Q2	0.615	Q3	0.400	Q2	46	Q3
OPER RES-GER	2.70	Q2	0.481	Q3	0.230	Q3	40	Q2
OPTIM CONTR APPL MET	1.80	Q3	0.387	Q4	0.189	Q4	62	Q3
OPTIM ENG	2.10	Q3	0.581	Q3	0.228	Q3	51	Q3
OPTIM LETT	1.60	Q4	0.623	Q3	0.202	Q4	60	Q3
OPTIM METHOD SOFTW	2.20	Q3	1.040	Q2	0.307	Q2	35	Q2
OPTIMIZATION	2.20	Q3	0.708	Q2	0.262	Q3	42	Q2
OR SPECTRUM	2.70	Q2	0.631	Q3	0.255	Q3	44	Q3
P I MECH ENG O-J RIS	2.10	Q3	0.352	Q4	0.176	Q4	63	Q3
PAC J OPTIM	0.20	Q4	0.122	Q4	0.086	Q4	85	Q4
PROBAB ENG INF SC	1.10	Q4	0.410	Q4	0.158	Q4	70	Q4
PROD OPER MANAG	5.00	Q1	1.693	Q1	0.616	Q1	16	Q1
PROD PLAN CONTROL	8.30	Q1	1.100	Q2	0.664	Q1	18	Q1
QUAL RELIAB ENG INT	2.30	Q3	0.444	Q3	0.202	Q4	48	Q3
QUAL TECH QUANT M	2.80	Q2	0.492	Q3	0.247	Q3	36	Q2
QUEUEING SYST	1.20	Q4	0.542	Q3	0.160	Q4	75	Q4
RAIRO-OPER RES	1.80	Q3	0.271	Q4	0.286	Q2	66	Q4
RELIAB ENG SYST SAFE	8.10	Q1	1.068	Q2	0.800	Q1	12	Q1
SAFETY SCI	6.10	Q1	1.008	Q2	0.518	Q2	19	Q1
SOCIO-ECON PLAN SCI	6.10	Q1	0.898	Q2	0.971	Q1	17	Q1
SORT-STAT OPER RES T	1.60	Q4	0.476	Q3	0.172	Q4	64	Q4
STUD INFORM CONTROL	1.60	Q4	0.204	Q4	0.142	Q4	83	Q4
SYST CONTROL LETT	2.60	Q2	1.130	Q2	0.340	Q2	38	Q2
SYSTEMS ENG	2.00	Q3	0.227	Q4	0.160	Q4	78	Q4
TECHNOVATION	12.50	Q1	1.914	Q1	1.204	Q1	1	Q1
TOP	1.80	Q3	0.609	Q3	0.204	Q3	61	Q3
TRANSP RES B-METH	6.80	Q1	2.208	Q1	0.771	Q1	7	Q1
TRANSPORT RES E-LOG	10.60	Q1	1.787	Q1	0.877	Q1	6	Q1
TRANSPORT SCI	4.60	Q2	1.818	Q1	0.577	Q1	20	Q1
Minimum	0.20		0.122		0.074		1	
Median	2.50		0.660		0.283		42	
Maximum	12.50		3.989		1.353		85	

15 journals out of a total of 85 are ranked in the Q1 category according to all 4 indicators in Table 2 – their names are in bold. The Q categories for all indicators are very close to each other for most journals but there are several exceptions. The most notable ones are the following:

- *International Journal of Information Technology & Decision Making* is in Q1 category according to the journal IF but it is rated as Q4 according to the AIS. DEA mode land aggregation procedure rates this journal as very good in the Q2 category.
- The reversed result holds for the journal *Mathematics of Operations Research*. It is rated as Q4 according to the journal IF but Q1 according to the AIS. This is probably because it is an American journal published by INFORMS, which has links to major ORMS journals.
- For the other journals, the Q categories are either the same across all four measures or differ by only one category.

Table 3 contains Spearman rank correlation coefficients between all pairs of measures including DEA model and aggregation procedure.

Table 3 – *Spearman rank correlation coefficients between all pairs of measures*

	IF	JCI	5-Y IF	Immediacy	AIS	DEA	Aggreg
IF	1.0000						
JCI	0.8986	1.0000					
5-Y IF	0.9700	0.8882	1.0000				
Immediacy	0.7528	0.7454	0.7450	1.0000			
AIS	0.7420	0.7961	0.8051	0.5550	1.0000		
DEA	0.8656	0.8748	0.9154	0.7900	0.8869	1.0000	
Aggreg	0.9376	0.9281	0.9564	0.7298	0.8884	0.9352	1.0000

The results in Table 3 show a very strong rank correlation between the rankings produced by the DEA model and aggregation procedure and all other metrics except the immediacy index. However, it should be noted that the immediacy index has the lowest rank correlation rate compared to other measures.

4 CONCLUSIONS

The aim of the paper was to analyze the most important scientometric measures and compare them with the two original approaches based on DEA methodology on one side and the aggregation procedure that solves integer optimization problem on the other side. The results show that the proposed approaches produce good results that are comparable with traditional metrics and can be applied in practice. Further research in this field could be focused on the application of other DEA models such as slack-based measure models, models with non-constant returns to scale technology etc. The aggregation procedure could be extended by minimization of the weighted maximum deviation of the final ranking and all partial rankings.

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References

[1] Andersen, P., Petersen, N.C. (1993). A procedure for ranking efficient units in data envelopment analysis. *Management Science*. 39(10), pp. 1261-1264. https://doi.org/10.1287/mnsc.39.10.1261

[2] Charnes, A., Cooper, W.W., Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research*. 2(6), pp. 429–444. https://doi.org/10.1016/0377-2217(78)90138-8

[3] Chen, K., Ren, X.T., Yang, G.L. (2021). A novel approach for assessing academic journals: Application of integer DEA model for management science and operations research field. *Journal of Informetrics*. 15(3). https://doi.org/10.1016/j.joi.2021.101176

[4] Halkos, G.E., Tzeremes, N.G. (2011). Measuring economic journals' citation efficiency: a data envelopment analysis approach. *Scientometrics*. 88, pp. 979-1001. https://doi.org/10.1007/s11192-011-0421-y

[5] Jablonsky, J. (2018). Ranking of countries in sporting events using two-stage data envelopment analysis models: a case of Summer Olympic Games 2016. *Central European Journal of Operations Research*. 26(4), pp. 951-966. https://doi.org/10.1007/s10100-018-0537-8

[6] Liao, H., Tang, M., Li, Z., Lev, B. (2019). Bibliometric analysis for highly cited papers in operations research and management science from 2008 to 2017 based on Essential Science Indicators. *Omega - International Journal of Management Science*. 88, pp. 223-236. https://doi.org/10.1016/j.omega.2018.11.005

[7] Lozano, S., Salmerón, J.L. (2011). Data envelopment analysis of OR/MS journals. *Scientometrics*. 64(2), pp. 133-150. https://doi.org/10.1007/s11192-005-0245-8

[8] Xu, Z., Cheang, B., Lim, A., Wen, Q. (2011). Evaluating OR/MS Journals via PageRank. *Interfaces*. 41(4), 375–388. http://doi.org/10.1287/inte.1110.0557

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MAXIMALLY DIVERSED IMPROVED POPULATION FOR EVOLUTIONARY METAHEURISTIC

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Abstract

Any solution of the min-sum p-location problem is given by an m-dimensional vector of m-p zeros and p units. A unit at the place of given component means that a facility should be located at the associated possible location. The quality metric of a solution is derived from mutual distances among serviced places and the set of located facilities. To apply an evolutionary metaheuristic, a starting population of the problem solutions must be generated. Quality of the population is evaluated according to two objectives. The first of them is the average objective function value of included system designs and the second one consists in the population diversity. Former suggested methods enable generation of maximally diverse population, but without any regard to objective functions of the included solutions. This contribution deals with such integer programming based method that was designed for improving the starting population so that it keeps diversity and contains better solutions.

Keywords: *Discrete location, p-location problems, uniformly distributed set of solutions*

JEL Classification: C61
AMS Classification: 90C27

1 INTRODUCTION

The p-location problems encompass a wide range of practical applications. The problem family comprises well-known issues such as the p-median and p-center problems, as well as their emergency service system design-specific variants (Doerner et al, 2005, Jánošíková and Žarnay, 2014, Marianov and Serra, 2002). Whenever a significant instance of mentioned problem needs to be solved, exact methods (Avella, Sassano and Vasil'ev, 2007, García et al., 2011) are frequently forsaken due to the unpredictable computational time they require to solve the associated complex linear programming problems. A multitude of metaheuristics are employed in order to derive a viable solution to the integer programming problem for practical purposes. In the family of problem-solving instruments, evolutionary metaheuristics, such as the genetic algorithm and scatter search approach, occupy a significant place (Rybičková, Burketová and Mocková, 2016, Gendreau and Potvin, 2010). Metaheuristics of this nature commence with an initial population of viable solutions, referred to as the population, and convert the elements from the current population into constituents of the new one. Metaheuristic approaches traverse a vast set of all feasible solutions in an effort to identify a satisfactory solution to the problem at hand. If the population becomes homogeneous, meaning that the solutions of the current population are comparable in terms of some metric, this search procedure may fail prematurely. In response to the loss of population diversity that can occur during the construction of the starting population or the evolutionary process, numerous strategies have been proposed, ranging from straightforward diversification processes to machine learning techniques such as the utilization of orthogonal arrays (Zhang and Leung, 1999).

This contribution deals with integer programming-based method for improving the starting population so that it keeps diversity and contains better solutions regarding given objectives associated with particular solutions of the p-location problems. In other words, we concentrate on such creation of a uniformly distributed set of issue solutions that can reflect a population that is as diversified as possible and may be crucial to all strategies that require maintaining population variety.

The structure of this conference paper takes the following form: The second chapter explains the uniformly distributed set of system designs and discusses the objective functions used to measure the solutions quality. In Section 3, we report on the suggested improving algorithm, the usability of which is studied making use of a small case study reported in Section 4. Finally, we provide the readers with a brief concluding section.

2 UNIFORMLY DEPLOYED SET OF PROBLEM SOLUTIONS

Such network problems fall into the category of p-location problems, which are often described as tasks involving the selection of p centers from a set of m potential center locations in order to minimize the associated objective function value. The potential center positions in this challenge match network nodes. Users' locations also correspond with the network nodes. The set $\boldsymbol{Y}$ of all feasible solutions of the p-location problem is defined by (1).

$$\boldsymbol{Y} = \left\{ \mathbf{y} \in \{0,1\}^m, \sum_{i=1}^{m} y_i = p \right\} \tag{1}$$

In connection with development of evolutionary metaheuristics, a demand for maximally diverse starting population of p-location problem solutions has appeared (Janáček and Kvet, 2019, Kvet and Janáček, 2019). Furthermore, the population cardinality had to exceed a given size. As the set $\boldsymbol{Y}$ of all feasible solutions consists of zero-one vectors, the difference of two vectors $\boldsymbol{y}$ and $\boldsymbol{x}$ can be measured by so called Hamming distance defined by (2).

$$H(\mathbf{x},\mathbf{y}) = \sum_{i=1}^{m} |x_i - y_i| \tag{2}$$

The value of distance between two elements of $\boldsymbol{Y}$ ranges from 0 to $2p$. The expression $p\text{-}H(\boldsymbol{y}, \boldsymbol{x})/2$ gives the number of possible facility locations occupied by centers in the both solutions. The uniform deployment problem can be established as a task to find such sub-set $\boldsymbol{S} \subset \boldsymbol{Y}$ that the inequality $H(\boldsymbol{y}, \boldsymbol{x}) \geq h$ holds for each $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{S}$ and $|\boldsymbol{S}| \geq u$.

There are two trivial cases of uniformly deployed set (*UDS*). The first case assumes h=2 and the second one sets $h = 2p$. Construction of $\boldsymbol{S}$ in the first case corresponds to enumeration of all p-tuples of the m locations. This set is too huge to be used as starting population and, in addition, the condition of minimal distance h need not be satisfied. In the case of h=2p, the i-th solution for i=1, …, $\lfloor m/p \rfloor$ consists of locations, subscripts of which can be obtained for k=1, …, p using the prescription $p*i+k$. The distance h is the biggest possible here, but the condition of minimal cardinality need not be satisfied. Thus, we focused on uniformly deployed p-location solution sets, which keep the minimal Hamming distance h=2(p-1), or 2(p-2) or 2(p-3) and closer values. There were developed several techniques for obtaining admissible *UDS* (Janáček and Kvet, 2019, Kvet and Janáček, 2019). Nevertheless, practical application of *UDS* mentioned above showed that these sets fulfil the demand of proper diversity, but the objective function values of the included solutions were very bad. This means that the used heuristic method had to expend big computational effort to obtain solutions of acceptable quality. This paper aims at ways of possible improvement quality of the *UDS* for *p*-location problem, with given objective function.

The definition of the presented objective function is based on mutual time-distances among users' locations and facility locations and it can take various forms. Within this contribution, the objective function will be proportional to average response time of the system to demands of individual users. It is assumed that *n* users is serviced by *p* facilities deployed in the set of *m* possible center locations. Each user *j* raises its demand randomly with frequency b_j. Due to randomness, the nearest facility may be occupied by a previous demand and thus the current users' demand is serviced from the nearest available facility. It is assumed that *r* nearest facilities participate on servicing a user with probability q_k , which gives the probability value that the *k*-th nearest facility is the closest available one. The time distance between a possible facility location *i* and a users' location *j* is denoted by t_{ij}. To complete the formulations of the objective functions, let the operation min_k return the *k*-th smallest element from the finite set *A* of reals. Then the objective function is defined by (3).

$$f\left(\boldsymbol{y}\right)=\sum_{j\in J} b_j \sum_{k=1}^{r} q_k \min_k \left\{ t_{ij} : i \in \left\{1,\dots,m\right\},\ y_i = 1 \right\} \tag{3}$$

3 IMPROVING ALGORITHM

The UDS improving process starts with an initial set of *noUDS* uniformly deployed solutions of the associated problem, where each deployed solution is described by list of *p* integer numbers ranging from 1 to *m*. One number of the list corresponds to possible service center location, which is to be equipped with a facility. Another representation of the solution is an *m*-dimensional vector of zeros and units, where the units are placed at the components, subscripts of which are given in the list. If the members of *UDS* are represented by vectors, the Hamming distance between members of each pair is greater than or equal to the given threshold *h*. It follows that a pair of vectors has at most $c = p = h/2$ units located at the same components. This relation can be formalized so that for each pair of the vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ representing members of *UDS* must satisfy that result of their dot product is less than or equal to the constant *c*.

The sequential improvement process follows the steps below, where *UDS* is presented by matrix $\boldsymbol{e}$. The element $e_{ti} \in \{0, 1\}$ takes the value of one if a facility is located at *i*-th possible service center location of the *t*-th solution of the current *UDS*.

1. The solution with worst objective function value is moved at the *noUDS* position and then it is excluded from the *UDS*.
2. The problem (4)-(7) is solved.

$$\mathit{Minimize} \quad f\left(\mathbf{y}\right) \tag{4}$$

$$\mathit{Subject\ to}: \quad \sum_{i=1}^{m} y_i = p \tag{5}$$

$$\sum_{i=1}^{m} e_{ti} y_i \le c \quad \mathit{for}\ t = 1,\dots,noUDS-1 \tag{6}$$

$$y_i \in \left\{0,1\right\} \quad \mathit{for}\ i = 1,\dots,m \tag{7}$$

3. Optimal solution of (4)-(7) is added to the current *UDS*.
4. If no better solution than the excluded one is obtained, then the process terminates. Otherwise, the process continues with step 1.

4 NUMERICAL EXPERIMENTS

The numerical experiments were focused on finding characteristics and efficiency of the suggested improving process. The associated benchmarks were created from the existing medical emergency system of the self-governing region Zilina of Slovakia The region consists of 315 dwelling places considered simultaneously as the users' locations and also possible service center locations. The shortest distances among the locations were obtained from the road network connecting these dwelling places. In the system, 29 service centers were considered. As a base of uniformly deployed sets submitted to the improving process an "ad hoc" uniformly deployed set SA0 was used (Janáček and Kvet, 2019, Kvet and Janáček, 2019). The set consists of 112 solutions with minimal Hamming distance 52. To construct individual cases for the numerical experiments, the set was ordered increasingly according to objective function values and *noS* best solutions were taken as initial *UDS* for the improving process. The cardinality *noS* was set to the values of 60, 40 and 20. Each instance was investigated for different minimal Hamming distances *h*, values of which were 52, 46 and 40. These minimal instances correspond to the following value of the constant *c* in constraints (6). These values of *c* were 3, 6 and 9 respectively. The experiments were performed on PC equipped with the Intel® Core™ i7 5500U 2.4 GHz processor and 16 GB RAM. To solve the problems (4)-(7) described in the previous sections by the branch-and-bound method embedded in IP solver, the optimization software FICO Xpress 7.3 was used. Due to extremely long time necessary to complete processing of the searching tree, the permitted computational time was limited by 1000 seconds. The objective function values were computed according to (3) for r =3 probability values $q_1 = 0.77063$, $q_2 = 0.16476$ and $q_3 = 0.06461$ (Janáček and Kvet, 2016, Kvet, 2014). The stopping rule of the process consists of two conditions, the first of them finishes the process if no improvement in the previous step has been achieved and the second one limits the number of performed steps by the threshold *MaxStep*.

During the individual runs the following parameters were observed:

- CT – computational time of the whole process given in seconds,
- Steps – the number of steps that were taken until the termination rule stopped the process,
- Sum – the sum of objective function values of all solutions included into *UDS*,
- Min – the minimal objective function value of the solutions,
- Max – the maximal objective function value of the solutions.

Each of the following tables includes column denoted by Staring V., where the values Sum, Min and Max of the starting *UDS* are reported.

Table 1 The results of experiments for *noS* = 60, *MaxStep* = 70

	Starting V.	*c* = 3	*c* = 6	*c* = 9
CT[s]	0	66037	55236	48799
Steps	0	67	65	61
Sum	4297173	3539334	2977842	2772502
Min	62111	45426	42127	42110
Max	78975	61969	51512	47227

Table 2 The results of experiments with cases of *noS* = 40, *MaxStep* = 50

	Starting V.	$c = 3$	$c = 6$	$c = 9$
CT[s]	0	42316	32289	29708
Steps	0	46	43	42
Sum	2764891	2250372	1956453	1834531
Min	62111	44942	42127	42110
Max	73534	59074	50093	46611

Table 3 The results of experiments with cases of *noS* = 20, *MaxStep* = 30

	Starting V.	$c = 3$	$c = 6$	$c = 9$
CT[s]	0	11337	10725	9014
Steps	0	22	22	21
Sum	1329234	1064392	958839	906421
Min	62111	42838	42110	42110
Max	69879	56127	49346	46116

5 CONCLUSIONS

A uniform deployment was developed in response to the requirement to examine an extremely large number of feasible solutions of any optimization problem that can only be resolved using a metaheuristic and for which the available exact approaches are unable to terminate the optimization process within an acceptable time interval.

The set of feasible system designs that is uniformly deployed may represent the population with maximal diversity, which is an attribute that evolutionary metaheuristics generally appreciate. A concept of uniformity based on the smallest possible Hamming distance between each pair of elements is discussed in the paper. A set of solutions is deemed to be deployed uniformly when the minimal Hamming distance between any two pairs of solutions is equal to or greater than a specified threshold, and when the set of solutions is maximal, meaning no additional solutions can be added.

This manuscript was aimed at developing such a method that could produce a uniformly deployed set of solutions, the diversity of which keeps good, and it contains solutions with satisfactory value of minimized objective function value. Suggested algorithm has been experimentally verified on a small dataset containing real-world problem instances. As reported in the computational study, the results are very satisfactory as far as the quality of the solutions set is concerned. If we analyze the objective function of the obtained solutions, it is very good from the minimization point of view.

A weak point of proposed method consists in its time complexity. The required computational time is enormously high. Thus, future development in this field should be focused on finding other possible ways to achieve comparatively good results in much shorter computational time.

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References

[1] Avella, P., Sassano, A. and Vasil'ev, I. (2007). Computational study of large scale p-median problems. *Mathematical Programming* 109, pp. 89-114.

[2] Doerner, K. F., Gutjahr, W. J., Hartl, R. F., Karall, M. and Reimann, M. (2005). Heuristic Solution of an Extended Double-Coverage Ambulance Location Problem for Austria. *Central European Journal of Operations Research*, 13(4), pp. 325-340.

[3] García, S., Labbé, M. and Marín, A. (2011). Solving large p-median problems with a radius formulation. *INFORMS Journal on Computing*, 23(4), pp. 546-556.

[4] Gendreau, M. and Potvin, J. (2010). *Handbook of Metaheuristics*, Montreal: Springer Science & Business Media, 648 p.

[5] Janáček, J. and Kvet, M. (2019). Uniform Deployment of the *p*-Location Problem Solutions. *Operations Research Proceedings 2019:* Selected Papers of the Annual International Conference of the German Operations Research Society (GOR), Dresden, Germany, September 4-6, 2019, pp. 315-321.

[6] Janáček, J. and Kvet, M. (2016). Min-max Optimization and the Radial Approach to the Public Service System Design with Generalized Utility. *Croatian Operational Research Review*, 7(1), pp. 49-61.

[7] Jánošíková, Ľ. and Žarnay, M. (2014). Location of emergency stations as the capacitated p-median problem. In: Quantitative Methods in Economics (Multiple Criteria Decision Making XVII). [online] Bratislava: Ekonóm, pp. 117-123. Available at: http://www.fhi.sk/files/katedry/kove/ssov/proceedings/Zbornik2014.pdf [Accessed 10th March, 2024].

[8] Kvet, M. (2014). Computational Study of Radial Approach to Public Service System Design with Generalized Utility. In: Proceedings of International Conference DT 2014, Žilina, Slovakia, pp. 198-208.

[9] Kvet, M. and Janáček, J. (2019). Usage of Uniform Deployment for Heuristic Design of Emergency System. *Operations Research Proceedings 2019*: Selected Papers of the Annual International Conference of the German Operations Research Society (GOR), Dresden, Germany, September 4-6, 2019, pp. 309-314.

[10] Marianov, V. and Serra, D. (2002). Location problems in the public sector, *Facility location - Applications and theory* (Z. Drezner ed.), Berlin, Springer, pp 119-150.

[11] Rybičková, A., Burketová, A. and Mocková, D. (2016). Solution to the locating – routing problem using a genetic algorithm. In: *SmaRTT Cities Symposiuum Prague (SCSP)*, pp. 1-6.

[12] Zhang, Q. and Leung, Y. W. (1999). An orthogonal genetic algorithm for multimedia multicast routing. IEEE Trans. Evol. Comput., vol. 3, pp. 53-62.

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THE HOME CARE CREW SCHEDULING PROBLEM: MODEL AND CASE STUDY

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Abstract

In the home care crew scheduling problem, a team of nurses needs to be assigned visits to clients' homes to provide them desired services. In long-term planning, regular daily rosters for nurses are designed with the goal of minimizing the distances travelled by nurses and their overtime work. At the operational level of planning, the objective is to minimize deviations from the regular plan. The problem is similar to the vehicle routing problem with time windows but includes additional dependencies between visits. Since the problem is NP-hard, solving practical-sized instances (with about 150 clients) to optimality within an acceptable time is not possible. However, mathematical programming can be helpful in daily management when the regular schedule must be modified due to the unexpected unavailability of personnel. In this paper, we present a mathematical programming model for this rescheduling problem. The applicability of the model is verified in real-life setting. The schedule of 51 nurses serving 146 clients in the city of Žilina is optimally adapted to substitute for 4 nurses withdrawing from work. The optimal schedule outperforms the manual schedule in terms of deviations from the regular timetable as well as the length of the nurses' routes.

Keywords: *Home care, Crew scheduling, Vehicle routing*

JEL Classification: JEL C61
AMS Classification: AMS 90B35, AMS 90C11

1 INTRODUCTION

The home care service is provided by nurses to elderly or disabled people who require assistance with everyday tasks such as getting out of bed, bathing, dressing, administering medicines, or meal preparation. The nurses do not need to have medical education. They are employees of the municipal office. The main purpose of the service is to enable senior citizens to remain in their homes for as long as possible, thereby reducing the demand for institutional care.

When a citizen applies for home care service, they specify the desired services, including the frequency, duration, and time windows of visits. The home care manager in the office then assigns these duties to nurses and creates their daily rosters. In Slovakia, the schedules are currently created and adapted manually without any software support. While this method is sufficient in small towns with a limited number of clients and nurses, designing a feasible schedule becomes a challenging task in large towns with hundreds of clients and tens of nurses. Additionally, creating a schedule that minimizes the time nurses spend travelling between clients is beneficial, as it allows more clients to be served. The demand for home care services is steadily increasing due to an aging population, making the need for an optimal schedule more urgent. Designing an optimal schedule surpasses the capabilities of a human planner. However, a near-optimal schedule can be created using mathematical programming and optimization procedures.

The home care crew scheduling problem (HCCSP) has attracted attention from researchers over the past 20 years. It represents a combination of employee scheduling and the vehicle routing problem with time windows. The HCCSP generalizes the travelling salesman problem that is well-known to be NP-hard. Therefore, the HCCSP is also NP-hard. Consequently, devising an optimal long-term or mid-term schedule from scratch is only possible for small instances. Solving practical-sized problems with hundreds of customers necessitates employing heuristic approaches such as clustering visits (Rasmussen, Justesen, Dohn and Larsen, 2012), a hybrid strategy combining constraint programming and large neighborhood search (Di Gaspero and Urli, 2014), or an adaptive large neighborhood search (Nickel, Schröder and Steeg, 2012).

A simpler situation arises in short-term planning when the regular long-term plan must be adapted due to unexpected circumstances, especially caused by nurses' withdraw from work. The duties of absent nurses need reassignment to working nurses to minimize overtime and deviations of visits from the regular timetable. This rescheduling problem can be optimally solved by using a mathematical programming model.

2 HOME CARE CREW RESCHEDULING PROBLEM FORMULATION

In this section, we concentrate on a mathematical programming formulation of the home care crew rescheduling problem. The problem consists in inserting non-covered visits into the regular daily schedule.

We have a set of clients who require home care services. A client may have multiple visits within one day, such as when the service is spread throughout the day (e.g., 2 hours in the morning and 2 hours at noon) or when the service requires two nurses working in parallel. Every visit is characterized by a time window, frequency (e.g., daily or weekly), and duration. The daily working time is limited for every nurse, typically 8 hours, although some nurses may only be available in the morning before attending to other tasks. Based on clients' demand and available staff, a planner establishes a weekly schedule, where each visit is assigned to a specified nurse and time slot, ensuring that each client is consistently visited by the same nurse. The weekly schedule results in daily routes for nurses. However, regular daily schedules can be disturbed by unexpected circumstances. For example, a client might cancel the service due to hospital admission, or a nurse may become sick. In such cases, the regular schedule must be adjusted. Cancelling the service is not an issue; these duties are simply removed from the nurses' routes. A more challenging scenario arises when some nurses are unable to work (e.g., due to sickness). In this situation, their duties need to be reassigned to the remaining nurses in a way that maximizes the level of service. Since clients are more sensitive to changes in timing than to alterations in the crew attending their homes, the primary goal is to minimize deviations of the visit starts from the regular timetable. The secondary goal is to minimize nurses' overtime work.

We formulate the rescheduling problem in terms of multi-objective mixed-integer mathematical programming. The following sets, parameters, and decision variables are used in the model.

Sets

K set of available nurses
$C = \{1, \dots, n\}$ set of visits

$V = C \cup \{0\}$	set of visits including an artificial visit 0 that serves as the start and end visit for each nurse
P	set of pairs of visits $(i,j) \in C \times C$, which require to start at the same time
R	set of pairs of visits $(i,j) \in C \times C$, which require the same nurse

Parameters

n	number of visits
$[e_i, l_i]$	time window for visit $i \in C$, i.e. the earliest and the latest possible start time of the visit
b_i	regular start time of visit $i \in C$
s_i	duration (service time) of visit $i \in C$
p_{ik}	coefficient taking value of 1, if visit $i \in C$ is regularly assigned to nurse k and nurse k is available, and 0 otherwise
t_{ij}	shortest travel time from visit $i \in V$ to visit $j \in V$ including duration of visit i; $t_{0i} = 0, t_{i0} = s_i$ for $i \in C$
L_k	working time of nurse $k \in K$

Decision variables

$$x_{ijk} = \begin{cases} 1, & \text{if nurse } k \in K \text{ travels directly from visit } i \in V \text{ to visit } j \in V \\ 0, & \text{otherwise} \end{cases}$$

y_i	start time of visit $i \in C$
z_k	overtime of nurse $k \in K$
u_i, v_i	positive and negative deviation of the start time of visit $i \in C$ from the regular time

The model can now be stated as follows:

$$\begin{array}{lrll}
\textit{minimize} & \sum_{j \in C}(u_j + v_j) & & (1) \\
\textit{minimize} & \sum_{k \in K} z_k & & (2) \\
\textit{minimize} & \sum_{k \in K} \sum_{i \in V} \sum_{\substack{j \in V \\ i \neq j}} t_{ij} x_{ijk} & & (3) \\
\textit{subject to} & \sum_{k \in K} \sum_{\substack{i \in V \\ i \neq j}} x_{ijk} = 1 & \textit{for } j \in C & (4) \\
& \sum_{\substack{i \in V \\ i \neq j}} x_{ijk} = \sum_{\substack{i \in V \\ i \neq j}} x_{jik} & \textit{for } j \in V, k \in K & (5) \\
& \sum_{j \in C} x_{0jk} \leq 1 & \textit{for } k \in K & (6) \\
& \sum_{\substack{i \in V \\ i \neq j}} x_{ijk} \geq p_{ik} & \textit{for } j \in C, k \in K & (7) \\
& \sum_{\substack{i \in V \\ i \neq j}} x_{ijk} = \sum_{\substack{i \in V \\ i \neq m}} x_{imk} & \textit{for } (j,m) \in R, k \in K & (8) \\
& \sum_{i \in V} \sum_{\substack{j \in V \\ i \neq j}} t_{ij} x_{ijk} \leq L_k + z_k & \textit{for } k \in K & (9) \\
& e_j \leq y_j \leq l_j & \textit{for } j \in C & (10) \\
& y_i = y_j & \textit{for } (i,j) \in P & (11) \\
& y_i + t_{ij} \leq y_j + T\left(1 - \sum_{k \in K} x_{ijk}\right) & \textit{for } i,j \in C, i \neq j & (12) \\
& y_j - b_j = u_j - v_j & \textit{for } j \in C & (13) \\
& x_{ijk} \in \{0,1\} & \textit{for } i,j \in V, k \in K & (14) \\
& y_j, z_j, u_j, v_j \geq 0 & \textit{for } j \in C & (15)
\end{array}$$

The objective functions are sorted based on their relevance. The highest priority is given to objective function (1) that minimizes deviations in the start times of visits from the regular schedule. Following that, objective function (2) focuses on minimizing nurses' overtime. While less critical, objective function (3) involves minimizing the total route length, which still holds significance. Constraints (4) ensure that every visit is covered, meaning it is assigned to exactly one nurse. Constraints (5) represent flow conservation constraints. Constraints (6) state that every nurse starts their daily route at most once. Constraints (7) ensure that a client will be visited by their regular nurse if the nurse is working on the given day. Constraints (8) ensure that if a client requires two separate visits during a day, then both visits will be performed by the same nurse. Constraints (9) set the overtime variable z_k if the working time of nurse k is exceeded. Constraints (10) ensure that time windows are respected. Constraints (11) are synchronization constraints ensuring that simultaneous visits will start at the same time. Constraints (12) ensure that travel times are respected, and the route forms a Hamiltonian cycle. T is a sufficiently large constant. Constraints (13) model the deviation $|y_j - b_j|$ of the start time of visit j from the regular time. Finaly, the obligatory constraints (14) and (15) specify the definition domains of the variables.

3 CASE STUDY

The municipal office in Žilina currently offers home care services to 153 clients. For testing purposes, we were provided with regular rosters for 55 nurses for a single day. On this day, the requirements of 146 clients are met within 174 visits. Services for the remaining 7 clients are omitted for that day. The average duration of a visit is 93 minutes. In addition to the regular assignment of visits, the rosters include irregular assignments due to the unavailability of 4 nurses. The remaining 51 nurses are tasked with substituting for them and covering 15 additional visits.

In terms of working hours, 5 nurses only work for one hour in the morning, while the remaining 46 nurses are available for an 8-hour shift. The working time of a nurse commences upon arrival at the first client's home and concludes when the last visit terminates. The time spent traveling from the nurse's home to the first client's residence and from the last client's location is not included in the working hours. Breaks for rest are not scheduled; it is assumed that nurses adjust their breaks according to their daily plans.

The test data lacks precise information on nurses' routes and visit schedules. Consequently, we had to compute them using the assignment of clients to nurses, visit specifications, and travel times between clients' homes. We computed both the regular schedule and the modified schedule based on this information.

We did not receive travel times between clients, so we had to estimate them based on the geographical positions of clients. Nurses have the option to travel by foot or bus. In this experimental stage, we did not utilize a digital street network to determine walking times or a digital timetable to calculate the shortest bus routes. For simplicity, we estimated walking and travel times using a correlation between direct distance and walking time. Initially, we derived a linear regression formula to estimate walking times. The regression calculation involved 80 data points, each consisting of two values: the spherical distance calculated using the haversine formula and the walking time between corresponding clients determined by the Google route planner. The regression function for the city of Žilina is represented by Eq. (16):

$$y = 0.015x + 1.0987, \tag{16}$$

where the independent variable x represents the spherical distance in metres, and the dependent variable y is the walking time in minutes. If the walking time exceeded 40 minutes, we assumed that the nurse would opt for a bus, setting the travel time to a fixed 40 minutes.

We determined the regular schedule solving the bi-objective model that minimizes nurses' overtime (2) and travel time (3), subject to constraints (4)-(12), (14), (15). In this model, variables u_i and v_i are omitted. We suppose that all nurses are available. In the optimal schedule, the total worktime for all nurses is 17,214.2 minutes, with a total travel time of 1,014.2 minutes. To fulfil the assigned duties, two nurses are required to work overtime, a total of 28.6 minutes.

The regular schedule needs adjustment due to the unavailability of 4 nurses. In calculating the routes and schedules for the adjusted plan, we set the coefficient p_{ik} to 1 if visit i is manually assigned to substituting nurse k. Based on personal communication with the planner, we understood that the reassignment aimed to minimize shifts in visit start times and reduce overtime. Therefore, we calculated the adjusted schedule by solving the model (1), (2), (4)-(15). The resulting plan shifts the visits of 11 clients by a total of 941.6 minutes (in absolute values).

Optimal reassignment and rescheduling can be achieved using the mathematical programming model presented. In this case, when nurse k does not work, the coefficient p_{ik} is set to 0 for all $i \in C$. It means that we let the solver find an optimal reassignment of her duties. We solved two variants of the model, differing in the objective functions. The first variant considered only objectives (1) and (2), while the second variant took into account all three objective functions. In both cases, we achieved a significant reduction in both the time deviation from the regular timetable and the total travel time, without an increase in overtime. A comparison of the rescheduled plans is provided in Table 1.

Table 1. Comparison of the rescheduled plans

Reassignment method	Objective function	Deviation from the regular timetable (min)	Travel time (min)	Overtime (min)	Number of rescheduled clients	Computing time (s)
Manual	Min deviations + overtime	941.6	1281.5	28.6	11	20
Model	Min deviations + overtime	165.4	1265.8	28.6	2	386
Model	Min deviations + overtime + travel time	190.4	1078.6	28.6	3	143

The computational experiments were performed on a personal computer equipped with the Intel Core i5 processor with 2.60 GHz and 16 GB RAM. The solver Xpress Optimizer 8.3 (64-bit, release 2017) was used to solve the MIP models.

As can be observed in Table 1, by prioritizing the minimization of time shifts and overtime work, the optimal schedule achieved nearly a 13-hour reduction in time shifts compared to manually reassigned duties. Additionally, this optimal plan reduced nurses' routes by 18.7 minutes, although trip lengths are not a primary concern in this model.

When introducing an objective function to minimize travel times, the optimal routes were further reduced by 187.2 minutes. Despite a slight increase of 25 minutes in visit shifts compared to the previous optimal plan, these results advocate for the application of the three-objective model (1)-(15) described in Section 2.

Importantly, the computing time of both models is highly favourable and well accepted in the operational management. In conclusion, the computational results demonstrate the successful application of mathematical programming in the development of daily routes and schedules for nurses providing home care services.

4 CONCLUSIONS

This study establishes that mathematical modelling proves beneficial in the operational management of home care services. Our future research will be dedicated to exploring methods for the automatic construction of robust regular long-term schedules with a focus on minimizing route length and overtime. It is acknowledged that this problem cannot be optimally solved using a mathematical programming approach, necessitating the use of heuristic methods and metaheuristics.

Among heuristic methods, matheuristics, which integrate mathematical programming into a metaheuristic framework, represent a modern and promising approach for optimizing complex problems. Notably, recent successes have been achieved with kernel search applied to the capacitated vehicle routing problem (Borčinová, 2022). In the realm of home care crew scheduling, the literature demonstrates the successful application of another metaheuristic called large neighborhood search (Pisinger and Ropke, 2010).

The goal of employing optimization algorithms is to provide those responsible for operational management with the foundation to construct the best possible plan. This plan aims to deliver the required services to customers with minimal time requirements for nurse transfers while adhering to practical constraints.

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References

[1] Borčinová, Z. (2022). Kernel Search for the Capacitated Vehicle Routing Problem. *Applied Sciences*, 12, p. 11421.
[2] Di Gaspero, L. and Urli, T. (2014). A CP/LNS Approach for Multi-day Homecare Scheduling Problems. In: Blesa, M.J., Blum, C., Voß, S. (eds) *Hybrid Metaheuristics*. HM 2014. Lecture Notes in Computer Science, vol 8457. Cham: Springer.
[3] Maniezzo, V., Boschetti, M. A. and Stützle, T. (2021). *Matheuristics: Algorithms and Implementations*. Cham: Springer.
[4] Nickel, S., Schröder, M. and Steeg, J. (2012). Mid-term and Short-term Planning Support for Home Health Care Services. *European Journal of Operational Research*, 219, pp. 574-587.

[5] Pisinger, D. and Ropke, S. (2010) Large Neighborhood Search. In: Gendreau, M., Potvin, J.-Y. (eds) *Handbook of Metaheuristics*. Cham: Springer.
[6] Rasmussen, M. S., Justesen, T., Dohn, A. and Larsen, J. (2012). The Home Care Crew Scheduling Problem: Preference-based Visit Clustering and Temporal Dependencies. *European Journal of Operational Research*, 219, pp. 598-610.

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SELECTION AND SCHEDULING OF TRANSPORT INFRASTRUCTURE PROJECTS FOR IMPLEMENTATION USING GENETIC ALGORITHMS

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Abstract

Development of transport infrastructure in the Czech Republic is carried out through the implementation of investment projects that can be financed from several sources. The largest source of funding for the development of transport infrastructure in the Czech Republic is the Operational Programme Transport (OPT), which is the focus of this article. Due to the limited budget of available funds a group of experts is established to decide on the selection of projects for implementation. By their selection, the experts decide on the funding of projects only in the following year, even though the OPT has a known budget for up to 7 years.

The authors of this article have long been involved in the issue of portfolio selection of transport infrastructure projects. This article extends the issue of project selection to the creation of a schedule for its financing within a defined timeframe which could potentially mean a more efficient development of transport infrastructure in the medium-term time horizon. Authors propose a solution to the problem using a genetic algorithm with a matrix encoding of chromosomes, which represents project selection and year of their financing.

Keywords: *Genetic Algorithms, Portfolio Selection, Scheduling, Transport Infrastructure Development*

JEL Classification: C630
AMS Classification: 90B50, 90C29, 68T20

1. INTRODUCTION AND LIRERATURE REVIEW

Transport infrastructure is a key element of modern society, having a major impact on, for example, the economy, employment, the mobility of people and their standard of living. Currently, the development of transport infrastructure is carried out through the implementation of investment projects. There are several sources of funding for transport infrastructure, with the largest source of funding in the Czech Republic being the Operational Programme Transport (OPT), which is the focus of this paper.

The selection of transport infrastructure projects is currently carried out by a group of experts whose task is to select the projects that are allowed to draw funds under the OPT for their implementation the following year. In previous publications, the authors of this paper have already dealt with the development of mathematical models to support decision-making in project selection. Since the OPT runs at regular seven-year intervals, it is suggested that the time aspect of the decision-making problem, which currently plays no role in the decision-making process, should be taken into account in terms of the disbursement of funds.

Since the authors of the article have long been dealing with the topic of project selection and scheduling in the context of transport infrastructure development, it is necessary to consider heuristic algorithms for solving the problem in addition to exact methods, since there are cases where exact methods cannot be applied in real applications due to their computational complexity. The genetic algorithm works with chromosomes that are coded as matrices that represent the actual schedule as well as the selection of transport infrastructure projects in the period under consideration.

Heuristic algorithms have been used in terms of project portfolio selection, for example in (Yu et al, 2010, p. 71-86) or (Cho and Moon, 2006, p. 61-70), where chromosomes were coded binary as a vector. In the context of scheduling, it is also possible to use permutation code when no selection is taking place and the operations are serialized, this way of coding has been used in (Liu et al, 2005, pp. 5699-5703) and (Chang et al, 2007). Coding of chromosomes in the form of a matrix has been addressed in (Chen and Bi, 2019) and (Wallet, Marchette and Solka, 1996). The matrix coding of chromosomes is very specific in both these publications and in this article, it also presents another specific form of chromosome coding and the related concept of genetic operators.

2. PROBLEM FORMULATION

Consider a set of projects I that are suitable to be implemented on the transport infrastructure in a predefined project financing period consisting of years of set J. Each project $i \in I$ has a value a_i that quantifies its benefit to society if it is implemented. Furthermore, for each project $i \in I$ a value c_i is defined that represents the financial cost of its implementation. Each year of the predefined project financing period $j \in J$ has a fixed amount of available funds for the implementation of projects b_j. The task is to determine which projects and in which years of the predefined project funding period should be financed, so that the amount of available funds is not exceeded and the benefit of the funded projects to society is maximized.

Assuming that the financing decision for project $i \in I$ in year $j \in J$ is modelled by the binary variable x_{ij}, the mathematical model of the problem can be written as follows:

objective function

$$f(x) = \sum_{i \in I} \sum_{j \in J} x_{ij} a_i \rightarrow max \tag{1}$$

subject to

$$\sum_{j \in J} x_{ij} \leq 1 \qquad for\ i \in I \tag{2}$$

$$\sum_{i \in I} x_{ij} \cdot c_i \leq b_j \qquad for\ j \in J \tag{3}$$

$$x_{ij} \in \{0; 1\} \qquad for\ i \in I,\ for\ j \in J \tag{4}$$

The objective function (1) ensures that the optimization criterion, i.e. the cumulative benefit of the projects to society, is maximized. The group of constraints (2) ensures that each project $i \in I$ is selected for funding at most once. The group of constraints (3) ensures that no more funds are spent on the implementation of projects than are allocated to a given year $j \in J$. The group of constraints (4) sets the definitional domain of the variables x_{ij} in the model.

3. APPLICATION OF THE GENETIC ALGORITHM

In the genetic algorithm, it is necessary to express not only the selection of a given project, but also the exact year of its financing. This can be ensured by encoding the chromosome in two dimensions, i.e., using a matrix in which the rows represent projects $i \in I$ and the columns represent years $j \in J$. While the validity of chromosomes with respect to the group of constraints (2) is obvious at first sight without any requirements for knowledge of the input parameters, and the validity of this group of constraints is not affected by properly set genetic operators, validity in terms of respect to the group of constraints (3) is more complicated. The validity of chromosomes in terms of satisfying the group of constraints (3) has to be calculated using the input parameters of the problem and can be violated during the generation of chromosomes and also during crossover and mutation. For easy understanding of the logic of the genetic algorithm, it is convenient to first explain the coding of chromosomes, the logic of decoding the fitness function, and the specific logic of genetic operators.

3.1. Chromosome coding

It is not uncommon for genetic algorithms to code chromosomes vectorially using binary code. Chromosomes can be coded in this way, for example, in location problems, where a value of 1 for a specific element of the vector indicates location selection, while a value of 0 indicates the opposite. This logic could be applied in the same way to the selection and scheduling. A chromosome could, for example, take the following form:

$$chromosome = X = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix},$$

Where the size of the matrix corresponds to $m \times n$, where $|I| = m$ and $|J| = n$. The elements of the matrix take the value 1 if project $i \in I$ will be financed in year $j \in J$, 0 if not.

3.2. Chromosome generation

Avoiding the creation of invalid chromosomes in terms of respecting the group of constraints (2) can already be assured in the chromosome generation. The group of constraints with respect to the matrix encoding of the chromosome can be interpreted in such a way that at most one element in each row of the matrix can have the value 1. Since it is true that $\sum_{j \in J} b_j < \sum_{i \in I} c_i$, not every project can be selected for funding and therefore the probability with which project $i \in I$ is selected for implementation must be determined. The probability can be calculated using the formula

$$p_{gen} = \frac{\sum_{j \in J} b_j}{\sum_{i \in I} c_i}. \tag{5}$$

Generation of initial chromosomes can be done as follows:

1. create a zero matrix X of size $m \times n$,
2. in each row $i \in I$ of matrix X with the probability p_{gen} randomly select one column $j \in J$ and set $X_{ij} = 1$

3.3. The concept of fitness functions and dealing with invalid chromosomes

The fitness function is designed in such a way that invalid chromosomes are penalized. The fitness function has the following form

$$f(X) = \sum_{i\in I}\sum_{j\in J^+}(X_{ij}a_i), where\ J^+ = \left\{ j \in J | b_j - \sum_{i\in I} x_{ij}c_i \geq 0 \right\}.$$

The fitness function will cause invalid chromosomes to have a significantly lower value, which will reduce their probability of their selection in the genetic selection operator. The penalty will be reflected in the non-counting of the contribution to society of all $i \in I$ projects financed in whichever years $j \in J$ the use of available funds is exceeded.

3.4. Selection

Since the goal is to maximize the criterion value, roulette-wheel selection was chosen. The purpose of selection operator is to create a new set (of population size) of chromosomes that will continue into the next genetic operators. For each chromosome in a population, the value of the fitness function is calculated and then the probability of its selection is calculated using the formula

$$p_i = \frac{f_i}{\sum_{l\in I} f_l} \qquad \text{for } i \in I. \qquad (6)$$

3.5. Crossover

The simplest way to crossover chromosomes is a single crossover that establishes a crossover point between 2 adjacent rows of chromosomes. If we set a crossover point between 2 adjacent columns of a chromosome, a large number of invalid chromosomes would be produced. The crossover point is chosen randomly at each crossover as a number from the interval $\langle 1; m-1 \rangle$. A crossover example is presented in Table 2 and Table 2.

		parent 1		
1	0	0	0	0
0	0	0	0	1
0	0	1	0	0
0	0	0	1	0
0	0	0	0	0
0	1	0	0	0

		parent 2		
0	0	0	0	1
1	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	0	0	1	0

Table 1 - crossover example - initial chromosomes

		child 1		
1	0	0	0	0
0	0	0	0	1
0	0	1	0	0
0	0	0	1	0
0	1	0	0	0
0	0	0	1	0

		child 2		
0	0	0	0	1
1	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	1	0	0	0

Table 2 - crossover example – chromosomes after crossover

3.6. Mutation

The mutation passes through each element of the chromosome and, if the mutation takes place, negates the value of the element. However, the mutation must be set in such a way that it does not produce multiple values of 1 in the same row. There are 3 different cases that can occur during mutation, presented in Table 3.

	Initial chromosome					Mutation	Mutated chromosome				
case 1	1	0	0	0	0		0	0	1	0	0
case 2	0	0	1	0	0	=>	0	0	0	0	0
case 3	0	0	0	0	0		0	0	1	0	0

Table 3 - mutation example (mutated element is colored)

4. COMPUTATIONAL EXPERIMENTS AND CONCLUSION

Computational experiments were performed to determine the suitable parameters settings of the genetic algorithm, specifically the number of iterations, population size and crossover and mutation probabilities. Genetic algorithm was run 10 times at each setting and during all runs, elitism was applied to ensure the best chromosome is always present in new populations. An example of one genetic algorithm run is presented in Figure 1. The results of the computational experiments in terms of suitable settings of the number of iterations and population size are presented in Figure 2.

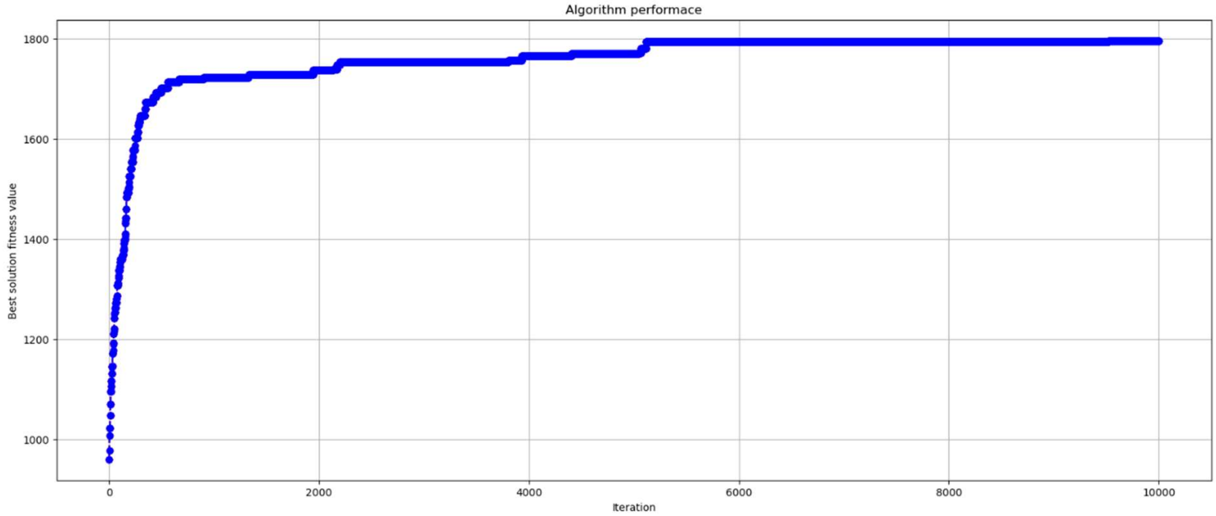

Figure 1 algorithm performance example

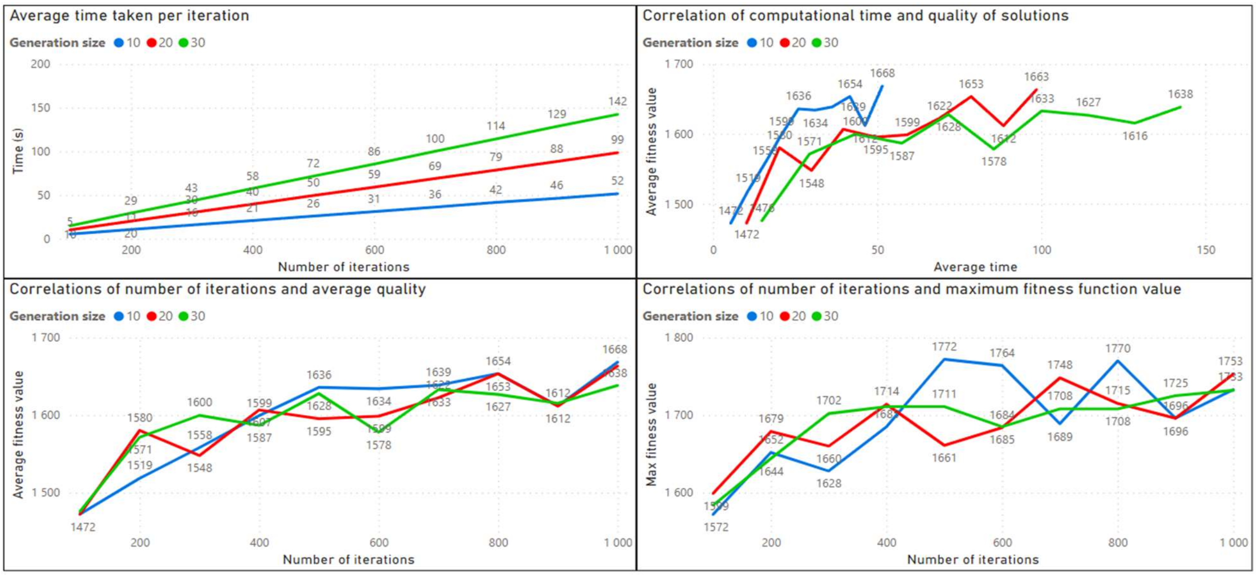

Figure 2 - results of computational experiments (computation time and quality of solutions)

The results show that increasing the population size does not improve the quality of the solutions obtained, while the computation time increases linearly. Therefore, in the computational experiments, the population size was further set to 10 chromosomes and the number of iterations was set to 1 000 (after this value the increase in solution quality decreased rapidly, as seen in Figure 1). In the second part of the computational experiments, the crossover probability and mutation probability were varied. The results obtained are presented in Table 4 and Table 5.

$p_{mutation}$ $p_{crossover}$	0,001	0,002	0,003	0,004	0,005
0,6	1745	1695	1591	1567	1562
0,7	1727	1644	1654	1579	1482
0,8	1731	1651	1672	1570	1556
0,9	1725	1640	1582	1544	1536

Table 4 - maximum fitness function value achieved at given mutation (columns) and crossover (rows) probabilities

$p_{mutation}$ $p_{crossover}$	0,001	0,002	0,003	0,004	0,005
0,6	1683	1553	1474	1435	1415
0,7	1662	1523	1526	1460	1412
0,8	1653	1524	1516	1426	1431
0,9	1655	1550	1485	1415	1387

Table 5 - average fitness function value achieved at given mutation (columns) and crossover (rows) probabilities

The optimal solution to the problem, which was obtained using a linear model, has an objective function value of 2 196. Thus, the best results in terms of the fitness function value of the genetic algorithm are approximately 80% of the value of the optimal solution. In the computational experiments, it was found that both the initial generation of chromosomes and both the crossover and mutation operators produce a large number of invalid chromosomes, which affects the quality of the final chromosomes. One of the possible solutions to this problem is to introduce an algorithm to check the validity of chromosomes, and if a chromosome is invalid, to alter the chromosomes to be valid. Since invalid chromosomes arise during the initial generation of chromosomes and after crossover and mutation, the algorithm would be applied after these operations.

In future research, the authors will focus on the implementation of an algorithm for controlling the validity of chromosomes and comparing the difference with respect to the quality of the solutions obtained and the computational complexity of the genetic algorithm. Furthermore, the authors aim to extend the problem by more aspects that affect it in real applications, such as financing of investment projects, which takes several years to implement, or the $n + 2$ funding rule, which allows to draw available funds from a certain year in the following 2 years.

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References

[1] Lean Yu, Shouyang Wang, Fenghua Wen & Kin Keung Lai. (2010) ‘Genetic algorithm-based multi-criteria project portfolio selection’, *Annals of Operations Research*, Volume 187, Issue 1, pp. 71-86.

[2] Cho Kyu-Kab, Moon Byung-Keun. (2006) ‘A method for selecting the optimal portfolio of performance improvement projects in a manufacturing system’, *International Journal of Industrial Engineering-theory Applications and Practice*, Volume 13, Issue 1, pp. 61-70.

[3] Liu, T; Liu, M; Zhang, YB; Zhang, L. (2005) ‘Hybrid genetic algorithm based on synthetical level of resource conflict for complex construction project scheduling problem’, *Proceedings of 2005 International Conference on Machine Learning and Cybernetics*, Volume 1-9, pp. 5699-5703

[4] Chang Pei-Chann, Chen Shih-Shin, Ko Qiong-Hui, Fan Chin-Yuan. (2007) ‘A genetic algorithm with injecting artificial chromosomes for single machine scheduling problems’, *2007 IEEE Symposium on Computational Intelligence in Scheduling*.

[5] Chen Kaiqi, Bi Weihong. (2019) ‘A new genetic algorithm for community detection using matrix representation method’, *Physica A-statistical Mechanics and its Applications*, Volume 535.

[6] Wallet, BC; Marchette, DJ; Solka, JL. (1996) ‘A matrix representation for genetic algorithms’, *Proceedings of The Society of Photo-Optical Instrumentation Engineers (SPIE)*, Volume 2756, pp. 206-214.

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EXPLORING THE PRIMARY FACTORS' ELASTICITIES OF SUBSTITUTION IN WAGE ANALYSIS OF IMMIGRATION

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Abstract

The article deals with the phenomenon of immigration in the Slovak Republic and analyzes its wage effects under different assumptions about the elasticities of substitution of the primary factors, investments, education, and qualification. Four different scenarios with various values for the parameters of elasticities and the volume of changes in primary factors are run. The results show that the wage effects are negative in every analyzed occupational group of workers as long as immigration is not accompanied by an increase in capital.

Keywords: *Migration, Computable General Equilibrium Model, Wage, Elasticity of Primary Factors Substitution*

JEL Classification: C68, J31
AMS Classification: 90-08

1 INTRODUCTION

The aim of this paper is to analyze the wage effects of immigration to the Slovak Republic resulting from the long-standing problems of labor shortages, which are mostly compensated by nationals of other countries. Nationwide, as of September 2023, there are more than 80,000 vacancies that are difficult to fill, especially in the automotive and agricultural sectors. The underrepresentation of occupations varies from region to region, but the most common shortages are for skilled workers in industry, doctors, healthcare professionals, nurses, programmers, warehouse workers, plumbers, and many other occupations [9].

This study uses a static computable general equilibrium (CGE) model with five occupational skill levels of immigrants and eleven industries: the Global Trade Analysis Project (GTAP) computable general equilibrium model, version 7 and the corresponding GTAP 11 database [1]. The reason for choosing this model, although there are more appropriate models for modeling migration effects (GMig or GMig2 models), is that it uses the most recent data, including 2017, and allows the most accurate disaggregation of the labor force. The experiments conducted in this paper are taken from Ježíková's thesis [5].

2 DATA

The GTAP 11 database includes 160 countries, covering 99.1% of world GDP and 96.4% of world population, which are categorized into fourteen aggregated regions for the purpose of this study: Serbia, Russia, Romania, South East Asia, North America, United Kingdom, Ukraine, European Union, Czech Republic, Hungary, Austria, Poland, Rest of the World, and Slovak Republic. The original 65 sectors are divided into eleven new sectors: Grains and crops, Livestock and meat products, Mining and extraction, Processed food, Textiles and clothing, Light manufacturing, Automotives, Heavy manufacturing, Utilities and construction, Transportation and communication, Other services. The primary factors use all available disaggregation of the database into five occupational categories: Agricultural and unskilled workers, Service workers, Clerks, Technicians/associate professionals, Officials/managers, and Land and Capital.

2.1 Elasticities

The model uses many types of elasticities, here we focus only on those related to the primary factors. There are different views on the values of the elasticity between the primary factors in production at the regional level, referred to as ESUBVA in this model. Borjas [2] assumed that workers with the same education but different level of experiences are imperfect substitutes in production. His research suggests that an increase in the labor force due to immigration leads to an increase in the labor force available for each skill class, despite accounting for favorable cross effects, which has a particularly negative impact on the wages of native workers in the lower-educated group. Ottaviano and Peri [6], like [2], found a significant degree of imperfect substitution between natives and immigrants, but their estimates differ quantitatively – OP1 approach. They also argued that labor and capital are complementary, and the growth rate of capital should be taken into account – OP2 approach. Edo and Rapaport [3] conclude that the impact of immigration on wages and employment of native workers within a given education group is more negative in states with low minimum wages and for workers with little education and experience. In the model, the ESUBVA is found in the equation that applies to the demand for endowment goods:

$$qfe_{i,j,r} = -afe_{i,j,r} + qva_{j,r} - ESUBVA_j \cdot \left(pfe_{i,j,r} - afe_{i,j,r} - pva_{j,r}\right) \qquad (1)$$

where:

$afe_{i,j,r}$	primary factor *i* augmenting technological change by industry *j* of region *r*,
$pfe_{i,j,r}$	firms' price for endowment commodity *i* in industry *j*, region *r*,
$pva_{j,r}$	firms' price of value added in industry *j* of region *r*,
$qfe_{i,j,r}$	demand for endowment *i* for use in industry *j* in region *r*,
$qva_{j,r}$	value added in industry *j* of region *r*.

The values for Slovakia are stated in Table *1*. The default values given by the GTAP 7 model vary in different production activities, column BASE with the highest value in Transport and communication.

Table 1: Elasticities values

Industries	BASE	OP1, OP2 and JEZ
Grains and crops	0.284	0.142
Livestock and meat products	0.522	0.261
Mining and extraction	0.200	0.100
Processed food	1.120	0.560
Textiles and clothing	1.260	0.630
Light manufacturing	1.260	0.630
Automotives	1.260	0.630
Heavy manufacturing	1.260	0.630
Utilities and construction	1.359	0.679
Transport and communication	1.599	0.799
Other services	1.260	0.630

Another elasticity that is important for modeling changes in the labor market is the elasticity of transformation for sluggish primary factor endowments, noted ETRAE. This parameter appears in the equation that distributes the sluggish endowments across sectors:

$$qoes_{i,j,r} = qo_{i,r} - endswslack_{i,r} - ETRAE_i \cdot \left(pm_{i,r} - pmes_{i,j,r}\right) \qquad (2)$$

where:

$endswslack_{i,r}$	slack variable in endowment market clearing condition,
$pm_{i,r}$	market price of commodity *i* in region *r*,
$pmes_{i,j,r}$	market price of sluggish endowment *i* used by industry *j* in region *r*,
$qo_{i,r}$	industry output of commodity *i* in region *r*,
$qoes_{i,j,r}$	supply of sluggish endowment *i* used by industry *j* in region *r*.

By definition, it’s value is non- positive. In the model used for our experiments, the values for all primary factors in Slovakia, including land, capital and five types of labor, are set to -0.2.

2.2 Primary factors across production sectors

Capital is the factor with the largest, more than 77%, overall representation across manufacturing sectors. The use of labor and capital is even across activities, with an average ratio of 30/70. For land, these tendencies are different, see Figure *1*. Agricultural production-oriented activities, most notably Grains and crops, are the largest consumers of a significant amount of land. The relatively constant labor-to-capital ratio is particularly low in Mining and extraction, which can be attributed to the high technological requirements of the mining machinery or drilling equipment of this activity.

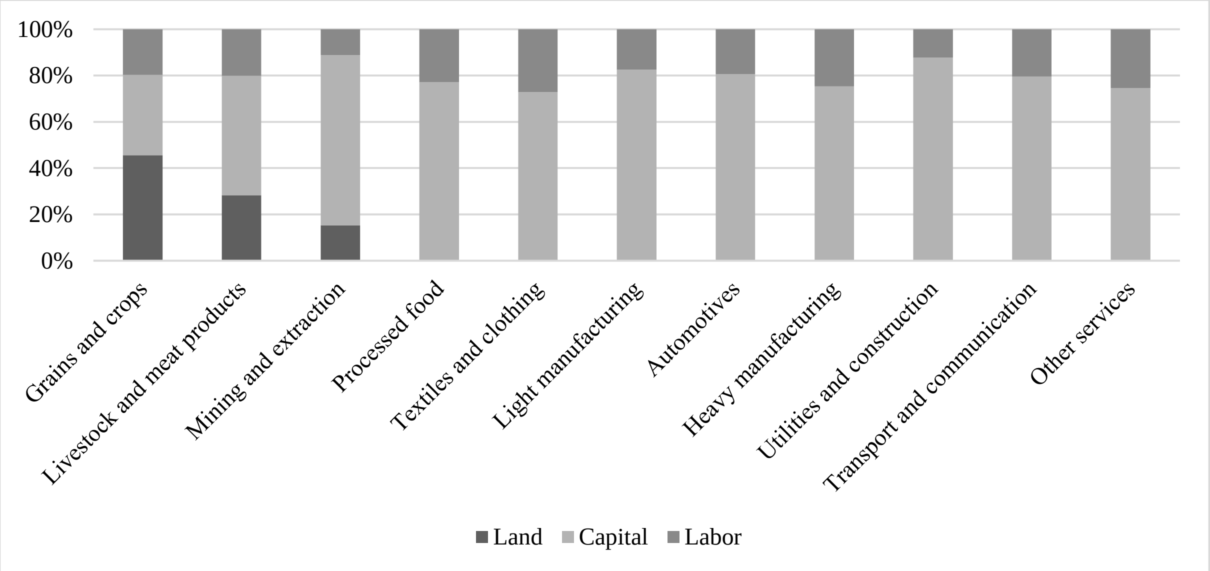

Figure 1: Shares of factors of production in production activities. Source: Authors' elaboration based on [1]

2.3 Educational structure of immigrants

The educational structure of immigrants in 2021 and 2022 is shown in Figure 2 (data counts for those older than 15 years). The most common immigrants are people with a secondary education and the second largest group consists of people with higher education. It is worth mentioning that about 43% of immigrants in 2021 had Slovak nationality, in 2022 it was 45 [8].

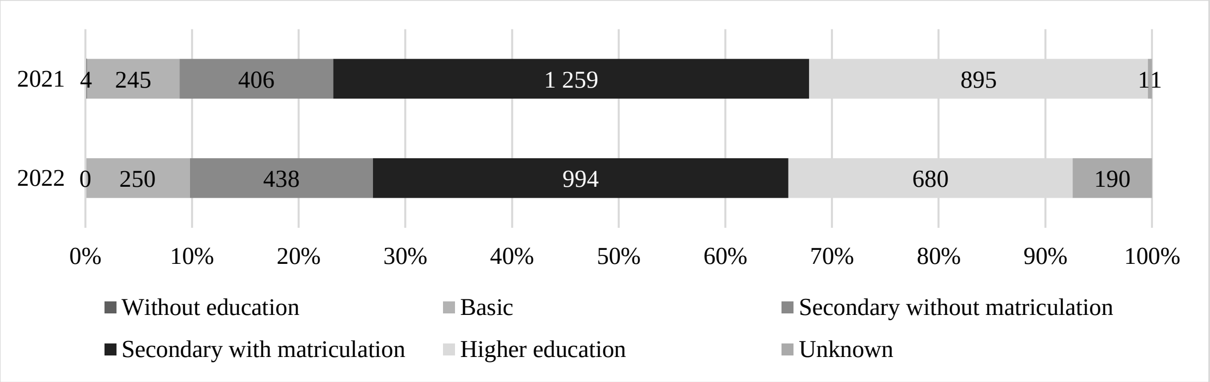

Figure 2: Educational structure of immigrants. Source: Authors' elaboration based on [8]

In view of these facts, it is worth being aware of the possibilities to cover the deficient areas of the labor market with these persons and to create initiatives that motivate persons with temporary residence, tolerated residence, work permit or information card to stay on the territory of the Slovak Republic. However, the statement of the Slovak Business Agency [7] is clear: "*The procedure for employing foreigners in professions with labor shortages is quite strictly regulated in Slovakia. On the one hand, it is about protecting the labor market in the Slovak Republic, on the other hand, such a strict procedure makes it impossible for many job seekers to find employment in Slovakia, which is criticized by employers' associations and organizations*." There is still a conservative attitude towards migration policy among policy makers, which means that many of the Ukrainian professionals who have received temporary protection in Slovakia due to the ongoing war are doctors, nurses and teachers working in low-skilled positions [4]. This approach is referred to as JEZ in our study.

3 MODELING AND RESULTS

The effects of immigration are introduced into the model as an exogenous shock representing an increase in the labor force without distinguishing the migrant's country of origin. The model is built for a medium-term estimation, i.e. it corresponds to about five years, therefore the results show the analyzed effects beyond this time horizon. The average annual development of migration flows over the five years 2018 – 2022 is used, resulting in a level of 4.2% [5]. Based on the available educational structure of foreign immigrants to the Slovak Republic (see Figure 2), the highest education of immigrants is assigned to five occupational groups as in the model and the above-mentioned shock of 4.2% is assigned according to the pattern in Tables 2, 3, 4, and 5. There are four scenarios for modeling the increase in immigration.

3.1 BASE scenario

In this scenario, the values of the elasticity of substitution between the primary factors are set as in the basic parameters of the GTAP 7 model, see Table 1, column BASE. The increase in the stock of immigrants is modeled in five occupational categories in relation to their representation in each education level (see Figure 1), as shown in Table 2.

The results of this experiment can be found in the last column of Table 2. Wages fall for all labor categories, especially in the most highly qualified group of Officials and managers (by 0.81%), and the effect is smallest for Agricultural and unskilled workers (by 0.18%). The higher is the increase in the labor supply, the higher is the drop in its wage (factor price), excluding the Service workers group. The price of capital and land increases for both primary factors.

Table 2: Distribution of shock across labor categories with low factor substitution

Primary factor	Change of factors		Change of factors price
Officials/managers	1.259%	⇨	-0.807%
Technicians/assoc. professionals	0.911%		-0.551%
Clerks	0.911%		-0.522%
Service workers	0.696%		-0.551%
Agricultur. and unskilled workers	0.424%		-0.177%
Sum	4.200%		
Capital			0.146%
Land			0.203%

3.2 OT1 scenario

The second scenario is based on the considerations of [6], especially on the complementarity of labor categories. The elasticity of factor substitution is therefore reduced by half of the original values for all activities to reach a level compatible with complementarity while maintaining the original ratios, see Table 1, column OP1.

Table 3: Distribution of shock across labor categories with complementarities in primary factors

Primary factor	Change of factors		Change of factors price
Officials/managers	1.259%	⇨	-1.500%
Technicians/assoc. professionals	0.911%		-1.000%
Clerks	0.911%		-0.955%
Service workers	0.696%		-0.588%
Agricultur. and unskilled workers	0.424%		-0.305%
Sum	4.200%		
Capital			0.415%
Land			0.285%

In this case, the increase in labor has an even more devastating impact on wages than in the previous scenario, underlining the role of capital and land as complementarities through an increase in their prices. Again, the Service workers wage is an exclusion which may suggest that it is workers in this occupational group that are in demand in the market in quantities corresponding to the introduced increase in supply, hence their increase has filled existing gaps in the market and has not caused an additional fall in wages with harder substitutability. The results can be found in Table 3.

3.3 OT2 scenario

With elasticity of substitution remaining the same as in the previous scenario, this scenario draws attention to a new element in the impact study of [6], in which the authors emphasize the role of physical capital and investment arising from growing markets due to increasing migration. Based on their assumption of 10% capital accumulation in the short term, this study assumes a value of 5% in the medium term, which represents the size of the additional shock to capital.

The simultaneous increase in inventories leads to an increase in labor wages at all occupation levels, with the increase being largest in Agricultural and unskilled workers compared to the

BASE and OP1 scenarios (see Table 4). This could be due to their high participation in industries such as Motor vehicles, Heavy and Light manufacturing which are characterized by an above average share of more than 75% in the use of capital, so that the decrease in its price causes a demand for these workers associated with higher wages. The price of capital has fallen due to the increase in supply.

Table 4: Distribution of shock across labor categories with capital increase

Primary factor	Change of factors	⇨	Change of factors price
Officials/managers	1.259%		1.647%
Technicians/assoc. professionals	0.911%		2.370%
Clerks	0.911%		2.181%
Service workers	0.696%		1.925%
Agricultur. and unskilled workers	0.424%		3.773%
Sum	4.200%		
Capital	5.000%		-4.123%
Land			6.692%

3.4 JEZ scenario

The fourth scenario is to illustrate the problem of recognizing the qualifications of immigrants described in the previous part of the paper and in [5]. The resulting fact is the employment of qualified immigrants in less qualified positions. To approximate this situation, the education levels are shifted down by one level, using an approach identical to the previous determination of an overall shock of 4.2%. In the scenario, the previous assumptions of complementarity and capital increase are applied.

The largest changes with the introduction of labor and capital increase are found in the top occupational group, the wages of Officials and managers, whose increase of 3.5% is mainly due to the smallest shock to this group, making these workers rarer and scarcer. The group of Service workers grew the most, which is reflected in the smallest wage increase for this group in Table 5. Despite the relatively large increase in the least skilled group, there is a significant improvement in their wage conditions, which could be due to their high participation in industries, as in the previous scenario. However, the shock-price disparity is now milder as these workers enter the workers market to a greater extent.

Table 5: Distribution of shock across shifted labor categories with capital increase

Primary factor	Change of factors	⇨	Change of factors price
Officials/managers	0.033%		3.481%
Technicians/assoc. professionals	0.646%		2.751%
Clerks	0.646%		2.532%
Service workers	1.788%		0.387%
Agricultur. and unskilled workers	1.084%		2.714%
Sum	4.200%		
Capital	5.000%		-4.154%
Land			6.692%

4 CONCLUSIONS

The impact of immigration in Slovakia is analyzed on the basis of an exogenous increase of 4.2% in the labor force, the size of which is based on the average annual development of migration flows, whereby the size of this shock is also distributed in such a way that it corresponds to the observed educational structure of the immigrants. This distribution is taken into account in the analysis of wage changes in the five occupational groups.

Four scenarios are considered under various assumptions about the values of the elasticity of substitution of the primary factors and different changes in the labor groups and capital. The first two scenarios with the increase in labor force from abroad showed negative effects on wages of native workers in each education or occupation category. Once the increase in capital is considered as a complementary primary factor to labor, the wage effects are positive in every occupational group. Taking into account the underestimation of immigrants' qualification in the Slovak labor market, the results are dominated by the observation of a positive impact of immigration on wages in all skill groups, which mainly arises from the application of the assumptions on capital accumulation and investment. Although the largest increases are observed for the less educated groups, a significant increase in the wages of Agricultural workers and unskilled workers is observed, due to their high participation in emerging industries such as Motor vehicles, Heavy and Light manufacturing. The smallest share of the most highly educated Officials and managers is reflected in their highest wage increase of almost 3.5%, which is due to their sporadic and important representation.

Further research will look at a more detailed analysis of the impact of immigration on economic sectors under various scenarios and also will distinguish the immigrants‘ country of origin.

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References

[1] Aguiar, A., Chepeliev, M., Corong, E. and van der Mensbrugghe, D. (2023). The Global Trade Analysis Project (GTAP) Data Base: Version 11. *Journal of Global Economic Analysis*, 7(2). doi: 10.21642/JGEA.070201AF.
[2] Borjas, G. J. (2003). The labor demand curve is downward sloping: Reexamining the impact of immigration on the Labor Market. Cambridge: National Bureau of Economic Research. *The Quarterly Journal of Economics*, 118(4), pp. 1335–1374.
[3] Edo, A. and Rapoport, H. (2019). Minimum wages and the labor market effects of immigration. *Labour Economics*, 61, 101753.
[4] European Commission. (2023). Slovakia: Continued growth in the employment and economic activity of migrants. Directorate-General for Migration and Home Affairs. Aailable at https://migrant-integration.ec.europa.eu/news/slovakia-continued-growth-employment-and-economic-activity-migrants_en [Accessed 12 April 2024].
[5] Ježíková, K. (2024). How Can Immigration Raise Wages? [Manuscript in preparation] Diploma thesis. Bratislava: Comenius University in Bratislava.
[6] Ottaviano, G. I. and Peri, G. (2012). Rethinking the effect of immigration on wages. *Journal of the European economic association*, 10(1), pp. 152-197.
[7] Slovak Business Agency. (2022). Analýza zmien na trhu práce a zamestnávania cudzincov na Slovensku v rokoch 2019 – 2021. Bratislava, 2022. Available at https://monitoringmsp.sk/wp-content/uploads/2022/10/Analyza-zmien-na-trhu-prace-

a-zamestnavania-cudzincov-na-Slovensku-v-rokoch-2019-2021.pdf [Accessed 12 April 2024].

[8] Štatistický úrad SR. (2023). Obyvateľstvo a migrácia: Pramenné dielo - Migrácia 2021, 2022. Available at https://slovak.statistics.sk/wps/wcm/connect/7874db89-ca64-4489-ab56-f3d10f995686/Pramenne_dielo_migracia_2021.zip?MOD=AJPERES&CVID=o5u0YtT&CVID=o5u0YtT&CVID=o5u0YtT&CVID=o5u0YtT&CVID=o5u0YtT&CVID=o5u0YtT&CVID=o5u0YtT&CVID=o5u0YtT&CVID=o5u0YtT&CVID=o5u0YtT&CVID=o5u0YtT&CVID=o5u0YtT&CVID=o5u0YtT&CVID=o5u0YtT [Accessed 12 April 2024].

[9] Ústredie práce, sociálnych vecí a rodiny. (2024). Zoznam zamestnaní s nedostatkom pracovnej sily. Redakčný systém KOXUS CMS. Available at www.upsvr.gov.sk/buxus/generate_page.php?page_id=806803 [Accessed 12 April 2024].

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PARAMETRIC MODELS IN FUNCTIONAL SPACE

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Abstract

A dataset observed in the functional space can easily be extended to a theoretical framework of classical parametric regression models. An estimation method of the unknown regression parameters with functional data is similar to its real-valued counterparts. A technique of the method of minimizing the least squares criterion is modified by defining an integral of the continuous function in its estimator. The objective of this paper is to show a theoretical framework of parametric regression models when data are observed in the functional space. The functionality in the model is possible when the response, covariate or both are observed as functions. The intention of our theoretical outline of these models is to evoke further research in this field that is applicable to high-frequency data. An automation in the data collection enables recording dataset that can be described by mathematical objects.

Keywords: *Functional data, Parametric regression, Ordinary least squares*

JEL Classification: C14, C55
AMS Classification: 91B82

1 INTRODUCTION TO FUNCTIONAL DATA

Let's consider that we observe a matrix of real numbers $\mathbf{X} = x_{ij}$ representing a p-dimensional column-wise covariate matrix, $j = 1, \dots, p$, recorded for sample units, $i = 1, \dots, n$. It follows that by assuming that the observed dataset comes from a functional process, the need dictates transforming the observed real-valued data into mathematical objects, or functions. A function is fitted into a matrix of real numbers x_{ij} across each sample unit i, and, therefore, a p-dimensional space is mapped into an infinite, or functional, space. A notation for such a functional covariate matrix is $\chi_i(t)$, where $i = 1, \dots, n$ now refers to a number of sample curves that are observed on a continuous domain $t \in T$. Theoretical concepts of functional spaces are provided by [5].

It follows that the basis spline expansion can be used to individually construct a continuous and sufficiently differentiable functions $\chi_i(t): R \to R$ for each sample unit i. The basis spline expansion is a linear combination of spline functions that are mathematically independent of each other and that have the property that any function can be approximated arbitrarily well by taking a weighted sum or linear combination of a sufficiently large number k of these functions. Without loss of generality, we consider the first sample unit $i = 1$. Hence, the first observation curve $\chi_i(t)$ can be expressed in terms of the basis spline expansion as [5]:

$$\chi_1(t) = \sum_{k=1}^{K} c_k B_{k,m}(t) \tag{1}$$

The spline functions defined in the $k \times 1$ vector, $B_{k,m}(t)$, are piecewise polynomials of order m that are automatically tied together at breakpoints, or knots, where $k = 1, \dots, K$ refers to a number of polynomials used in the expansion, which determines a degree of smoothing of the original real-valued data. The c_k is a vector of unknown parameters that needs to be estimated. The derivatives of spline curve up to order $m - 2$ also match up at knots. The interested reader should consult [1], [2] or [3] for more theoretical details related to basis splines, or other types of spline functions.

The application of basis spline smoothing to real data can be done through the familiar technique of fitting statistical models to data by minimizing the sum of squared errors, or the least squares method, which leads to an estimation of unknown parameters c_k. It follows, omitting the subscript m to simplify the notation, $B_k(t) = B_{k,m}(t)$, that the estimated parameters are derived as:

$$\hat{c}_k = \left(B_k(t)^{\mathrm{T}} B_k(t)\right)^{-1} B_k(t)^{\mathrm{T}} x_{1p} \tag{2}$$

where x_{1j} is a vector of observations for the first sample unit from the covariate matrix x_{ij} and a subscript T refers to a transpose. Hence, $\chi_1(t)$ in eq. (1) can be rewritten as:

$$\chi_1(t) = \sum_{k=1}^{K} \hat{c}_k B_k(t) = \left(B_k(t)^{\mathrm{T}} B_k(t)\right)^{-1} B_k(t)^{\mathrm{T}} x_{1p} \tag{3}$$

$\chi_1(t)$ is now observable on a continuous domain t, and can be discretised at any values on t. The number of polynomials $k = 1, \dots, K$ is defined as in eq. (1). Naturally, the estimation of curves for each sample unit can be extended to functional covariate matrix $\chi_i(t)$, where the least squares method is expanded to estimate a matrix of parameters $\mathbf{C} = c_{ik}$, noting that the same $B_k(t)$ is used across all curves.

2 FUNCTIONAL PARAMETRIC MODELS

The statistical analysis normally comprises of identifying a structure and (co)variability in the response variable and the covariate matrix, where either or both can take a functional form. In the following subsections, we show a structure of the theoretical framework for parametric regression models when data are functional and briefly discuss its estimation procedures.

2.1 Functional response

In this subsection, the features of the functional matrix of observed curves $\boldsymbol{\chi}(t) = \chi_i(t)$, as defined in the previous section, are used to fit a regression model by constructing a set of binary independent variables in the design matrix $\mathbf{Z} = z_{i(c),j}$, with a dimension of $n \times (C+1)$. The number of curves in each group, $c = 1, \dots, C$, is denoted by N_C, i.e. the i^{th} sample curve falls in the c^{th} group. Hence, the columns $c = 1, \dots, C+1$ of the design matrix $\mathbf{Z}$ are constructed as a binary indicator that represents a category of individual observed curves, with ones in the first column that serves for the estimation of the mean function across all curves. In this case, the statistical technique of Analysis of Variance (ANOVA) extended in the functional space addresses the problem because the values of the independent variables are 0's or 1's coding the group of the observed curves. The ANOVA model can be formalised as [5]:

$$\boldsymbol{\chi}(t) = \mathbf{Z}\boldsymbol{\beta}(t) + \boldsymbol{\varepsilon}(t) \tag{4}$$

where $\boldsymbol{\beta}(t) = \left(\beta_0(t), \beta_1(t), \dots, \beta_c(t)\right)^{\mathrm{T}}$ are unknown functional parameters that need to be estimated. The function $\beta_0(t)$ represents the overall mean across all N sample curves, where $N = \sum_{c=1}^{C} N_C$, and $\beta_1(t), \dots, \beta_c(t)$ are the functions of specific effects which represent departures from the overall mean specific to each group c. The error function $\boldsymbol{\varepsilon}(t) = \varepsilon_{i(c)}(t)$ are the unexplained variations specific to the i^{th} sample curve within group c that are assumed to be independently and identically distributed with 0 mean and a constant variance σ^2.

The regression functions $\boldsymbol{\beta}(t)$ can also be expressed in terms of the basis spline expansion, $\boldsymbol{\beta}(t) = b_{(c+1)k_\beta}\theta_{k_\beta}(t)$, where k_β is the number of basis splines in the expansion and $c_{(c+1)k_\beta}$ is a $(c+1) \times k_\beta$ matrix of coefficients that corresponds to a number of parameters in $\boldsymbol{\beta}(t)$.

After some rearrangements, the functional form of the ordinary least squares method leads to the following estimated parameters $\widehat{\mathbf{B}} = \hat{b}_{(c+1)k_\beta}$ [5]:

$$vec(\widehat{\mathbf{B}}) = \left(\mathbf{J}_{\theta_{k_\beta}\theta_{k_\beta}} \otimes (\mathbf{Z}^{\mathrm{T}}\mathbf{Z})\right)^{-1} vec\left(\mathbf{Z}^{\mathrm{T}}\mathbf{C}\mathbf{J}_{B_k(t)\theta_{k_\beta}}\right) \tag{5}$$

where $vec(.)$ denotes a vectorised matrix, $\otimes$ is the Kronecker product, $\mathbf{J}_{\theta_{k_\beta}\theta_{k_\beta}} = \int \theta_{k_\beta}\theta_{k_\beta}{}^T(t)dt$ and $\mathbf{J}_{B_k(t)\theta_{k_\beta}} = \int B_k(t)\theta_{k_\beta}{}^T(t)dt$ are the derivatives of basis spline functions over its domain t; refer to [4], [5], [6] for detailed derivation.

2.2 Functional covariate

In this subsection, a linear model is defined by a set of curves in its covariate, but the response variable is real-valued which contrasts with the previous section where the response was functional. In addition to the previous section, we introduce a real-valued response variable y_i that is observed for each sample unit $i = 1, \dots, n$. The regression model takes the following form [5]:

$$y_i = \alpha + \int \left(\chi_i(t)\right)^T \beta(t)dt + \varepsilon_i \tag{6}$$

where ε_i is a vector of random errors, α is an unknown point-wise intercept and $\beta(t)$ is a regression coefficient function observable on a continuous domain t. The model in eq. (6) is a functional extension of a linear regression model where the usual summation of $\sum_{j=1}^{p}\beta_j x_{ij}$ is replaced by integration over a continuous domain t. Similarly, the regression coefficient function $\beta(t)$ is decomposed in terms of a set of basis spline functions $\beta(t) = \sum_{k=1}^{K_\beta} b_k\theta_k(t) = \theta_k(t)'b_k$, with $\theta_k(t)$ a vector of basis spline functions of length $K_\beta \times 1$ and a corresponding vector of unknown coefficients, b_k.

After some algebra and rearrangements, the functional form of the ordinary least squares method leads to the following estimated parameters [5]:

$$\left(\hat{\alpha}, \hat{b}_1, \dots, \hat{b}_{k_\beta}\right)^{\mathrm{T}} = (\mathbf{Z}^{\mathrm{T}}\mathbf{Z})^{-1}\mathbf{Z}^{\mathrm{T}}y_i \tag{7}$$

where $\mathbf{Z} = [1 c_{ik} \int B_k(t)\theta_k(t)^T dt]$ is a $n \times (K_\beta + 1)$ matrix; refer to [4], [5], [6] for detailed derivation.

2.3 Functional response and functional covariate

In this subsection, we consider that both the response and the covariate matrix are observed in the functional space. Here, the notation for the functional response is $\Upsilon_i(t)$, where $i = 1, \dots, n$ refers to the number of sample curves that correspond to $\chi_i(t)$. The model takes the following form:

$$\Upsilon_i(t) = \int \left(\chi_i(t)\right)^T \beta(t)dt + \varepsilon_i(t) \tag{8}$$

It is important to note that in eq. (8) the response $\Upsilon_i(t)$ and the covariate $\chi_i(t)$ are both functions of the same domain t and the influence of $\chi_i(t)$ on $\Upsilon_i(t)$ is only at time t. A more complicated model structure with the influence of $\chi_i(t)$ through a range of its values on $\Upsilon_i(t)$ can also be defined. Both of these options are discussed in [5] for details.

Similarly, a decomposition of the functional response, the functional covariate and the functional regression parameter can be carried out. It follows that we need to minimise the following least sum of squared errors criterion [4]:

$$LSSE = \int \left(\Upsilon_i(t) - \left(\chi_i(t)\right)^T \beta(t)dt + \varepsilon_i(t)\right)^2 \tag{9}$$

The procedure for deriving the solution to eq. (9) is similar to the previously defined functional parametric models.

3 CONCLUSIONS

The paper introduces a theoretical framework for parametric regression models applied to functional data. By considering data observed in a functional space, the traditional parametric regression model is extended to accommodate functions as responses, covariates, or both. The methodology involves transforming real-valued data into mathematical objects, specifically functions, and utilizing techniques such as basis spline expansion for modelling. Estimation methods similar to those used in classical parametric regression, such as ordinary least squares, are adapted for functional data. The proposed framework provides a foundation for statistical analysis in scenarios where data are described by mathematical functions, enabling further research particularly applicable to high-frequency data. This framework holds promise for various practical applications, especially in fields dealing with data collected at high frequencies, such as finance, healthcare monitoring, and signal processing. By embracing the functional data perspective, researchers and practitioners can gain deeper insights and develop more accurate models for analysing complex and dynamic datasets. This paper thus contributes to advancing methodologies for analysing functional data and encourages exploration in this emerging field with its diverse potential applications. Hence, the authors' main intention is to present theoretical concepts of functional parametric models that can stem further empirical research, particularly, in the econometric modelling.

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References

[1] Anderson, T.W. (2003). *An Introduction to Multivariate Statistical Analysis*. Third edition. New York: Wiley.
[2] De Boor, C. (2001). *A Practical Guide to Splines*. Revised Edition. New York: Springer.
[3] Ferraty, F. and Romain, Y. (2011). *The Oxford Handbook of Functional Analysis*. Oxford: University Press.
[4] Ramsay, J. O. and Silverman, B. W. (2002). *Applied Functional Data Analysis Methods and Case Studies*. Springer.
[5] Ramsay, J. O. and Silverman, B. W. (2005). *Functional Data Analysis*. Second Edition. Springer.
[6] Ramsay, J. O., Hooker G. and Graves, S. (2009). *Functional Data Analysis with R and MATLAB*. Use R!, Springer.

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SIMULATION MODEL FOR THE SELECTION OF A SUPPLIER OF ELECTRICITY TO HOUSEHOLDS IN THE CZECH REPUBLIC

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Abstract

Electricity is one of the most important raw materials that fundamentally affect our daily functioning. Since 2002, the Czech Republic has transformed its electricity market, culminating in 2006 with the possibility for households to choose their supplier and product. In the following years, other suppliers entered the market and electricity production using solar cells on houses or industrial buildings slowly became popular. This option can cover household consumption, especially during summer days, and even generate surpluses that suppliers are obliged to buy and distribute through their grid. The energy market has been dealt a big blow by events in the world, particularly in 2022, which have caused, among other things, large fluctuations in electricity prices. All of these aspects have a major impact on household electricity costs and it is therefore important to choose the best (cheapest) product from the large number of products on offer. In this paper, we consider the choice of product from an electricity supplier in dual tariff rate D25d (for households) using a simulation model. For this tariff, data for 70 different products for each of the three distribution regions in the Czech Republic (Liberec region, South Bohemia region and Prague) for the year 2023 was obtained from a web-based calculator.

Keywords: *tariff rate D25d, electricity, electricity suppliers, simulation, year 2023*

JEL Classification: C44, C63, O13
AMS Classification: 90C15

1 INTRODUCTION – MOTIVATION TO DEAL WITH THE PROBLEM

Electricity is one of the basic raw materials used on a daily basis by human society. From the 1950s to about the 1990s, there was mainly a steep increase in its consumption. In recent years, consumption has peaked (Energy Regulatory Office, 2024) – see Figure 1.

Figure 1 *Long-term trends in electricity consumption in the Czech Republic (1919-2022) (FG Forrest, a. s., no date)*

In contrast, electricity prices have experienced very turbulent times, and in 2022 the state even had to step in with regulations. If a household did not have a fixed electricity price or its fixation ended during 2022, its costs increased by leaps and bounds (Kurzy.cz, no date) – see Figure 2.

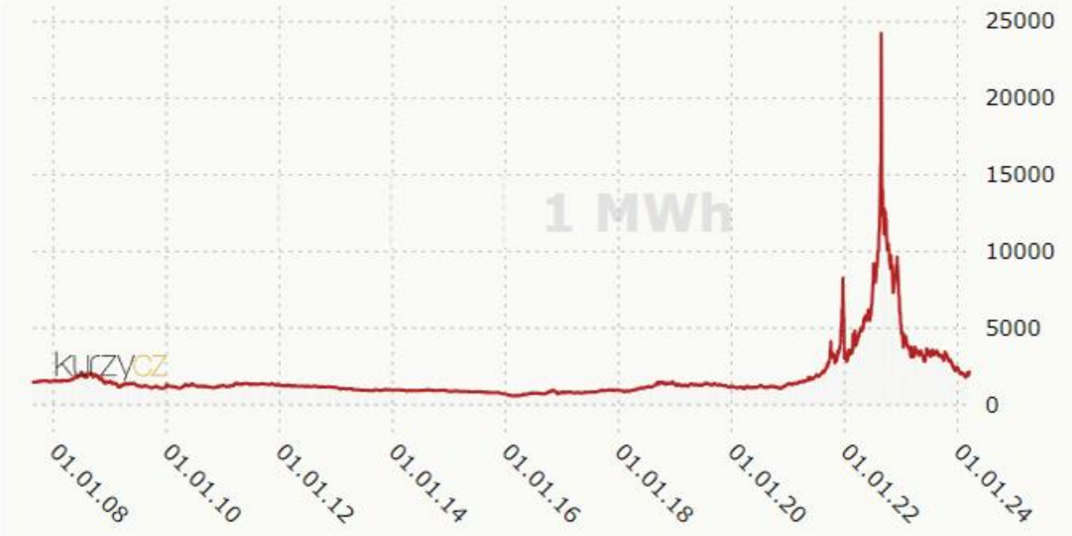

Figure 2 *Electricity prices (in CZK per 1MWh/year) - trend from 2008 to 2024 (Kurzy.cz, no date)*

In addition to the cost of electricity, the cost of housing, fuel and food has risen over the past few years. For these reasons, it is important for households to choose a supplier of high quality with the most cost-effective product according to their requirements and needs. Based on the similar work (Kuncová, 2020, p.12-20), (Kuncová, 2019, pp.7-14), (Kuncová, 2015, pp.714-720)[9] and (Kuncová and Sekničková, 2014, pp.542-547) a Monte Carlo simulation is used to iteratively generate monthly consumption values and calculate annual ones. The simulation model is suitable mainly because it can generate and also account for variable consumption values. The random inputs are generated from a normal distribution and the classical MS Excel is used to implement the model without the need to use any of the add-ins. In the final section, the results will be compared with previous analyses, (Kuncová, 2020, p.12-20), (Kuncová, 2019, pp.7-14), (Kuncová, 2015, pp.714-720) and (Kuncová and Sekničková, 2014, pp.542-547).

The Czech Republic is divided into three distribution regions - the capital city of Prague is administered by the distributor PRE, the South Bohemia, South Moravia, Vysočina region except Havlíčkobrodsko and Prostějovsko and Zlín regions except Vsetín are administered by EG.D (formerly E.ON) and the rest of the Czech Republic is administered by ČEZ (Konecny, 2020). These distributors are also the owners of the transmission grid, and their job is to distribute the energy, but they don't offer any product, that's the job of the supplier. Households can choose the supplier, but not the distributor, who is linked to the distribution network in the territory. In 2023, there were 76 electricity suppliers operating in the country. Each household is assigned a tariff by the supplier according to the way it uses electricity. In this paper, we focus on households that use electricity for storage water heating with an eight-hour low tariff, lower consumption and a main circuit breaker size of 3x25A. For these households, there is a two-tariff (dual-tariff) rate D25d, which has 2 periods (high tariff, low tariff) during the day (Srovnání elektřiny a plynu 2024, 2024) and (TZB-info, Kalkulátor cen energií, 2024).

The final price depends on several factors - it includes the price for consumption (in our case divided into high and low tariffs), fixed rates or taxes. For simplicity, the price of electricity can be divided into items whose price is set annually by the Energy Regulatory Office (ERÚ) (Energy Regulatory Office, 2024).

It covers:

- monthly lease for the circuit breaker,
- price per megawatt hour (MWh) in high tariff (HT),
- price per megawatt hour in low tariff (LT),
- price per system services,
- price for the support of the renewable energy purchase,
- charges for the electricity market operator,
- electricity ecological tax (28.30 CZK per 1 MWh).

The second part of the total price is given by the electricity supplier. It covers:

- fixed monthly fee for the selected product,
- price per megawatt hour (MWh) in high tariff (HT),
- price per megawatt hour in low tariff (LT),

The final price is increased by VAT that is 21%.

2 SIMULATION MODEL AND DATA

Simulation is a method that is used to map more complex systems or systems that are in the preparation stage and therefore cannot be tested for their response, functionality or identify weaknesses directly in operation. Currently, simulations are widely used especially for manufacturing systems. The use of simulation for a manufacturing system was discussed in (Mourtzis, 2020, pp.1927-1949). Furthermore, simulation can be used in planning, optimization and operational control of production flows and technologies (Drastich, 2017, p.23-26), in mass service models (Hu, Barnes and Golden, 2018, p. 7-49)[5], but also in situations where random fluctuations caused by dynamic environmental or human factors need to be modeled (Xu *et al.* 2021, pp.368-380). In conjunction with electricity, Monte Carlo simulation is often used, for example, to test the stability of distribution networks (Caballero-Pena et al., 2022) or to generate aggregate market demand for electricity (López-Gonzales et al., 2020). The simulation model is based on generating monthly consumption (based on a normal distribution with a given mean and 20% standard deviation) and determining the annual consumption of a selected household. The formula for calculating the total annual cost is as follows (Kuncová, 2019, pp.7-14):

$$COST_{ij} = (1 + VAT) \cdot \left[12 \cdot \left(mf_{ij} + mf_j\right) + p_{LT} \cdot gc \cdot \left(ph_{ij} + ph_j\right) + p_{HT} \cdot gc \cdot \left(pl_{ij} + pl_j\right) + gc \cdot (os + t)\right]$$

where

i … product, $i = 1, \ldots, 70$,
j … distributor, $j = 1, \ldots, 3$,
VAT … value added tax (0.21 for 2023),
mf … suppliers' fix monthly fee,
gc … simulated annual consumption in MWh,
ph … price in high tariff per 1 MWh,
pl … price in low tariff per 1 MWh,
p_{HT} … percentage of the consumption in high tariff (0.45),
p_{LT} … percentage of the consumption in low tariff (0.55),
os … price for other services per 1 MWh,
t … electricity tax per 1 MWh (t = 28.3 CZK).

As in previous articles (Kuncová, 2020, p.12-20), (Kuncová, 2019, pp.7-14), (Kuncová, 2015, pp.714-720) and (Kuncová and Sekničková, 2014, pp.542-547) the products are compared for tariff rate D25d with an electricity consumption of about 10 MWh per year (or monthly about 800-900 kWh), 45% of energy in the high tariff and 55% in the low tariff and with a 3x25A circuit breaker.

Table 1 *Average suppliers' prices, fees and distributors' prices, fees for year 2023 [CZK] (TZB-info, Kalkulátor cen energií, 2024)*

distrib. region	suppliers' fix monthly fee	suppliers' price in high tariff per 1 MWh	suppliers' price in low tariff per 1 MWh	fix monthly fee per circuit breaker	distributors' price in high tariff per 1 MWh	distributors' price in low tariff per 1 MWh	distributors' price for other services
EG.D	73,75	1857,93	1609,16	163	1747,68	203,40	113,53
PRE	74,99	1867,77	1591,92	157	1407,54	106,08	
ČEZ	66,40	1683,25	1480,63	162	1766,76	179,98	

The average supplier rates are very similar in all three distribution districts, but the PRE distributor is significantly cheaper than the other two. It can therefore be expected that the average costs in this distribution part will be slightly lower than in the rest of the Czech Republic. For the simulation, 70 products were selected for both new and existing customers. The simulated consumption has been generated for each month from the normal distribution with 20% of the average taken as the standard deviation. In all Monte Carlo simulations 30 experiments have been tried to randomly select consumption for each month and afterwards the annual costs are calculated based on the formula mentioned above.

3 CALCULATION EXPERIMENTS WITH THE MODEL

The comparison of products from suppliers was carried out on the basis of 30 simulation experiments separately for each distribution area of the Czech Republic. Based on the repeated simulation experiments, the ranking of the products was determined. The 5 most expensive and 5 cheapest products for each distribution part of the Czech Republic are presented in Tables 2, 3 and 4 respectively.

Table 1 *Simulation model results – cheapest and most expensive products for D25d distributor EG.D year 2023 [CZK]*

Product no. (rank)	**Distributor**	**Supplier / Product name**	**Average annual costs**
51 (1)	EG.D	Pražská plynárenská / DŮVĚRA 3R	30 243
42 (2)	EG.D	Innogy / Elektřina Top	31 118
9 (3)	EG.D	CENTROPOL ENERGY / Moje Energie-Optimum	31 487
67 (4)	EG.D	X Energie / XE RELAX	31 760
46 (5)	EG.D	PARC4U / eYello CZ/Yello Legado	32 577
…	…	…	…
56 (66)	EG.D	Rodinná energie / FIX36	44 960
5 (67-68)	EG.D	CENTROPOL ENERGY / FIXNĚ ONLINE na 1 rok	45 513
6 (67-68)	EG.D	CENTROPOL ENERGY / FIXNĚ ONLINE na 3 roky	45 513
11 (69)	EG.D	ČEZ Prodej / ELEKTŘINA – NA 2 ROKY NA MÍRU	46 112
14 (70)	EG.D	ČEZ Prodej / ELEKTŘINA JEN PRO VÁS smlouva na 2 roky	46 557

Table 2 *Simulation model results – the cheapest and the most expensive products for D25d distributor PRE year 2023 [CZK]*

Product no. (rank)	Distributor	Supplier / Product name	Average annual costs
52 (1)	PRE	Pražská plynárenská / KOMPLET	26 747
44 (2)	PRE	OBECNÍ PLYNÁRNA, s.r.o. / TŘI	27 557
30 (3)	PRE	Energie ČS / Produkt On-line	27 745
14 (4)	PRE	ČEZ Prodej / ELEKTŘINA – NA 3 ROKY V AKCI	28 993
43 (5)	PRE	Lumius / FIX 3	29 269
…	…	…	…
25 (66-68)	PRE	E.ON Energie / Ceník Variant PRO	43 086
56 (66-68)	PRE	Rodinná energie / FIX36	43 086
5 (66-68)	PRE	CENTROPOL ENERGY / FIXNĚ ONLINE na 1 rok	43 086
38 (69)	PRE	EPET / KOMFORT varianta Partner	43 776
67 (70)	PRE	X Energie / XE RELAX	44 227

Table 3 *Simulation model results – the cheapest and the most expensive products for D25d distributor ČEZ year 2023 [CZK]*

Product no. (rank)	Distributor	Supplier / Product name	Average annual costs
42 (1)	ČEZ	Innogy / Elektřina Top	30 186
3 (2)	ČEZ	CENTROPOL ENERGY / DUVERA START	30 212
54 (3)	ČEZ	Pražská plynárenská / DŮVĚRA 3R	30 480
10 (4)	ČEZ	CENTROPOL ENERGY / Moje Energie-Optimum	31 735
44 (5)	ČEZ	Lumius / FIX 3	31 907
…	…	…	…
41 (66)	ČEZ	Innogy / elektřina Garance 36	40 293
20 (67)	ČEZ	ČEZ Prodej / Elektřina smlouva na 3 roky	40 484
4 (68)	ČEZ	CENTROPOL ENERGY / FIXNĚ na 3 roky	40 621
34 (69)	ČEZ	Energie ČS / SPORO FIX 36	41 609
50 (70)	ČEZ	Pražská energetika / DPI 2022	45 149

The results show that the cheapest products are in the area of the distributor PRE, which is due to the lowest distribution fees in the Czech Republic. In the distribution area of EG.D, the annual prices of the cheapest products are around CZK 30 000 and the most expensive ones around CZK 47 000. For this distribution part the annual savings between the cheapest and the most expensive supplier is 35% (around 16 thousand CZK). In the PRE distribution part, the prices of the cheapest products are around CZK 27 000 and the most expensive around CZK 44 000. For this distribution part, the annual savings between the cheapest and the most expensive supplier is almost 40% (around CZK 17 thousand). In the distribution part of CEZ, the prices of the cheapest products are around CZK 30 000 and the most expensive around CZK 45 000. For this distribution part, the annual saving between the cheapest and the most expensive supplier is 33% (around CZK 15 000).

In all cases, Pražská plynárenská was the only supplier ranked in one of the top five positions (first place in the distribution part of PRE and EG.D and 4th place in the CEZ part). Suppliers Innogy and Lumius placed their products in two of the three distribution networks in one of the first places (Innogy 2nd place in EG.D and 1st place in CEZ and Lumius 5th place in PRE and CEZ). Another potentially interesting supplier appears to be Centropol Energy, which placed in the distribution parts of EG. D and CEZ in 3rd and 2nd and 4th place with its products, but in the distribution part of PRE it did not rank at all in the observed top positions, and moreover in all three distribution parts it ranked with its ONLINE products in some of the last positions. In all three distribution parts, as expected, the products offered by the distributors themselves, which are among the most expensive, also ranked last.

4 CONCLUSIONS

The complexity of the retail electricity market in the Czech Republic makes it difficult for consumers to effectively compare costs due to the large number of suppliers and differences in products. The use of Monte Carlo simulation proves to be beneficial for product price comparisons that reflect the variability of consumption. Compared to the articles (Kuncová, 2019, pp.7-14), (Kuncová, 2015, pp.714-720) and (Kuncová and Sekničková, 2014, pp.542-547), there have been quite interesting changes. None of the suppliers that ranked at the top in 2014, 2015 and 2019 are on the top rungs for 2023. For some suppliers this is due to their collapse (Bohemia Energy), other suppliers (e.g. Elimon) did not rank because there was no fixed price in their products and therefore in turbulent times such as 2022 and 2023 for energy prices they became completely unsuitable. On the contrary, a recurring situation in all the articles is that the products of the distributors themselves, acting as suppliers, are ranked last. When comparing the average annual costs, not much has changed in the case of the cheapest products, in all papers the average cost is between 27-33 thousand crowns, but there are quite significant differences between the least favourable products, where in the mentioned papers it was between 36-42 thousand crowns. In 2023, the average cost of the least favourable products ranged between 40-47 thousand crowns. The phenomenon for the supplier Centropol Energy confirms that careful product selection is key in reducing costs, as one supplier may have products that are both very advantageous for a given consumption, but also extremely disadvantageous.

Significant differences in costs between regions were again observed, with suppliers from the CEZ region generally having higher prices, which is similar to the previous research findings. Our analysis also found that PRE consistently offered the most cost-effective options. Another possible way of future research could be to combine simulation with optimization either to find the minimum/maximum consumption so that the given product is more suitable than all others or to find the value of the optimal consumption at a given limit of the annual electricity costs in one of the optimization solvers (e.g. Xpress IVE). Furthermore, individual products could start to be differentiated according to the time period for which their price lists are valid, and according to this period and household preferences, products could be weighted and thus better positioned in the simulations and potentially purchased.

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References

[1] Caballero-Pena, J., Cadena-Zarate, C., Parrado-Duque, A. and Osma-Pinto, G. (2022). Distributed energy resources on distribution networks: A systematic review of modelling, simulation, metrics, and impacts. *International Journal of Electrical Power & Energy Systems*, 138, 107900.

[2] Drastich, A. (2017). Optimization of material flow by simulation methods. *Acta logistica*, 4(4), pp. 23-26.

[3] Energy Regulatory Office (2024). Available at: https://eru.gov.cz [Accessed 15 March 2024]

[4] FG Forrest, a. s. (no date). Energetika v ČR, Skupina ČEZ – O společnosti. [online] Available at: https://www.cez.cz/cs/pro-media/cisla-a-statistiky/energetika-v-cr [Accessed 10 March 2024].

[5] Hu, X., Barnes, S. and Golden, B. (2018). Applying queueing theory to the study of emergency department operations: a survey and a discussion of comparable simulation studies. *International transactions in operational research*, 25(1), pp. 7-49.

[6] Konecny, M. (2020). Trh s elektrickou energií v České republice, *Optimal Energy*. [online] Available at: https://optimal-energy.cz/2020/01/10/trh-s-elektrickou-energii-v-ceske-republice/ [Accessed 20 March 2024].

[7] Kuncová, M. (2020). Volba dodavatele elektřiny v ČR v roce 2020 z pohledu nákladů na spotřebu elektřiny pro domácnosti a pro podnikatelský maloodběr – simulační model, *Trendy v podnikání*, 10(3), pp. 12–20.

[8] Kuncová, M. (2019). Electricity supplier selection by a household in the Czech Republic in 2017 and 2018 – Monte Carlo simulation approach. In: *The 7th International Workshop On Simulation For Energy, Sustainable Development & Environment*. Lisabon, 18.09.2019 – 20.09.2019. Genoa : DIME Universita, 2019, pp. 7–14.

[9] Kuncová, M. (2015). Methods for the electricity supplier selection – case study of the Czech Republic. *International Journal of Mathematical Models and Methods in Applied Sciences*, 9, pp. 714-720.

[10] Kuncová, M. and Sekničková, J. (2014). Analysis of the efficiency of the electricity supplier selection depending upon the price changes. In: *Mathematical Methods in Economics* [online] Olomouc, 10.09.2014 - 12.09.2014. Olomouc : Palacký University in Olomouc, 2014, pp. 542-547.

[11] Kurzy.cz (no date). Elektřina – historický vývoj ceny elektřiny po letech, minima, maxima, průměr. 1 mwh - měna CZK, Kurzy měn, akcie cz online a komodity, investice online - Kurzy.cz. [online)] Available at: https://www.kurzy.cz/komodity/cena-elektriny-graf-vyvoje-ceny/historie-czk-1MWh [Accessed 18 March 2024].

[12] López-Gonzales, J. L., Castro Souza, R., Leite Coelho da Silva, F., Carbo-Bustinza, N., Ibacache-Pulgar, G. and Calili, R. F. (2020). Simulation of the energy efficiency auction prices via the markov chain monte carlo method. *Energies*, 13(17), pp. 4544.

[13] Mourtzis D. (2020) Simulation in the design and operation of manufacturing systems: state of the art and new trends, *International Journal of Production Research*, 58(7), pp. 1927-1949.

[14] Srovnání elektřiny a plynu 2024 (2024). Ceny energie. [online] Available at: https://www.cenyenergie.cz/ [Accessed 10 March 2024].

[15] TZB-info, Kalkulátor cen energií (2024). Dodávka elektrické energie – porovnání nabídek, info. [online] Available at: https://kalkulator.tzb-info.cz/cz/dodavka-elektricke-energie-porovnani-nabidek [Accessed: 18 March 2024].

[16] Xu S. *et al*. (2021). Agent-based modeling and simulation for the electricity market with residential demand response, in *CSEE Journal of Power and Energy Systems*, 7(2), pp. 368-380.

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SEQUENTIAL HEURISTIC APPROACH TO IMPROVEMENT OF UNIFORMLY DEPLOYED SET OF DESIGNS

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Abstract

The previous research in the field of the p-location problem characteristics provided the family of associated utilities with constructive methods of creating set of uniformly deployed problem solutions. The quality of generated location deployment was measured by minimal quarantined Hamming distance among all included solutions unless any problem objective is taken into account. Within this contribution, we suggest and study a heuristic approach capable to improve the objective function values of the individual solutions submit to the condition that the minimal Hamming distance among the improved solutions cannot drop below a given threshold.

Keywords: *Discrete location, p-location problems, uniformly deployed set of solutions*

JEL Classification: C61, C63
AMS Classification: 90C27

1 INTRODUCTION

The weighted p-median model is used to describe and simulate the majority of public service system design problems that have a min-sum quality criterion (Avella, Sassano and Vasil'ev, 2007, Doerner et al, 2005, García et al., 2011, Jánošíková and Žarnay, 2014, Marianov and Serra, 2002). The significant feature of public service systems with randomly occurring demands, which function as queuing stochastic systems, is not reflected in this method. Because of this method of operation, a demand that arises at random is met by the closest accessible service center rather than the closest one, which may be located considerably farther away from the demand location. The concept of generalized disutility has been established in order to incorporate the new feature into related models (Janáček and Kvet, 2016, Kvet, 2014). Estimating the likelihood that the closest center, the next closest center, and so on until the r-th nearest center are the closest accessible centers is the basis of the generalized disutility. After estimating these probabilities, one can construct a linear integer programming model and, for medium-sized instances of the problem, use a common commercial integer programming solver (IP-solver) to find the optimal solution of the p-location problem with generalized disutility. Large instances of the issue provide a challenge to experts and researchers in charge of the instance remedies. Professionals look to the wide range of heuristic approaches due to the erratic time of the branch-and-bound based method included in the commercial IP-solvers.

Since many heuristic approaches as genetic algorithms and other evolutionary techniques are based on processing an initial population of solutions, it is necessary to construct the starting set as precisely as possible to keep good advantages of the population and to avoid troubles following from insufficient population (Rybičková, Burketová and Mocková, 2016, Gendreau and Potvin, 2010). This contribution is concentrated on a study and development of such heuristic approach, which could run fast even with large instances, and which would be able to produce a uniformly deployed set *UDS* of solutions of good quality. Once the uniformly deployed is constructed, it may be used repeatedly for different problems.

The remaining part of the paper is organized as follows: After the first chapter with the scientific background of our research, the notion of uniformly deployed set is discussed. Furthermore, the objectives which measure and evaluate the quality of the set are reported. In the third section, the heuristic approach of *UDS* improving is suggested. The last two sections are devoted to the results of performed experiments and to the final conclusions related to the obtained results.

2 NOTION OF UNIFORMLY DEPLOYED SET

The mathematical models, we are interested in, are based on locating given number p of facilities among the associated network to optimize given objective. The set of nodes in which a facility may be located is usually denoted by I and it contains m elements. The set $\boldsymbol{Y}$ of all feasible solutions of the problem can be expressed by (1). In the expression (1), the symbol $\mathbf{y}$ denotes the vector of location variables y_i defined for each candidate location i from I. The variable y_i takes the value of one if a facility is to be located at i and it takes the value of zero otherwise. The integer value of p limits the number of located facilities.

$$\boldsymbol{Y} = \left\{ \mathbf{y} \in \{0,1\}^m, \sum_{i=1}^{m} y_i = p \right\} \tag{1}$$

It must be realized that the set of all feasible solutions of real-sized instances is extremely large and it cannot be processed completely solution by solution. That is the reason, why the common exact methods usually fail. Another weakness of the most of exact methods consists in their unpredictable memory or time demands. Therefore, several heuristic approaches have been recently developed to obtain a good solution in a short period.

The uniformly deployed set of solutions denoted by *UDS* can be defined as such subset of $\boldsymbol{Y}$ that meets certain requirements to be applicable as an initial population for different heuristic and metaheuristic solving approaches (Janáček and Kvet, 2019, Kvet and Janáček, 2019, Janáček and Kvet, 2024).

A need for the initial population of p-location issue solutions to be maximally diversified has emerged in relation to the development of evolutionary metaheuristics. Moreover, the cardinality of the population had to be greater than a certain value. Since the set $\boldsymbol{Y}$ of all possible solutions is made up of zero-one vectors, so-called Hamming distance, which is defined by (2), may be used to calculate the difference between two vectors, $\mathbf{y}$ and $\mathbf{x}$.

$$H(\mathbf{x},\mathbf{y}) = \sum_{i=1}^{m} |x_i - y_i| \tag{2}$$

Except the request for sufficient cardinality of *UDS* and its maximal diversity, another requirement may occur. In this study, we focus on such *UDS* creation, in which good solutions are contained. Since we assume that the *UDS* will be used for a concrete service system design problem with a specific objective function, the elements of *UDS* may be evaluated also by the associated quality criterion. Let us define it in a mathematical way. The objective function that is being given can have several forms and its definition is dependent on the mutual time-distances between the locations of users and the facilities. The objective function in this contribution will be proportional to the system's average response time to each user's request. It is believed that p facilities deployed in the set of m potential center locations service n users. The users' locations set is denoted by J. With frequency b_j, each user j boosts their demand at random. Owing to chance, the closest facility can already be in use by a prior demand; in this case, the current users' needs are met by the closest facility that is available. The likelihood that the *k*-th nearest facility is the closest one that is available is given by the assumption that the *r* nearest facilities participate in servicing a user with probabilities q_k for k=1, …, *r*. The time interval t_{ij} represents the distance in time between a potential facility location *i* and a user's location *j*. Let's say the operation $min_k\{A\}$ returns the *k*-th smallest element from the finite set *A* of reals to finish the objective function definitions. Then, (3) defines the objective function.

$$f(\boldsymbol{y})=\sum_{j\in J}b_j\sum_{k=1}^{r}q_k\,\min\nolimits_k\left\{t_{ij}:i\in\{1,\dots,m\},\,y_i=1\right\} \tag{3}$$

The process of *UDS* construction and improvement will be analyzed in the following section.

3 HEURISTIC APPROACH TO UDS IMPROVING

The problem of *UDS* improving can be simply formulated in the following way. A starting set *E* of *noE* *p*-tuples of different integers is given. The integers are chosen from the set of integers starting from 1 and ending at *m* so that each pair of *p*-tuples from *E* contains at most *c* common integers. Each *p*-tuple ***e*** has its objective min-sum function value *f*(***e***) and evaluation of $E=\{\boldsymbol{e}^1, \dots, \boldsymbol{e}^{noE}\}$ is given by (4).

$$F(E)=\sum_{i=1}^{noE}f\left(\boldsymbol{e}^i\right) \tag{4}$$

The objective is to find such a set $\underline{E}=\{\underline{\boldsymbol{e}}^1, \dots, \underline{\boldsymbol{e}}^{noE}\}$ of cardinality *noE* that its evaluation $F(\underline{E})$ is minimal, and the condition of maximal number *c* of common integers is kept.

As even simplified problem of exact minimizing *f* on the set of all *p*-tuples is hard combinatorial problem, a heuristic approach seems to be a rational approach, which can yield considerably improved set $\underline{E}$ in acceptable time.

To solve the above formulated problem, we suggest a heuristic, which proceeds according to sequential scheme starting with an initial set *E* of *p*-tuples. The scheme proceeds maximally in *MStep* steps. The sequential scheme is described by the following steps.

0. Set Step at 0.
1. Find the worst evaluated *p* –tuple $\boldsymbol{e}^w$ of the current *E* and exclude $\boldsymbol{e}^w$ from *E*. Set $f^w=f(\boldsymbol{e}^w)$.
2. Perform the neighborhood search starting with $\boldsymbol{e}^w$ subject to constraints that only such neighbors are taken into account, which have at most *c* common integers with *noE*-1 *p*-tuples od the reduced set *E*. Denote the resulting *p*-tuple by $\boldsymbol{e}^b$. Set $f^b=f(\boldsymbol{e}^b)$ and *Step* =*Step*+1.
3. If *Step* = *MStep* or $f^b=f^w$ then terminate, otherwise insert $\boldsymbol{e}^b$ into *E* and go to 1.

The neighborhood search inspects the neighborhood of a current *p*-tuple $\boldsymbol{e}^c$, which consists of all *p*-tuples differing from $\boldsymbol{e}^c$ in exactly one integer and, furthermore, which has at most *c* most *c* common integers with each *p*-tuple of the reduced *E*.

If the *p*-tuples in the reduced *E* are represented by *m*-dimensional zero-one vectors $\boldsymbol{e}^i=[e^i_1, \dots, e^i_m]$ and ***y*** is a list of *p* integers corresponding to the tested *p*-tuple neighboring the current $\boldsymbol{e}^c$, the following constraints (5) must be satisfied.

$$\sum_{k=1}^{p} e^{i}_{y(k)} \le c \quad for\ i = 1,\ldots,noE-1 \tag{5}$$

To recognize that the system (5) of constraints is not satisfied, it is enough to stop the process of verification at the first constraint, which is not fulfilled. In addition, verification of an individual inequality can be made much smarter, if it is considered that the tested p-tuple $\boldsymbol{y}$ differs from the current p-tuple $\boldsymbol{y}^c$ in one integer only. Let assume that $y(k) = y^c(k)$ for $k = 1, \ldots, p$, $k \neq t$ and $y(t) \neq y^c(t)$. See the expression (6).

$$\sum_{\substack{k=1\\k\neq t'}}^{p} e^{i}_{y(k)} = \sum_{\substack{k=1\\k\neq t'}}^{p} e^{i}_{y^c(k)} \le c - e^{i}_{y^c(t)} \tag{6}$$

It follows that (7) holds.

$$\sum_{k=1}^{p} e^{i}_{y(k)} \le c - e^{i}_{y^c(t)} + e^{i}_{y(t)} \tag{7}$$

It means that only such inequality i need not be satisfied, where $e^{i}_{y^c(t)} = 1$ and $e^{i}_{y(t)} = 0$. This way the process of verification of swap operation feasibility can be considerably reduced.

4 NUMERICAL EXPERIMENTS

The purpose of the numerical experiments was to determine the properties and effectiveness of the proposed improvement procedure. The corresponding criteria were developed using the Žilina, Slovakia, self-governing region's current medical emergency system. The area is made up of 315 residential areas that are simultaneously considered as potential service center locations and user locations as well. The road network that connects these residences provided the shortest distances between the destinations. The number of 29 service centers were taken into account in the system. An "ad hoc" uniformly deployed set SA0 was utilized as the foundation for uniformly deployed sets submitted to the improvement process (Janáček and Kvet, 2019, Kvet and Janáček, 2019). There are 112 solutions in the set, and their minimal Hamming distance is 52. The collection was progressively sorted by objective function values to create individual cases for the numerical experiments, and the *noS* best answers were chosen as the starting *UDS* for the process of improvement. The values of 60, 40, and 20 were assigned to the cardinality *noS*. The smallest Hamming distances (*h*) for each case were examined; the values of *h* were 52, 46, and 40. The following value of constant *c* corresponds to these minimal examples. These were 3, 6, and 9 for *c*, in that order. The trials were conducted on a PC with a 16 GB RAM and an Intel® CoreTM i7 5500U 2.4 GHz processor. The optimization program FICO Xpress 7.3 was utilized to solve the problems that were covered in the preceding sections using the branch-and-bound approach built into the IP solver. Due to extremely long time necessary to complete processing of the searching tree, the permitted computational time was limited to 1000 seconds. The objective function values were computed according to the expression (3) for r =3 probability values q_1 = 0.77063, q_2 = 0.16476 and q_3 = 0.06461. The stopping rule of the process consists of two conditions, the first of them finishes the process if no improvement in the previous step has been achieved and the second one limits the number of performed steps by the threshold *MStep*. During the individual runs the following parameters were observed:

- CT – computational time of the whole process given in seconds,
- Steps – the number of steps that were taken until the termination rule stopped the process,
- Sum – the sum of objective function values of all solutions included into *UDS*,
- Min – the minimal objective function value of the solutions,
- Max – the maximal objective function value of the solutions.

Each of the following tables includes column denoted by Staring V., where the values Sum, Min and Max of the starting *UDS* are reported. The results of the suggested heuristic are reported in the rows denoted by "Heuristic approach". To enable comparison to the exact method (Janáček and Kvet, 2024), the rows denoted by "Exact approach" are given.

Table 1 The results of experiments with cases of *noE* = 60, *MStep*=70

		Starting V.	*c* =3	*c* =6	*c* =9
Exact approach	CT[s]	0	66037	55236	48799
	Sum	4297173	3539334	2977842	2772502
Heuristic approach	CT[s]	0	1	23	31
	Steps	0	49	61	61
	Sum	4297173	3772590	3021244	2811557
	Min	62111	50386	42129	42117
	Max	78975	67378	52277	48659

Table 2 The results of experiments with cases of *noE*= 40, *MStep*=50

		Starting V.	*c* = 3	*c* = 6	*c* = 9
Exact approach	CT[s]	0	42316	32289	29708
	Sum	2764891	2250372	1956453	1834531
Heuristic approach	CT[s]	0	2	16	19
	Steps	0	39	41	41
	Sum	2764891	2363854	1991042	1857798
	Min	62111	48543	42143	42132
	Max	73534	63978	52181	47796

Table 3 The results of experiments with cases of *noE* = 20, *MStep*=30

		Starting V.	*c* = 3	*c* = 6	*c* = 9
Exact approach	CT[s]	0	11337	10725	9014
	Sum	1329234	1064392	958839	906421
Heuristic approach	CT[s]	0	3	9	10
	Steps	0	21	21	21
	Sum	1329234	1084892	971641	918870
	Min	62111	43438	42117	42117
	Max	69879	57520	50210	47440

5 CONCLUSIONS

The *p*-location problem features have been the subject of prior study, which has given the family of related utilities useful techniques for developing a collection of uniformly deployed problem solutions. Unless any challenge objective is taken into consideration, the minimal quarantined

Hamming distance among all included solutions was used to assess the quality of the created location deployment. In this work, we proposed and examined a heuristic method that can enhance the objective function values of the individual solutions, provided that the least Hamming distance between the enhanced solutions stays over a predetermined threshold. Based on performed experiments we can conclude that we have constructed a very effective approach providing excellent results and performing very fast. The main scientific goal was fulfilled.

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References

[1] Avella, P., Sassano, A. and Vasil'ev, I. (2007). Computational study of large scale p-median problems. *Mathematical Programming* 109, pp. 89-114.

[2] Doerner, K. F., Gutjahr, W. J., Hartl, R. F., Karall, M. and Reimann, M. (2005). Heuristic Solution of an Extended Double-Coverage Ambulance Location Problem for Austria. *Central European Journal of Operations Research*, 13(4), pp. 325-340.

[3] García, S., Labbé, M. and Marín, A. (2011). Solving large p-median problems with a radius formulation. *INFORMS Journal on Computing*, 23(4), pp. 546-556.

[4] Gendreau, M. and Potvin, J. (2010). *Handbook of Metaheuristics*, Montreal: Springer Science & Business Media, 648 p.

[5] Janáček, J. and Kvet, M. (2024). Maximally diversed improved population for evolutionary metaheuristic. *Quantitative Methods in Economics 2024*: proceedings, in print

[6] Janáček, J. and Kvet, M. (2019). Uniform Deployment of the *p*-Location Problem Solutions. *Operations Research Proceedings 2019:* Selected Papers of the Annual International Conference of the German Operations Research Society (GOR), Dresden, Germany, September 4-6, 2019, pp. 315-321.

[7] Janáček, J. and Kvet, M. (2016). Min-max Optimization and the Radial Approach to the Public Service System Design with Generalized Utility. *Croatian Operational Research Review*, 7(1), pp. 49-61.

[8] Jánošíková, Ľ. and Žarnay, M. (2014). Location of emergency stations as the capacitated p-median problem. In: Quantitative Methods in Economics (Multiple Criteria Decision Making XVII). [online] Bratislava: Ekonóm, pp. 117-123. Available at: http://www.fhi.sk/files/katedry/kove/ssov/proceedings/Zbornik2014.pdf [Accessed 10th March, 2024].

[9] Kvet, M. (2014). Computational Study of Radial Approach to Public Service System Design with Generalized Utility. In: Proceedings of International Conference DT 2014, Žilina, Slovakia, pp. 198-208.

[10] Kvet, M. and Janáček, J. (2019). Usage of Uniform Deployment for Heuristic Design of Emergency System. *Operations Research Proceedings 2019*: Selected Papers of the Annual International Conference of the German Operations Research Society (GOR), Dresden, Germany, September 4-6, 2019, pp. 309-314.

[11] Marianov, V. and Serra, D. (2002). Location problems in the public sector, *Facility location - Applications and theory* (Z. Drezner ed.), Berlin, Springer, pp 119-150.

[12] Rybičková, A., Burketová, A. and Mocková, D. (2016). Solution to the locating – routing problem using a genetic algorithm. In: *SmaRTT Cities Symposiuum Prague (SCSP)*, pp. 1-6.

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SPATIAL COMPETITION: A SCENARIO OF LANZAROTE ISLAND WITH OR WITHOUT REGULATORY INTERVENTION

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Abstract

Game theory is a discipline of applied mathematics focused on decision making in situations involving more than one subject. Game theory encompasses scenarios involving conflict among various entities, such as individuals, companies, states, and political parties, each operating under specific assumptions and rules with clearly specified assumptions and rules. The objective of the article is to present a spatial mathematical model of duopolistic type applied to the regions of the island of Lanzarote, a part of the Canary Islands archipelago. Within this model, a regulatory entity intervenes, aiming to support specific regions through the implementation of targeted regional development policies. The study delves into various scenarios, each shedding light on nuanced dynamics arising from the interplay of economic agents, market demand, and governmental interventions. Scenario analysis serves as a vital tool in deciphering optimal strategies for business localization, considering factors such as consumer demand, and regulatory influence.

Keywords: *Game theory, geographical space, spatial modeling, regulator, duopoly.*

JEL Classification: D190, D900, L220
AMS Classification: 91A10, 91A05, 91A70

Introduction

The object of study of game theory is to understand, explain and predict the results of a possible interaction of an individual with his environment, if we assume that the intention of the actions of an individual can be modified by the rational behavior of another individual or environment, whose manifestations are also rational and calculable by models.

One of the applications of game theory deals with competition for space where each company is inclined to locate its facilities in a place that maximizes its profits and minimizes the cost (Sequeira Lopez & Čičková, 2019). In such a way it can lead to an outcome with not so attractive regions having a low fall, either in employment or in the income of the population of these regions. An intervention of a regulatory body that intends in some way to encourage the development of these regions is in this case necessary (Sequeira Lopez a Čičková, 2019).

In this article we present a game theory model that includes the intervention of a regulatory body applied to the island of Lanzarote.

1 LANZAROTE – GENERAL INFORMATION, PREFERRED REGIONS

The island of Lanzarote, located in the easternmost of the Canary Islands, has an approximate population of one hundred and fifty-six thousand inhabitants. The island is divided into seven municipalities, with the most populated one of Arrecife which is also the capital.

The level of employment in each region is shown in the graph below.

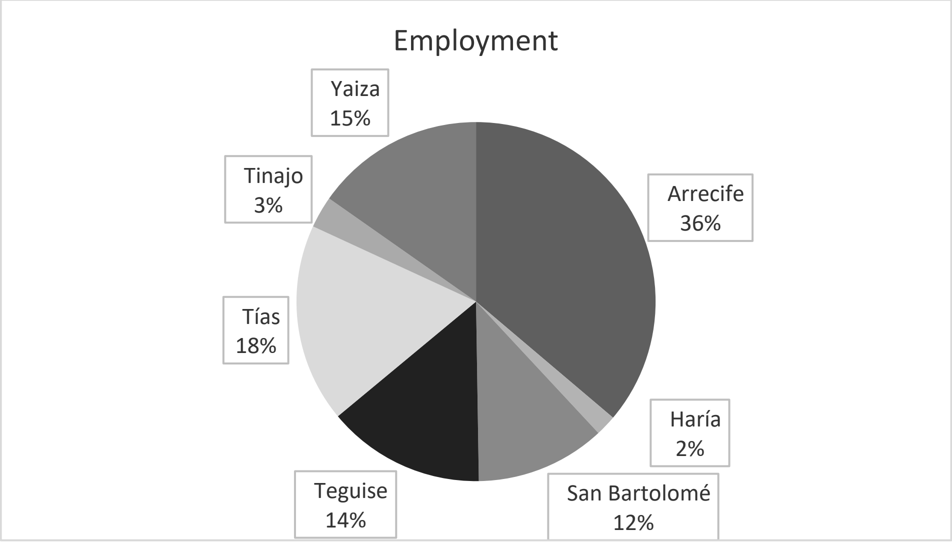

Figure 1 Employment of the island in 2022[1]

To provide a more detailed view of the distribution of economic activity by municipalities of the island, a summary of key indicators throughout the island is included in the following table.

	Population (g)		**Employment (2022)**		**Income per capita (€) 2022**
Municipality	**Nº**	**%**	**Nº**	**%**	
Arrecife	64,497	41.29	22,307	36.19	10,145
Haría	5,365	3.43	1,159	1.88	12,244
San Bartolomé	19,058	12.20	7,204	11.69	11,940
Teguise	23,044	14.75	8,767	14.22	12,509
Tías	20,801	13.32	11,038	17.91	10,793
Tinajo	6,447	4.13	1,763	2.86	11,189
Yaiza	16,977	10.87	9,406	15.26	10,468
LANZAROTE	**156,189**	**100**	**61,644**	**100**	**10,948**

Table 1 Key economic indicators of Lanzarote in 2022[2]

The table and graphs highlight the situation of Arrecife, pointing out its high percentage of population considering that it is the capital of the island and the seat of government, but its employment rate and per capita income are relatively lower compared to other municipalities on the island. We can correctly identify the need for Arrecife to focus on boosting its development and attracting companies to improve its employment rate and increase its average per capita income.

2 CASE STUDY

Let´s assume two companies entering the market. The potential locations are the following seven towns in various regions of Lanzarote: 1. Arrecife, 2. Teguise, 3. San Bartolome, 4. Tías, 5. Tinajo, 6. Haría, 7. Yaiza.

These towns are considered nods where the company can be located. These towns then represent nodes of the graph in set $V = \{1,2, \dots ,7\}$.

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[1] https://www.gobiernodecanarias.org/istac/

[2] https://www.gobiernodecanarias.org/istac/

These nodes 1 – Arrecife and 7 – Yaiza are considered preferred nodes of the regulator because they have a lower income per capita. The V set is also a closed set of customers. It can be assumed that the demand of each of the considered cities reflects the population. Therefore, demand is represented by a vector: $\mathbf{g} = (64; 23; 19; 20; 6; 5; 17)^T$ where g_i , $i \in V$ represents the population of each city in Table 1 rounded down to thousands of people.

Assume there are two companies (players) $P = \{1,2\}$. These companies have the flexibility to position their branches in any node within the set $V = \{1, 2, \dots, 7\}$, which also serves as the location of consumers. The goal of both players is to maximize the number of served nodes (Sequeira Lopez, 2019). Consider that both players offer a homogeneous product at the same price. We also assume that the consumer always makes a purchase, that is we do not consider lost demand.

In addition to the price of the product that customers pay when buying, they also bear the transport costs of the journey they make, if they decide to buy from the company (if they are in another district). We will not consider the cost of transport within one area. The unit cost is represented by parameter $p_1 = p_2 = 15$. Then t is a value based on average costs of transport including fuel and amortization. The matrix of shortest distances representing the shortest distances between cities is then as follows: $\boldsymbol{D} = d(i,j), i \in V,$ where $t = 0.5$ denotes the transportation cost factor.

$$\mathbf{D} = \begin{bmatrix} 0 & 8 & 8 & 15 & 19 & 25 & 28 \\ 8 & 0 & 10 & 17 & 7 & 17 & 30 \\ 8 & 10 & 0 & 7 & 11 & 27 & 20 \\ 15 & 17 & 7 & 0 & 18 & 34 & 13 \\ 19 & 7 & 11 & 18 & 0 & 34 & 31 \\ 25 & 17 & 27 & 34 & 34 & 0 & 47 \\ 28 & 30 & 20 & 13 & 31 & 47 & 0 \end{bmatrix}$$

Then cost matrix for Player 1 can be represented by the cost of transportation from one node to the other plus the price of the product denoted as $n_{ij}^{(1)}$. Similarly, for Player 2, we define a cost matrix considering the transportation cost from one node to another, plus the product cost, represented as $n_{ij}^{(2)}$. The cost matrices of the first and second player consider the total shipping costs from the i-th customer to the respective company as well as the purchase price of the product.

Furthermore, we assume that if the Player 1 places his branch in $j-$th node, he acquires a customer from the i-th node only if $n_{ij}^{(1)} < n_{ij}^{(2)}$ for $i, j = 1,2, \dots, n$. In the event that $n_{ij}^{(1)} = n_{ij}^{(2)}$, the players share the demand equally. Otherwise, the customer from the i-th node is served by the other player.

We consider three scenarios:
Scenario number 1: We assume a situation without any regulation, where there is no government involvement or influence. Companies (players) make decisions solely based on minimizing their operational costs. In such a scenario, each company will analyze the costs associated with operating in each town. We assume that companies are likely to concentrate their operations in the most populated area (potentially Arrecife in the case of Lanzarote) because it offers a larger customer base and potentially lower operational costs due to economies of scale. This could lead to uneven development across the island, neglecting less populated areas like Yaiza, which might have a lower income level despite having a significant

employee base. To analyze this scenario, we employ the mathematical model developed by Lopez and Čičková (Sequeira Lopez & Čičková, 2018).

Scenario number 2: Similar to scenario 1, two companies are evaluating the possibility of opening branches in different towns. This time, they consider not only the operational costs in each town but also the population size, represented by the potential customer base. Each town is assigned a weight based on its population, defined in Table 1 as (**g**), with higher population areas assigned higher weights. However, there is a fixed demand threshold, beyond which customers are not considered potential customers for a particular company. In this scenario, companies may still opt for the most populous area (Arrecife) due to its high weight and large customer base. However, they may also consider other towns with significant populations that fall within the customer travel threshold. This scenario could result in a more balanced distribution of companies compared to scenario 1, where only costs were considered. To calculate this situation, we can utilize another mathematical model proposed by Lopez and Čičková (Sequeira Lopez & Čičková, 2019).

Scenario number 3: Like the previous scenarios, we consider two companies contemplating opening branches in various towns, considering the preferences of the local regulator. The government designates specific nodes (locations) for development and may offer incentives to encourage companies to locate there, with a preference for nodes 7 – Yaiza, as mentioned earlier.

Consumers are divided into two groups: Myopic Consumers and Socially Aware Consumers.
Myopic Consumers: These consumers only consider the price of the product or service and are not aware of the potential negative consequences (externalities) of buying from companies located outside the regulator's preferred nodes.
Socially Aware Consumers: These consumers consider the regulator's recommendations and are willing to purchase from companies located in the preferred nodes, even if it might cost slightly more due to potential transportation costs. For this scenario we assume that high social awareness can significantly influence the success of the regulator's development goals. More customers buying from companies in the preferred node (Yaiza) could increase demand there, making it a more attractive location for companies overall. This can lead to a more balanced distribution of economic activity compared to scenarios 1 and 2. From the mathematical perspective we can utilize the model proposed by Lopez and Čičková (Sequeira Lopez & Čičková, 2019).

By analyzing the interaction between economic dynamics, consumer demand, and governmental regulations, these scenarios offer insights into strategically positioning businesses for optimal results. The pricing of services/products is standardized at fifteen units, with an additional constant transportation cost of 0.5 units.

Scenario	Player 1 strategy	Player 2 strategy	Play value
Scenario No. 1	Node 1	Node 1	3,5
Scenario No. 2	Node 2 (0,5) Node 3 (0,5)	Node 2 (0,5) Node 3 (0,5)	2
Scenario No. 3	Node 1 (0,052) Node 2 (0,066) Node 3 (0,011) Node 7 (0,871)	Node 1 (0,021) Node 2 (0,016) Node 3 (0,023) Node 6 (0,94)	5,773

Table 2 Results for different scenarios.

The results were calculated using CONOPT solver version 4.33.

In scenario 1 (Cost Minimization) companies prioritize Arrecife, the most populated city, reflecting the current situation. However, Arrecife has a lower income level. With the second scenario (Weighted Nodes) something similar happens; the only difference is that in this case the options are extended to the city of Tinajo which is node 5. Tinajo has an income per capita of 11,189 € surpassing by a little more than a thousand € the city of Arrecife. We must also consider the population of Tinajo which represents only 4 % of the total population of the island. In fact, companies consider both cost and demand, leading to a potential inclusion of Tinajo (higher income per capita but lower population).

And finally in scenario 3 (Social Awareness) high social awareness (87 %) leads to a more balanced distribution, with Yaiza (the central node with lower income) receiving some support. In this context, decision is divided among four cities: Arrecife, Teguise, San Bartolomé and Yaiza. The purpose of supporting the central node, which is the city of Yaiza, with an acceptance level of 87 %, would be partially achieved. Fifteen percent of the island's employees work in this city, which like Arrecife has one of the lowest income per capita.

A greater willingness of the population to use services and products that support sustainable development and community well-being is implied by a high level of social awareness. Therefore, implementing policies and actions focused on improving the economic and social conditions of cities such as Arrecife and Yaiza can considerably help the equitable and sustainable development of the island.

In short, when society is more aware, people are willing to use services and products that foster sustainable development and community well-being. Consequently, the equitable and sustainable development of the island can be significantly contributed to through the implementation of policies and actions aimed at improving the economic and social conditions of cities such as Yaiza.

3 CONCLUSION

The presented scenarios illustrate how the presence of a regulatory body could impact the situation, but also emphasize the limitations of relying solely on regulations. However, a collaborative approach combining strong regulation with citizen accountability offers a more comprehensive solution. By fostering a sense of shared responsibility, citizens can actively contribute to positive outcomes.

In summary, the research conducted here brings to light the complexities of spatial effects permeating in decision-making processes of economic activity distribution between regions. As the key regulation tools which can help to steer companies in the required direction, their isolated impact is, however, proven to be limited. The prerequisite of the overall governance strategy that strives for comprehensive regulation by active citizen engagement is the basis of sustainable and fair development.

The research requires further investigation to analyze the multifactor that impairs regional development and offers appropriate sustainable solutions for those in emerging regions. This implies the research of the performances and feasibilities thereof of green incentive policies that shall attract investment for economic development as well as a strong community identity.

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Sources

[1] Gobierno de canarias, (2023) Instituto Canario de Estadística, https://www.gobiernodecanarias.org/istac/. Available at: https://www.gobiernodecanarias.org/istac [Accessed: 25 March 2024].

[2] Myerson, R. B. 1991. Game Theory: *Analysis of Conflict*. Harvard University Press, 1991. ISBN-13: 978-0674341166. DOI: https://doi.org/10.2307/j.ctvjsf522.

[3] Sequeira Lopez, A. J. - Čičková, Z. 2018. Price Policy in Games in Spatial Competition. Quantitative Methods in Economics: Multiple Criteria Decision Making XIX: Proceedings of the International Scientific Conference: 23rd May - 25th May 2018, Trenčianske Teplice, Slovakia. Trenčianske Teplice, Slovakia: Letra Edu, 2018. ISBN 978-80-89962-07-5.

[4] Sequeira Lopez, A. J. 2019. Spatial Competition with Considering the Weight of the Nodes: Case Study. Ekonomické rozhľady: vedecký časopis Ekonomickej univerzity v Bratislave. Bratislava: Ekonomická univerzita v Bratislave, 2019. Zv. 48, 4, s. 463-476. ISSN 0323-262X.

[5] Sequeira Lopez, A. J. - Čičková, Z. 2019. Spatial Competition with Regulatory Intervention. LOGIC 2019. Logistics International Conference. LOGIC 2019: Proceedings of the 4th Logistics International Conference, 23-25 May 2019, (Belgrade, Serbia). Belgrade, Serbia: University of Belgrade, Faculty of Transport and Traffic Engineering, 2019. s. 92-99. ISBN 978–86–7395–402–8.

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COMPARISON OF DJIA STOCKS BASED ON FINANCIAL INDICATORS AND ESG VALUES

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Abstract

Currently, when investing, the emphasis is increasing not only on financial indicators of stocks but also on ESG (environmental, social and corporate governance) indicators. In the paper, the authors compared stocks based on financial indicators of return and risk, as well as ESG indicators. The analysis was carried out using the PROMETHEE II method, based on which an arrangement of alternatives was obtained. First, the stocks were examined using financial indicators, resulting in an alternative arrangement. Subsequently, stocks were examined and arranged based on ESG information. In conclusion, the authors compared the commonality of the arrangement using these two sets of criteria, i.e. whether the current trend towards ESG-based funds impacts financial indicators.

Keywords: *Stocks, Financial Indicators, ESG*

JEL Classification: *G11, C60*

AMS Classification: *91B30, 90C90*

1 INTRODUCTION

Environmental investment is oriented towards those investments where most or all revenues and profits come from green business activities. The field of modern environmental investment can also include the use of ESG indicators, which reflect the behaviour of individual companies in terms of socially responsible investment (SRI - *socially responsible investment*)[1] or investing taking into account *environmental, social and governance criteria (ESG* - environmental, social and corporate governance[2]). This corresponds to the European Commission's Action Plan on Financing Sustainable Growth, which aims to redirect finance flows towards sustainable investments, integrate sustainability as a normal part of risk management, increase transparency and emphasise a focus on the long term.

ESG is determined by three main factors that influence a company's sustainability:

- Environmental factor ("E"), which focuses on the environmental impacts of resource consumption, such as carbon footprint, wastewater discharges and other activities affecting the environment.
- The Social Factor ("S") evaluates collaboration with the communities in which the organization operates, its internal policies regarding inclusion, the company's relations with employees, customers and other stakeholders, health and safety in the workplace, product safety and quality, consumer protection, and other social factors.
- The risk management factor ("G") is determined by internal procedures that lead to effective decision-making and regulatory compliance.

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[1] Socially responsible investing (SRI) is an investment that is considered socially responsible due to the nature of the business that the company conducts.

[2] Environmental criteria take into account how a company protects the environment, including corporate policies. Social criteria examine how a company manages relationships with employees, suppliers, customers and the communities in which it operates. Management deals with corporate management, executive remuneration, audits, internal controls and shareholder rights.

ESG can thus be seen as an alternative approach to risk management. As demand for environmental and social change grows over the next few decades, investors are increasingly putting their capital into companies that they think will make these changes. ESG is one of the important factors in the decision-making of investors and banks.

Based on ESG, we can observe a trend in investing in which investors look not only at financial indicators but also at the sustainability of investments for the future. In the paper, the authors analyze whether the impact of sustainability indicators affects the profitability and riskiness of stocks. The analysis used financial data of stocks and the values of their ESG indicators, using the PROMETHEE II method, to organize profitability based on three financial indicators: return, CVaR and CDaR risk ratio. In the next part, they used the above method using four criteria ESG, E, S, G. In conclusion, they compared the consistency of the arrangement in these cases.

In the first part, the authors define the financial indicators used in the analysis. They then define ESG indicators. The PROMETHEE II method was used for the analysis, which is described in the next section. Finally, they present the ranking of stocks from the DJIA index, based on the financial indicators used over the past 3 years (2021.03.01 – 2024.03.01) and current values of ESG indicators, analyzing the consistency of the arrangement, i.e. how ESG indicators affect financial indicators.

The PrometheeTools R library was used to analyze the formulated problem.

2 FINANCIAL INDICATORS

2.1 Historical yield simulation

The calculation of the return on assets is generally based on the construction of a discrete random variable X (represented by the vector **r**) using price changes. If the price change is defined as relative to some initial price, then such a change is called yield. Let P_t be the price at time $t = 1, 2, \ldots T$, where T is the length of the monitored period. The rate of return between periods t and $t-1$ can be expressed as the relative rate of return r_t for the same time period (Pekár et al., 2022):

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{1}$$

When formulating certain models, the measure of the cumulative yield of the asset is also used, at time t it can be expressed as

$$y_t = \frac{P_t - P_1}{P_1} \tag{2}$$

This method's disadvantage is that the investigated period can be characterized by atypicality, which can lead to non-representative data. Therefore, selecting historical data according to time relevance, current correlation, and volatility is advisable. Some critics consider this method not very reliable in terms of revenue distribution and logical inconsistency.

2.2 Expected asset return

Assuming the representation of the random variable X by discrete values (vector **r**), the expected rate of return can be calculated as a geometric mean. For the expected return $EG(\boldsymbol{r})$

($\mathbf{r} = r_1, r_2, \ldots r_t$) of an individual asset owned for the T period, with the exact significance of individual observations, the following generally applies (Pekár et al., 2022):

$$EG(\boldsymbol{r}) = \left(\prod_{t=1}^{T}(1 + r_t)\right)^{\frac{1}{T}} - 1 \tag{3}$$

where r_t is the return of the discrete random variable in the t-th state, $t = 1, 2, \ldots T$. T is the total number of random states. The expected return of an asset as a geometric mean, based on the value of the cumulative return, can be expressed as

$$E = (1 + y_T)^{\frac{1}{T}} - 1 \tag{4}$$

2.3 Risk measure: Conditional Value at Risk

Conditional Value at Risk (CVaR), a very often used risk measure for financial analyses, can be defined as an expected loss exceeding the Value at Risk (VaR), so CVaR only considers higher loss values than the VaR value. The CVaR value is defined for a given confidence level α (Pekár et al., 2022):

$$CVaR_\alpha(X) = E(L(X)|L(X) \geq VaR_\alpha), \tag{5}$$

where X denotes the random variable representing the return, $L(X)$ denotes the loss function of the random variable X and is the value at risk. Assuming the existence of a discrete random variable X, represented by the vector $r = (r_1, r_2, \ldots r_T)$, where T is the number of components, the CVaR risk measure can be defined as

$$\begin{aligned} CVaR_\alpha(X) &= VaR_\alpha - \frac{1}{\alpha}E[|\boldsymbol{r} + VaR_\alpha|_-] \\ CVaR_\alpha(X) &= VaR_\alpha + \frac{1}{\alpha}\sum_{t=1}^{T} max(-(r_t + VaR_\alpha), 0) \end{aligned} \tag{6}$$

2.4 Risk measure based on Drawdown

Drawdown is defined as the biggest decline in asset return. In this section, we will describe the Drawdown and Conditional Drawdown at Risk.

The risk rate of Drawdown (*DD)* at the considered time is defined as the difference between the maximum value of the asset (portfolio) in previous periods and its present value at the time t, which takes the values from the analyzed reference period T. Cumulative yields of the j-th asset, $j = 1, 2, \ldots n$, in each period $t = 1, 2, \ldots T$ are denoted as y_{jt}. The value $DDj(t)$ of j-th asset (portfolio), $j = 1, 2, \ldots n$, in time t, $t = 1, 2, \ldots T$ can be calculated as the difference between the maximum value in the v-*th* period ($v = 1, 2, \ldots t$) and the value in time t. According to (Geboers et al., 2023) it is possible to calculate as:

$$DD_j(t) = \max_{v=1,2,\ldots t} y_{jv} - y_{jt} \tag{7}$$

For the data in the observed time interval $t = 1, 2, \ldots T$ according to (Geboers et al., 2023) the Conditional Drawdown (CDaR) can be defined for j-th asset, $j = 1, 2, \ldots n$, as:

$$CDaR_j(T)_\alpha = \min_{DaR}\left\{DaR + \frac{1}{(1-\alpha)T}\sum_{t=1}^{T}\left[DD_j(t) - DaR\right]^+\right\} \tag{8}$$

Where DaR is the Drawdown (DD) threshold of the portfolio, this value can only be exceeded by observations. Notation + in formula (6) means, only the positive values are taken into account$(1-\alpha)T$.

3 ESG INDICATORS

Bloomberg provides a variety of proprietary scores that investors can use to assess company or government disclosure and performance on a wide range of ESG and thematic issues. Bloomberg's ESG (Pyles, 2020) and thematic scores can be integrated into company research and portfolio construction. Bloomberg ESG scores measure a company's management of financially material ESG issues. Financial materiality is defined as the issues that can have a negative or positive impact on a company's financial performance, such as revenue streams, operating costs, cost of capital, asset value and liabilities.

Access unparalleled breadth and depth of coverage across 15,000 companies in over 100 countries, including as-reported data, derived ratios and sector-and country-specific fields going back to 2006.

Financial materiality is defined as the issues that can have a negative or positive impact on a company's financial performance, such as revenue streams, operating costs, cost of capital, asset value and liabilities. Bloomberg identifies financial material issues based on proprietary research, which is shared transparently and based on an assessment of the probability, magnitude and timing of the impact. Bloomberg scores measure best-in-class performance within peer groups. The scores consider the disclosure of quantitative data as a dimension of performance. Each indicator is scored using a quantitative methodology, considering normalization, polarity and the type of a field. Scores range between 0-10, with higher scores indicating a better management of material issues.

The scores are based on publicly available, company-disclosed data, and do not rely on estimates or an analyst's opinion. As a result, the scores can be updated in a timely manner as companies release new data throughout the year. Scores also consider the disclosure of quantitative data as a dimension of performance.

4 PROMETHEE II METHOD

Promethee class methods (Brans and Smet, 2016) were developed in the early 1980s of the 20th century, and since then, various modifications have been implemented. These methods are used in decision-making and to establish decision-making scenarios in areas such as business, government institutions, transportation, health, and education.

Instead of pointing to the "right" decision, the Promethea method helps decision-makers find the alternative that best suits their goals. The basic elements of the Promethee method were first introduced by Brans in 1982 (Brans and Smet, 2016).

In the paper, we focus on the use of the PROMETHEE II method, resulting in a complete sequence of net *flow induced* alternatives to the set *Y*:

$$\begin{cases} y' \, P^{II} \, y'', & ak \quad \Phi(y') > \Phi(y'') \\ y' \, I^{II} \, y'', & ak \quad \Phi(y') = \Phi(y'') \end{cases} \tag{9}$$

Thus, the result is an arrangement of alternatives based on net flow values, with two types of relation, and that we are indifferent in comparing two alternatives in the case of an identical net flow value Φ, or we prefer an alternative with a higher net flow value Φ to an alternative with a lower net flow value.

5 DJIA STOCK INDEX ANALYSIS

In our analysis, we selected the following stocks included in the Dow Jones Industrial Average (DJIA) index, namely 3M (MMM), Amazon.com, Inc (AMZN), American Express Company (AXP), Amgen Inc (AMGN), Apple Inc. (AAPL), Boeing (BA), Caterpillar Inc. (CAT), Chevron Corporation (CVX), Cisco Systems (CSCO), The Coca-Cola Company (KO), The Walt Disney Company (DIS), Dow Inc. (DOW), Goldman Sachs (GS), The Home Depot (HD), Honeywell International Inc. (HON), IBM (IBM), Intel (INTC), Johnson & Johnson (JNJ), JPMorgan Chase (JPM), McDonald's (MCD), Merck & Co. (MRK), Microsoft (MSFT), Nike, Inc. (NKE), Procter & Gamble (PG), Salesforce, Inc. (CRM), The Travelers Companies (TRV), UnitedHealth Group (UNH), Verizon Communications (VZ), Visa Inc. (V), Walmart (WMT).

Table 1: Net flow values and ranking of financial and ESG indicators

Alternative	Φ Fin	Ranking Fin	Φ ESG	ESG ranking
AAPL	0.00318	10	0.202266	6
AMGN	0.004589	7	0.14452	10
AMZN	-0.00565	25	-0.35371	28
AXP	0.000315	19	-0.07176	21
BA	-0.01476	30	-0.247	23
CAT	0.002427	13	-0.2523	24
CRM	-0.00722	26	-0.19191	22
CSCO	0.000737	18	0.162232	9
CVX	-0.00075	22	0.042579	15
DIS	-0.00929	27	-0.29452	26
DOW	-0.0013	23	0.125546	11
GS	0.001372	15	0.002552	19
HD	0.001199	16	0.100843	13
HUNT	0.002598	12	0.029607	16
IBM	0.003333	9	0.191164	7
INTC	-0.00978	29	0.232314	5
JNJ	0.006535	1	0.484503	1
JPM	2.90E-04	20	-0.27198	25
KO	0.003048	11	-0.03258	20
MCD	0.005141	5	-0.38555	29
MMM	-0.00931	28	0.355783	2
MRK	0.005804	2	0.264061	4
MSFT	0.001451	14	0.190368	8
NKE	-0.00372	24	0.017439	18

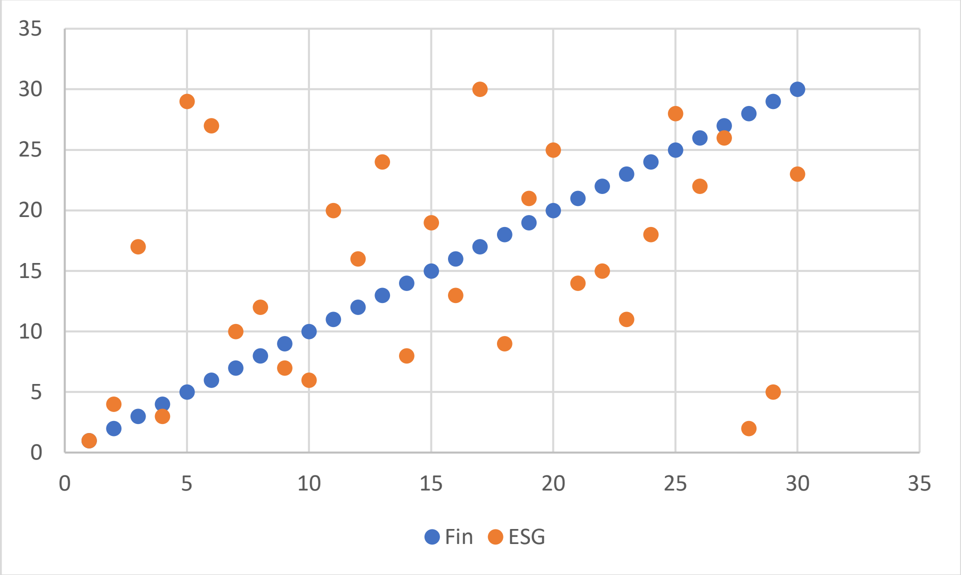
PG	0.005794	3	0.017943	17
TRV	0.000755	17	-0.65746	30
UNH	0.005721	4	0.349713	3
VZ	0.003422	8	0.119746	12
V	-0.00071	21	0.078149	14
WMT	0.004757	6	-0.35256	27

Source: The authors

As already mentioned in the analysis, the PROMETHEE II method was used, performing calculations for financial and ESG indicators. Table 1 shows the net flow values and the order of actions.

Ranking matching analysis can be performed on the basis of correlation analysis. In Figure 1, the authors show the ranking of stocks based on financial and ESG indicators.

Figure 1: Ranking of financial and ESG indicators



Source: The authors

In the correlation analysis, the ranking of individual stocks was compared, with a correlation of 0.2378198. The result is a weak positive correlation, which can be interpreted as meaning that the trend in the development of financial and ESG indicators does not follow the same trend, although the positiveness of the correlation indicates a slight agreement in the tendency of ESG indicators to influence investors. However, this analysis should be carried out on a larger group of assets, as it is currently possible to compare the trend of funds containing sustainable investments considering ESG indicators, which causes a higher demand for assets of this type, which increase their price, i.e. return on investment.

6 CONCLUSION

The increasing popularity of sustainable investments puts pressure on firms to improve indicators that consider environmental, social, and governance criteria. In the paper, the authors examined whether this pressure affects financial indicators. As can be seen from the results, a slight positive trend exists, but it is not so pronounced that investors focus only on assets that take this trend into account to a high degree.

The analysis was conducted on shares of the DJIA stock index, while a larger group of assets will be analyzed in future research. Assets from the same segment will also be analyzed, which should demonstrate whether the introduction of these criteria into company processes affects its financial indicators.

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References

1. Brans, JP. and De Smet, Y. (2016). PROMETHEE Methods. In: Greco, S., Ehrgott, M., Figueira, J. (eds) *Multiple Criteria Decision Analysis. International Series in Operations Research & Management Science*, vol 233. New York, Springer, pp.187-219.
2. Geboers, H., Depaire, B. and Annaert, J., (2023). A review on drawdown risk measures and their implications for risk management. *Journal of Economic Surveys*, *37*(3), pp.865-889.
3. Pekár, J., Brezina, I. and Reiff, M., (2022). Determining the investors' strategy during the COVID-19 crisis based on the S&P 500 stock index. *Strategic Management-International Journal of Strategic Management and Decision Support Systems in Strategic Management*. Online First. Availeble at: https://smjournal.rs/index.php/home/article/view/354
4. Pyles, M.K., (2020). Examining portfolios created by bloomberg ESG scores: Is disclosure an alpha factor? *The Journal of Impact and ESG Investing*, *1*(2), pp.39-52.

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APPLICATION OF THE CAPACITATED LOT SIZING PROBLEM TO THE FAST MOVING CONSUMER GOODS INDUSTRY

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Abstract

In this paper we apply the well-known Capacitated Lot Sizing Model (CLSP) to the real business environment of a cigarette manufacturer dealing with dynamic demand and capacity. The challenge comes from the fact that we face a multi-period, multi-machine (resource), multi-SKU problem and each SKU can be marketed in a standard or promotional variant. They should not meet in the market and the planning department is expected to plan for a proper phase-in and phase-out. The aim is to obtain a production plan that makes the detailed scheduling problem feasible. On the capacity side, we are dealing with a multi-resource problem and the resources are not homogeneous. The planner can choose between a network of independent manufacturing and packaging machines and a set of tightly coupled manufacturing and packaging machines.

Keywords: *CLSP, Multi-period, Multi-machine, Capacity, Dynamic*

JEL Classification: C61
AMS Classification: 90C11

1 INTRODUCTION

Due to space limitations, we will limit ourselves to a basic overview. Dynamic lot sizing has enjoyed popularity since (Wagner and Whitin, 1958) who proposed the solution algorithm. Since the CLSP is known to be NP-hard, many algorithms have been proposed based on Lagrangian relaxation (Chen and Chu, 2003) or Dantzig-Wolfe decomposition (Degraeve and Jans, 2007) have been proposed to speed up the branch-and-bound method for problems of real size. CLSP is often analysed as part of a scheduling problem, as in (Gupta and Magnusson, 2005), where the setup costs are sequence dependent. Since the CLSP is practically motivated by the needs of the industry, (Ramya *et al.*, 2019) provides a variety of models that have been practically applied in different industries. The basic CLSC model implemented in this paper is taken from (Fiala, 2002).

2 BUSINESS PROBLEM

The real world application will now be studied in the Philip Morris CR a.s. environment. Philip Morris is a cigarette manufacturing company that produces different types of cigarette products, including Full Flavour (FF) and Light (LT) versions. These two main categories are available in different lengths such as King Size (KS) or Regular Size (RS) or Hundreds (HU). All the characteristics described so far refer to the cigarette itself. The cigarettes are then bundled and packed in different types of packs. You can get a BOX pack or a SOFT pack containing either 10 or 20 loose cigarettes. The bundle of loose cigarettes, together with the pack, forms a brand. In addition to these standard packs, manufacturers sometimes produce special promotional brands.

2.1 Demand Planing – Standard and Promo Products

These special promotions often have some kind of pack modification, for example, a prize coupon may be inserted into the pack (INSERT) or the foil on the outside of the pack may have some special drawings on it (Printed Foil - OPP). The nature of marketing promotions is such that the special version has a fixed launch date, total quantity and initial volume. Once launched, the standard version of the brand is banned from sale and cannot be sold again until the stock of the promotional version is exhausted. In addition, the stock level of the standard brand must reach approximately 50% of its safety level before the launch of the promotional version. The next diagram shows the evolution of a promotional campaign interacting with its standard counterpart.

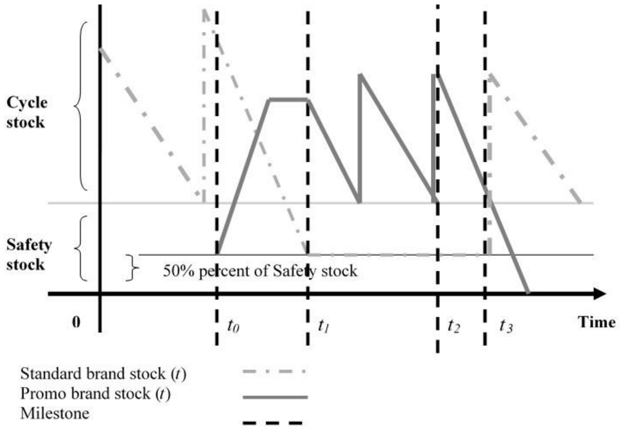

Figure 1 Demand Problem Structure

There are four milestones in the model. At t_0, production of the promo version must start in order to build up the initial stock (initial volume parameter) in relation to the available capacity of the required resource. At t_1, the promo starts to be sold and its stock level must remain above the safety stock level derived from its standard counterpart until t_2 which indicates a time when the total planned quantity is equal to the quantity produced so far. From that moment on, the safety stock policy can be violated and the stock must be completely used up. The moment when the promo stock crosses the safety level t_3 should indicate the moment when the production of the standard can be initialised as well as its sales.

2.2 Capacity planning and its flow-shop nature

The formulation of the capacity problem is as follows. The production of a brand consists of two main operations: Manufacturing (or making) and Packaging (or packing), As mentioned above, by a manufacturing characteristic we mean the length of a filter (i.e. FF, LT) and the format of a cigarette (i.e. KS, RS, HU). In total, we can construct the following strings in the manufacturing part: FF KS, LT KS, FL HU, and FL RS. FL refers to the usage one type of filter, regardless of the FF or LT format of the cigarette. The packaging process is more complicated with regard to special promotional activities. The basic characteristics of the pack are the type of pack (BOX, SOFT) and the number of loose cigarettes contained in the pack (20, 10, etc.). Thus, the basic pack strings could be formulated as follows: BOX OF 20, BOX 10, SOFT 20, SOFT 10. Let us call these CORE strings. The promo action causes an extension of the packaging process, for example by adding some coupons to the pack (INSERT - PIN) or by wrapping a pack in a special printed film (OPP). The PIN and OPP extensions are added to the CORE packing string to form the NON-CORE strings. Hence, the resources shown in Figure 2 could be grouped in the following categories:

1) Linked machine: a machine that treats manufacturing (or making) and packaging as a single process and does not allow them to be separated.

2) Free standing maker: a device that produces loose cigarettes of a specific format and length, which are then fed to the packer in the last stage.
3) Free standing packer: a device which receives loose cigarettes and packs them into a specific type of pack.

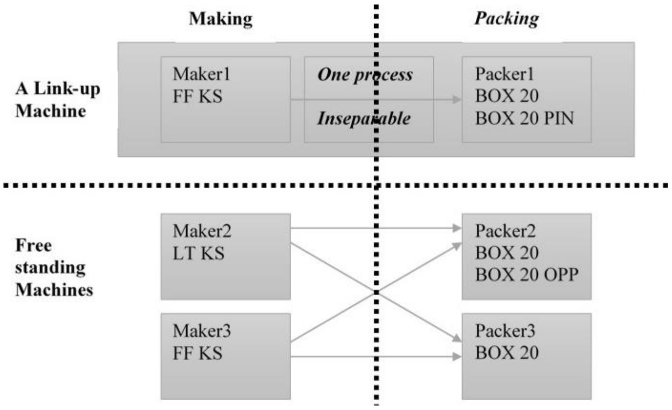

Figure 2 Capacity Planning Problem

In a typical factory, this diversity can be found in the coexistence of linked and stand-alone machines. The requirements of the special promo variants are met by special installation kits on the regular machines. As shown above, the Packer 1 can deliver a special pack with an insert. However, this will also take up the regular BOX20 capacity.

3 CLSP MODEL

The planning horizon spans T planning periods ($t = 1,2,\dots,T$). The product set $N = \{b_i;\ i = 1,2,\dots,N\}$ must be structured further in the following way. Let us now define the three main subsets of N : 1) promo brands $NP \subset N$, $NP = \{b_{ij};\ i = 1,2,\dots,NP, j = 1,2,\dots,NSP\}$, 2) standard brands with promo, $NSP \subset N$, $NSP = \{b_i;\ i = 1,2,\dots,NP, j = 1,2,\dots,NSP\}$, and 3) standard brands without promo $NS \subset N, NS = \{b_i;\ i = 1,2,\dots,NS\}$ such that $NP \cup NSP \cup NS = N$ and $NP \cap NSP \cap NS = \{\}$.
Next, consider a set M of M making characteristics $M = \{m_l; l = 1,2,\dots,M\}$ and a set P of P packing characteristics $P = \{p_j; j = 1,2,\dots,P\}$. Let us further specify a subset of P called $PC \subset P$, $PC = \{p_j; j = 1,2,\dots,PC\}$ containing only the CORE packing strings as stated above. The total count of all characteristics will yield a set S of S strings ($S = M \times P$). Let us now create a matrix $S_{M\times P} = [s_{lj}]$ where s_{lj} represents a combination of *l-th* making string and *j-th* packing string. Now, we can define a subset $SL \subset S$ which contains link-up resource strings. Having created the set of all characteristics we have to link them to the actual brands in the portfolio. This can be done with the following mapping Table 1 where the rows represent the members of the set $S = \{s_k;\ k = 1,2,\dots,S\}$ and the columns represent the product $i, (i = 1,2,\dots,N)$.

String / brand	b_1	b_2	b_3	b_4	b_5	...	b_N
FFKSBOX20	1	1	1				
FFKSBOX20PIN			1				
LTKSBOX20				1	1		
LTKSBOX20PIN							
...							
S^{th} string							

Table 1 Capacity string and Brand mapping

Note that b3 is promo version of b2 and thus is linked to both of the FFKSBOX20 strings. We will now define the two concrete sets of packing strings SP_1, SP_2 as $SP_1 = \{FFKSBOX20, LTKSBOX20\}$, $SP_2 = \{FFKSBOX20PIN, LTKSBOX20PIN\}$ and two sets of strings to control the making process SM_1, SM_2 as $SM_1 = \{FFKSBOX20\}$, $SM_2 = \{LTKSBOX20\}$. The model variables and parameters are summarized in Table 2:

CLSP Model Notation

D_{it}	Demand forecast
SSt_{it}	Safety stock level
c_i	Unit costs of holding a unit of inventory of a product $\forall b_i \in N$
s_i	Setup or/and production costs $\forall b_i \in N$
max_lot_i	Maximum production lot of a product i, $\forall b_i \in N$
min_lot_i	Minimum (technological) production lot of a product i, $\forall b_i \in N$
I_{i0}	, Starting inventory level of a product i, $\forall b_i \in N$ at time 0
$ratio$	Target percentage of the level of the standard brand's safety stock before the launch of a promo variant. / $ratio$ = 0.5
$Launch\_p_i$	Period when the promo starts to sell $\forall b_{ij} \in NP_j, \forall j (j = 1, 2, \dots, NSP)$
$Init\_v_i$	Min. promo stock on hand before launch $\forall b_{ij} \in NP_j, \forall j (j = 1, 2, \dots, NSP)$
$Total\_v_i$	Total ordered quantity for the promo action $\forall b_{ij} \in NP_j, \forall j (j = 1, 2, \dots, NSP)$
x_{it}	Quantity of a product *i* produced at time *t*
z_{it}	Zero-one variable indicating a production of a product *i* at time *t*
I_{it}	Inventory of a product *i* at the end of a period *t*
k_{it}	Binary variable indicating that total production of a promo brand $b_{ij} \in NP_j, \forall j (j = 1, 2, \dots, NSP)$, fulfilled the order at time *t*
y_{it}	Continuous variable indicating the share of sales forecast between a core brand and all of its promo variants $b_{ij} \in NP_j, \forall j (j = 1, 2, \dots, NSP)$, $0 \leq y_{it} \leq 1$.
W_FMFF_{jt}	Free standing making capacity $\forall t, s_j \in SM_1$ $(j = 1, 2, \dots, PC)$
W_FMLT_{jt}	Free standing making capacity $\forall t, s_j \in SM_2$ $(j = 1, 2, \dots, PC)$
$W\_FPBOX20_{jt}$	Free standing packing capacity $\forall t, s_j \in SP_1$ $(j = 1, 2, \dots, M)$
$W\_FPBOX20PIN_{jt}$	Free standing packing capacity $\forall t, s_j \in SP_2$ $(j = 1, 2, \dots, M)$
LU_{jt}	Link-up resource capacity $\forall s_j \in SL$
FM_{jt}	Free standing making capacity $\forall m_j \in M$
FP_{jt}	Free standing packing capacity for all strings $\forall p_j \in P$
FMP_{jt}	Free-standing making & packing cap. $\forall s_j \in SF \subset S$, $FMP_{jt} = \min_{i \in M, j \in P} [m_{it}, p_{jt}]$

Table 2 CLSP Variables and Parameters

Last mapping we must provide contains combinations of strings and brands $\forall b_i \in N\ (i = 1,2,\dots,N)$ for the respective subsets of strings as follows:
$\boldsymbol{A}^{(1)} = \left[a_{ji}^{(1)}\right] \forall s_j \in S$, $\boldsymbol{A}^{(2)} = \left[a_{ji}^{(2)}\right] \forall s_j \in SM_1$, $\boldsymbol{A}^{(3)} = \left[a_{ji}^{(3)}\right] \forall s_j \in SM_2$, $\boldsymbol{A}^{(4)} = \left[a_{ji}^{(4)}\right] \forall s_j \in SP_1$, $\boldsymbol{A}^{(5)} = \left[a_{ji}^{(5)}\right] \forall s_j \in SP_2$.

3.1 CLSP MILP optimization model

In the following section we will give the definition of the Capacitated Lot-sizing model in the Mixed Integer Linear Programming formulation

$$\sum_{t=1}^{T} \sum_{i=1}^{N} (c_i\, I_{it} + s_i\, z_{it}) \quad \rightarrow \quad min \quad \text{s.t.}$$

$$x_{it} + I_{i,t-1} - I_{i,t} = D_{it}, \forall t \in T, b_i \in NS \tag{1}$$

$$x_{it} + I_{i,t-1} - I_{i,t} = D_{it}, \forall t \in T, b_i \in NS \tag{2}$$

$$x_{it} + I_{i,t-1} - I_{i,t} = D_{it} \sum_{\forall j \in NP_i} \left(1 - y_{jt}\right),\ \forall t \in T, b_i \in NSP \tag{3}$$

$$x_{it} + I_{i,t-1} - I_{i,t} = D_{it} \sum_{\forall j \in NP_i} \left(1 - y_{jt}\right),\ \forall t \in T, b_i \in NSP \tag{4}$$

$$x_{it} + I_{i,t-1} - I_{i,t} = y_{it} D_{it} \quad , \forall t \in T, b_{ij} \in NP_j, \forall j (j = 1, 2, \dots, NSP) \tag{5}$$

$$\sum_{t=1}^{T} x_{it} = Total_{v_i}, \forall b_{ij} \in NP_j, \forall j (j = 1, 2, \dots, NSP) \tag{6}$$

$$\sum_{t=1}^{\tau < Launch\_p_i} x_{it} \geq Init\_v_i, \forall b_{ij} \in NP_j, \forall j (j = 1, 2, \dots, NSP), \tau \in \{1,2, \dots, T\} \tag{7}$$

$$Total_{v_i} - \sum_{t=1}^{\tau} x_{it} \geq 1\text{-}k_{i,\tau},\ \forall \tau \in T, b_{ij} \in NP_j, \forall j (j = 1, 2, \dots, NSP) \tag{8}$$

$$\sum_{t=1}^{\tau} x_{it} \geq k_{i,\tau} Total_{v_i}, \forall \tau \in T, b_{ij} \in NP_j, \forall j (j = 1, 2, \dots, NSP) \tag{9}$$

$$I_{jit} = ratio \times SSt_{jit},\ \forall\ t \in T,\ b_j \in NSP, b_{ij} \in NP_j, j = 1, 2, \dots, NSP, i = 1, 2, \dots, NP_j \tag{10}$$

$$y_{i,t+1} \leq y_{i,t}, \forall\ t \in T, b_{ij} \in NP_j, j = 1, 2, \dots, NSP, i = 1, 2, \dots, NP,\ t \geq Launch\_p_i \tag{11}$$

$$\sum_{t=1}^{\tau<Launch\_p_i} y_{it} = 0\ ,\ \forall b_{ij} \in NP_j,\ i = 1,2,\dots,NP, \forall j(j = 1,2,\dots,NSP) \tag{12}$$

$$\sum_{t=1}^{\tau<Launch\_p_i} y_{it} = 0, \forall b_{ij} \in NP_j,\ i = 1,2,\dots,NP, \forall j(j = 1,2,\dots,NSP) \tag{13}$$

$$\sum_{t\geq Launch\_p_i}^{\tau} D_{it}(1-y_{i\tau}) = (1-y_{i\tau})Total_{v_i}\ \forall\tau \in T\ , b_{ij} \in NP_j, \forall j(j = 1,2,\dots,NSP) \tag{14}$$

$$\sum_{i=1}^{N} a_{ji}^{(1)} x_{it} \leq LU_{jt}\ +\ FMP_{jt}\ , \forall t \in T, j \in S \tag{15}$$

$$\sum_{i=1}^{N} a_{ji}^{(2)} x_{it} \leq LUFF_{jt}\ +\ FMFF_{jt} \times W\_FMFF_{jt}\ , \forall t \in T, j \in PC \tag{16}$$

$$\sum_{i=1}^{N} a_{ji}^{(3)} x_{it} \leq LULT_{jt}\ +\ FMLT_{jt} \times W\_FMLT_{jt}\ ,\ \forall t \in T, j \in PC \tag{17}$$

$$\sum_{i=1}^{N} a_{ji}^{(4)} x_{it} \leq LUBOX20_{jt}\ +\ FPBOX20_{jt} \times W\_FPBOX20_{jt},\ \forall t \in T, j \in M \tag{18}$$

$$x_{it} \leq z_{it} \times max\_lot_i\ ,\ \forall t \in T, b_i \in N \tag{19}$$

$$x_{it} \geq z_{it} \times min\_lot_i\ ,\ \forall t \in T, b_i \in N \tag{20}$$

$$I_{it} = SSt_{it}\ ,\ \forall t \in T, b_i \in N \tag{21}$$

$$I_{it} \geq SSt_{it}\ (1\text{-}k_{it}\ )\ , \forall t \in T, b_{ij} \in NP_j, j = 1,2,\dots,NSP, i = 1,2,\dots,NP, t \geq Launch_{p_i} \tag{22}$$

$$I_{it} \geq SSt_{it}\ \sum_{\forall j\in NP_i}\left(1-y_{jt}\right),\ \forall t \in T, b_{ij} \in NP_i, i = 1,2,\dots,NSP, t \neq Launch_{p_i} \tag{23}$$

$$x_{it} \geq 0, I_{it} \geq 0, y_{it}{\geq}0\ ,\ k_{it}\{0,1\}, z_{it}\{0,1\}\ ,\ \forall t \in T, b_i \in N\ , \tag{24}$$

$$W\_FMFF_{jt} \geq 0, W\_FMFF_{jt} \geq 0, \forall t \in T, j \in PC \tag{25}$$

$$W\_FPBOX20_{jt} \geq 0, W\_FPBOX20PIN_{jt} \geq 0\ , \forall t \in T, j \in M \tag{26}$$

The objective is to minimise total cost, which is the sum of holding and production set-up costs. Constraints (1) to (3) are inventory equations for standard brands without promotions, with promotions and with promo brands. Constraint (4) ensures that no more than the demanded quantity of each promo variant is produced, (5) says that we build up sufficient stock before the launch. Constraints (6), (7) make it possible to completely deplete the promo stock. Constraint (8) reduces the level of the standard brand each time a promo variant is about to be sold. Constraint (9) allows for a gradual depletion of the promo variant stock. Equations (10) and (11) ensure that no promo is sold before its launch and that the standard brand is blocked from selling while its promo variant is selling. The set of constraints (12)-(16) deals with the flow shop capacity planning environment, then (17) and (18) impose constraints on the minimum and maximum production lots per brand. Constraints (19) to (21) control the safety stock policy for the standard brand without promotion, the promotion brands and the standard brands with promotion. The last block of constraints consists of the non-negativity conditions and the integrality conditions on the binary variables.

3.2 Computational results

In this section we apply the MILP CLSP model to a set of simulation data presented in Table 3. In this case we are dealing with a portfolio of five brands, out of which four are standard and there is one promotional variant PTB-00 to a standard brand PTB. The whole model (1)-(25) was coded in GAMS and solved using its MIP solver.

	Holding cost	setup cost	Max_Lot	Min_Lot
MLB	.2	30	80	8
PTB	.35	40	80	8
PTB-00	.35	40	80	8
PTC	.4	45	40	8
PTH	.15	20	50	4

Table 3 Brand parameters

The results of the brand performance in the eight-week planning horizon can be analysed in Table 4. We can see that production does not take place in every period, but reflects the set-up costs and, most importantly, we can see how PTB-00 and PTB interact. PTB-00 starts selling in planning period four and runs out of stock in period six. Therefore, before PTB-00 is launched, we need to build up sufficient pipeline stock and reduce the safety stock of its standard variant PTB.

	Production	Demand	Inventory	Safety stock	Inventory - Safety stock	Total Cost
MLB .1	60,0	30,0	30,0	20,0	10,0	36,0
MLB .2	25,0	30,0	25,0	20,0	5,0	35,0
MLB .3	65,0	30,0	60,0	20,0	40,0	42,0
MLB .4	80,0	40,0	100,0	20,0	80,0	50,0
MLB .5		40,0	60,0	20,0	40,0	12,0
MLB .6		40,0	20,0	20,0		4,0
MLB .7	70,0	40,0	50,0	20,0	30,0	40,0
MLB .8		30,0	20,0	20,0		4,0
PTB .1	40,0	60,0	20,0	20,0	0,0	47,0
PTB .2	80,0	70,0	30,0	30,0		50,5
PTB .3	55,0	70,0	15,0	30,0	***-15,0***	45,3
PTB .4		100,0	15,0	30,0	***-15,0***	5,3
PTB .5	43,7	100,0	13,5	30,0	***-16,5***	44,7
PTB .6	80,0	60,0	56,1	30,0	26,1	59,6
PTB .7		60,0	18,7	30,0	***-11,3***	6,5
PTB .8	61,3	50,0	30,0	30,0		50,5
PTB-00.1		60,0		20,0	***-20,0***	
PTB-00.2	40,0	70,0	40,0	30,0	10,0	54,0

	Production	Demand	Inventory	Safety stock	Inventory - Safety stock	Total Cost
PTB-00.3	40,0	70,0	80,0	30,0	50,0	68,0
PTB-00.4	50,0	100,0	30,0	30,0		50,5
PTB-00.5	70,0	100,0	45,2	30,0	15,2	55,8
PTB-00.6		60,0	22,6	30,0	***-7,4***	7,9
PTB-00.7		60,0		30,0	***-30,0***	
PTB-00.8		50,0		30,0	***-30,0***	
PTC .1	8,0	15,0	13,0	8,0	5,0	50,2
PTC .2	30,0	15,0	28,0	8,0	20,0	56,2
PTC .3	30,0	25,0	33,0	8,0	25,0	58,2
PTC .4	40,0	25,0	48,0	8,0	40,0	64,2
PTC .5		35,0	13,0	8,0	5,0	5,2
PTC .6	30,0	20,0	23,0	8,0	15,0	54,2
PTC .7		15,0	8,0	8,0		3,2
PTC .8	15,0	15,0	8,0	8,0		48,2
PTH .1	21,0	5,0	26,0	5,0	21,0	23,9
PTH .2		5,0	21,0	5,0	16,0	3,2
PTH .3		5,0	16,0	5,0	11,0	2,4
PTH .4	20,0	6,0	30,0	5,0	25,0	24,5
PTH .5		7,0	23,0	5,0	18,0	3,5
PTH .6		8,0	15,0	5,0	10,0	2,3
PTH .7		5,0	10,0	5,0	5,0	1,5
PTH .8		5,0	5,0	5,0		0,8

Table 4 Results Summary – Brand Performance

Table 5 illustrates how the capacity shortage spills over from the linked machines into the network of independent manufacturers and packers, using their idle capacity.

Link - up string cap.	Avail.	Used	Idle
String / time period	Cap.	Cap.	Cap.
FFKSBOX20 .1	80	100	***-20,0***
FFKSBOX20 .2	120	145	***-25,0***
FFKSBOX20 .3	120	160	***-40,0***
FFKSBOX20 .4	120	130	***-10,0***
FFKSBOX20 .5	120	113,71	6,3
FFKSBOX20 .6	80	80	
FFKSBOX20 .7	80	70	10,0
FFKSBOX20 .8	80	61,29	18,7
FFKSBOX20PIN.2	40	40	
FFKSBOX20PIN.3	40	40	
FFKSBOX20PIN.4	60	50	10,0
FFKSBOX20PIN.5	70	70	0,0
FFKSBOX20PIN.6	60		60,0
FFKSBOX20PIN.7	80		80,0
FFKSBOX20PIN.8	30		30,0
LTKSBOX20 .1	10	29	***-19,0***
LTKSBOX20 .2	15	30	***-15,0***
LTKSBOX20 .3	15	30	***-15,0***
LTKSBOX20 .4	30	60	***-30,0***
LTKSBOX20 .6	15	30	***-15,0***
LTKSBOX20 .8	30	15	15,0

Free make - Full Flavor FF			
period	Used	Avail.	Idle
1	20	40	20
2	25	40	15
3	40	40	
4	10	40	30
5		40	40
6		40	40
7		40	40
8		40	40

Free pack - KSBOX20			
period	Used	Avail.	Idle
1	40	40	
2	40	40	
3	60	60	
4	60	60	
5	60	60	
6	60	60	
7	60	60	
8	60	60	

Free make - Lights			
period	Used	Avail.	Idle
1	20	20	
2	15	15	
3	15	15	
4	30	30	
6	15	15	
8	30	30	

Table 5 Results Summary - Capacity Utilization

4 CONCLUSIONS

In this paper, we have presented how traditional CLSP can address a practical but complex medium-term capacity planning problem, taking into account the dynamic business environment and frequent product placement promotions that add complexity to the planner's task. We have shown how to approach capacity planning when resources are not homogeneous and when brands could be allocated to more than one resource. GAMS proved to be a powerful tool despite the limited size of the problem. A possible extension could be the use of fuzzy variables or the extension with a scheduling optimisation problem.

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References

[1] Chen, H. and Chu, C. (2003). *Supply chain planning with order/setup costs and capacity constraints - A new Lagrangian relaxation approach*. Journal of Systems Science and Systems Engineering - J SYST SCI SYST ENG, p. 1748 vol.2. Available at: https://doi.org/10.1109/ROBOT.2003.1241846.

[2] Degraeve, Z. and Jans, R. (2007). *A New Dantzig-Wolfe Reformulation and Branch-and-Price Algorithm for the Capacitated Lot Sizing Problem with Set Up Times*. Operations Research, 55, pp. 909–920. Available at: https://doi.org/10.1287/opre.1070.0404.

[3] Fiala, P. (2002). Modelování a analýza produkčních systémů. 1. vyd. Professional Publishing,.

[4] Gupta, D. and Magnusson, T. (2005). *The capacitated lot-sizing and scheduling problem with sequence-dependent setup costs and setup times*. Computers & Operations Research, 32, pp. 727–747. Available at: https://doi.org/10.1016/j.cor.2003.08.014.

[5] Ramya, R. et al. (2019). *Capacitated Lot Sizing Problems in Process Industries*. Available at: https://doi.org/10.1007/978-3-030-01222-9.

[6] Wagner, H. and Whitin, T. (1958). *Dynamic Version of the Economic Lot Size Model*. Management Science, 50, pp. 1770–1774. Available at: https://doi.org/10.1287/mnsc.1040.0262.

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SIMULATION OF VEHICLE ROUTES IN WASTE COLLECTION PROBLEM

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Abstract

In this paper, a modified algorithm of the nearest neighbor for vehicle routing problems with time windows and vehicle capacity is analyzed to minimize fuel consumption and therefore CO_2 emissions. The proposed algorithm was tested on Solomon's instances and the obtained route plan was simulated in the Flexsim 2021 software. The main advantage of the proposed algorithm, which deals with the routing of vehicles for the collection of waste, is that it unites the goals of transport companies on the one hand and society on the other.

Keywords: *vehicle routing problem, simulation modeling, waste collection*

JEL Classification: C630, C610
AMS Classification: 65K05, 90B06

1 INTRODUCTION

The green vehicle routing problem (Green-VRP) represents an extension of the classic routing problem where environmental criteria are taken into account, like minimizing carbon dioxide emissions (CO_2), fuel consumption, and noise level. Given that the road transport sector mainly uses vehicles that use oil as a fuel (internal combustion engine vehicles), which has a very harmful effect on the environment, the application of green routing is necessary to maintain this sector in the future. Namely, in recent years, the transportation sector has been faced with many challenges where requests for environmentally friendly practices have increased like targeting a 40% reduction in greenhouse gas (GHG) emissions by 2030 in the European Union (EU) (Asghari and Al-e, 2021). This target poses a challenge for the transportation sector responsible for 20% of total GHG emissions (Asghari and Al-e, 2021).

Green-VRP routes are designed to consider both environmental and financial objectives (da Costa et al., 2018). Namely, the amount of CO_2 emitted by a vehicle is proportional to fuel consumption which is influenced by several factors like distance traveled, average driving speed and acceleration, load, etc. (da Costa et al., 2018). Both internal combustion vehicles (ICV) and alternative fuel vehicles (AFV) are considered, dividing Green-VRP into two separate subclasses: ICV-based Green-VRP and AFV-based Green-VRP (Matijević, 2023). The environmental aspect of AFV-based GVRP, which is the subject of this paper, is implicit and comes from using AFVs in transport (Matijević, 2023).

Review papers on Green-VRP can be found in Park and Chae (2014), Mahmudul et al. (2017), Asghari and Al-e (2021), etc. Kabadurmus and Erdogan (2023) formulated Green-VRP with multi-depot, multi-tour, heterogeneous fleet and split deliveries as a Mixed Integer Linear Programming (MILP) model to minimize the total carbon emissions. A Genetic Algorithm (GA), employing a MILP model and using niching and constraint handling techniques, is also developed to solve the problem effectively. Prakash and Pushkar (2022) proposed a time window-based Green-VRP solution with an exact routing approach. The authors tested the proposed method on a set of three reference problem specimens (C-101, R-101 and RC-11).

In the case of collecting waste, if two clients are at a similar distance, the client from whom a smaller amount of waste needs to be collected should be served first, as this ensures that the remaining part of the route is covered by the vehicle with less load. In this way, fuel consumption is reduced, because a less loaded vehicle consumes less fuel. Introducing new constraints and variables in the vehicle routing problem to take into account environmental objectives as well as financial objectives further complicates the problem and makes it difficult to formulate and solve it. To solve the problem, a modified algorithm of the nearest neighbor was applied. This algorithm was tested on Solomon instances c103, c104, c201 and c202. To validate the new algorithm, the obtained route plans were simulated in the software tool Flexsim 2021.

The rest of the paper is organized as follows. Section two presents a description of the problem and proposed algorithm for routing vehicles for waste collection with time windows to minimize fuel consumption. Section three presents the numerical results as well as simulation analysis of obtained vehicle routes while concluding remarks are given in section four.

2 DESCRIPTION OF THE PROBLEM

In this section, a modified algorithm of the nearest neighbor to minimize fuel consumption, and thus reduce negative impacts on the environment is presented. The algorithm is used to solve the problem of routing vehicles for waste collection with time windows. The criterion of fuel consumption minimization can be considered both an economic and an environmental criterion, because by minimizing fuel consumption, the costs of the transport company, as well as CO_2 emissions, are reduced. This algorithm partly uses the logic of the nearest neighbor algorithm that takes into account time windows, vehicle load, and fuel consumption. As a measure of the distance between nodes, the fuel consumption obtained by applying formula (1) is used (Xiao et al.,2012), and accordingly, it is best to visit the node that is close and has a smaller waste quantity to collect. The steps of Algorithm I are following:

Step 1: between all pairs of nodes calculate the fuel consumption:

$$U_{ij}^{f} = \rho_0 + \frac{\rho^{*}-\rho_0}{Q} Q_j \qquad (1)$$

where

U_{ij}^{f} – a measure of distance that represents fuel consumption between nodes i and j in the route f depending on the waste quantity in node j,
Q_j – the waste quantity that has to be collected in node j,
Q – the capacity of the vehicle (the maximal weight the vehicle could carry),
ρ^{*} – the full-load fuel consumption rate,
ρ_0 – the no-load fuel consumption rate.

Step 2: Route formation starts from the node that represents the base node by finding the node that is closest to the base according to fuel consumption which is not served. Then that node should be included in the route and called the current node, and the waste quantity that should be collected in that node should be added to the collected waste quantity in the route. It is still necessary to calculate the moment when the vehicle leaves the base which is D(0)=0. After this step, the partial route consists of the base node and its closest node.

Step 3: In this step, it is necessary to calculate the moment when the vehicle leaves the current node after collecting the waste. It is calculated by adding the travel time from the previous node to the current node to the moment of leaving the previous node and then finding the maximum between the obtained value and the earliest start of service in that node. It is necessary to add the waste collection time in the current node to the obtained value.

Step 4: In step four, it is necessary to find the closest unserved node according to the fuel consumption to the current node, which can be served concerning the load limit and the service time interval. If there is such a node, include it in the partial route, add the waste quantity generated in that node to the quantity of collected waste in the route, declare it as the current node, and return to step three. Otherwise, the route ends and the vehicle returns to base.

Step 5: Verify that all nodes are served. If all nodes are served, finish with the algorithm, otherwise go back to step two and start creating a new route.

After generating the routes using Algorithm I, the total fuel consumption is calculated according to the formula (2) (Xiao et al.,2012):

$$C_{fuel} = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{fuel}^{ij} x_{ij} = \sum_{i=1}^{n}\sum_{j=1}^{n} c_0 \rho_{ij} d_{ij} x_{ij} \tag{2}$$

where:

n – total number of nodes that need service,

$x_{ij} = \begin{cases} 1, & if\ vehicle\ uses\ arc\ (i,j) \\ 0, & otherwise \end{cases}$,

c_{fuel}^{ij} - the cost of fuel consumption on arc (*i, j*),

c_0 – unit fuel cost,

ρ_{ij} – fuel consumption rate from node i to node *j,*

d_{ij} – length of the arc (*i, j*).

3 NUMERICAL RESULTS

In this section, the results of solving the problem of routing waste collection vehicles with limited capacity, respecting time windows, will be presented. Solomon's instances are used as the benchmark for problems that include time window constraints containing geographical data randomly generated and denoted as problem sets: R1, C1 and RCl. In problem set R1 data are randomly generated by a random uniform distribution, clustered in the problem set C1, and mix of both in the problem set RC1. We tested the proposed Algorithm I on all of Solomon's instances, but chose instances c103, c104, c201 and c202 to present results because the advantages of the proposed approach are best shown in these four instances. To use any of the Solomon instances, the distance between the nodes, the travel time between the nodes, and the vehicle's capacity must be determined. When it comes to vehicle capacity, the recommended value for these instances was taken, namely 200 units for instances c103 and c104, and 700 units for instances c201 and c202. To determine the distance between nodes, it is recommended to use the Euclidean distance, while the temporal distance between nodes is taken to be equal to 2 times the spatial distance. In this way, it was defined that the average speed of a vehicle during waste collection is 30 km/h. That is, to cross one unit of length, it takes two units of time [Xiao et al., 2012].

When applying the proposed Algorithm I, the consumption fuel rate of an empty vehicle will take the value 2, and the consumption rate of a full vehicle will take the value 3 because the fuel consumption of an empty vehicle is one-third less than the fuel consumption of a full vehicle, so accordingly these rates take the previously defined values [European Commission, 2009]. In Solomon's instances, the number of clients is 100. The base node is marked with 1, and the clients are marked with numbers from 2 to 101. It is necessary to determine the distances between all pairs of nodes, based on their (x, y) coordinates by applying the formula for calculating the Euclidean distance between two points.

After determining the distance matrix, travel time, and vehicle capacity as previously described, then the formation of vehicle routes can begin. It is necessary to find such routes that will allow all 100 nodes in the network to be visited and the waste generated in each node to be collected.

Each route should start and end at node 1, which is both a depot and a place for waste disposal. When creating routes, it is also necessary to take into account the vehicle's carrying capacity and pre-defined time windows. The total amount of waste collected on one route must not exceed the capacity of the vehicle.

The results of testing Algorithm I are shown in Table 1. Table 1 shows the total consumption, total time and total distance traveled during the implementation of the defined set of routes for each instance. It can be seen from the table that by applying Algorithm I in the case of instances c103 and c104, when the carrying capacity of the vehicle is smaller (200 units), a larger number of routes is generated than in the case of instances c201 and c202 when the carrying capacity is higher (700). The last column of the table shows the average consumption per 100 units of distance traveled. Given that the fuel consumption of the vehicle that realizes the routes is between 20 and 30 units, depending on the load, the quality of the generated set of routes is better when the average consumption during the realization of all routes is closer to 20 units. From Table 1, it can be seen that the average consumption for each of the instances is less than or equal to 23.30 units, which makes each of these solutions very good.

Table 1. Results of testing Algorithm I

Instances	**Fuel consumption**	**Distance**	**Time of route realization**	**Number of routes**	**Fuel consumption on 100 units**
c103	430.0332	1899.617	20750.07	20	22.64
c104	385.619	1655.34	17075.13	16	23.30
c201	360.7464	1700.601	34034.58	13	21.21
c202	377.192	1759.414	40177.53	15	21.44

Then, the obtained route plans for the four instances are simulated in the software tool Flexsim 2021. The simulation aims to perform an analysis of the obtained solution in real conditions where stochastics is included by defining distributions for service time and vehicle speed. In the theoretical case, the service time lasts a fixed 90-time units, while in the case of simulations, the defined triangular distribution for the service time is shifted to the left [45, 90, 110]. In this way, it is defined that the service time can take some value from the mentioned interval, where the central value is 90 time units. A triangular distribution was also used for the speed of the vehicle, where the speed was taken to be in the interval [0.33; 0.5; 0.58]. In the case of speed, the distribution is also shifted to the left, and the reason for this is that in most cases the speed will be lower than the average, given that the vehicles are moving in urban conditions. In the theoretical example, it was assumed that the speed of a vehicle is 0.5 units of distance per unit of time, i.e. approximately 30 km/h.

The inclusion of stochastics in the implementation of the route plan can lead to the breaking of time windows. For example, if according to the plan it was obtained that the vehicle arrives at the client's service before the end of the time window, due to a lower speed of the vehicle or a longer stay at the previous client, the vehicle can break through the time window. In this way, the client remains unserved. In that situation, the vehicle should be rerouted again to find a better route. However, with the route plans defined by Algorithm I for 4 instances and each of the plans tested for 20 replications, only once happened that the client was not served and the vehicle was late returning to the depot.

Table 2 shows the time intervals of service, travel and waiting, based on 20 replications of simulating the obtained route plan by Algorithm I for each of the 4 instances (c103, c104, c201 and c202). The table shows the minimum, average and maximum time spent on each of the three activities for each instance. The total time spent on the service during the implementation of the route plan for instance c103 ranges from 7985.92 to 8314.37 time units. The time spent traveling ranges from 1824.13 to 1920.37 time units, while the total time spent by vehicles waiting during the implementation of the route plan, for instance, c103 ranges from 8899.08 to 9168.92-time units.

Table 2. Time intervals of service, travel and waiting

Instances	Service [min]	Service [avg]	Service [max]	Travel [min]	Travel [avg]	Travel [max]	Waiting [min]	Waiting [avg]	Waiting [max]
C103	7985.92	8140.62	8314.37	1824.13	1859.42	1920.37	8899.08	9069.82	9168.92
C104	8100.00	8245.35	8347.33	1587.35	1615.10	1642.07	5449.60	5579.53	5727.87
C201	5892.13	6067.02	6281.63	1425.90	1505.82	1561.05	19566.35	19746.02	19938.27
C202	7943.08	8088.70	8274.77	1665.08	1735.83	1783.58	29162.17	29337.32	29437.18

The simulation analysis was performed based on 20 replications of simulating the generated route plans for each instance. Figure 1 shows the percentage distribution of the average time (obtained based on 20 replications) spent on travel, service, and waiting, for each instance. In instances c103, c201, and c202, most of the time is spent waiting for service, while in instance c104, most time is spent serving clients. In the route plans obtained for instances c201 and c202, vehicles spend over 70% of their time waiting at nodes to provide service. So it can be concluded that the route plans obtained for those instances are not efficient, because most of the time the vehicles are not productive, because the time spent in the waiting state is lost time. One of the reasons why instances c201 and c202 spend most of their time in a waiting state is because the capacity of the vehicles is larger, and therefore the number of clients served per route is greater. In this way, it is more difficult for the vehicle to ideally fit into the clients' time windows, so there is a waiting time.

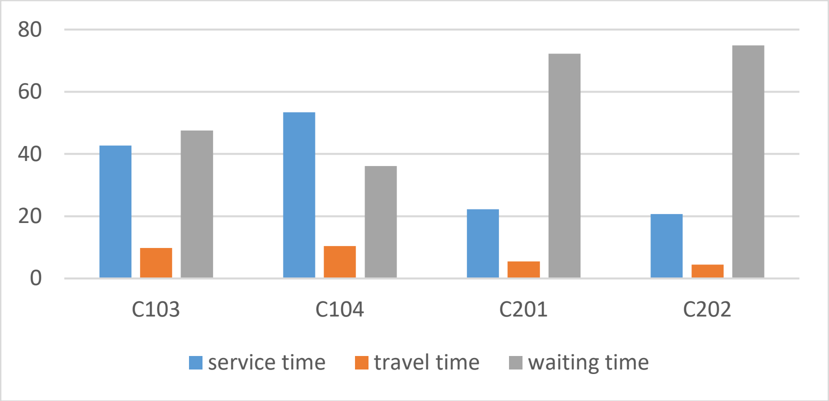

Figure 1. Distribution of the average time spent on travel, service, and waiting

One of the ways in which the time spent in the waiting state can be reduced is that the vehicles start the realization of the route later in situations when the left limit of the time window of the first client in the route is not the zero moment. In this way, the vehicle will avoid the waiting state at the first client and affect the reduction of the total time spent waiting, which can be seen from Figure 2. The time spent in waiting was reduced in all instances, but mostly in c201 and c202. With these two instances, the time spent in the waiting state was reduced by about 20%.

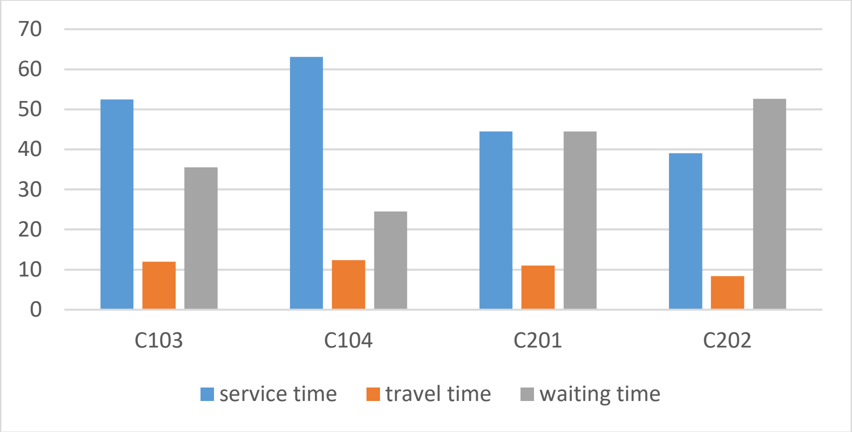

Figure 2. Distribution of the average time spent on travel, service, and waiting without waiting time at first client

4 CONCLUSIONS

Green routing, i.e. the routing problem that aims to define vehicle routes in such a way as to minimize the harmful impact on the environment, is a young field of research that has become popular in the last 10 years. In the case of collecting waste, if two clients are at a similar distance, the client from whom a smaller amount of goods needs to be collected should be served first, as this ensures that the remaining part of the route is covered by the vehicle with less load. In this way, fuel consumption is reduced, because a less loaded vehicle consumes less fuel. To solve the problem, a modified algorithm of the nearest neighbor Algorithm I was presented. This algorithm was tested on Solomon instances c103, c104, c201 and c202. In order to validate Algorithm I, the obtained route plans were simulated in the software tool Flexsim 2021. In this way, it was examined how the solution provided by Algorithm I behaves in practice. The simulation concluded that the obtained route plans are good and are not sensitive to practical situations such as: longer or shorter service time, higher or lower vehicle speed, etc. A future upgrade of the Algorithm I could be taking into account the time window of the first client in the route and thus start the implementation of the route plan in a timely manner, and not at the zero moment. Also, Algorithm I can be improved by introducing a new step related to vehicle rerouting in real conditions if it is determined that the vehicle cannot arrive to visit the next client in the existing route, due to the time window. Namely, in addition to the direct minimization of fuel consumption, by minimizing the time the vehicle spends in waiting and traveling states, an indirect reduction of fuel consumption and thus CO_2 emissions is achieved. The savings achieved by minimizing the time of use of the fleet allow vehicles to be engaged in other tasks during that time, thus increasing their efficiency.

References

[1] Asghari, M. and Al-e, S.M.J.M., 2021. Green vehicle routing problem: A state-of-the-art review. *International Journal of Production Economics*,231, p.107899.

[2] da Costa, P.R.D.O., Mauceri, S., Carroll, P. and Pallonetto, F., 2018. A genetic algorithm for a green vehicle routing problem. *Electronic notes in discrete mathematics*, 64, pp.65-74.

[3] European Commission, (2009). Press release [online] Available at: http://europa.eu/rapid/pressReleasesAction.do?reference=IP/09/1379 [Accessed 20.9.2022.]

[4] Mahmudul, H.M., Hagos, F.Y., Mamat, R., Adam, A.A., Ishak, W.F.W., and Alenezi, R., 2017. Production, characterization and performance of biodiesel as an alternative fuel in diesel engines–a review. *Renewable and Sustainable Energy Reviews*, 72, pp. 497–509.
[5] Matijević, L. 2023. Metaheuristic approaches for the green vehicle routing problem. *Yugoslav Journal of Operations Research,* 33(2), pp.153–198.
[6] Kabadurmus, O., Erdogan, M.S. 2023. A green vehicle routing problem with multi-depot, multi-tour, heterogeneous fleet and split deliveries: a mathematical model and heuristic approach. *Journal of Combinatorial Optimization*, 45, 89.
[7] Park, Y., and Chae, J., 2014. A review of the solution approaches used in recent G-VRP (Green Vehicle Routing Problem). *International Journal of Advanced Logistics,* 3(1–2), pp.27–37.
[8] Prakash, R. and Pushkar, S., 2022. Green vehicle routing problem: Metaheuristic solution with time window. *Expert Systems*, p.e13007.
[9] Xiao, Y., Zhao, Q., Kaku, I., and Xu, Y., 2012. Development of a fuel consumption optimization model for the capacitated vehicle routing problem. *Computers & Operations Research*, 39(7), pp.1419–1431.

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ANALYSIS OF DIFFERENT SIMULATION TECHNIQUES FOR SOLVING QUEUEING THEORY PROBLEMS

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Abstract

This study deals with applying and analysing various simulation techniques in solving a problem from queueing theory (mass service system). The motivation for this study is to understand how different characteristics are obtained when using different probability distributions to generate input data. The starting point for this study is unpaired data on the number of units entering the system per unit time and the service time of requests. We modelled the changing intensity of input requirements using step change in intensity and polynomial regression. The service time was modelled by empirical, triangular, and gamma distribution. Combining these approaches, we created six models of a mass service system and identified their characteristics. The characteristics obtained by each model varied very little within statistical variation.

Keywords: *Queuing theory, Simulation methods, Probability distribution*

JEL Classification: C630, L840
AMS Classification: 68U20

1 INTRODUCTION

Queuing theory is a discipline that uses mathematical modelling to determine the characteristics of mass service processes. In mass service processes, units (requests) enter the system and wait in a queue until they are served. The occurrence of these units is random. As a result of the limited number of service lines (service channels), the problem of request accumulation and queuing may occur. One of the tasks of queueing theory is to find the optimal number of operator lines in which service requests do not wait too long, and the operator lines are sufficiently busy.

Queuing theory problems can be solved analytically or by simulation. For both approaches, a good specification of the variables is essential when building the model. These are mainly the statistical characteristics of the input request stream and the characteristics of the request service time. In addition, the arrangement and number of serving channels, queuing discipline, and queuing mode must be specified. Only a limited number of simpler problems can be solved analytically, where an exponential distribution can model the interval between the input of two consecutive requests and the service time. Especially for service times, the assumption of an exponential distribution is often violated. Analytical approaches to queuing systems are described in Ross (2010).

More complex models or models where service times or intervals between consecutive customer arrivals do not have an exponential distribution cannot be solved analytically. A simulation approach can be used to calculate the characteristics. The output characteristics are determined based on the simulated passage of units through the modelled system. Simulation techniques use a random number generator uniformly distributed on the intervals from zero to one. The generated random numbers represent the values of probability distribution functions that model the intervals between entering requests, service times, customer impatience, etc. A description of some selected simulation techniques used in queuing systems is given, for example, by Stewart (2009). Several studies have been published on the use of queuing models to solve a variety of problems. Examples of these applications include their use in telecommunication networks (De Boer 2000) or banking (Madani 2013). Some studies use queuing models as a tool for solving issues related to improving customer service in the field of retailing (Xing et al. 2015), healthcare (Wang et al. 2019) or general public services (Xian et al. 2016).

The aim of this study is to get some idea of how different characteristics in the mass-servicing model are obtained when different probability distributions are used to generate the input data. In fact, the motivation for this study was students' questions about what distribution to use when solving queueing theory problems and whether it would be a mistake to use a distribution that does not completely correspond to the real situation.

2 DATA

The mass service system is in operation 9 hours a day. We have data for five days on the number of customers entering the system and the time taken to serve them. The number of customers entering per hour is shown in Table 1.

Table 1 The number of customers

day	hour 1	2	3	4	5	6	7	8	9	Σ
1	10	20	25	29	19	25	23	29	14	194
2	13	12	28	34	15	12	26	22	17	179
3	15	31	20	37	25	26	34	34	13	235
4	20	25	26	22	12	19	29	31	11	195
5	9	10	34	25	19	20	37	25	9	188
mean	13,4	19,6	26,6	29,4	18	20,4	29,8	28,2	12,8	
var	19,3	39,8	25,8	38,3	24	31,3	32,7	22,7	14,2	

The data in Table 1 shows that the number of entering customers varies within time. The lowest entry intensity is in the first and ninth hours of operation, while high intensities are reached in the fourth, seventh and eighth hours. These differences are statistically significant. In the test of the null hypothesis H_0: $\mu_1 = \mu_2 = \cdots = \mu_9$ against the hypothesis H_A: non-H_0 (ANOVA) the *p-value* <0.001. The differences between days are not statistically significant and can be explained by random variation. On average, 22.04 customers enter the system per hour.

The distribution of the service time distribution is shown in Figure 1.

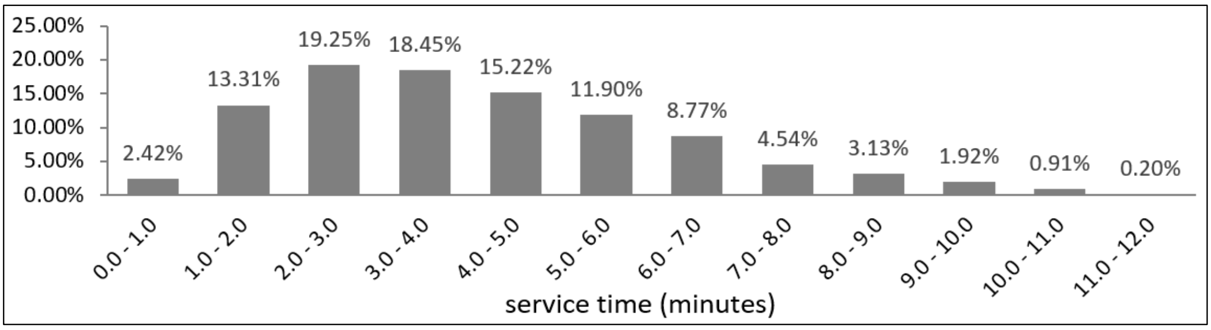

Figure 1: Distribution of service time

Customer service times do not show any daily or hourly differences. The distribution of the service time is positively skewed, with mean service time *E(X)* = 4.15 and variance of service time *D(X)* = 4.70.

3 MODELS FORMATION

An analytical solution cannot be used to calculate the characteristics of this mass service system. This is prevented by the variable intensity of the input units, and the service time does not have an exponential distribution. We use a simulation of the servicing process to determine the system's characteristics.

3.1 Modelling customer arrivals

When modelling customer arrivals, we assume that the entry of units into the system satisfies the conditions of a Poisson process. Thus, we will assume that the interval between the inputs of two consecutive customers can be described by an exponential distribution, where the parameter of this distribution λ is a function of time. We will model the changes in the parameter λ in two ways: by a step change and by a continuous change by polynomial.

For the step change modelling of the parameter λ, we use the average values from Table 1. The parameter λ will vary stepwise according to formula (1):

$$\begin{gathered} \lambda(t) = 13.4 \quad for \quad 0.00 \le t \le 1.00 \\ \lambda(t) = 19.6 \quad for \;\; 1.00 \le t \le 2.00 \\ \ldots \\ \lambda(t) = 12.8 \quad for \;\; 8.00 \le t \le 9.00\,, \end{gathered} \tag{1}$$

where λ is the number of customers entering per hour, and *t* is the time.

We used a fifth-degree polynomial regression function to model the continuous parameter λ. The regression polynomial fit of the values from Table 1 is shown in Figure 2.

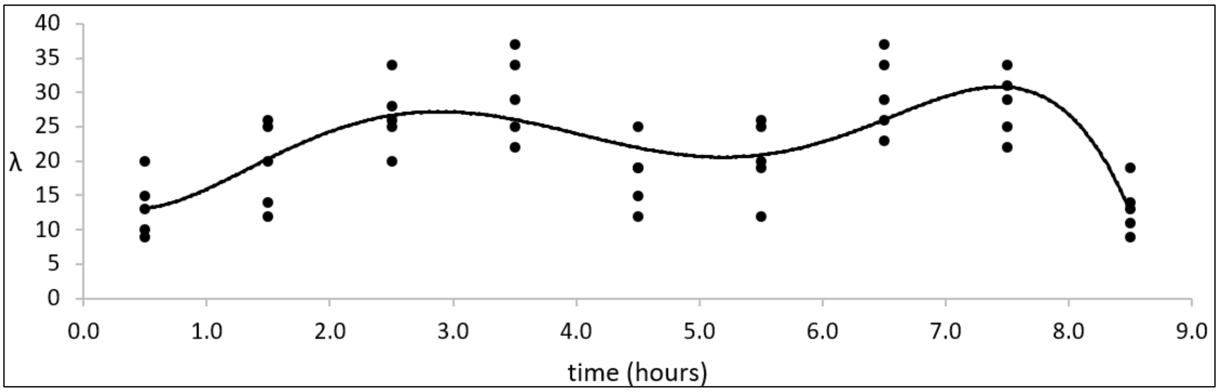

Figure 2: Regression polynomial of the fifth degree

The parameter λ is modelled by the relation (2):

$$\begin{gathered} \lambda(t) = -0.0531t^5 + 1.057t^4 - 7.196t^3 + 18{,}78t^2 - 11.81t + 15{,}14 \; for \; 0{,}00 \le t \le 8{,}75 \\ \lambda(t) = 0 \; for \; t \ge 8.75 \end{gathered} \tag{2}$$

The polynomial regression takes a negative value for $t \ge 8.75$. Upon reflection, we set λ = 0 in the model for $t \ge 8.75$. This setting can model the real situation well since the number of customers entering the system before closing time can be expected to be low.

3.2 Modelling of service time

The service time will be modelled by an empirical distribution exactly reproducing the values in Figure 1, a triangular distribution and a gamma distribution.
The probability distribution of the empirical distribution is given in Table 2.

Table 2 Empirical distribution

i	1	2	3	4	5	6	7	8	9	10	11	12
t	0,00 – 1,00	1,00 – 2,00	2,00 – 3,00	3,00 – 4,00	4,00 – 5,00	5,00 – 6,00	6,00 – 7,00	7,00 – 8,00	8,00 – 9,00	9,00 – 10,00	10,00 – 11,00	11,00 – 12,00
$p(x_i)$	0,024	0,133	0,193	0,184	0,152	0,119	0,088	0,045	0,031	0,019	0,009	0,002
$F(x_i)$	0,024	0,157	0,350	0,534	0,686	0,805	0,893	0,939	0,970	0,989	0,998	1,000

Service time is calculated according to the relationship (3):

$$t = x_{i,d} + \frac{RND - F(x_i)}{p(x_i)}, \tag{3}$$

where *RND* is a generated random number representing the value of the distribution function, *i* is the label of the interval that belongs to the value of the *RND* distribution function, $x_{i,d}$ is the lower bound of the interval, $p(x_i)$ is the probability of the *i-th* interval, $F(x_i)$ is the value of the distribution function at the beginning of the interval. For example, for *RND* = 0.6 the service time is calculated by:

$$t = 4.00 + \frac{0.6 - 0.534}{0.152} = 4.43 \text{ minutes}.$$

The triangular distribution *TRI* (*a*, *b*, *c*) is uniquely determined by the minimum value (parameter *a*), the most frequent value (parameter *b*) and the maximum value (parameter *c*). The basic characteristics of this distribution are given by the relations:

$$E(X) = \frac{a + b + c}{3} \tag{4}$$

$$D(X) = \frac{a^2 + b^2 + c^2 - ab - ac - bc}{18} \tag{5}$$

In setting the parameters of the triangular distribution, we preferred the mean to the variance and the parameters *a* and *b*, to the parameter *c*. Thus, according to the preferences and Figure 1, we have to calculate the parameter *c*:

$$4.15 = \frac{0 + 2.5 + c}{3} \rightarrow c \simeq 10 \rightarrow TRI(0, 2.5, 10)$$

With these parameters, *D(X)*=4.5, which is very similar to the empirical variance. A comparison of the empirical distribution function with the distribution function *TRI* (0, 2.5, 10) is seen in Figure 3.

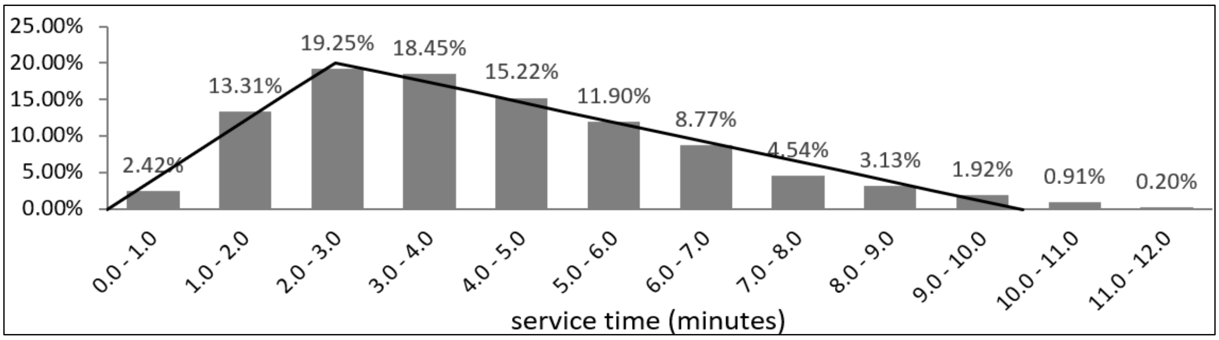

Figure 3: *TRI* (0, 2.5, 10) fitting

The chi-square goodness-of-fit test was used to test the goodness-of-fit of the distributions according to Figure 1 and *TRI* (0, 2.5, 10). The test statistic is equal to 38.04 and the *p-value* is less than 0.001, so from the statistician's point of view, we reject the hypothesis of equality of distribution. There are significant differences at the tails of the distribution only.

The gamma distribution Γ (k, d) has two parameters k and d. The parameter k determines the shape of the distribution. When $k < 1$ the distribution takes its maximum value at zero, just like the exponential distribution. The exponential distribution is a special case of the gamma distribution when $k = 1$. If $k > 1$ the density function is concave with a peak near zero. The parameter d specifies the scale, similar to the parameter of the exponential distribution. The basic characteristics of this distribution are given by the relations:

$$E(X) = kd \tag{6}$$

$$D(X) = kd^2 \tag{5}$$

Estimates of the parameters k and d are obtained by inserting the sample statistics of the empirical service time into formulas (5) and (6).

$$d = \frac{D(X)}{E(X)} = \frac{4.70}{4.15} = 1.13 \qquad k = \frac{E(X)}{d} = 3.67$$

A comparison of the empirical distribution function with the distribution function Γ (3.67, 1.13) is shown in Figure 4.

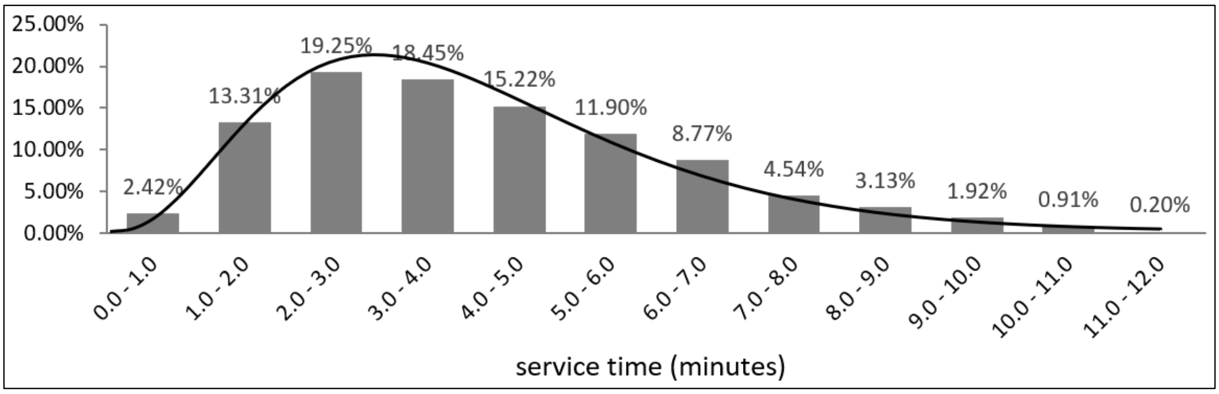

Figure 4: Γ (3.67, 1.13) fitting

The chi-square test statistic of the goodness-of-fit test is 9.8 and the *p-value* is 0.27. Thus, the service times are modelled by the gamma distribution well.

4 EXPERIMENTAL RESULTS

In total, six models of mass service were analysed, combining two models of customer input with three models of service time. For each model, the simulation was run under the following conditions: number of service channels 2, FIFO queuing mode, the number of queue seats is unlimited, all customers entering the system before closing time are served, all customers are patient, 300 repetitions of the simulation run were performed. In the models, we observed the following characteristics: number of customers entering the system (denoted by N), total waiting time of customers (TW), average waiting time (AWT), average number of customers in the queue (AQ), and the service capacity utilization (SU). The means ($\bar{x}$) and standard deviations (s) of these characteristics are shown in Table 3.

Table 3 Characteristics of the models

model	customer entry	service time	char.	N	TW	AWT	AQ	SU
M1	pol.	EPM	$\bar{x}$	200.7	22:03:35	0:06:26	2.43	78.3%
			s	13.8	15:07:48	0:04:02	1.64	5.5%
M2	pol.	TRI	$\bar{x}$	199.3	20:53:21	0:06:07	2.30	77.9%
			s	14.4	14:16:08	0:03:47	1.54	5.7%
M3	pol.	Γ	$\bar{x}$	199.8	21:22:02	0:06:16	2.36	78.2%
			s	14.3	14:20:24	0:03:54	1.56	5.8%
M4	step	EPM	$\bar{x}$	199.2	20:38:58	0:06:05	2.28	77.6%
			s	14.6	12:01:01	0:03:14	1.32	5.9%
M5	step	TRI	$\bar{x}$	199.5	21:51:39	0:06:25	2.41	78.1%
			s	14.4	13:49:28	0:03:43	1.51	5.6%
M6	step	Γ	$\bar{x}$	198.6	20:32:42	0:06:03	2.27	77.5%
			s	14.1	12:44:18	0:03:25	1.39	5.6%

The values in Table 3 show that the differences in the means for the observed characteristics of the mass service are non-significant relative to their standard deviation and can be explained by random variation. A two-sided test of the hypothesis H_0: $\mu_{Min} = \mu_{Max}$ was conducted for all five characteristics. All *p-values* were greater than 0.05, again confirming the non-significance of the differences between the models.

5 CONCLUSIONS

In this study, we investigated how different characteristics in a mass service system are obtained, when solving the simulation problem using different models to generate the input data. We obtained similar operator characteristics in all six models. The differences were statistically non-significant and can be explained by random variation. The study suggested that as long as the probability distributions used to generate the service times have the same mean, similar shape, and the differences are on the tails, it does not matter which distribution we use. Similarly, when modelling the changing intensity of the input units, it is not so important whether it is modelled by a continuous regression polynomial or a discrete regression polynomial, where the intensity changes by jumps after certain intervals. However, these conclusions cannot be generalized and always depend on the nature of the input data.

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References

[1] De Boer, P. T. (2000). *Analysis and Efficient Simulation of Queueing Models of Telecommunication Systems*. Enschede: Centre for Telematics and Information Technology.

[2] Madani, N., Roudsari, A. H., Wong, K. Y. and Galankashi, M. R. (2013). Modeling and Simulation of a Bank Queuing System. In: *5th International Conference on Computational Intelligence, Modelling and Simulation*, pp. 209-215. doi: 10.1109/CIMSim.2013.41.

[3] Ross, S. M. (2014). *Introduction to probability models*. 11th ed. Elsevier. doi: 10.1016/B978-0-12-407948-9.00014-1.

[4] Stewart, W. R. (2009). *Probability, Markov Chains, queues and simulation: the mathematical basis of performance modeling*. Princeton University Press.

[5] Xian, T. C., Hong, C. W. and Hawari, N. N. (2016). Modeling and Simulation of Queuing System for Customer Service Improvement: A Case Study. *AIP Conference Proceedings*, 1782(1), Article Number: 040020. doi: 10.1063/1.4966087.
[6] Xing, W. J., Li, S. B. and He, L. H. (2015). Simulation Model of Supermarket Queuing System. In: *34th Chinese Control Conference*. Hangzhou, pp. 8819-8823. doi: 10.1109/ChiCC.2015.7261032
[7] Wang, X., Gong, X., Geng, N., Jiang, Z. and Zhou, L. (2019). Metamodel-based simulation optimisation for bed allocation. International Journal of Production Research. doi: 10.1080/00207543.2019.1677962.

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ANALYSIS OF CONVERGENCE IN EUROPEAN UNION ECONOMICS AND EUROPEAN MONETARY UNION ECONOMICS

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Abstract

In the paper, we examine the convergence of member countries of the European Union and the European Monetary Union. We answer whether the differences in economic activity between member states are decreasing and whether there is an assumption that one day the states will be equally prosperous. For this purpose, we estimate the convergence specification in which the average annual GDP growth depends on the starting value of GDP. We use both cross-section and panel data. According to the first question, we test whether the convergence parameter is nonpositive; according to the second, we test whether the intercept parameters are equal. According to the results, income differences across all countries are narrowing but not disappearing; income differences between the 12 monetary union countries are not decreasing.

Keywords: *convergence, cross-section data, panel data, European Union*

JEL Classification: C12, C23, O47
AMS Classification: 62P20, 91B62

1 INTRODUCTION

One of the priorities of the central policy of the European Union is equalizing the differences in economic activity between different European regions and states. Today, the official databases of the Eurostat portal of the European Statistical Office already provide sufficiently long databases of the development of real activity so that we can check some convergence hypotheses and answer whether and how the differences between countries are being reduced. The database is long enough to apply panel data methods with 2 to 3 multi-year periods. In the methodological part, we formulate two convergence questions. The first is if differences in economic activity between countries are decreasing. The second is if there is a future period S in which countries will be equally rich unless other fundamental changes occur. We will formulate and test the hypotheses relating to the questions for 26 EU countries and 12 EMU countries.

We use the β concept to test the hypotheses. Barro (2008, chapter 5), Baumol (1986), DeLong (1988), Barro (1991), Barro and Sala-i-Martin (1991, 1992a, 1992b and 2004, chapter 11) used this concept to verify the convergence of the economies. Our study is distinguished by the estimation using panel data. In the methodological part, we will propose a test of the second hypothesis, which assumes the complete elimination of differences between different countries in the future. The test corresponds to the non-significance of fixed effects in the specification.

2 METHODS

Considering the growth theory by Barro and Sala-i-Martin (2003), we differ between the convergence questions. The first question is whether poor countries grow sufficiently fast while rich countries grow slower. If so, the wealth difference between countries in a group decreases over time. However, even if poorer countries grow faster, the second question is whether their GDP per capita will reach the level of rich countries in the distant future. If so, countries' GDPs per capita converge to the same level over a sufficiently long time *S*.

Let us formalize the convergence hypotheses from both questions. The following expression expresses a part of the growth theory we are interested in:

$$y_{it} = g(x_{iS} - x_{it}), \tag{1}$$

where y_{it} denotes the average annual per capita growth rate of country *i* in period *t*, x_{it} is the GDP per capita in country i in the beginning year of period *t*, and x_{iS} is the GDP per capita in country *i* after sufficiently long time *S*. The period *t* lasts several years and is precisely specified, while the future time *S* is unknown. The greater the difference $x_{iS} - x_{it}$, the greater the average growth rate; $g'(x_{iS} - x_{it}) > 0$.

We do not know the GDP per capita (steady state) level x_{iS} (in period *S*). However, according to the growth theory, its determinants are technology level, technology growth rate, discount factor, population growth rate, and depreciation rate. If these determinants are the same for countries in a country group, then these countries will be equally rich in period *S*.

We can rewrite the growth function into the celebrated $\beta - convergence$ econometric specification

$$y_{it} = \beta_{0i} + \beta_1 x_{it} + u_{it}, \tag{2}$$

where the β_{0i} parameter captures the growth determinants of the *i*-th country in the period *t*, β_1 is the convergence parameter, and u_{it} is the stochastic term.

Let us move on to formulating hypotheses regarding the questions. Let us repeat that we ask if the wealth differences between the various countries in a group are shrinking. If so, the countries with low initial GDP per capita levels grow faster, and $\beta_1 < 0$. We form the first hypothesis as $\beta_1 \geq 0$. Rejecting the hypothesis, we reject the idea that the wealth differences between the countries stay the same or even increase.

The second question is if all countries' GDP per capita in a group will be the same in time S. According to the second hypothesis, $\beta_{0i} = \beta_0 \forall i$. If we do not reject it, all the countries per capita GDPs will converge to the same level, ceteris paribus.

3 DATA

Our analysis is based on annual data published by EUROSTAT. The essential data are the real gross domestic product in millions of euros taken from the table GDP and main components (output, expenditure and income). The other important ones are the population data taken from the table Population on 1 January by age and sex. All the data were gathered from 26 EU countries over several different periods. The consistent data are published for almost all EU countries (currently 27 countries) except Malta, which has published its GDP only since 2000, so we did not include it in the analysis. We divided the 29 years of data in two ways: the first into two fifteen-year periods from 1995 to 2009 and from 2009 to 2023, and the second into three ten-year periods from 1995 to 2004, from 2004 to 2013 and from 2013 to 2022.

In the model, we use variables that are not part of the data but can be calculated using existing variables. First, we create the variables representing real GDP in euros per capita in all periods. Then, we create the dependent variables as the average growth of real GDP in euros per capita according to the relations:

$$y = (GDPpc\_initial\_period / GDPpc\_final\_period)^{(1/T)} - 1 \tag{3}$$

where *T* equals 15 when the sample is divided into two periods and 10 when divided into three periods, the explanatory variable is the logarithm of real GDP in euros per capita in the initial period. So, for example, for 29 years divided into three periods, the dependent variable in the middle period (2004-2013) is the average growth of real GDP in euros per capita for the years 2004 to 2013, and the explanatory variable is the logarithm of real GDP in euros per capita in 2004.

The euro area was founded by 11 countries in 1999, to which Greece joined two years later. We used data from 1999 to 2023 for 12 countries in the case of the EMU. We used the same data preparation method as in the case of the EU. We divided the 25 years of data in two ways: the first into two thirteen-year periods from 1999 to 2011 and from 2011 to 2023, and the second into three nine-year periods from 1999 to 2007, from 2007 to 2015 and from 2015 to 2023.

4 RESULTS

Table 1 shows the estimation results of individual equations for the EU. The results obtained for 10-year periods are on the left side, and the results for 15-year periods are on the right.

Table 1: EU – The Results of Estimation of Individual Equations for Individual Periods

```
|==============================================|=====================|
|                     y04        y13       y22  |   y09         y23   |
|             period  (1)        (2)       (3)  |   (1)         (2)   |
|----------------------------------------------|---------------------|
|l_HDPpc95          -0.010***                  | -0.013***           |
|                   (0.003)                    | (0.002)             |
|l_HDPpc04                    -0.014***        |                     |
|                             (0.003)          |                     |
|l_HDPpc09                                     |            -0.011***|
|                                              |            (0.003)  |
|l_HDPpc13                              -0.010** |                   |
|                                       (0.004) |                    |
|Constant            0.124***  0.146*** 0.125***|  0.146***   0.123***|
|                   (0.031)   (0.032)  (0.043) | (0.020)    (0.030)  |
|----------------------------------------------|---------------------|
|Observations         26        26        26   |  26          26     |
|R2                  0.271     0.431     0.194  |  0.595       0.344  |
|Residual Std. Error 0.014     0.012     0.014  |  0.009       0.011  |
|F Statistic         8.932*** 18.185*** 5.762** | 35.299***  12.561***|
|==============================================|=====================|
 Note:                                      *p<0.1; **p<0.05; ***p<0.01
```

Source: own computations

Table 2 for the euro area shows the same order of results as Table 1 for the EU.

Table 2: EMU – The Results of Estimation of Individual Equations for Individual Periods

```
|==============================================|=====================|
|                     y07        y15       y23  |   y11         y23   |
|             period  (1)        (2)       (3)  |   (1)         (2)   |
|----------------------------------------------|---------------------|
|l_HDPpc99           0.004                     |  0.007**            |
|                   (0.006)                    | (0.003)             |
|l_HDPpc07                     0.009           |                     |
|                             (0.009)          |                     |
|l_HDPpc11                                     |            -0.006   |
|                                              |            (0.007)  |
|l_HDPpc15                              -0.002 |                     |
|                                       (0.007) |                    |
|Constant           -0.026    -0.102    0.030  | -0.063**    0.072   |
|                   (0.066)   (0.089)  (0.075) | (0.026)    (0.078)  |
|----------------------------------------------|---------------------|
|Observations         12        12        12   |  12          12     |
|R2                  0.046     0.109     0.006  |  0.440       0.057  |
|Residual Std. Error 0.009     0.012     0.012  |  0.384       0.011  |
|F Statistic         0.479     1.229     0.061  |  7.866**     0.600  |
|==============================================|=====================|
 Note:                                      *p<0.1; **p<0.05; ***p<0.01
```

Source: own computations

Tables 3 and 4 show the results of estimating fixed effect models for EU panel data, first for three 10-year periods and then for two 15-year periods. We can see also the results of pool F test, which does not reject the null hypothesis with Fixed Effect Model in both cases.

Table 3: EU – The Results of Estimation of Panel Data Fixed Effect Model for 3 Periods

```
Oneway (individual) effect Within Model    Balanced Panel: n = 26, T = 3, N = 78

Coefficients:
           Estimate Std. Error t-value  Pr(>|t|)
X        -0.0342718  0.0073703   -4.65 2.384e-05 ***

Total Sum of Squares:    0.013014
Residual Sum of Squares: 0.0091394
R-Squared:               0.29774
F-statistic:             21.6222 on 1 and 51 DF, p-value: 2.3839e-05

pooltest F statistic null hypothesis: FEM is preferred
                         F = 1.8633, df1 = 25, df2 = 51, p-value = 0.02998
```

Source: own computations

Table 4: EU – The Results of Estimation of Panel Data Fixed Effect Model for 2 Periods

```
Oneway (individual) effect Within Model    Balanced Panel: n = 26, T = 2, N = 52

Coefficients:
           Estimate Std. Error t-value  Pr(>|t|)
X        -0.0245955  0.0040874 -6.0174 2.761e-06 ***

Total Sum of Squares:    0.002464
Residual Sum of Squares: 0.0010064
R-Squared:               0.59157
F-statistic:             36.2095 on 1 and 25 DF, p-value: 2.7614e-06

pooltest F statistic null hypothesis: FEM is preferred
                         F = 3.8341, df1 = 25, df2 = 25, p-value = 0.0006551
```

Source: own computations

Finally, Tables 5 and 6 present the results of fixed-effect models for the euro area panel data, first for three 9-year periods and then for two 13-year periods. We can also see the results of the pool F test, which does not reject the null hypothesis with the Fixed Effect Model (FEM) in the first case. The test result for two periods rejects the null hypothesis, and FEM is not preferred. Since FEM was not preferred, we supplemented our analysis with the Hausman test, which does not reject the null hypothesis of a preference for the Random Effect Model.

Table 5: EMU – The Results of Estimation of Panel Data Fixed Effect Model for 3 Periods

```
Oneway (individual) effect Within Model    Balanced Panel: n = 12, T = 3, N = 36

Coefficients:
          Estimate Std. Error t-value Pr(>|t|)
X        -0.084552   0.019435 -4.3506 0.000235 ***

Total Sum of Squares:    0.005482
Residual Sum of Squares: 0.0030072
R-Squared:               0.45144
F-statistic:             18.9277 on 1 and 23 DF, p-value: 0.00023498

pooltest F statistic null hypothesis: FEM is preferred
                         F = 2.9627, df1 = 11, df2 = 23, p-value = 0.0135
Hausman Test null hypothesis: REM is preferred
                         chisq = 0.22491, df = 1, p-value = 0.6353
```

Source: own computations

Table 7 shows the results obtained by estimating the Random Effect Model. The parameters are statistically insignificant in this case.

Table 6: EMU – The Results of Estimation of Panel Data Fixed Effect Model for 2 Periods

```
Oneway (individual) effect Within Model    Balanced Panel: n = 12, T = 2, N = 24

Coefficients:
          Estimate Std. Error t-value Pr(>|t|)
X        -0.010253   0.028392 -0.3611   0.7249

Total Sum of Squares:    0.0010989
Residual Sum of Squares: 0.001086
R-Squared:               0.011715
F-statistic:             0.130396 on 1 and 11 DF, p-value: 0.72486

pooltest F statistic null hypothesis: FEM is preferred
                         F = 1.1802, df1 = 11, df2 = 11, p-value = 0.3942
```

Source: own computations

Table 7: EMU – The Results of Estimation of Panel Data Random Effect Model for 2 Periods

```
Oneway (individual) effect Random Effect Model (Swamy-Arora)
Balanced Panel: n = 12, T = 2, N = 24

Effects:           var   std.dev  share
idiosyncratic 9.873e-05 9.936e-03 0.879
individual    1.360e-05 3.688e-03 0.121
theta: 0.1146

Coefficients:
               Estimate Std. Error z-value Pr(>|z|)
(Intercept) -0.0209807  0.0549222 -0.3820   0.7025
X            0.0029761  0.0052944  0.5621   0.5740

Total Sum of Squares:    0.0021257
Residual Sum of Squares: 0.0020956
R-Squared:               0.014159
Chisq:                   0.315977 on 1 DF, p-value: 0.57404
```

Source: own computations

5 CONCLUSIONS

The paper tested the convergence hypotheses in 26 European Union and 12 European Monetary Union countries. In the first step, we estimated the $\beta - convergence$ specification in EU countries in the entire time range. We rejected the hypothesis of a nonpositive convergence coefficient, indicating that the income differences between countries have been decreasing over time.

Then, we split the time range into two parts. Using the panel data, we estimated the specification. We again rejected the hypothesis of a nonpositive convergence coefficient, but we also rejected the hypothesis that each country's intercept parameters are the same. In our interpretation, we rejected the hypothesis that the countries' wealth will be the same in the distant future.

The same result comes from the same procedure after splitting the time range into three parts. In this case, dummy variables capturing time effects were statistically significant. According to the first hypothesis, this result may mean that countries' convergence speed has not stayed the same. However, it may correspond to the fact that the beginnings and ends of the three time periods fell on different phases of the business cycle.

After repeating the panel data estimation of the convergence specification for the 12 European Monetary Union countries, we could not reject the hypothesis that the convergence parameter is positive in both cases. We have no evidence that the income differences between countries are decreasing.

According to our estimates, even if the countries adopted a common currency, their differences in economic activity do not decrease. From this point of view, the membership of countries in the European Union is more successful, even if the idea that all countries will one day be equally rich is not yet realistic. Even here, however, we should be more careful in our conclusions. Let us be aware that the European Union consists of two groups of countries, post-communist and the others. In the period under review, post-communist countries underwent a massive transformation from a centrally planned economic system to a market economy. The transformation resulted in structural changes, thanks to which the steady state in the countries increased and approached the steady state of Western European countries. The reduction of income differences between post-communist and Western European countries may not have resulted from integrating the countries into the EU but from the transformation of post-communist countries. This idea is supported by the indication that we cannot confirm the convergence of the 12 countries of the monetary union, which belong only to the Western European group. In addition, part of the economic transformation of post-communist countries was their integration into Western European market structures, including entry into the European Union. Therefore, we do not reject the idea that the integration of post-communist countries is not the cause of convergence according to the first hypothesis but a consequence of their transformation.

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References

[1] Barro, R. J. (1991). Economic Growth in a Cross Section of Countries. *Quarterly Journal of Economics* 106(2), pp. 407-443.
[2] Barro, R. (2008). *Macroeconomics: a modern approach.* Mason: Thomson South Western.
[3] Barro, R. J. and Sala-i-Martin, X. (1991). Convergence across States and Regions. *Brookings Papers on Economic Activity* 1, pp. 107-182.
[4] Barro, R. J. and Sala-i-Martin, X. (1992a). Convergence. *Journal of Political Economy* 100(2), pp. 223-251.
[5] Barro, R. J. and Sala-i-Martin, X. (1992b). Regional Growth and Migration: A Japan–United States Comparison. *Journal of the Japanese and International Economies* 6(4), pp. 312-346.
[6] Barro, R. J. and Sala-i-Martin, X. (2004). *Economic growth, 2nd ed.* Cambridge: The MIT Press.
[7] Baumol, W. J. (1986). Productivity Growth, Convergence, and Welfare: What the Long-Run Data Show. *American Economic Review* 76, pp. 1072-1085.
[8] DeLong, J. B. (1988). Productivity Growth, Convergence, and Welfare: Comment. *American Economic Review* 78, pp. 1138-1154.
[9] Eurostat, (2016). Database – Eurostat. [online] Available at: http://ec.europa.eu/eurostat/data/database [Accessed 21 March 2024].
[10] Szomolányi, K., Lukáčik, M. and Lukáčiková, A. (2011). The β-convergence of the EU27 Countries. In: *29th International Conference on Mathematical Methods in Economics 2011*. Praha: Professional Publishing, pp. 287-291.

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