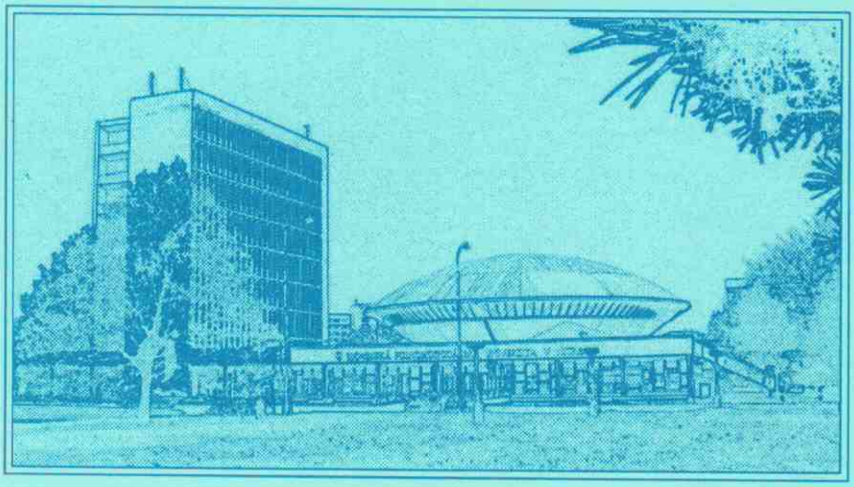
QUANTITATIVE METHODS IN ECONOMICS
(MULTIPLE CRITERIA DECISION MAKING XI)



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Technical Editor: Valéria Magáthová, Slovak Agricultural University in Nitra
Web Editor: Martin Lukáčik, Economic University in Bratislava

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A TRAFFIC CONTROL DECISION MODEL IN THE ROAD LANE CLOSURE CASE

ONDREJ BARTL
UNIVERSITY OF ŽILINA, SLOVAKIA

1. Introduction

Transport systems sometimes require to have closed one of two directions for traffic by reason of maintenance, repair or reconstruction. Only one path then remains to be used by transport means in both directions. The rules to control the traffic process should be available in such a case. If the total consumption of time spent by transport means waiting in queues on both sides of the closure sector is to be minimized, then two alternative approaches can be used. One is based on the employment of programme control, the other represents feedback control of a dynamic system which corresponds to the traffic process under conditions of closure. The core idea of the former approach is to model the traffic situation evolution as a discrete accumulation process with a fixed length of the accumulation cycle. The evolution of the number of transport means that wait on each side of the closure sector at the end of the accumulation cycle is represented by a discrete-time Markov chain with an infinite state space. The technique of probability generating functions can be used to determine performance measures of the accumulation process. It enables us to find the optimal duration of the accumulation cycle and its (fixed) division into two lengths of periods which are successively used to release groups of waiting transport means for motion through the closure sector in both directions [3]. The latter approach is represented by sequential decision making on the optimal number of transport means released to move through the closure sector with respect to the current amount of transport means waiting on both sides of the closure area at the decision epoch. It requires to build the corresponding semi-Markov decision model of the traffic control process. The details are given in the text that follows. Each approach represents some approximation to the real world phenomena. According to the specific conditions and traffic features in the site where a closure is planned, one of them may be chosen to control the traffic situation in the case of a one-directional closure occurrence.

2. Road lane closure and traffic control

Let us consider a road with two lanes, i.e. only one lane is used for each direction. If one lane has to be closed between two points then all the traffic in the closure sector must be performed in the other (not closed) lane. It means that groups of vehicles from one and the other direction are alternately released to move through the closure sector along the open lane. The concept of feedback control may be employed to govern traffic in the closure sector. Then decisions on the group size have to be made at successive decision epochs that are given by the points in time when transits of vehicle groups through the closure sector are completed. A decision on how many vehicles should form the group in the just activated direction is taken after having reviewed the current situation in queue lengths on both sides of the closure sector. When the transit of the vehicle group through the closure sector is finished, the opposite direction is activated and a similar decision on the group size is made. Traffic lights guarantee that no further vehicle may enter the closure sector in neither direction while a specified group of vehicles is moving inside the sector.

3. Semi-Markov decision model of traffic control for a closure occurrence

Due to computational feasibility, a finite-state-space model is suitable to be built. Thus, the queue lengths in the model are limited in contrast to the real situation in front of the closure sector on each sector side. Then the probabilistic description of the traffic situation evolution in the model approximates real behaviour with some differences. The higher the queue limits, the less the distortion between the model and the real situation.

For the modelling purposes, the dynamics of vehicle motion in the closure sector is simplified. A constant time interval is considered for each vehicle to pass through the closure sector. The distance between consecutive vehicles moving in the closure sector is represented by a constant time gap. A safety time interval is incorporated into the model when directions for motion of vehicle groups are changed. If no vehicle is released to move through the closure sector in a given direction, then some very short switching time interval is considered to elapse in order to get into the epoch when the situation in the opposite direction may be reviewed.

One direction (no matter which) can be denoted as the main direction and referred to as a 1-direction. The other direction is then the opposite direction referred to as a 0-direction. The traffic intensity is supposed to be describable by mutually independent Poisson processes with rates $\lambda_0 > 0$ vehicles arriving per unit time in the 0-direction and $\lambda_1 > 0$ vehicles arriving per unit time in the 1-direction. The queue limits are L^0 vehicles waiting in the 0-direction and L^1 vehicles waiting in the 1-direction, where $0 < L^l < \infty, l = 0,1.$ Any vehicle that arrives at the closure area in the 0-direction when the queue length is filled up to the limit L^0 is considered as rejected. The same is valid for any vehicle arriving in the 1-direction when the queue limit L^1 is filled up. Such a vehicle does not join the corresponding queue in the model construction as if being held somewhere outside the closure area. This lack of coincidence between the model and the reality can be partially eliminated by a proper modification of the expected cost function that reflects the cost associated with the waiting of vehicles in the queues in front of the closure sector.

The process of the traffic situation evolution in the road lane closure case can be regarded as a stochastic discrete event dynamic system. State-dependent traffic control in the closure sector may then be modelled by a semi-Markov decision model. To determine a cost-effective policy for taking (control) actions, the corresponding semi-Markov decision model has to be built. The following text specifies the model cornerstone elements.

The time set of the system, when the system behaviour is subject to control, is the set $T = [0, \infty)$. Time points when the current situation in the system is reviewed and the subsequent decision on an action is made are decision epochs. They can be numbered by non-negative integers starting with zero used for the initial decision made at time $T_0 \equiv 0$. Decision epochs T_1, T_2, ... are points in time, when transits of vehicle groups through the closure sector in their directions are completed. Each transit completion is a discrete event that generates a new decision epoch occurrence, at which the group size in the opposite direction is determined and the corresponding transit begins. Time intervals between two consecutive decision epochs are review intervals or stages. Their ordinal numbers are non-negative integers, where zero denotes the initial review interval. The set of ordinals used for numbering stages as well as decision epochs is the set $E = \{0,1,...\}$. The set $T_E = \{T_0, T_1,...\}$ is the set of decision epochs. Review intervals $\tau_n = T_{n+1} - T_n, n \in E,$ and decision epochs $T_n, n \in E,$ are (non-negative) random variables.

At any time each of both directions can be in an active or passive mode of operation. A direction is said to be active at time $t \in T$ if it is being used for transit through the closure sector at time t. At the same time the other direction is passive. The modes of both directions are interchanged at decision epochs.

The state of the system at time $t \in T$ is a vector random variable $\boldsymbol{X}(t)$ with components $X^0(t), X^1(t), X^2(t)$, where $X^l(t)$ is the number of vehicles waiting in the queue in front of the closure sector in the l-direction, $l = 0,1$, at time t and $X^2(t)$ is the number of the active direction at time t. In the model construction, state variables $X^l(t)$, $l = 0,1$, follow the constraints $0 \le X^l(t) \le L^l$, l = 0,1, for each $t \in T$. The system state at the nth decision epoch is $\boldsymbol{X}_n = (X_n^0, X_n^1, X_n^2) \equiv (X^0(T_n), X^1(T_n), X^2(T_n)) = \boldsymbol{X}(T_n), n \in E.$ The state space is given by the set $S = \{(i^0, i^1, i^2) : i^0 \in S^0, i^1 \in S^1, i^2 \in S^2\} = \{\boldsymbol{i} : \boldsymbol{i} \in S^0 \times S^1 \times S^2\}$, where $S^l = \{0,1,...,L^l\}$, l = 0,1, and $S^2 = \{0,1\}$.

Traffic in the closure sector is controlled by taking actions at decision epochs. Let $A(t)$ denote the action valid at time $t \in T$, i.e. an action under realization at time t as the result of the last decision made up to time t (with t included). Then the action $A_n \equiv A(T_n)$ chosen by the decision at the nth decision epoch, $n \in E$, is the transit of A_n vehicles through the closure sector in the active direction. Hence, the quantity A_n prescribes how many vehicles have to move (in a group) from one to the other end of the closure sector in the direction that is active in the nth stage, i.e. in the review interval between decision epochs T_n and $T_{n+1}, n \in E.$ The principle to release at least one of waiting vehicles in the active direction is employed in the traffic control. It should contribute to avoiding situations when some vehicle waits too long in front of the closure sector. If the queue in the active direction is empty at the current decision epoch, no transit is obviously made in this direction and after a (very short) switching time interval the opposite direction becomes active. Thus, the set $U_{\boldsymbol{i}}$ of feasible actions in state $\boldsymbol{i} = (i^0, i^1, i) \in S$ is given by:

$$U_{\boldsymbol{i}} = U_{i^0,i^1,i} = \begin{cases} \{0\}, & i^0 = 0, i^1 \in S^1, i = 0, \\ \{1,2,...,i^0\}, & i^0 \in S^0 \setminus \{0\}, i^1 \in S^1, i = 0, \\ \{0\}, & i^0 \in S^0, i^1 = 0, i = 1, \\ \{1,2,...,i^1\}, & i^0 \in S^0, i^1 \in S^1 \setminus \{0\}, i = 1. \end{cases} \tag{1}$$

The set of all possible actions in the system states with the l-direction in the role of the current active direction is $U^l = \bigcup_{(i^0,i^1) \in S^0 \times S^1} U_{i^0,i^1,l} = \{0,1,...,L^l\}, l = 0,1.$ The action space U is stated by $U = \bigcup_{(i^0,i^1,i) \in S} U_{i^0,i^1,i} = \bigcup_{\boldsymbol{i} \in S} U_{\boldsymbol{i}} = U^0 \cup U^1 = \{0,1,..., \max(L^0, L^1)\}.$

Specific traffic conditions in some closure systems can require to admit decisions with no vehicle released to move through the closure sector in the active direction in spite of the non-empty queue in such a direction at the current decision epoch. Then it is necessary to register the number of consecutive rejections to dispatch vehicles in the given direction in a continuous series of equal decisions. Therefore an extended notation for the system state values must be used in the model. State $\boldsymbol{i}$ can then be expressed in the form $\boldsymbol{i} = \left(i^0 . i_r^0, i^1 . i_r^1, i\right)$, where i^l is the current number of vehicles in the queue of the l-direction, $l = 0,1$, i_r^l is the current number of successive rejections for transit of vehicles in the l-direction through the closure sector in an uninterrupted sequence of rejections, $l = 0,1$, and i is the number of the currently active direction. This refined notation is only supposed to be employed for low numbers of vehicles waiting in the queue of a direction, i.e. for $i^l \in S_e^l = \{1,2,\dots,s^l\}, 0 < s^l < L^l, l = 0,1$. The values of i_r^l are considered to be limited by the (small) maximal number s_r^l with $0 < s_r^l < \infty, l = 0,1$, i.e. $i_r^l \in S_r^l = \{0,1,\dots,s_r^l\}, l \in \{0,1\}$. Otherwise the principle to release at least one of waiting vehicles in the active direction has to be kept. Then the set of feasible actions if the l-direction ($l = 0,1$) is the active direction at a current decision epoch is then given by

$$U_{i^0,i^1,l} = \begin{cases} \{0\}, & i^l = 0,\ i^{1-l} \in S^{1-l}, \\ \{0,1,\dots,i^l\}, & 0 < i^l \le s^l,\ 0 \le i_r^l < s_r^l,\ i^{1-l} \in S^{1-l}, \\ \{1,2,\dots,i^l\}, & 0 < i^l \le s^l,\ i_r^l = s_r^l,\ i^{1-l} \in S^{1-l}, \\ \{1,2,\dots,i^l\}, & s^l < i^l \le L^l,\ i^{1-l} \in S^{1-l}, \end{cases}$$

for each state $\left(i^0, i^1, l\right) \in S = S^0 \times S^1 \times S^2$.

Duration of a review interval between two consecutive decision epochs is expressed by the stage length function $\tau(\boldsymbol{i}, a)$, where $\boldsymbol{i} \in S$ is the state of the closure system at a current decision epoch and $a \in U_{\boldsymbol{i}} \subset U$ is the subsequent action. If the current decision is to release the group of $a > 0$ vehicles for motion through the closure sector in the active direction, then the function τ approximately reflects the corresponding transit time. If no vehicle is dispatched in the active direction, then the function τ represents the duration of a very short switching interval to activate the opposite direction. Time consumption associated with the transit of $a \ge 0$ vehicles in the l-direction is given by $T^l(a) = \operatorname{sgn}(a)\left(m^l + (a-1)d^l + s^l\right) + \left(1 - \operatorname{sgn}(a)\right)h^l$ for $l = 0,1$. The sign function $\operatorname{sgn}(a)$ is equal to 1 if $a > 0$, to 0 if $a = 0$, to -1 if $a < 0$. The constant m^l represents the transit time of one vehicle to pass through the closure sector in the l-direction. The constant d^l is the time gap between two vehicles passing consecutively through the closure sector in the l-direction. The constant s^l is the safety time interval after the (theoretical) end of the transit of a vehicle group in the l-direction to guarantee that this transit is fully completed. The constant h^l is the switching time interval required to elapse before activating the opposite direction after no vehicle has been dispatched in the l-direction. All the constants are non-negative real numbers. Using $T^l(a)$, the stage length function can be stated by $\tau(\boldsymbol{i},a)=\tau(i^0,i^1,i,a)=T(i,a)=iT^1(a)+(1-i)T^0(a)$ for $\boldsymbol{i}=(i^0,i^1,i)\in S$ and $a\in U_{i^0,i^1,i}\subset U$. The duration of the nth stage is a non-negative random variable $\tau_n\equiv\tau(X_n^0,X_n^1,X_n^2,A_n)$, $n\in E=\{0,1,...\}$. The expected stage length function $\overline{\tau}(\boldsymbol{i},a)$ determines the conditional expectation of stage duration given that the system state at the beginning of a stage is $\boldsymbol{i}$ and the subsequent action is a. The length of a review interval is a deterministic variable in the model construction if the state and the action at the review interval origin are specified. Then:

$$\begin{aligned}
&\overline{\tau}(\boldsymbol{i},a)=\overline{\tau}(i^0,i^1,i,a)=E\left[\tau(X_n^0,X_n^1,X_n^2,A_n)\middle|X_n^0=i^0,X_n^1=i^1,X_n^2=i,A_n=a\right]\\
&=E\left[T(X_n^2,A_n)\middle|X_n^0=i^0,X_n^1=i^1,X_n^2=i,A_n=a\right]=E[T(i,a)]=T(i,a)=iT^1(a)+(1-i)T^0(a)\\
&=i\left[\operatorname{sgn}(a)(m^1+(a-1)d^1+s^1)+(1-\operatorname{sgn}(a))h^1\right]\\
&\quad+(1-i)\left[\operatorname{sgn}(a)(m^0+(a-1)d^0+s^0)+(1-\operatorname{sgn}(a))h^0\right],\boldsymbol{i}=(i^0,i^1,i)\in S,a\in U_{i^0,i^1,i}\subset U,n\in E.
\end{aligned}\tag{2}$$

Let $Z^l(t)$ denote the number of vehicles that arrive in front of the closure sector in the l-direction (l = 0,1) within a t time unit interval. Then the number of vehicles arriving in the l-direction during the nth review interval, $n\in E$, is given by the non-negative random variable $Z_n^l=Z^l(T_{n+1}-T_n)$. Note that $T_{n+1}-T_n=\tau_n\equiv\tau(X_n^0,X_n^1,X_n^2,A_n)=T(X_n^2,A_n),n\in E.$ Hence, $Z_n^l=Z^l(T(X_n^2,A_n)),n\in E,l\in\{0,1\}$.

Behaviour of the closure system is described by the following transition equation:

$$\begin{aligned}
X_{n+1}^0&=\min\{X_n^0-(1-X_n^2)A_n+Z_n^0,L^0\},n\in E,\\
X_{n+1}^1&=\min\{X_n^1-X_n^2A_n+Z_n^1,L^1\},n\in E,\\
X_{n+1}^2&=1-X_n^2,n\in E.
\end{aligned}\tag{3}$$

Transition equation (3) reflects the fact that limits L^0 and L^1 are considered in the model for queue lengths, which leads to rejections of vehicles arriving in the case of queue saturation.

Stochastic dynamics of the closure system evolution is specified by transition probabilities

$$\begin{aligned}
&p(\boldsymbol{i},\boldsymbol{j},a)\equiv p_{i,j}(a)=p(i^0,i^1,i,j^0,j^1,j,a)=P\{\boldsymbol{X}_{n+1}=\boldsymbol{j}\,|\,\boldsymbol{X}_n=\boldsymbol{i},A_n=a\}\\
&=P\left\{X_{n+1}^0=j^0,X_{n+1}^1=j^1,X_{n+1}^2=j\,\middle|\,X_n^0=i^0,X_n^1=i^1,X_n^2=i,A_n=a\right\},\\
&\boldsymbol{i}=(i^0,i^1,i)\in S,\boldsymbol{j}=(j^0,j^1,j)\in S,a\in U_{i^0,i^1,i}\subset U,n\in E=\{0,1,...\}.
\end{aligned}\tag{4}$$

To determine transition probabilities (4), we denote particular events in (4) by symbols:

$$A=\{X_{n+1}^0=j^0\}, B=\{X_{n+1}^1=j^1\}, C=\{X_{n+1}^2=j\}, D=\{X_n^0=i^0, X_n^1=i^1, X_n^2=i, A_n=a\}.$$

Then:

$$\begin{aligned} p(\boldsymbol{i},\boldsymbol{j},a) &= P(ABC|D)=P(ABCD)/P(D)=P(A|BCD)P(B|CD)P(C|D)P(D)/P(D) \\ &= P(A|BCD)P(B|CD)P(C|D)=P(A|D)P(B|D)P(C|D), \end{aligned} \quad (5)$$

where the last equality follows since the events *A*, *B*, *C* are independent. It is the consequence of mutual independence of arrival processes in the 0-direction and 1-direction and a change of the active direction. As vehicles are supposed to arrive in the *l*-direction according to the Poisson process with rate λ_l, $l=0,1$, the probability of *k* arrivals in the *l*-direction during an interval with duration of *t* time units is $p_k^l(t)\equiv P\{Z^l(t)=k\}=e^{-\lambda_l t}(\lambda_l t)^k/k!,\ k=0,1,\ldots,\ t\geq 0$, $l=0,1$. Using (5) we can state transition probabilities (4) as follows:

$$\begin{aligned} & p(\boldsymbol{i},\boldsymbol{j},a)=p(i^0,i^1,i,j^0,j^1,j,a)=p(i^0\xrightarrow{i,a}j^0)\,p(i^1\xrightarrow{i,a}j^1)\,p(i\xrightarrow{a}j), \\ & \boldsymbol{i}=(i^0,i^1,i)\in S, \boldsymbol{j}=(j^0,j^1,j)\in S, a\in U_{i^0,i^1,i}\subset U, \end{aligned} \quad (6)$$

where

$$p(i^l\xrightarrow{i,a}j^l)\equiv P\{X_{n+1}^l=j^l\,|\,X_n^l=i^l, X_n^{1-l}=i^{1-l}, X_n^2=i, A_n=a\}$$

$$=\begin{cases} \sum\limits_{k=L^l-i^l+[1-l+(2l-1)i]}^{\infty} p_k^l(T(i,a)), & j^l=L^l, \\ p^l_{j^l-i^l+[1-l+(2l-1)i]a}(T(i,a)), & i^l-[1-l+(2l-1)i]a\leq j^l<L^l, \\ 0, & 0\leq j^l<i^l-[1-l+(2l-1)i]a, \end{cases} \quad (7)$$

$$0\leq i^l\leq L^l, l=0,1, i\in S^2=\{0,1\}, a\in U_{i^0,i^1,i}\subset U, n\in E,$$

and

$$\begin{aligned} & p(i\xrightarrow{a}j)\equiv P\{X_{n+1}^2=j\,|\,X_n^0=i^0, X_n^1=i^1, X_n^2=i, A_n=a\}=\begin{cases}1, & j=1-i, \\ 0, & j=i,\end{cases} \\ & i\in S^2=\{0,1\}, i^0\in S^0, i^1\in S^1, a\in U_{i^0,i^1,i}\subset U, n\in E. \end{aligned} \quad (8)$$

The specification of transition probabilities has to be modified if the extended representation of state values is considered. Roughly speaking, when the *l*-direction is a current active direction (i.e. $l=i$), transitions between state values with an extension are made with non-zero probabilities only from $i^l.i_r^l$ to $j^l.(i_r^l+1)$ if the current action is $a=0$ and the transition $i^l\to j^l$ is possible, and from $i^l.i_r^l$ to $j^l.0$ if the current action is $a>0$ and the transition $i^l\to j^l$ is possible. When the *l*-direction is a current passive direction (i.e. $l=1-i$), transitions between state values with an extension are made with non-zero probabilities only from $i^l.i_r^l$ to $j^l.i_r^l$ if the transition $i^l \to j^l$ is possible and the current action is $a \geq 0$. Details are omitted here.

The consumption of time spent by vehicles waiting in the queues on both sides of the closure sector is expressed in a transformed form by a cost function. If the system state at a current decision epoch is $\boldsymbol{i}$ and the subsequent action is a then the corresponding cost incurred during a review interval is given by the cost function $C(\boldsymbol{i}, a)$. The cost associated with the waiting of vehicles in the queues on both closure sector sides in the nth stage ($n \in E$) is a random variable $C_n \equiv C(\boldsymbol{X}_n, A_n)$. The mean cost incurred in any stage is specified by the expected cost function

$$\begin{aligned}
\overline{C}(\boldsymbol{i}, a) &= \overline{C}(i^0, i^1, i, a) = E\left[C(X_n^0, X_n^1, X_n^2, A_n) \middle| X_n^0 = i^0, X_n^1 = i^1, X_n^2 = i, A_n = a\right] \\
&\equiv E\left[C_n \middle| \boldsymbol{X}_n = (i^0, i^1, i), A_n = a\right] = E\left[W_n + R_n \middle| \boldsymbol{X}_n = (i^0, i^1, i), A_n = a\right] \\
&= E\left[W(X_n^0, X_n^1, X_n^2, A_n) \middle| X_n^0 = i^0, X_n^1 = i^1, X_n^2 = i, A_n = a\right] \\
&\quad + E\left[R(X_n^0, X_n^1, X_n^2, A_n) \middle| X_n^0 = i^0, X_n^1 = i^1, X_n^2 = i, A_n = a\right] \\
&= \overline{W}(i^0, i^1, i, a) + \overline{R}(i^0, i^1, i, a), \boldsymbol{i} = (i^0, i^1, i) \in S, a \in U_{i^0, i^1, i} \subset U, n \in E = \{0,1,\ldots\},
\end{aligned} \tag{9}$$

where the random variables W_n and R_n represent the waiting cost and the rejection cost respectively.

The expected waiting cost function $\overline{W}$ reflects the expected cost associated with the waiting of vehicles in the queues with finite queue limits $L^l, l = 0,1,$ during a review interval. To state the function $\overline{W}$, we need some auxiliary functions and variables. The number of allowed registrations of new vehicles in the queue of the l-direction is given by the function $N^l(i^l, i, a)$ and the number of vehicles dispatched to move through the closure sector in the l-direction by the function $M^l(i, a)$, $l = 0,1$. Taking i^l, i, a as the realizations of random variables X_n^l, X_n^2, A_n (for any specified $n \in E$), we have $N^l(i^l, i, a) = L^l - i^l + [1 - l + (2l - 1)i]a, l = 0,1,$ and $M^l(i, a) = [1 - l + (2l - 1)i]a, l = 0,1.$ That is, $N^0(i^0, i, a) = L^0 - i^0 + (1 - i)a,$ $N^1(i^1, i, a) = L^1 - i^1 + ia,$ $M^0(i, a) = (1 - i)a,$ $M^1(i, a) = ia$ for $i^0 \in S^0, i^1 \in S^1, i \in S^2 = \{0,1\}, a \geq 0.$ Recall that i indicates the current active direction. As vehicles arrive in front of the closure sector in the l-direction according to the Poisson process $\{K^l(t), t \geq 0\}$ with rate $\lambda_l, l = 0,1$, the time of the kth arrival in the l-direction after a current decision epoch is the Erlang-k random variable Y_k^l with parameters $k \in \{1,2,...\}$ and λ_l, whose probability density function is denoted by $g_k^l(y), y \geq 0$. Let $w^l > 0$ be the cost incurred per vehicle waiting in the queue of the l-direction per unit time, $l = 0,1$. Using a real-valued function $I(y,t)$ such that $I(y,t) = t - y$ for $y \leq t$ and $I(y,t) = 0$ for $y > t$, we can write

$$\begin{aligned}
\overline{W}(i^0,i^1,i,a) &= \overline{W}(\boldsymbol{i},a) = E\left[\sum_{l=0}^{1} w^l \int_{T_n}^{T_n+T(X_n^2,A_n)} X^l(t)dt \,\middle|\, X_n^0 = i^0, X_n^1 = i^1, X_n^2 = i, A_n = a\right] \\
&= \sum_{l=0}^{1} E\left[W_{\text{que}}^l(X_n^l, X_n^2, A_n) + W_{\text{reg}}^l(X_n^l, X_n^2, A_n) \,\middle|\, X_n^0 = i^0, X_n^1 = i^1, X_n^2 = i, A_n = a\right] \\
&= \sum_{l=0}^{1} \left\{E\left[W_{\text{que}}^l(X_n^l, X_n^2, A_n) \,\middle|\, \boldsymbol{X}_n = (i^0,i^1,i), A_n = a\right] + E\left[W_{\text{que}}^l(X_n^l, X_n^2, A_n) \,\middle|\, \boldsymbol{X}_n = (i^0,i^1,i), A_n = a\right]\right\} \\
&= \sum_{l=0}^{1} \left[\overline{W}_{\text{que}}^l(i^l,i,a) + \overline{W}_{\text{reg}}^l(i^l,i,a)\right], \boldsymbol{i} = (i^0,i^1,i) \in S, a \in U_{i^0,i^1,i} \subset U, n \in E,
\end{aligned} \tag{10}$$

where

$$\begin{aligned}
\overline{W}_{\text{que}}^l(i^l,i,a) &= E\left[w^l\left[X_n^l - M^l(X_n^2,A_n)\right]T(X_n^2,A_n) \,\middle|\, X_n^0 = i^0, X_n^1 = i^1, X_n^2 = i, A_n = a\right] \\
&= w^l\left[i^l - M^l(i,a)\right]T(i,a), \boldsymbol{i} = (i^0,i^1,i) \in S, a \in U_{i^0,i^1,i} \subset U, n \in E, l \in \{0,1\},
\end{aligned} \tag{11}$$

and

$$\begin{aligned}
\overline{W}_{\text{reg}}^l(i^l,i,a) &= E\left[\sum_{k=1}^{N^l(X_n^l,X_n^2,A_n)} w^l I(Y_k^l, T(X_n^2,A_n)) \,\middle|\, X_n^0 = i^0, X_n^1 = i^1, X_n^2 = i, A_n = a\right] \\
&= E\left[\sum_{k=1}^{N^l(i^l,i,a)} w^l I(Y_k^l, T(i,a))\right] = \sum_{k=1}^{N^l(i^l,i,a)} w^l \int_0^\infty E\left[I(Y_k^l, T(i,a)) \,\middle|\, Y_k^l = y\right] g_k^l(y)dy \\
&= \sum_{k=1}^{N^l(i^l,i,a)} w^l \left\{T(i,a) - \frac{k}{\lambda_l}\left[1 - p_k^l(T(i,a))\right] + \left(\frac{k}{\lambda_l} - T(i,a)\right)\sum_{j=0}^{k-1} p_j^l(T(i,a))\right\}, \\
&\boldsymbol{i} = (i^0,i^1,i) \in S, a \in U_{i^0,i^1,i} \subset U, n \in E, l \in \{0,1\}.
\end{aligned} \tag{12}$$

The function $\overline{W}_{\text{que}}^l$ reflects the expected waiting cost associated with vehicles staying in the queue of the l-direction at the beginning of a current review interval. The function $\overline{W}_{\text{reg}}^l$ represents the expected waiting cost associated with vehicles arriving during a current review interval and being registered as new ones in the queue of the l-direction within the queue limit of L^l waiting vehicles.

The expected rejection cost function $\overline{R}$ reveals the expected cost incurred during a current review interval in connection with the rejection of vehicles arriving when the related queue is filled up to its queue limit. As rejection is a virtual construction made in the model because of computational feasibility, the expected rejection cost function should be designed so as to build the model as close to the reality on the road as possible. Therefore the rejection cost function R consists of two terms R_{wcd} and R_{rej}. The former states the difference between the waiting cost function for the case of infinite queues and the case with finite queue limits. The latter represents a penalty cost function associated with rejecting vehicles when queues are saturated up to their (finite) limits. Then:

$$\begin{aligned}
&\overline{R}\left(i^0,i^1,i,a\right)=\overline{R}(\boldsymbol{i},a)=E\left[R_{\mathrm{wcd}}\left(X_n^0,X_n^1,X_n^2,A\right)+R_{\mathrm{rej}}\left(X_n^0,X_n^1,X_n^2,A\right)\middle|\boldsymbol{X}_n=\left(i^0,i^1,i\right),A_n=a\right]\\
&=E\left[\sum_{l=0}^{1}R_{\mathrm{wcd}}^l\left(X_n^l,X_n^2,A_n\right)+\sum_{l=0}^{1}R_{\mathrm{rej}}^l\left(X_n^l,X_n^2,A_n\right)\middle|X_n^0=i^0,X_n^1=i^1,X_n^2=i,A_n=a\right]\\
&=\sum_{l=0}^{1}\left\{E\left[R_{\mathrm{wcd}}^l\left(X_n^l,X_n^2,A_n\right)\middle|\boldsymbol{X}_n=\left(i^0,i^1,i\right),A_n=a\right]+E\left[R_{\mathrm{rej}}^l\left(X_n^l,X_n^2,A_n\right)\middle|\boldsymbol{X}_n=\left(i^0,i^1,i\right),A_n=a\right]\right\}\\
&=\sum_{l=0}^{1}\left[\overline{R}_{\mathrm{wcd}}^l\left(i^l,i,a\right)+\overline{R}_{\mathrm{rej}}^l\left(i^l,i,a\right)\right],\ \boldsymbol{i}=\left(i^0,i^1,i\right)\in S,a\in U_{i^0,i^1,i}\subset U,n\in E.
\end{aligned}\tag{13}$$

The expected waiting cost difference function $\overline{R}_{\mathrm{wcd}}^l$ related to the l-direction represents an addition to the expected waiting cost function $\overline{W}_{\mathrm{reg}}^l$ for a finite queue limit in the l-direction to get the expected waiting cost function $\overline{W}^l$ with an unlimited queue length in the l-direction, l = 0,1. The function $\overline{W}^l$ gives the expected cost associated with the waiting of arriving vehicles in the queue in front of the closure sector in the l-direction during a review interval in accordance with the real situation on the road. The number of arrivals by time $t\geq 0$ in the Poisson arrival process for the l-direction is represented by the Poisson distributed random variable $K^l(t)$ with mean $\lambda_l t$. The equation $E\left[K^l(u+t)-K^l(u)\right]=E\left[K^l(t)\right]$ holds for any $u\geq 0$ and $t\geq 0$ in the Poisson process of arrivals in the l-direction, l = 0,1. Hence,

$$\begin{aligned}
&\overline{W}^l\left(i^l,i,a\right)=E\left[\int_0^{T\left(X_n^2,A_n\right)}w^l\left[K^l\left(T_n+t\right)-K^l\left(T_n\right)\right]dt\,\middle|X_n^0=i^0,X_n^1=i^1,X_n^2=i,A_n=a\right]\\
&=E\left[\int_0^{T(i,a)}w^l\left[K^l\left(T_n+t\right)-K^l\left(T_n\right)\right]dt\right]=\int_0^{T(i,a)}w^lE\left[K^l\left(T_n+t\right)-K^l\left(T_n\right)\right]dt=\int_0^{T(i,a)}w^lE\left[K^l(t)\right]dt \quad (14)\\
&=\int_0^{T(i,a)}w^l\lambda_l t\,dt=w^l\lambda_l\frac{\left[T(i,a)\right]^2}{2},\ \boldsymbol{i}=\left(i^0,i^1,i\right)\in S,a\in U_{i^0,i^1,i}\subset U,T_n\in T,n\in E,l\in\{0,1\}.
\end{aligned}$$

Then the function $\overline{R}^{l}_{\text{wcd}}$ is given by

$$\begin{aligned}
\overline{R}^{l}_{\text{wcd}}\left(i^{l},i,a\right) &= E\left[W^{l}\left(X^{l}_{n},X^{2}_{n},A_{n}\right)-W^{l}_{\text{reg}}\left(X^{l}_{n},X^{2}_{n},A_{n}\right)\middle| X^{0}_{n}=i^{0},X^{1}_{n}=i^{1},X^{2}_{n}=i,A_{n}=a\right]\\
&= E\left[W^{l}\left(X^{l}_{n},X^{2}_{n},A_{n}\right)\middle| \boldsymbol{X}_{n}=\left(i^{0},i,a\right),A_{n}=a\right]-E\left[W^{l}_{\text{reg}}\left(X^{l}_{n},X^{2}_{n},A_{n}\right)\middle| \boldsymbol{X}_{n}=\left(i^{0},i,a\right),A_{n}=a\right] \qquad (15)\\
&= \overline{W}^{l}\left(i^{l},i,a\right)-\overline{W}^{l}_{\text{reg}}\left(i^{l},i,a\right),\ \boldsymbol{i}=\left(i^{0},i^{1},i\right)\in S, a\in U_{i^{0},i^{1},i}\subset U, n\in E, l\in\{0,1\}.
\end{aligned}$$

The expected penalty rejection cost function $\overline{R}^{l}_{\text{rej}}$ related to the *l*-direction reflects the expected penalty cost incurred in a review interval because new vehicles are rejected to be registered in the queue of the *l*-direction if they arrive when the queue limit L^{l} is filled up. Let $r^{l}(i)\geq 0$, $l = 0,1$, be the penalty cost incurred per vehicle rejected to join the queue in the *l*-direction (expressed in terms of the number of the current active direction given by *i*). The penalty cost in the form $r^{l}(i)=w^{l}t^{l}(i)$ can represent an approximation of the total cost associated with the waiting of a rejected vehicle in the next review intervals till the possibility to pass through the closure sector. The factor $t^{l}(i)$ is a delay function expressing an estimate of the waiting time spent by a rejected vehicle in the next review intervals till the possibility to be dispatched to move through the closure sector in the *l*-direction. The function $t^{l}(i)$ can be considered, for example, in the form $t^{l}(i)=\left(2t_{1-l}+t_{l}\right)\left(1-|l-i|\right)+\left(t_{1-l}+t_{l}\right)|l-i|$, $i\in S^{2}=\{0,1\}$, $l\in\{0,1\}$, where $t_{k}, k\in\{0,1\}$, is an estimate of the time that a vehicle in a queue spends by waiting during the transit of a group of vehicles through the closure sector in the *k*-direction. The group size in the estimate is supposed to be sufficient to prevent unlimited queue lengths from tending to infinity. A possible form of the estimate can be, e.g., $t_{k}=m^{k}+s^{k}+\lambda_{k}T''d^{k}$, $k=0,1$, with an estimate $T''=\left(1+\lambda_{0}d^{0}+\lambda_{1}d^{1}\right)\left(m^{0}+s^{0}+m^{1}+s^{1}\right)$ of the cycle length for the transit time of vehicle groups in one and the other direction. The structure of the estimate T'' is given by $T''=t''_{0}+t''_{1}$ with $t''_{k}=m^{k}+s^{k}+\lambda_{k}T'd^{k}$, $k=0,1$, and $T'=m^{0}+s^{0}+m^{1}+s^{1}$. Superscripts as well as subscripts in the expressions above denote the number of a direction (with 1 used for the main direction and 0 for the opposite one). Having determined penalty costs $r^{l}(i)$, $i\in S^{2}=\{0,1\}$, $l\in\{0,1\}$, in some way and using an indicator function $J(y,t)$ such that $J(y,t)=1$ for $y\leq t$ and $J(y,t)=0$ for $y>t$, we can write

$$\overline{R}^l_{\text{rej}}\left(i^l,i,a\right)=E\left[r^l\left(X_n^2\right)\sum_{k=N^l\left(X_n^l,X_n^2,A_n\right)+1}^{\infty}J\left(Y_k^l,T\left(X_n^2,A_n\right)\right)\Big|X_n^0=i^0,X_n^1=i^1,X_n^2=i,A_n=a\right]$$

$$=\int_0^{\infty}E\left[r^l(i)\sum_{k=N^l\left(i^l,i,a\right)+1}^{\infty}J\left(Y_k^l,T(i,a)\right)\Big|Y_k^l=y\right]g_k^l(y)dy,\ \boldsymbol{i}=\left(i^0,i^1,i\right)\in S,a\in U_{i^0,i^1,i}\subset U,n\in E,l\in\{0,1\},$$

which yields

$$R^l_{\text{rej}}\left(i^l,i,a\right)=r^l(i)\left\{E\left[Z^l\left(T(i,a)\right)\right]-N^l\left(i^l,i,a\right)+\sum_{j=0}^{N^l\left(i^l,i,a\right)-1}\left[N^l\left(i^l,i,a\right)-j\right]p_j^l\left(T(i,a)\right)\right\},\tag{16}$$
$$\boldsymbol{i}=\left(i^0,i^1,i\right)\in S,a\in U_{i^0,i^1,i}\subset U,l\in\{0,1\}.$$

Note that $E\left[Z^l\left(T(i,a)\right)\right]=\sum_{k=0}^{\infty}k\,p_k^l\left(T(i,a)\right)=\sum_{k=0}^{\infty}ke^{-\lambda_lT(i,a)}\frac{\left[\lambda_lT(i,a)\right]^k}{k!}=\lambda_lT(i,a),l\in\{0,1\}.$

Joining (15) and (16) together gives the final expression for the expected rejection cost function $\overline{R}$:

$$\overline{R}\left(i^0,i^1,i,a\right)=R(\boldsymbol{i},a)=\sum_{l=0}^{1}\left\{r^l(i)E\left[Z\left(T(i,a)\right)\right]+\frac{1}{2}w^l\lambda_l\left[T(i,a)\right]^2+\frac{w^l}{2\lambda_l}\sum_{k=0}^{N^l\left(i^l,i,a\right)}k^2p_k^l\left(T(i,a)\right)\right.$$
$$-\left[w^l\left(T(i,a)+\frac{1}{2\lambda_l}\right)+r^l(i)\right]\sum_{k=0}^{N^l\left(i^l,i,a\right)}k\,p_k^l\left(T(i,a)\right)-\left(1-\sum_{k=0}^{N^l\left(i^l,i,a\right)}p_k^l\left(T(i,a)\right)\right)\tag{17}$$
$$\left.\times\left[w^l\left(T(i,a)-\frac{N^l\left(i^l,i,a\right)+1}{2\lambda_l}\right)+r^l(i)\right]N^l\left(i^l,i,a\right)\right\},\ \boldsymbol{i}=\left(i^0,i^1,i\right)\in S,a\in U_{i^0,i^1,i}\subset U.$$

Traffic situation in the closure area is controlled by dispatching groups of waiting vehicles to move through the closure sector by turns in one and the other direction. A decision on the group size (for the current active direction) is made with respect to the number of vehicles in the queues on both sides of the closure sector at the current decision epoch. Actions to control the closure system evolution at decision epochs are thus chosen according to the current state of the system at the corresponding decision epoch. After reviewing the system state, an action is always selected with certainty, not by chance. Such a prescription for action selection is a deterministic Markovian control rule. A contingency plan for taking action at all decision epochs over the planning horizon is a policy. The policy is a sequence of control rules. If δ_n denotes a control rule for selecting actions in all system states at the *n*th decision epoch, $n\in E,$ and π a policy to control the system evolution, then $\pi=\{\delta_n\}_{n\in E}$. The policy consisted of deterministic Markovian control rules is a deterministic Markovian policy referred to as a pure Markov policy.

The closure system is supposed to act under considered arrival and economic conditions for a relatively long period of time. An infinite planning horizon is therefore the proper representation of such long-term operation of the system. The evolution of the closure system governed in accordance with a feedback control scheme is modelled by the semi-Markov decision process $\{\boldsymbol{X}(t), t \geq 0\}$ with the expected stage length function (2) and the transition probabilities (4) specified in detail by (6), (7), (8). The system behaviour satisfies the Markovian property:

$$\begin{aligned}
&P\{\boldsymbol{X}_{n+1} = \boldsymbol{j} | \boldsymbol{X}_n = \boldsymbol{i}, A_n = a, \boldsymbol{X}_{n-1} = \boldsymbol{i}_{n-1}, A_{n-1} = a_{n-1}, \ldots, \boldsymbol{X}_0 = \boldsymbol{i}_0, A_0 = a_0\} \\
&= P\{\boldsymbol{X}_{n+1} = \boldsymbol{j} | \boldsymbol{X}_n = \boldsymbol{i}, A_n = a\}, \\
&\forall n \in E = \{0,1,\ldots\}, \forall T_{n+1}, T_n, \ldots, T_1 \in T = [0,\infty), T_0 \equiv 0, \\
&\forall \boldsymbol{j}, \boldsymbol{i}, \boldsymbol{i}_{n-1}, \ldots, \boldsymbol{i}_1, \boldsymbol{i}_0 \in S, \forall a, a_{n-1}, \ldots, a_1, a_0 \in U, \\
&\text{where } \boldsymbol{X}_n \equiv \boldsymbol{X}(T_n), A_n \equiv A(T_n), T_n \in T, n \in E.
\end{aligned} \tag{18}$$

Stationary arrival, transit and economic conditions of the closure system under consideration justify the employment of a stationary pure Markov policy

$$d^\infty = \{\delta_n\}_{n=0}^\infty \text{ with } \delta_n = d \text{ for all } T_n \in T = [0,\infty) \text{ and } n \in E = \{0,1,\ldots\}, \text{ where } d : S \to U,$$

to control the system evolution over the infinite planning horizon. The stationary pure Markov policy d^∞ is the sequence $(d, d, \ldots)$ of decision functions $d : S \to U$. Denoting the set of all decision functions by $\Delta = \{d \,|\, d : S \to U\}$, the set of feasible decision functions is given by $D = \{d \in \Delta : d(\boldsymbol{i}) \in U_{\boldsymbol{i}} = U_{i^0,i^1,i} \subset U, \boldsymbol{i} = (i^0, i^1, i) \in S\}$. A stationary pure Markov policy is feasible if a feasible decision function $d \in D$ is repeatedly used to select actions at all decision epochs. If a policy $\pi = d^\infty$, where $d \in D$, controls the system evolution then the action chosen at the *n*th decision epoch is $A_n = d(\boldsymbol{X}_n) \in U_{\boldsymbol{X}_n} = U_{X_n^0, X_n^1, X_n^2} \subset U$, $n \in E$, with $d(\boldsymbol{X}_n) = d(X_n^0, X_n^1, X_n^2) \equiv d(X^0(T_n), X^1(T_n), X^2(T_n))$.

The criterion of interest, which measures the quality of the system performance over the infinite planning horizon when actions are prescribed by a stationary pure Markov policy, is represented by the long-run expected average cost per unit time as follows:

$$a^{d^\infty}(\boldsymbol{i}) = \lim_{t\to\infty} E\left[\frac{C(t)}{t} | \boldsymbol{X}_0 = \boldsymbol{i}, \pi = d^\infty\right], \boldsymbol{i} \in S, d \in D, \tag{19}$$

where $C(t)$ denotes the total cost associated with the system operation by time $t \geq 0$. Using random variables $C_n \equiv C(\boldsymbol{X}_n, A_n)$ and $\tau_n \equiv \tau(\boldsymbol{X}_n, A_n)$, the alternative criterion formulation can be used:

$$a^{d^\infty}(\boldsymbol{i}) = \lim_{m\to\infty} \frac{E\left[\sum_{n=0}^{m-1} C_n \middle| \boldsymbol{X}_0 = \boldsymbol{i}, \pi = d^\infty\right]}{E\left[\sum_{n=0}^{m-1} \tau_n \middle| \boldsymbol{X}_0 = \boldsymbol{i}, \pi = d^\infty\right]}, \boldsymbol{i} \in S, d \in D. \quad (20)$$

The alternative criterion value can be obtained for each initial state $\boldsymbol{i} \in S$ of the system according to

$$a^{d^\infty}(\boldsymbol{i}) \equiv a^{d^\infty}(i^0, i^1, i) = \lim_{m\to\infty} \frac{E\left[\sum_{n=0}^{m-1} C(X_n^0, X_n^1, X_n^2, A_n) \middle| X_0^0 = i^0, X_0^1 = i^1, X_0^2 = i, \pi = d^\infty\right]}{E\left[\sum_{n=0}^{m-1} \tau(X_n^0, X_n^1, X_n^2, A_n) \middle| X_0^0 = i^0, X_0^1 = i^1, X_0^2 = i, \pi = d^\infty\right]}$$

$$= \frac{\sum_{j^0\in S^0} \sum_{j^1\in S^1} \sum_{j\in S^2} \overline{C}(j^0, j^1, j, d(j^0, j^1, j)) \varphi_{i^0,i^1,i}^{d^\infty}(j^0, j^1, j)}{\sum_{j^0\in S^0} \sum_{j^1\in S^1} \sum_{j\in S^2} \overline{\tau}(j^0, j^1, j, d(j^0, j^1, j)) \varphi_{i^0,i^1,i}^{d^\infty}(j^0, j^1, j)}, \boldsymbol{i} = (i^0, i^1, i) \in S, d \in D, \quad (21)$$

where

$$\varphi_{\boldsymbol{i}}^{d^\infty}(\boldsymbol{j}) \equiv \varphi_{i^0,i^1,i}^{d^\infty}(j^0, j^1, j) = \lim_{m\to\infty} \frac{1}{m} \sum_{n=0}^{m-1} P\left\{X_n^0 = j^0, X_n^1 = j^1, X_n^2 = j \middle| X_0^0 = i^0, X_0^1 = i^1, X_0^2 = i, \pi = d^\infty\right\}$$

$$\equiv \lim_{m\to\infty} \frac{1}{m} \sum_{n=0}^{m-1} p_{i,j}^{(n)}(d^\infty), \boldsymbol{i} = (i^0, i^1, i) \in S, \boldsymbol{j} = (j^0, j^1, j) \in S, d \in D,$$

(22)

is the Cesaro limit of n-step transition probabilities $p_{i,j}^{(n)}(d^\infty)$ when n approaches infinity and the stationary pure Markov policy d^∞ governs the system evolution.

Let

$$a^*(\boldsymbol{i}) = \min_{d\in D} a^{d^\infty}(\boldsymbol{i}), \; \boldsymbol{i} \in S. \quad (23)$$

A stationary pure Markov policy $(d^*)^\infty$ is said to be optimal if

$$a^{(d^*)^\infty}(\boldsymbol{i}) = a^*(\boldsymbol{i}) \text{ for all } \boldsymbol{i} \in S. \quad (24)$$

Efficient methods [1], [2] represented by modified version of the methods designed for discrete-time Markov decision processes are available to determine an optimal stationary pure Markov policy in decision-making problems modelled by semi-Markov decision processes.

The application of an optimal stationary pure Markov policy to traffic control in the road lane closure case results in obtaining the minimal long-run expected average cost per unit time. It means that the consumption of time spent by vehicles waiting in the queues on both sides of the closure sector is minimized. The optimal stationary pure Markov policy

$\left(d^*\right)^\infty$ prescribes the most appropriate sizes of vehicle groups to move through the closure sector with respect to the numbers of vehicles waiting on each sector side. Traffic situation during the closure of a road lane can thus be directed according to a cost-effective feedback control procedure, which provides an alternative to choose besides the usual concept of programme control with a fixed cycle length.

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Ondrej Bartl

Department of Special Technologies, Faculty of Management Science and Informatics, University of Žilina, Veľký diel, 010 26 Žilina, Slovakia, tel.: +421-41-5134133, tel./fax: +421-41-5254613, e-mail: bartl@kst.fri.utc.sk

APPLICATION OF ORDINAL INFORMATION IN DECISION MODELS

HELENA BROŽOVÁ
CZECH UNIVERSITY OF AFRICULTURE PRAGUE, CZECH REPUBLIC

Abstract

This paper deals with special ways of data setting in decision models and with the possibility of their solving.

Payoffs and probabilities of states of nature are a quantitative crisp estimation in typical decision model applications. The quality of model results depends on quality of these estimations. Suppose now, that both estimations needn't be particular numbers and the decision-maker knows only preference information either for payoffs or for probabilities. That means, ordinal information for both data sets can be known.

ORESTE method for multiple attribute model solving can be used for decision models solving according to certain parallelism between these two types of models.

1. Introduction

The main problems of mathematical methods application can be explained by two questions: How to choose the proper model and solving method and how to set all data for model quantification. In this paper we would like to answer the second question particularly for decision models.

The decision theory and decision models enable the decision-maker to make rational decisions. There are many principles for decision model solving, but all of them need crisp data assessing. This may often lead to unsatisfactory results because of the accuracy of data estimation.

For this reason we suggest using methods for multiple attribute decision making for decision model solving whenever the crisp quantification is difficult or impossible. This idea is based on the facts of some similarity of both models.

We can see in the literature, that some simple methods as maximin, maximax or dominance are used in decision theory and multiple attribute decision making too. We want to show that more sophisticated methods as ORESTE are also applicable. These methods can be used in the case of special type of decision model input data. The first part of this paper describes the different kinds of decision model input information and special type of decision situations and possibility of their solving. The practical application of this idea is in the second part.

2. Input information types and solving possibility of decision models

Decision model can be easily conformable to multiple attribute model. List of alternative is in both model definitions, attributes correspond with states of nature and payoffs can be considered as attribute evaluation. Both models have the same matrix form so the payoff matrix can be taken as a criterion matrix of multiple attribute model with unique criterion the values of which differ according to the state of nature. **Table 1** shows the similarity of elements of both models.

		States of nature/Criteria			
		s_1/K_1	**s_2/K_2**	**.....**	**s_n/K_n**
Alternatives	**a_1**	v_{11}	v_{12}	.....	v_{1n}
	a_2	v_{21}	v_{22}	.....	v_{2n}
	.....	.....	.....	.....	.....
	a_m	v_{m1}	v_{m2}	.....	v_{m3}
Probabilities/Preferences		p_1	p_2	.....	p_n

Table 1: Correspondence of decision model and multiple attribute model elements

The input information for multiple attribute models can have a different character and the solving methods differ according to type of preference information.

It is not necessary to know the quantitative estimation of input data. Qualitative or soft preference information can be used in these models and this information type is less demanding for the decision-maker to assess than the quantitative information. Preference information sometimes may not be indicated at all. The preference information can be expressed in following ways:

- no information – importance of attributes in not known
- nominal information - importance of attributes is expressed by standard level of each attribute,
- ordinal information - relative importance of attributes or alternatives is set by ordinal data,
- cardinal information - relative importance of attributes or alternatives is assessed by cardinal data.

Standard form of decision model and its solving methods needs cardinal data for payoffs. Cardinal data for probabilities of states of nature have to be assessed in the case of decision under risk or can be missing in the case of decision under uncertainty. Quality of results depends on quality of input data, but this data is often an expert estimation and depends on the degree of judgement skill. Therefore it is necessary to make sensitivity analysis for model results.

Solving methods split into two main parts – the principles for decision under risk and the principles for decision under uncertainty.

Suppose now, that both data estimations needn't be crisp numbers. Generally there are six possible types of decision model definitions according to the available type of information. The possibilities are ordinal or cardinal information for payoffs and no information, ordinal or cardinal information for possibilities of states of nature.

Following table shows different characters of decision model in relation with type of input information.

		Information about payoffs	
		Ordinal	**Cardinal**
Information about probabilities of states of nature	**No information**	Decision model with qualitative estimation of payoffs under uncertainty	**Standard decision model under uncertainty**
	Ordinal	Decision model with qualitative estimation of payoffs and states of nature probabilities	Decision model with qualitative estimation of states of nature probabilities
	Cardinal	Decision model with qualitative estimation of payoffs under risk	**Standard decision model under risk**

Table 2: Type of input data and decision model character

From this view decision under uncertainty can be characterised as no information about the state of nature and cardinal information about payoffs. Decision under risk includes cardinal information about states of nature as well as about payoffs. These types of decision model are widely used and there are many methods for their solution.

Ordinal information of states of nature means that their probabilities could not be set exactly but we know which state of nature is more probable and which one is less probable. Corresponding decision situation with ordinal information about states of nature and cardinal information about payoffs lies between decision under risk and decision under uncertainty. We think, that this character of decision situation is very frequent in practice. These problems can be solved by lexicographic method, for instance, or ordinal information about states of nature can be transformed to cardinal information using some method for assessing weights.

Last three types of decision model are remarkably different. Decision situations with ordinal information about payoffs and no information, ordinal or cardinal information about states of nature could not be solved using classical methods for decision models solving. If payoffs could be in ordinal form, decision-maker could set only preference order or preference for all pair of alternatives and state of nature. This would be the possibility, how to deals with data, which are very imperfect or really soft.

Multiple attribute decision-making methods can be used for selection of the best alternative in these cases. For instance ORESTE method can be used for models with ordinal information about states of nature and payoffs.

3. Application of ORESTE method for decision model solving

Management of DOAGRA s.r.o. wants to select the optimal further organisation of machine repairing, because costs of own repair shop seem to be very high.

There are three main strategies - alternatives, the last one can be split into five alternatives:

- to keep the repair shop with no organisational changes and offer services for external clients;
- to expand the repair shop and offer services for external clients;
- to inactivate the repair shop (to sell it) and use external services – there are five companies, which offer repair shop services (Fronk, s. r. o., Agro Domažlice, a.s., Karpem, s. r. o., Bodas, a. s. and ZD Draženov.

The consequence of each alternative depends on the number of own repaired machines and on the expectant demand of external clients. While the estimation of the number of own repaired machines and its probability is known, there is no quantitative estimation of the demand of external clients and its probability.

Four states of nature on the first level have been identified – less than 40, 40 – 60, 60 – 80 and more than 80 reparations of own machine. Their probabilities have been calculated from the frequency of repairs made in previous years.

Four states of nature on the second level have been assumed – less than 40, 40 – 50, 50 – 60 and more than 60 reparations of external client machine. Probabilities of these states of nature could not be estimated, because there is no experience of the previous period. However, it is possible to set an ordinal information. Preferences of these states of nature have been set by simple method for pairwise comparison.

Preferences	< 40	40-50	50-60	> 60	SUM
< 40	1	0	0	1	**2**
40-50	1	1	1	1	**4**
50-60	1	0	1	1	**3**
> 60	0	0	0	1	**1**

Table 3: Pairwise comparison

Altogether there are sixteen states of nature on the main level. Their probabilities are unknown, only ordinal information about them is known, because global preference of states of nature can been calculated as a multiple of probabilities of the first level states of nature and results of pairwise comparison of the second level states of nature.

		States of nature	AI	AII	AIII	AIV	BI	BII	BIII	BIV	CI	CII	CIII	CIV	DI	DII	DIII	DIV
		Global preferences	0,074	0,148	0,111	0,037	0,23	0,46	0,345	0,115	1,38	2,76	2,07	0,69	0,316	0,632	0,474	0,158
		Criteria type	max	max	max	max	max	max	max	max	max	max	max	max	max	max	max	max
			Risk factor - number of internal repair															
		States of nature I	**< 40**				**40-60**				**60-80**				**> 80**			
		Probabilities	**3,70%**				**11,50%**				**69,00%**				**15,80%**			
			Uncertainty factor - number of external repair															
		States of nature II	**< 40**	**40-50**	**50-60**	**> 60**	**< 40**	**40-50**	**50-60**	**> 60**	**< 40**	**40-50**	**50-60**	**> 60**	**< 40**	**40-50**	**50-60**	**> 60**
		Preferences	**2**	**4**	**3**	**1**	**2**	**4**	**3**	**1**	**2**	**4**	**3**	**1**	**2**	**4**	**3**	**1**
Alternatives	Inactivate repair shop (external service using)	**Fronk, s. r. o.**	1 240 177	1 240 177	1 240 177	1 240 177	1 215 959	1 215 959	1 215 959	1 215 959	1 250 485	1 250 485	1 250 485	1 250 485	1 456 820	1 456 820	1 456 820	1 456 820
		Agro Domažlice, a.s.	1 140 479	1 140 479	1 140 479	1 140 479	1 188 075	1 188 075	1 188 075	1 188 075	1 371 801	1 371 801	1 371 801	1 371 801	1 458 170	1 458 170	1 458 170	1 458 170
		Karpem, s. r. o.	1 319 399	1 319 399	1 319 399	1 319 399	1 271 864	1 271 864	1 271 864	1 271 864	1 208 485	1 208 485	1 208 485	1 208 485	1 273 962	1 273 962	1 273 962	1 273 962
		Bodas, a. s.	1 248 766	1 248 766	1 248 766	1 248 766	1 256 864	1 256 864	1 256 864	1 256 864	1 187 485	1 187 485	1 187 485	1 187 485	1 221 191	1 221 191	1 221 191	1 221 191
		ZD Draženov	1 241 132	1 241 132	1 241 132	1 241 132	1 231 642	1 231 642	1 231 642	1 231 642	1 252 396	1 252 396	1 252 396	1 252 396	1 253 105	1 253 105	1 253 105	1 253 105
	Continue repair shop (external repair)	**Capacity keeping**	868 245	1 108 683	1 347 802	1 395 510	859 869	1 028 266	1 028 266	1 028 266	538 614	538 614	538 614	538 614	48 961	48 961	48 961	48 961
		50% expanding	946 762	1 197 015	1 443 765	1 688 071	924 535	1 174 637	1 421 765	1 666 701	904 753	1 153 872	1 400 648	1 645 609	891 321	1 139 256	1 213 261	1 213 261

Table 4: Original decision table

Payoffs - costs or revenue for each alternative depend on the number of reparations of own machine and also on the demand of the external client. Therefore, payoffs have been estimated for combination of intervals, which correspond with number of internal and external reparations. This estimation can be used as ordinal information rather then cardinal, because the real payoff values may differ around this estimation and we can only suppose that the order of these values does not change.

Decision model with described estimation of payoffs and states of nature probabilities have been constructed for this problem. **Table 4** shows the decision table of this situation.

Because this estimation has been taken only as ordinal information, this model data have been transformed, so payoffs are ranked in the order of their expected values (**Table 5**).

		States of nature	AI	AII	AIII	AIV	BI	BII	BIII	BIV	CI	CII	CIII	CIV	DI	DII	DIII	DIV
		Preference	2	5	3	1	7	10	9	4	14	16	15	13	8	12	11	6
		Criteria type	min	min	min	min	min	min	min	min	min	min	min	min	min	min	min	min
Alternatives	Inactivate repair shop (external service using)	**Fronk, s. r. o.**	4	4	6	6	4	4	5	5	3	3	4	4	2	2	2	2
		Agro Domažlice, a.s.	5	6	7	7	5	5	6	6	1	1	2	2	1	1	1	1
		Karpem, s. r. o.	1	1	3	3	1	1	2	2	4	4	5	5	3	3	3	3
		Bodas, a. s.	2	2	4	4	2	2	3	3	5	5	6	6	5	5	5	5
		ZD Draženov	3	3	5	5	3	3	4	4	2	2	3	3	4	4	4	4
	Continue repair shop (external repair)	**Capacity keeping**	7	7	2	2	7	7	7	7	7	7	7	7	7	7	7	7
		50% expanding	6	5	1	1	6	6	1	1	6	6	1	1	6	6	6	6

Table 5: Ordinal model definition

The model has been solved by ORESTE method. **Table 6** shows results of model solving. The best alternative is to inactivate the repair shop and use external services from Agro Domažlice, a.s.. Additional matrix of preference relations between pairs of alternatives enables further analysis of this result.

	ORESTE method		Additional matrix of preference relations						
	ri values	Order	Fronk, s. r. o.	Agro Domažlice, a.s.	Karpem, s. r. o.	Bodas, a. s.	ZD Draženov	Capacity keeping	50% expanding
Fronk, s. r. o.	861,5	3	Indifferent	Worse	Indifferent	Better	Indifferent	Better	Better
Agro Domažlice, a.s.	845,5	1	Better	Indifferent	Better	Better	Indifferent	Better	Better
Karpem, s. r. o.	862,5	4	Indifferent	Worse	Indifferent	Better	Indifferent	Better	Better
Bodas, a. s.	920	5	Worse	Worse	Worse	Indifferent	Worse	Better	Indifferent
ZD Draženov	853,5	2	Indifferent	Indifferent	Indifferent	Better	Indifferent	Better	Better
Capacity keeping	1065	7	Worse	Worse	Worse	Worse	Worse	Indifferent	Worse
50% expanding	920	5	Worse	Worse	Worse	Indifferent	Worse	Better	Indifferent

Table 6: Result of model solving

4. Conclusion

This paper examined a possibility of different type of input information for decision model and possibility of its solving. Decision models can be solved when their data estimation is either ordinal or cardinal.

Generally there are six possible types of decision situations according to the type of information available - ordinal or cardinal information for payoffs and no information, ordinal or cardinal information for possibilities of states of nature. The standard models describe two of them - decision under risk and under uncertainty.

Because data estimation is often difficult, decision model with ordinal information as for payoffs as for probabilities of states of nature can be very useful in real situations. The solving methods of multiple attribute model as ORESTE can be used for its solving.

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RNDr. Helena Brožová, CSc.
Dept. of Operations Research and System Analysis, Faculty of Economics and Management, Czech University of Agriculture, 165 21 Praha 6 – Suchdol, e-mail: brozova@pef.czu.cz

THE NEW ROLE OF STATISTICS IN LOCAL PUBLIC ADMINISTRATION

LUCIA BUZZIGOLI
DIPARTIMENTO DI STATISTICA "G. PARENTI", ITALY

Abstract

Statistical methods are gaining a new role inside the reformed public administration in Italy. In particular, the law provides for the use of statistical information systems in local management control together with a set of indicators which are considered of great importance for decision making support.

The paper presents the working project that the Department of Statistics "G. Parenti" of the University of Florence is making together with the Municipality of Florence, aiming at the reorganization of an efficient informative system inside the administrative structure.

1. Introduction

Over the last few years, governments all over Europe have been engaged in projects of administrative improvement. In Italy, too, a number of laws from the 90s onwards have radically changed the relationship between the political sphere and the administrative one and have produced significant changes in the public management style.

This paper aims to describe the role of statistics in this new framework and stems from a working project involving the Department of Statistics "G. Parenti" and the Municipality of Florence. The content of the paper can be summarised as follows.

The next paragraph (§2) synthetically describes the long and difficult process of reform that states the foundation of a new model of public management in Italy. Our attention is devoted to the administration at the level of local government and to the new system of checks and controls to evaluate the quality and quantity of public services.

The new role of statistics and the extent to which it can help to formalise, standardise and structure the massive amount of data held in administrative offices is presented in §3.
§4 gives a brief presentation of the working project with the description of its various steps.
Finally, the paper ends with some concluding remarks (§5).

2. The reform of Public Administration in Italy

All over the world governments are undertaking a substantial process of administrative and managerial improvement. This process has also been labelled as "global governance reform" to stress the fact that, despite some national differences, all governments have to face the same challenges and the same paradigm of change (Lynn and Stein, 2000).

Italy has not been immune to the international trend: in particular, the modernisation process under way in our country is connected to the European one, which is going towards the definition of standards for institutional change.

In Italy the main purpose of the reform was to overcome the complex and rigid system of forms and procedures, norms and dispositions, derogations and exceptions, which regulated the civil service in the past. This transformation involved all the levels of public administration, but in our case we are interested in the effect of these new practices on local government (Ladu, 2000).

The decennial process of reform began with the Parliamentary Act no.142/90, which, at local level, differentiated the roles and the functions of political organs (which plan the overall strategy) from those of the administrative ones (which have the responsibility to carry out the goals determined by the political level). The successive legislative decree no.23/93 (modified by no.80/99) established that each department in the local administration has an amount of resources – and a corresponding budget - to gain the predetermined objectives and that the department have the consequent responsibility of gaining those objectives.

Finally, two laws of 1997 (Bassanini laws) introduced the need of computerised systems inside administrations and the principles of good management and customer satisfaction (i.e. adjustment of services to the needs of the customers) as fundamentals of a modern government.

In synthesis, it is clear that the objective of legislation was to move from a highly bureaucratised society whose main principle was that of 'legality' to a new model of public management where the method of evaluation is the result of the administrative activity, that must prove how well public services are provided.

Moreover, the rapidly merging information technologies (from Internet to database theory and information systems reengineering), which have introduced new means of data storing, managing and transferring, have improved the access of citizens to information, transformed the way the organizations are structured, and, in one word, intensified the process of modernization of the new public administration model.

3. The role of statistics: using information in an effective way

All organizations need information to perform their mission and use, often inadvertently, information systems. Local governments, in particular, cannot work without a proper informational basis (Martelli, 2002).

One of the main subjects of public administration reform in Italy was the set of internal checks and controls that are needed to oversee the system and to evaluate the services delivered. The law explicitly refers to a unified statistical information system that contains the quantitative information to be used for management control, strategic control and evaluation (law no.59/1997; Dipartimento della Funzione Pubblica, 2001).

Local administrations are in possession of a lot of data that derive directly from their daily activities. To help local governments in their decision making process these data must be translated into information by means of appropriate methods. Statistics can offer the working methodologies and criteria that are essential to apply measurement to the entire organization as a system.

From a statistical viewpoint administrative data have a number of specific qualities: the collected information is often very rich, with a broad coverage (for example, the population register) and, last but not least, is inexpensive (because it derives directly from administrative activity). Nevertheless, the production of statistics is a secondary use of administrative data: this means that the measuring procedure is often out of the statistician’s control and that administrative concepts are not necessarily in harmony with statistical ones. Therefore, the translation from administration to statistics involves several aspects (Statistics Denmark, 2000), because it is well known that statistical concepts, classification systems, and other statistical aspects must respect a wide range of criteria.

The most important question concerns validity and relevance: can administrative data be used for estimating the concepts that are sought for in statistics? Are these data relevant?

The next requirement concerns reliability (do the data faithfully reflect reality?) and precision (are data recorded with a degree of precision suited to the needs of the statistic?).

Other aspects of great importance are connected to the temporal dimension - comparability over time (changes in legislation can alter data definition or content) and timeliness of information - and scalability issues (must the results be aggregated in some sense?).

Finally, internal comparability is essential to form an integrated system of data.

In this context the role of metadata is essential in facilitating sharing, querying and understanding the content of statistical information over the lifetime of data (UNECE Secretariat, 2000). An important issue in this respect is metadata quality (i.e. the degrees to which metadata serve their purpose). Strategies for the management, control and nurturing of metadata through metadata collection, production, storage and dissemination processes must necessarily be properly designed. The standardization of methods will be of great help and the dissemination of "best practices" could contribute to better data quality.

Using statistics to address policy and government issues means also conducting effective performance measurement and evaluation, which are essential to determine what policies have been achieved and whether the goals have been met. Outcome-based management is not a new subject in public sector: numerous alternative definitions and classifications of performance indicators are available in the literature and research has shown the critical role of these indicators in building an overall framework for performance management at local level (National Center for Public Productivity, 1997).

Traditional schemes present the distinction between productivity, efficiency, effectiveness and accountability, but sometimes these terms are used synonymously, without a clear distinction. The multidimensionality of these measurements raises various questions but it gives evidence of the separate dimensions of institutional performance, that – due to its complexity - cannot be combined into a single index.

Unfortunately, when we move from theoretical frameworks to real case studies there is the risk that what is measured is not always what is really needed but only what is at present available and measurable in that particular administrative context.

4. The working project with the Municipality of Florence

The project stems from the cooperative work of the Department of Statistics of the University of Florence and the Municipality of Florence and its objective is twofold: first of all, it represents a significant effort to provide a coherent description of the current informative system of the Municipality; secondly, it will be used to identify the critical areas of the system, where statistical methodology can give a substantial aid in better tracking administrative activities and providing policy makers and others with data that describes their activities and use of services.

The project work is organised in three steps, which are not independent, but strictly related to each other.

The first, long and complex step aims to describe the Municipality organization itself in order to identify the relationships among the various levels of political and administrative structures, to describe the decision process and to determine, as a consequence, the informational needs. This component could be labelled as the "informal human system" of the general information system (Land and McGregor, 2002), and is generally organised on a hierarchical basis. This is helpful to draw the diagram of departments and offices of the Municipality.

At the same time a new professional position has been created inside the administrative organization by the general director of the Municipality: the statistical referee. This employee (one for each department) has the duty to help his administrative office to deal with those aspects which are strictly connected to the statistical ones (the first duty will be to give assistance to the department during the survey on the informative sources that will be held in the second step). A training period is required that adequately educates and prepares these new experts: the Department of Statistics has organised a series of lectures, which are under way at the moment, on basic statistical subjects in order to share with these people a common language and a common basis of knowledge.

In the second step we are going to survey the data which are readily available inside the various departments to see how these data are organised in order to produce information and to form the basis of the decision making system (Martelli, 2002). This sort of 'data census' involves two different levels.

The first level analyses the flow of information that comes to Management Control from the various departments and the one that from Management Control goes back to the departments, because in the organization diagram the Management Control Area plays a fundamental role. A database of indicators has been created taking information from the administrative documents: this will be very useful to assess what is currently measured and to decide what, on the contrary, should be measured to improve the monitoring of the process.
The second level aims to survey all the informative sources which are available inside the various departments. This exhaustive survey will be made by means of an electronic questionnaire, which reports an analytical description of each (not necessarily electronic) file in terms of metadata: this means that the output will go directly into a database, which will be studied together with the diagram of the organization obtained in the first step.

In summary, the first two phases will form a clear report on the current situation of the informative system in the Municipality of Florence, which is a fundamental framework to identify the critical areas and the forward targets.

The third step is the logical conclusion of all this work and could be labelled as "analysis and action": starting from a critical analysis of all the collected data and of the organization we will try to spot weaknesses and threats, strengths and opportunities of the information system. Particular attention will be given to set of indicators used by the various departments and by Management Control Area, in order to find whether they do add value to the information system or not; this analysis will probably cause the refinement of existing indicators, the removal of the unnecessary ones and the introduction of new indicators that are important in the effective use of local monitoring systems, in order to define a coherent framework of statistical measures. Naturally, this phase can be performed only in strict coordination with Municipality departments.

5. Concluding remarks

As we have seen, important challenges result from the introduction of management principles and techniques inside the public administration. One of these challenges relates to a coherent and substantial use of statistical methods in the administrative context. The standardization of terms and methods and the use of common approaches can be difficult, therefore the risk is that the reform process remains of a formal nature, instead of producing substantial changes.

In this sense only the analysis of existent administrative realities with their real data comparability problems, collection effort burdens, etc., can give an accurate picture of what can/must be done to translate the legal regulations in effective government instruments.

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Lucia Buzzigoli

Dipartimento di Statistica "G. Parenti", Viale Morgagni 59, 50134 Italy, Tel.: +39 055 4237258, Fax: +39 055 4223560, e-mail: lucia@ds.unifi.it

DISCRETE MODELS OF PRODUCTION FUNCTION

MARTIN DLOUHÝ, JAN PELIKÁN
UNIVERSITY OF ECONOMICS PRAGUE, CZECH REPUBLIC

1. Introduction

Production functions model relations between quantities and combinations of inputs and quantities and combinations of output. Quantitative approaches to production function (econometrics and methods of operational research) serve as tools for setting the optimal structure of inputs and outputs. The efficiency evaluation of production units is a one way to performance improvement. In this paper, we present one class of production function models, which by their mathematical formulation belong to discrete models of mathematical programming (Pelikán, 2001). These models, by their mathematical way, identify inefficient production units, measure the level of inefficiency, and show the possibilities for improvement.

2. Models

The oldest discrete model is the FDH (Free Disposable Hull), which was first formulated by Deprins, Simar a Tulkens (1984). The basic motivation is to ensure that efficiency evaluation of production unit is affected from only actually observed units. The production possibility set in FDH is non-convex, meanwhile in Data Envelopment Analysis (Cooper, Seiford, Tone, 2000, Fiala, 2000) is convex. There are no a priory economic grounds to believe that the production set is convex. See discussion in the *Journal of Productivity Analysis* on this point (Thrall, 1999, Cherchye, Kuosmanen, Post, 2000).

Let us suppose that there is a reference set of n production units, which consume m inputs and produce p outputs. Each unit is characterised by vectors (x_k, y_k) and its performance is compared to the reference set defined with the matrices X a Y. A common technique to evaluate performance is by means of a radial distance measure given an orientation in the production space. In the input-oriented model, the objective is input minimisation. In the output-oriented model, we maximise output at a given level of input. A mathematical formulation of both FDH programs follows. Input-oriented model:

Minimise $\varphi + \varepsilon\,(1s^{+} + 1\,s^{-})$

s. t.

$$Y\lambda - s^{+} = y_k$$

$X\lambda + s^- = x_k\varphi$

$s^+ \geq 0, s^- \geq 0, 1\lambda = 1, \lambda \in \{0, 1\}.$

Output-oriented model:

Maximise $\theta - \varepsilon (1s^+ + 1 s^-)$

s. t.

$Y\lambda - s^+ = y_k\theta$

$X\lambda + s^- = x_k$

$s^+ \geq 0, s^- \geq 0, 1\lambda = 1, \lambda \in \{0, 1\}.$

The programs are solved by mixed binary programming with variables λ, s^+, s^- and variable φ or θ according to model orientation. The program has to be solved *n* times for each production unit. The value of objective function measures the level of efficiency. Generally, the FDH efficiency scores are higher than scores obtained with DEA, since the production unit cannot be dominated by convex combination of other units in the FDH model. Finally, we show that the discrete model differs from the continuous one only in condition for vector λ. The conditions for DEA model with constant returns to scale (known as CCR model) are

$s^+ \geq 0, s^- \geq 0, \lambda \geq 0.$

The extension of FDH is Free Replicability Hull (FRH). The model allows integer replications of observations. Binary condition for λ is replaced by integrality condition. The program is solved by mixed integer programming. Input-oriented model:

Minimise $\varphi + \varepsilon (1s^+ + 1 s^-)$

s. t.

$Y\lambda - s^+ = y_k$

$X\lambda + s^- = x_k\varphi$

$s^+ \geq 0, s^- \geq 0, \lambda \in Z_+$ (nonnegative integer).

Output-oriented model:

Maximise $\theta - \varepsilon (1s^+ + 1 s^-)$

s. t.

$Y\lambda - s^+ = y_k\theta$

$X\lambda + s^- = x_k$

$s^+ \geq 0, s^- \geq 0, \lambda \in Z_+$ (nonnegative integer).

The ERH (Elementary Replicability Hull) model allows integer replications of individual observations, but their combinations are rejected. The condition for vector λ is:

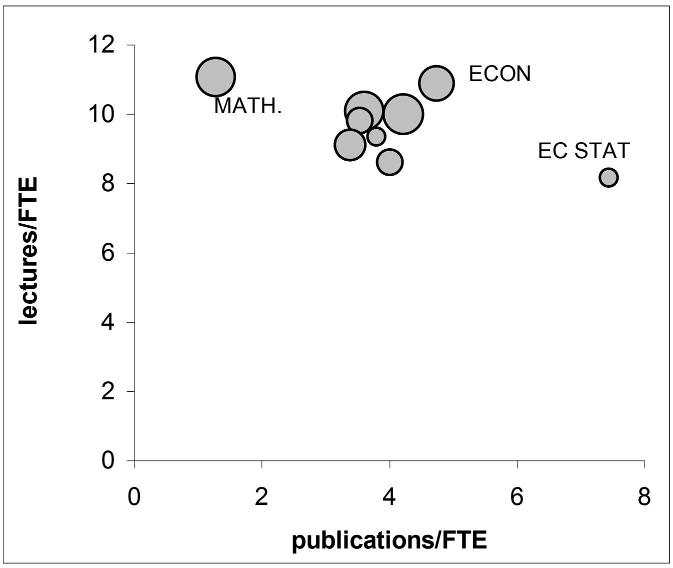
$\forall h\ \forall j \neq h\ \ \lambda_h \lambda_j = 0,\ \lambda_h \in Z_+$.

The necessary computations of FDH, ERH and FRH models or DEA models can be programmed in the software for mathematical programming (Jablonský, 1999, 2001) or in the specialised software. The enumeration algorithms are available for FDH, see e.g. Cherchye, Kuosmanen and Post (nondated).

3. Performance of University Departments

In this section, the FDH model is applied to performance evaluation of university departments at the Faculty of Informatics and Statistics, University of Economics, Prague. The data are from 2001. The input is average number of teaching staff in full-time equivalents. The outputs are total number of publication items in the university database and total number of lectures per week. The performance of university departments in the research and teaching activities is shown in Figure 1. The size of the circle represents the size of department in FTE. We observe three clusters: 1) the middle cluster of seven departments with department of econometrics (ECON) as a leader, 2) large department of mathematics (MATH), oriented on teaching, 3) small, research oriented department of economic statistics (EC STAT).

Figure 1: Performance of Department in Full-Time Equivalents



DEA (constant returns to scale) and FDH models were used for efficiency evaluation. The results are summarised in Table 1. In the case of constant return to scale, the DEA efficiency score is the same for both input- and output-oriented models. FDH is a less discriminatory method, therefore the FDH scores are higher than DEA scores. DEA identifies some inefficiency, but FDH does not (independently on model orientation). The FDH is more conservative in blaming the production units for inefficiency and allows that observed DEA-inefficiency may be caused by the scale.

Table 1: Efficiency Scores

Department	**DEA**	**FDH**	**FDH**
	both	**input**	**output**
Demography	83.71	100.00	100.00
Econometrics	100.00	100.00	100.00
Economic Statistics	100.00	100.00	100.00
Philosophy	80.81	100.00	100.00
Information Technology	92.42	100.00	100.00
Information Engineering	89.84	100.00	100.00
Mathematics	100.00	100.00	100.00
System Analysis	83.47	100.00	100.00
Statistics	92.02	100.00	100.00

4. Conclusion

We presented production function models, which, according to their mathematical formulations, belong to the category of discrete models. As a small illustrative example, we made evaluation of university departments. The measures of input and output were simplified, we do not have data on the number PhD students, the type and quality of teaching. The results may serve as basis for discussion. For a dynamic view on the performance of department, the Malmquist index (e. g. Färe, Grosskopf,, Lindgren, Roos, 1994) is one alternative. Some computations and discussion may be found in Dlouhý (2002).

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Martin Dlouhý, Jan Pelikán

Dpt. Econometrics, University of Economics Prague, W. Churchill Sq. 4, 130 67 Prague, Czech Republic, e-mail: dlouhy@vse.cz, pelikan@vse.cz

INTERTEMPORAL EFFICIENCY MEASURES IN WHEAT PRODUCTION INDUSTRY

PETER FANDEL
SLOVAK AGRICULTURAL UNIVERSITY IN NITRA, SLOVAKIA

Abstract

The paper presents results of analysis of efficiency and productivity change in the sector of wheat production in the period of years of 1996-2000. In the analysis intertemporal method of nonparametric Malmquist indices of TFP has been applied which is based on calculation of distance functions as reciprocal values of output orientated Farrell's technical efficiency measures. In the paper there are calculated indices of technical efficiency change, technical change and index of total factor productivity change. The analysis was performed on data of a 28 farms sample, which represents farms of all growing regions of Slovakia. Results show that farms struggle with the problems of decreasing efficiency and productivity. Within the reference period only 18% of farms were able to improve technical efficiency. Technological progress (innovation) was found in 7% of farms. Total factor productivity growth was found in 7% of farms.

1. Introduction

Wheat production industry passed in years 1996 - 2000 a difficult period with variation in both acreage and yields. Natural conditions were the most significant factors of the variation. But there were also technological factors of which influence on efficiency and productivity is the subject of the presented analysis.

2. Methodology

The aim of the paper is to analyse efficiency and productivity change in wheat production industry within the period of years 1996 - 2000. The analysis was performed on data of 28 farms, mostly cooperative farms sampled from all production regions of Slovakia. The basic methodology applied in the analysis was non-parametric methods of technical efficiency and total factor productivity analysis.

Non-parametric methods of productivity analysis are based either explicitly or implicitly on a Malmquist TFP index, which is under certain conditions equivalent to Tornquist index. The seminal paper in this field are Nishimizu a Page (1982) a Färe, Grosskopf, Norris and Zhang (1994). The first of these papers uses the Aignera and Chu

(1968) linear programming methods applied to construct parametric production frontiers and subsequently measure TFP growth as sum of an efficiency change component and a technical change component. The Färe, Grosskopf, Norris and Zhang (1994) paper takes the Malmquist index of TFP growth, defined in Caves, Christensen and Diewert (1982), and illustrates how component distance function can be estimated using DEA-like methods. They also showed how the resulting TFP indices could be decomposed into technical efficiency change and technical change components. The basic difference between these two approaches is that Nishimizu a Page (1982) use parametric methods while Färe, Grosskopf, Norris a Zhang (1994) use non-parametric methods.

The Malmquist index is defined using distance functions. They allow one to describe a multi-input, multi-output production technology without the need to specify a behavioural objective (cost minimisation or profit maximisation). It is possible to define input distance functions and output distance functions. It this paper we use output distance function, which considers a maximal proportional expansion of the output vector, given an input vector. The output distance function is applied in the methodology developed by Färe, Grosskopf, Norris and Zhang (1994).

Malmquist TFP index measures the TFP change between two data points by calculating the ratio of the distances of each data point relative to a common technology. Following Färe, Grosskopf, Norris and Zhang (1994) Malmquist output-orientated TFP change index between period s (the base period) and period t is given by

$$M_o(y_s, x_s, y_t, x_t) = \left[\frac{d_o^s(y_t, x_t)}{d_o^s(y_s, x_s)} \times \frac{d_o^t(y_t, x_t)}{d_o^t(y_s, x_s)}\right]^{1/2}, \quad (1)$$

where the notation $d_o^s(x_t, y_t)$ represents the distance from the period t observation to the period s technology. A value of M_o greater than one indicates positive TFP growth from period s to period t while a value less than one indicates a TFP decline. Equation (1) in fact is a geometric mean of two TFP indices. The first is evaluated with respect to period s technology and the second with respect to period t technology.

An equivalent way of writing the productivity index is

$$M_o(y_s, x_s, y_t, x_t) = \frac{d_o^t(y_t, x_t)}{d_o^s(y_s, x_s)}\left[\frac{d_o^s(y_t, x_t)}{d_o^t(y_t, x_t)} \times \frac{d_o^s(y_s, x_s)}{d_o^t(y_s, x_s)}\right]^{1/2}, \quad (2)$$

where the ratio outside the square brackets measures the change in the output-orientated measure of Farrell technical efficiency between period s and t. That is, the efficiency change is equivalent to the ratio of the Farrell technical efficiency in period t to the Farrell technical efficiency in period s. The remaining part of the index in equation (2)is a measure of technical change. It is the geometric mean of the shift in technology between the two periods, evaluated at x_t and also at x_s.

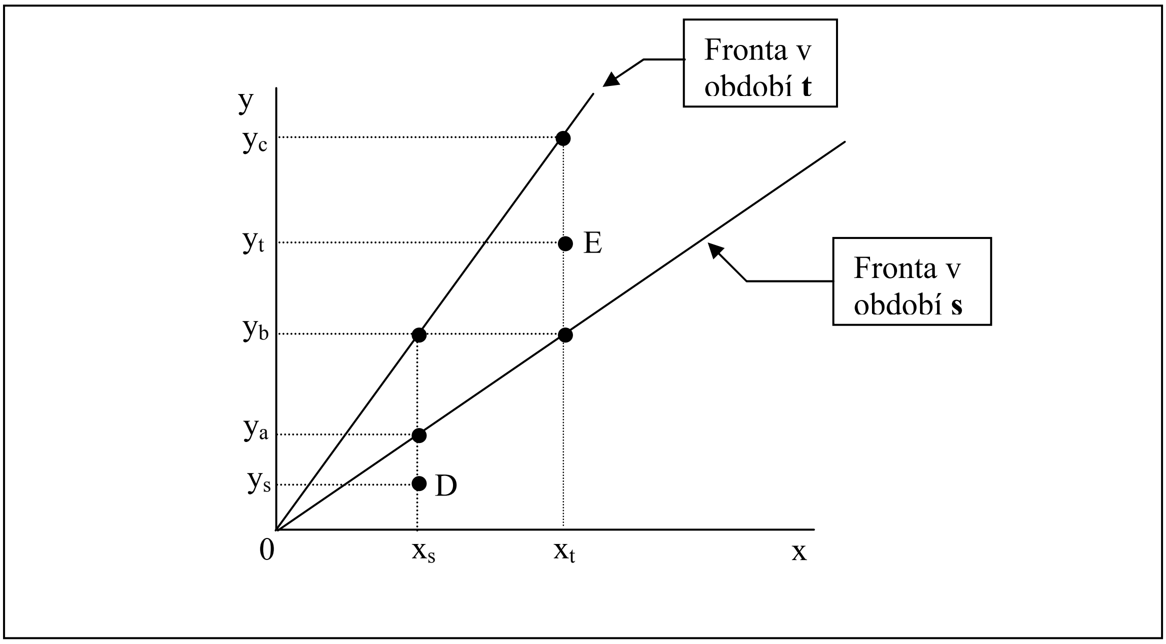

Figure 1 Malmquist productivity indices

Thus the two terms in equation (2) are:

$$\text{Efficiency change} = \frac{d_o^t(y_t, x_t)}{d_o^s(y_s, x_s)} \tag{3}$$

$$\text{Technical change} = \left[\frac{d_o^s(y_t, x_t)}{d_o^t(y_t, x_t)} \times \frac{d_o^s(y_s, x_s)}{d_o^t(y_s, x_s)}\right]^{1/2} \tag{4}$$

This decomposition is illustrated in Figure 1. The described technology assumes constant returns to scale involving a single input and a single output. The firm produces at the points D and E in periods s and t, respectively. In each period firm is operating bellow the technology for that period. Hence there is technical inefficiency in both periods. Using equation (3) and (4) we obtain:

$$\text{Efficiency change} = \frac{y_t / y_c}{y_s / y_a} \tag{5}$$

$$\text{Technical change} = \left[\frac{y_t / y_b}{y_t / y_c} \times \frac{y_s / y_a}{y_s / y_b} \right]^{1/2} \tag{6}$$

In empirical applications the four distance measures which appear in (1) should be calculated for each firm in each pair of adjacent time periods. There are a number of different methods that could be used to measure the distance functions, which make up the Malmquist TFP index. In the paper we use DEA-like linear programming methods. Following Färe, Grosskopf, Norris a Zhang (1994), and given that suitable panel data are available, we can calculate the required distances using DEA-like linear programming programs. For the i-th firm, we must calculate four distance functions to measure TFP change between two periods. This requires the solving of four linear programming problems. The required LP problems under constant returns to scale technology are:

$$[d_o^t(y_t, x_t)]^{-1} = \max_{\varphi, \lambda} \varphi,$$
subject to
$$\begin{aligned} \varphi y_{it} - Y_t\lambda &\leq 0 \\ X_t\lambda &\leq x_{it} \\ \lambda &\geq 0, \end{aligned} \tag{7}$$

$$[d_o^s(y_s, x_s)]^{-1} = \max_{\varphi, \lambda} \varphi,$$
subject to
$$\begin{aligned} \varphi y_{is} - Y_s\lambda &\leq 0 \\ X_s\lambda &\leq x_{is} \\ \lambda &\geq 0, \end{aligned} \tag{8}$$

$$[d_o^t(y_s, x_s)]^{-1} = \max_{\varphi, \lambda} \varphi,$$
subject to
$$\begin{aligned} \varphi y_{is} - Y_t\lambda &\leq 0 \\ X_t\lambda &\leq x_{is} \\ \lambda &\geq 0, \end{aligned} \tag{9}$$

$$[d_o^s(y_t, x_t)]^{-1} = \max_{\varphi, \lambda} \varphi,$$
subject to
$$\begin{aligned} \varphi y_{it} - Y_s\lambda &\leq 0 \\ X_s\lambda &\leq x_{it} \\ \lambda &\geq 0, \end{aligned} \tag{10}$$

Notation:

y_{it} is M×1 vector of outputs of the i-th firm in period t

x_{it} is K×1 vector of inputs of the i-th firm in period t

Y_t is N×M matrix of outputs of N firms in period t

X_t is N×K matrix of inputs of N firms in period t

λ is K×1 vector of constants

φ is scalar

If there are T times and N firms, N×(3T-2) LPs need to be solved.

3. Results and discussion

The efficiency and productivity analysis was performed on a sample of 28 farms within the period of years of 1996 - 2000. From the size point of view the smallest farm was of 49 ha and the largest one was of 1649 ha. Average acreage of wheat in the farm sample was 523 ha.

In accordance with the DEA rools we used:

1 variable for output (wheat production)

6 variables for inputs (wheat acreage/land, seeds, fertilizers, labour, other direct costs, indirect costs)

Table 1 shows statistics of used variables.

As a result of solving model (7) to (9) there are output-orientated measures of technical efficiencies which are equal to distance functions ($TE_o = 1/ \varphi$). Distance functions values are then used to calculate Malmquist indices. They are presented in the table 2, 3 a 4. For each farm three measures are listed:

- technical efficiency change [TECH]
- technical change [TCH]
- TFP productivity change[TFPCH]

We interpret Malmquist indices or their components in such a way that index lesser than one represents regressive change in farm performance, index greater than one represents progress in performance, and index equal to one means that there is no change in performance. As the Malmquist productivity index is given as a product of technical efficiency change and technical change they may behave contradictory. For example Malmquist productivity index greater than one (e.g. 1.25) may have a component of efficiency change less than (e.g. 0.5) and a component of technical change greater than one (e.g. 2.5). Then we interpret the components following Färe, Grosskopf, Norris, Zhang, 1994 this way:

- The component of efficiency change greater than one is an indicator of improvement in farm technical efficiency.
- The component of technical change greater than one indicate innovation

Thus decomposition provides alternative way of testing the productivity growth convergent ion and simultaneously enables to identify innovation, what is impossible by traditional measures.

Table 1 Minimum, mean, maximum, and standard deviation of output values in the period of years 1996-2000

		1996	1997	1998	1999	2000
Output 1 **Wheat prod. [t]**	Min.	325	253	253	174	160
	Max.	8767	9119	8563	8805	7756
	Mean	2815	2772	2613	1752	1989
	St.dev.	2241	2311	2145	1914	1937
Input 1 **Land [ha]**	Min.	80	65	65	49	70
	Max.	1602	1500	1511	1375	1649
	Mean	571	535	543	381	539
	St.dev.	421	382	388	324	398
Input 2 **Seeds [000 Sk]**	Min.	142	154	62	112	118
	Max.	2521	3340	3782	3764	3455
	Mean	922	998	1127	811	924
	St.dev.	626	725	833	794	767
Input 3 **Fertilizers [000 Sk]**	Min.	120	86	4	12	138
	Max.	6689	7628	7350	4992	9392
	Mean	1540	1724	1635	993	1359
	St.dev.	1533	1681	1778	1158	1848
Input 4 **Labour [000 Sk]**	Min.	5	1	1	1	1
	Max.	1915	1604	1417	737	1778
	Mean	276	259	249	200	301
	St.dev.	419	328	313	232	396
Input 5 **Other direct costs [000 Sk]**	Min.	302	354	261	312	417
	Max.	13550	16428	15686	14468	18773
	Mean	3691	4247	4520	3289	4578
	St.dev.	3177	3730	3984	3220	4533
Input 6 **Indirect costs [000 Sk]**	Min.	145	128	117	75	238
	Max.	4542	7010	4826	5078	5317
	Mean	1589	1768	1679	1377	1612
	St.dev.	1253	1717	1403	1372	1407
Number of farms		28	28	28	28	28

Table 2 Malmquist indices - 1996-1997

Year 1996-1997	Malmquist indices		
	TECH	TCH	TFPCH
Geom. mean	0,978	0,984	0,962
Minimum	0,694	0,709	0,641
Maximum	1,284	2,089	2,089
Stand. deviation	0,116	0,238	0,278

Table 3 Malmquist indices - 1997-1998

Year 1997-1998	Malmquist indices		
	TECH	TCH	TFPCH
Geom. mean	1,050	0,991	1,041
Minimum	0,857	0,773	0,697
Maximum	1,273	4,033	4,033
Stand. deviation	0,109	0,592	0,595

Table 4 Malmquist indices - 1998-99

Year 1998-99	Malmquist indices		
	TECH	TCH	TFPCH
Geom. mean	0,874	1,034	0,903
Minimum	0,532	0,708	0,403
Maximum	1,220	2,753	2,753
Stand. deviation	0,174	0,459	0,507

Table 5 Malmquist indices - 1999-2000

Year 1999-2000	Malmquist indices		
	TECH	TCH	TFPCH
Geom. mean	1,077	0,695	0,749
Minimum	0,556	0,231	0,218
Maximum	1,878	1,050	1,305
Stand. deviation	0,304	0,192	0,255

As it is evident from table 2 in year-to-year comparison of farm performance in years 1996 and 1997 a mean technical efficiency decline was 2.2%, technical change declined by 1.6% and productivity decline by 3.8%.

In a detailed analysis of the results of the comparison of years 1996 and 1997 in individual farms index of technical efficiency varies in the range of $\langle 0.694; 1.284 \rangle$, technical change index varies in range of $\langle 0,709; 2,089 \rangle$ and productivity change index varies in range of $\langle 0,641; 2,089 \rangle$. It means in percent that productivity varies form decline of 35.9% to increase of 108.9% .

Comparison of years 1997 and 1998 (table 3) shows that mean growth in performance was better. We observe 5% technical efficiency increase, 0.9% technical change decrease, and 4.1% productivity increase. Indices varied in following ranges:

technical efficiency change: $ZTE \in \langle 0,857; 1,273 \rangle$

technical change: TZ $\in \langle 0{,}773; 4{,}033 \rangle$

productivity change: ZTFP $\in \langle 0{,}697; 4{,}033 \rangle$

Comparison of years 1998 and 1999 (table 4) shows 9.7% productivity decrease, 12.4% technical efficiency decrease, and 3.4% progress in technical change.

Table 5 shows comparison of year 2000 and 1999. Year 2000 was extremely dry what caused low wheat yields. In performance measures it was reflected by 24.1% TFP productivity decrease, but the technical efficiency increased by 7.7%. Table 6 summarize comparison of all adjacent years.

Table 6 Malmquist indices - mean values for all farms

Year	Malmquist indices		
	TECH	TCH	TFPCH
1996-1997	0,978	0,984	0,962
1997-1998	1,050	0,991	1,041
1998-1999	0,874	1,034	0,903
1999-2000	1,077	0,695	0,749
Geom. mean	0,992	0,915	0,907

From the table 6 it is evident that within reference period there was yearly 0.8% decrease in technical efficiency, 8.5% decrease in technology, and 9.3% decrease in productivity.

In table 7 we present cumulative indices for all years and all farms. They were aggregated using geometric mean and the subsequent indices were converted into cumulative (chained) indices. Cumulative index of technical change 0.701 indicates that overall regress in wheat production technology was 29.9% and TFP productivity decrease was 32.3%.

Table 7 Cumulative Malmquist indices for all years and farms

Year	Cumulative Malmquist indices		
	TECH	TCH	TFPCH
1996	1,000	1,000	1,000
1997	0,978	0,984	0,962
1998	1,027	0,975	1,001
1999	0,898	1,008	0,904
2000	0,967	0,701	0,677

Cumulative indices over years 1996 – 2000 are plotted in figure 2

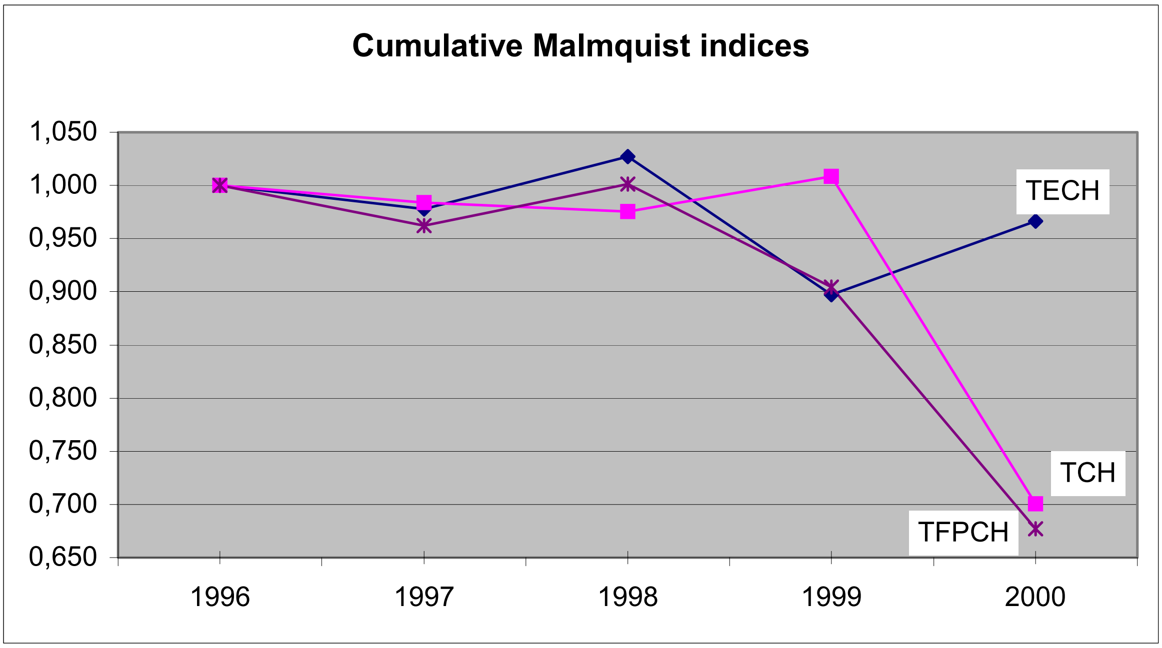

Figure 2 Cumulative Indices of Technical Efficiency Change, Technical Change and TFP Change

In table 8 we present summary of all Malmquist indices. As it is evident mean values are equal to means listed in table 6.

Table 8 Statistics of all Malmquist indices for all years and farms

	Cumulative Malmquist indices		
	TECH	TCH	TFPCH
Geom. mean	0,992	0,915	0,907
Minimum	0,532	0,231	0,218
Maximum	1,878	4,033	4,033
Stand. deviation	0,191	0,385	0,403

Malmquist indices statistics shows the best and the worst results in the sample. Maximum value of technical efficiency change (1.878) indicates that best farm increased technical efficiency by 87.8%. As far as the technical change and TFP productivity change it was 303.3% increase. The worst individual results were 46.8% technical efficiency decrease, 78.2% TFP productivity decrease and 76.9% regress in technical change.

4. Conclusions

Productivity analysis on a sample of farms showed that wheat production in most of farms is connected with problems of regress or stagnation both in efficiency and productivity measures. Calculated indices of technical efficiency change proof that only 18% of farms were able to increase technical efficiency. Technology innovation (technology progress) reached only 7% of farms and productivity growth only 7% of farms.

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Peter Fandel

Department of Statistics and Operations Research, Faculty of Economics and Management, Slovak Agricultural University in Nitra, A.Hlinku 2, 949 76 Nitra, tel.: +421-37-6508 176, e-mail: Peter.Fandel@uniag.sk

QUANTITATIVE ANALYSIS OF THE TAX SYSTEM OF THE SLOVAK REPUBLIC

MICHAL FENDEK
UNIVERSITY OF ECONOMICS BRATISLAVA, SLOVAKIA

Abstract

The purpose of this paper is to present possible solutions that provide quantitatively oriented part of economic theory in solving problems concerning regulation of the government fiscal policy with the emphasis to income modelling problems of the government budget as well as analysis of expected effects of using fiscal policy instruments.

For such a solution we use recent knowledge of econometric modelling and software products for economic and mathematical modelling. Traditional analyses of the development in macroeconomic quantities on the basis of econometric models will be enriched by optimization models of economic policy on the basis of linear and nonlinear multi-criterial optimization.

As a result we will receive a hierarchical model structure which allows us to analyse the development in returns of the tax system of SR beyond the horizon 1995 - 2001 and optimize their development for medium term strategy of the finance policy in 2001 - 2006. Solutions of the project will contain scenarios for forecasts and analysis of the development in tax incomes.

1. Introduction

Positive results in the development of Slovak economy during first years of its existence as an independent state indicate a good trend in economy on one side, but on the other side they require a systematic and qualified work in conducting economic policy, which will guarantee a relative stability of this development.

The share of government in management of economy is not negligible in the developed countries. In general it is possible to describe four basic functions of government:

1. to create legal basis for functioning of economic processes;
2. to formulate macroeconomic stabilization policy with the purpose to eliminate tendencies to growth of unemployment and increase of inflation;
3. to generate and allocate resources for financing general economic activities of the society via regulation of taxes and government expenditures;
4. to make redistribution of resources via social transfers.

It can be said that macroeconomic regulation of market economy finally consists in affecting the aggregate demand and supply by instruments of fiscal and monetary policy.

Success as well as failure of the government budget policy is determined first of all by effectiveness of its fiscal policy and by effects of this policy on development, stagnation or decline of the economic environment.

The present tensive situation in resources of the economy lays stress on guaranty of effective expenditures of means. It is also useful to evaluate assumed effect of taken measures with the help of models or to compare or to evaluate alternative variants. Together with this preference requirements for exact analytic approaches in this project we will consider possibilities of using model approaches in evaluation and comparison of instruments effectiveness used by fiscal policy of SR at the present time as well as from a view point of perspective measures.

It is shown that recent instrument of government budget policy which are gradually adapted to market conditions represent a key instrument of macroeconomic regulation with relatively short horizon of return.

With a certain simplification it is possible to say that government realises fiscal policy by regulation of the budget government expenditures via tax system.

The government of SR is systematically considering problems of increasing the effectiveness of its fiscal policy instruments and possibilities of the budget incomes mobilisation. However it is not possible to evaluate effects of measures introduced by the government of SR on taxes in the present time but it is possible for a long-time expected effects. Therefore it is necessary to consider information about results in the development of the budget incomes for a longer period than one year and then to carry out following analyses and forecasts of the whole system of the SR economic development indicators. On the basis of certain model calculations we can predict some effects of these measures of course taking into account the character of used econometric and optimisation methods, which use functions describing the economic development on its historical basis.

Consequences of these measures must be carefully analysed and their effects must be evaluated with respect to income part of the budget.

2. Models of tax system revenues of Slovak Republic

During 1998 – 2002 at the Department of Operation Research and Econometrics at the University of Economics in Bratislava in co-operation with Ministry of Finance of SR econometric and an optimisation model has been worked out. They were used in the analysis and forecasts of the development of Slovak tax system. For the construction of these models and computer calculations we used programming system SORITEC for econometric models and programming systems for solving problems of linear and nonlinear optimisation GAMS and SOLVER for EXCEL. Experiences from this stage of the research indicate on the effectiveness of these quantitative methods for utilising instruments of fiscal policy.

The purpose of the project will be to use model approaches and economic-mathematical methods in the analysis of effects of the government tax policy. We will also pay attention to applications of theoretical concepts of microeconomic models of tax regulation as well as macroeconomic models of tax regulation.

In the construction of the macroeconomic model of SR and its forecasting and optimisation applications we used data from the next sources:

- quarterly data on macroeconomic aggregate for 1995 - 2001 published by the Statistical Bureau of the Slovak Republic

- quarterly and annual data characterising the tax system of SR for 1993 - 2001 received from the Ministry of Finance

3. Forecasting, optimisation and analysis of tax system

System of models for analysis, optimisation and forecasting of the tax system's incomes maybe effectively used in the following areas:

(1) To analyze the development of basic macroeconomic indicators of Slovak economy using database from years 1995 - 2001 and on this basis to construct a macroeconomic econometric model.

(2) On the basis of the econometric model to formulate a typical forecast of the economic development in the horizon 2002 - 2006 under variant formulated assumptions on the development and consequences of the effect of fiscal policy instruments.

(3) With the use of optimization models of goal programming constructed on the basis of estimated functions of the econometric model to analyze the development in the tax systems and economy of SR under variant defined goals of the economic policy.

(4) To simulate effects from realized steps in the area of the tax system and to consider substitution possibilities of returns from the individual taxes.

The results of the incomes analysis and prognosis of the Slovakia's tax system are shown in the Table 1 including their illustrations in Figure 1.

Development and forecasting of the incomes growth rate of the single elements of the Slovakia's tax system are shown in the Table 2 including their illustrations in Figure 2.

The a.m. tables approach the following abbreviations of the single elements of the Slovakia's tax system:

D – total tax incomes, in billion SKK

DPFO – personal income tax, in billion SKK

DPPO – corporate income tax, in billion SKK

DKV – capital gains tax, in billion SKK

DM – property tax, in billion SKK

DPH –value added tax, in billion SKK

DSPOT – excise tax, in billion SKK

DCEST – vehicle excise tax, in billion SKK

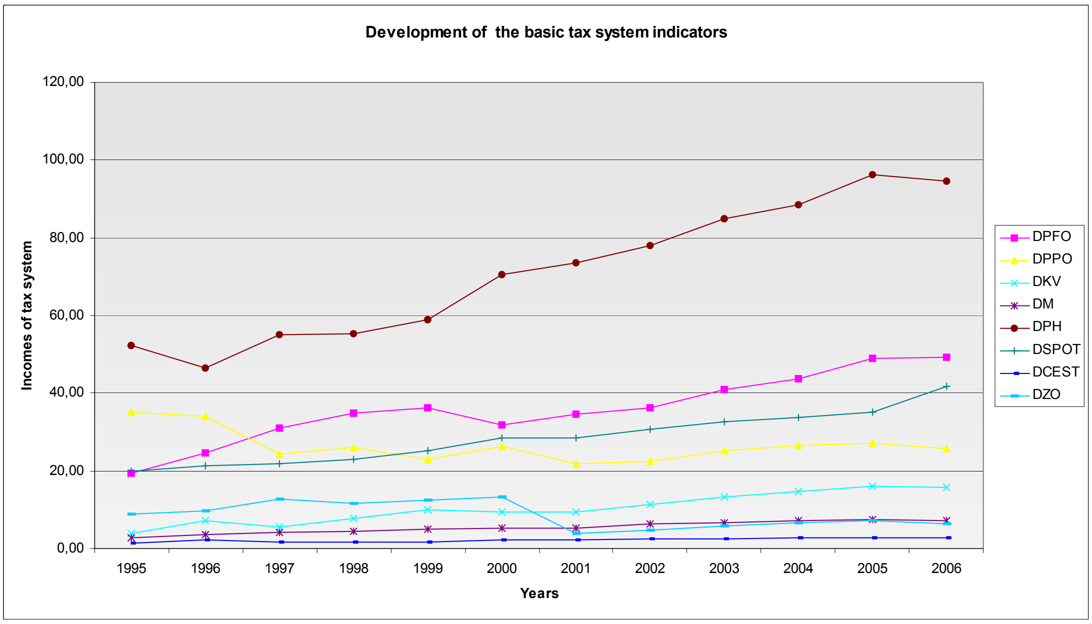
DZO - tax on imports, in billion SKK

Tab 1 Development of the basic tax system indicators

B. Development of basic indicators of tax system

Ukazovateľ	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
PROGNÓZA												
D	143,75	148,73	156,31	164,82	172,27	187,21	179,34	191,77	211,77	223,38	240,73	243,16
DPFO	19,33	24,48	30,93	34,92	36,17	31,84	34,62	36,17	40,97	43,69	48,96	49,13
DPFOZA	12,97	16,44	20,35	24,02	25,74	20,88	23,13	23,39	26,46	29,91	32,97	34,11
DPFOSC	3,07	3,52	5,29	5,36	4,65	4,52	4,61	6,44	8,27	7,71	10,30	10,10
DPFOOB	3,29	4,52	5,29	5,55	5,79	6,44	6,89	6,34	6,24	6,07	5,69	4,92
DPPO	35,23	33,93	24,37	26,07	22,98	26,35	21,73	22,49	25,19	26,46	26,97	25,77
DPPOSR	33,94	33,01	23,59	24,70	22,02	25,12	20,21	21,79	24,13	25,12	25,43	24,18
DPPOOB	1,29	0,92	0,78	1,36	0,97	1,23	1,51	0,69	1,06	1,34	1,54	1,59
DKV	3,92	7,14	5,65	7,63	9,84	9,45	9,53	11,24	13,26	14,68	16,01	15,69
DM	2,86	3,68	4,16	4,50	4,89	5,23	5,33	6,26	6,71	7,16	7,59	7,05
DPH	52,31	46,48	54,94	55,25	58,94	70,59	73,57	77,97	84,77	88,42	96,17	94,69
DSPOT	19,97	21,25	21,87	23,07	25,16	28,45	28,40	30,60	32,68	33,77	35,07	41,63
PALIVA	11,78	11,47	13,39	12,97	14,93	17,61	18,17	19,80	21,58	22,40	23,52	30,39
PIVO	1,32	1,19	1,09	0,99	1,17	1,31	1,35	1,28	1,30	1,33	1,35	1,41
TABAK	2,86	3,17	3,32	4,55	4,21	4,79	4,59	5,49	5,89	6,25	6,53	6,63
VINO	0,40	0,44	0,38	0,38	0,40	0,38	0,30	0,34	0,35	0,34	0,34	0,34
LIEH	3,89	4,29	3,70	4,18	4,46	4,37	4,00	3,69	3,56	3,45	3,34	2,87
DCEST	1,38	2,11	1,58	1,72	1,73	2,12	2,24	2,36	2,52	2,69	2,83	2,85
DZO	8,75	9,66	12,81	11,66	12,53	13,18	3,92	4,69	5,67	6,51	7,12	6,34
CLO	5,44	5,82	5,25	5,24	3,93	3,68	3,55	4,69	5,67	6,51	7,12	6,34
DOVPR	3,31	3,83	7,45	6,29	8,43	9,42	0,29	0,00	0,00	0,00	0,00	0,00

Graph 1 Development of the basic tax system indicators



Tab 2 Incomes growth rates of the single elements of the Slovakia's tax system

6	Ukazovateľ	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
42													
43	**Incomes growth rate of the single elements of the Slovakia's tax system**												
44	D		3,46%	5,10%	5,44%	4,52%	8,67%	-4,20%	2,44%	10,43%	5,48%	7,77%	1,01%
45	DPFO		26,62%	26,35%	12,92%	3,58%	-11,99%	8,76%	13,62%	13,26%	6,65%	12,05%	0,35%
46	DPFOZA		26,72%	23,81%	18,00%	7,18%	-18,89%	10,78%	12,00%	13,16%	13,04%	10,21%	3,48%
47	DPFOSC		14,74%	50,19%	1,24%	-13,22%	-2,83%	1,98%	42,71%	28,29%	-6,76%	33,61%	-1,95%
48	DPFOOB		37,32%	16,99%	5,02%	4,23%	11,32%	6,96%	-1,54%	-1,66%	-2,65%	-6,21%	-13,64%
49	DPPO		-3,70%	-28,16%	6,95%	-11,82%	14,64%	-17,55%	-14,67%	12,02%	5,03%	1,94%	-4,44%
50	DPPOSR		-2,75%	-28,53%	4,71%	-10,86%	14,10%	-19,55%	-13,25%	10,70%	4,11%	1,25%	-4,92%
51	DPPOOB		-28,75%	-15,07%	74,47%	-29,21%	26,98%	23,32%	-43,70%	53,62%	26,10%	14,89%	3,53%
52	DKV		82,31%	-20,87%	34,98%	29,06%	-3,95%	0,84%	18,87%	18,01%	10,70%	9,07%	-2,03%
53	DM		28,87%	12,91%	8,20%	8,66%	6,89%	1,91%	19,76%	7,22%	6,66%	5,98%	-7,03%
54	DPH		-11,15%	18,19%	0,57%	6,69%	19,75%	4,22%	10,46%	8,73%	4,30%	8,77%	-1,54%
55	DSPOT		6,42%	2,94%	5,48%	9,07%	13,07%	-0,18%	7,54%	6,80%	3,33%	3,87%	18,70%
56	PALIVA		-2,57%	16,67%	-3,09%	15,13%	17,90%	3,18%	12,44%	8,99%	3,79%	5,01%	29,22%
57	PIVO		-10,05%	-8,74%	-9,21%	18,35%	11,84%	3,52%	-1,99%	1,91%	1,74%	1,73%	4,55%
58	TABAK		11,03%	4,57%	37,26%	-7,56%	13,70%	-4,16%	14,74%	7,20%	6,16%	4,50%	1,46%
59	VINO		11,56%	-14,37%	0,84%	3,96%	-3,96%	-22,81%	-12,12%	3,11%	-1,00%	-0,09%	-0,16%
60	LIEH		10,30%	-13,92%	13,03%	6,66%	-1,87%	-8,46%	-15,50%	-3,52%	-3,13%	-3,40%	-14,07%
61	DCEST		53,54%	-25,25%	8,69%	0,82%	22,48%	5,57%	11,27%	6,84%	6,75%	5,32%	0,53%
62	DZO		10,31%	32,70%	-8,98%	7,47%	5,16%	-70,24%	-64,42%	20,99%	14,79%	9,33%	-10,98%
63	CLO		7,02%	-9,79%	-0,21%	-24,94%	-6,38%	-3,60%	27,32%	20,99%	14,79%	9,33%	-10,98%
64	DOVPR		15,72%	94,27%	-15,63%	34,03%	11,75%	-96,97%	########	NA	NA	NA	NA

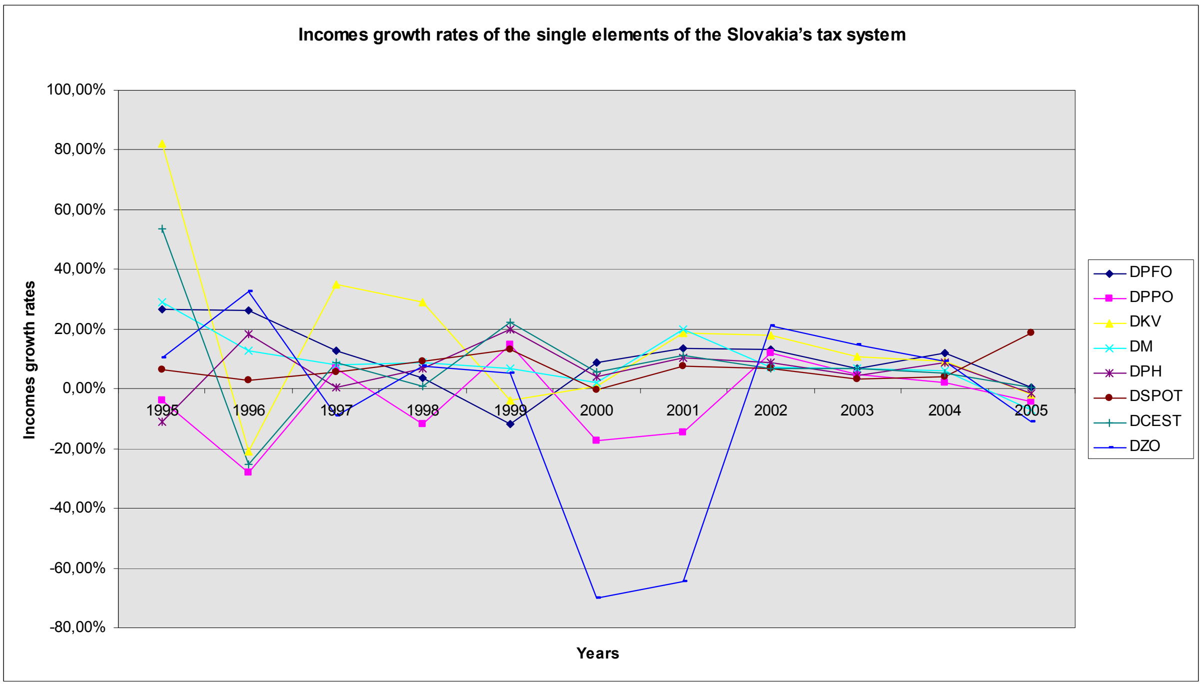
4. Conclusion

The system of models supporting the analysis, forecasting and optimisation of tax system revenues has been presented in the paper and the results can be specified in the next three main areas:

(1) Preparation of the model apparatus for support of fiscal policy instruments with accent to the analysis of the state and development of the tax system effectiveness in Slovak

Republic in connection with expected integration process of SR into European economic space.

Graph 2 Incomes growth rates of the single elements of the Slovakia's tax system



(2) Preparation and verification of the hierarchical structure of optimization and forecast models of fiscal policy instruments with the purpose to deep economic ties between model quantities.

(3) Development and utilization of methodological apparatus of econometric and optimization models in analysis, forecasting, optimization and effectiveness measuring of returns of the tax systems of SR.

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Prof. Ing. Michal Fendek, CSc.
Katedra operačného výskumu a ekonometrie EU v Bratislave, Dolnozemská 185235 Bratislava, e-mail: fendek@euba.sk

MEAN SOJOURN TIME IN STATE *k*, *k*=0,1,…, FOR THE M|G|∞ QUEUEING SYSTEM
(Exact and approximated expressions)

MANUEL ALBERTO M. FERREIRA
UNIDE/ISCTE, LISBOA, PORTUGAL

1. Introduction

In a M|G|∞ queue system λ is the Poisson process arrival rate, α is the mean service time, $G(\cdot)$ is the service time distribution function and there are infinite servers.

When we consider practical situations to apply this model we do not want necessarily the physical presence of infinite servers. But we only guarantee that when a customer arrives at the system it always finds immediately a server available. Other situations occur when there is no distinction between a customer and its server.

So, often, it is very important to manage a group of servers, in order to guarantee that the system works as an infinite server queueing system as it was designed. For this purpose it is important to know the mean sojourn time in state k, $k = 0,1,\ldots$. Here, by state k, we mean that there are k customers in the system, or, that is the same, k servers occupied.

Unhappily, only for M|M|∞ (exponential service time) queueing systems we know that mean. But as it was proposed in (Ramalhoto and Girmes, 1977) we will consider that M|G|∞ systems are well approximated by a Markov Renewal Process. And so we will consider the mean sojourn time in state k, $k = 0,1,\ldots$ for that process as a good approximation to the ones of the M|G|∞ queueing systems.

Then we are going to show some results to the mean sojourn time in state k, $k = 0,1,\ldots$ distribution function for the Markov Renewal Process considered.

2. Mean Sojourn Time in State *k, k*=0,1,2,… for the Markov Renewal Process

Calling m_k the mean sojourn time in state k for the Markov Renewal Process we have

$$m_k = \int_0^\infty e^{-\lambda t}\left[\frac{\int_t^\infty [1-G(x)]dx}{\alpha}\right]^k dt,\ k = 0,1,\ldots \qquad (1).$$

We have

$$m_0 = \frac{1}{\lambda} \tag{2}$$

as it happens with any queueing system with Poisson process arrival, and

$$m_k \le \frac{1}{\lambda} \tag{3}$$

because $\alpha^{-1}\int_t^\infty [1-G(x)]dx \le 1$. So, the mean sojourn time in any state does not exceed the one of the state "0".

By Schwartz's inequality $m_k^2 \le \int_0^\infty e^{-2\lambda t}dt \int_0^\infty \left[\frac{\int_t^\infty [1-G(x)]dx}{\alpha}\right]^{2k} dt =$

$= \frac{1}{2\lambda\alpha^{2k}}\int_0^\infty \left[\int_t^\infty [1-G(x)]dx\right]^{2k} dt = \frac{1}{2\lambda\alpha^{2k}}\frac{2k\alpha^2}{2}\left(\gamma_s^2+1\right)\frac{\alpha^{2k-1}b_{2k-1}}{2k(2k+1)} \le \alpha\frac{\gamma_s^2+1}{2\lambda(2k+1)}$, where γ_s is the service time variation coefficient, because according to (Sathe, 1985) $\int_0^\infty \left[\int_t^\infty [1-G(x)dx]\right]^n dt = \frac{n\alpha^2}{2}\left(\gamma_s^2+1\right)\frac{\alpha^{n-1}b_{n-1}}{n(n-1)}$ with $b_n \le 2,\ n = 0,1,\ldots$. So

$$m_k \le \alpha\sqrt{\frac{\gamma_s^2+1}{2\rho(2k+1)}},\ k = 1,2,\ldots \tag{4}$$

We have also $m_k \le \int_0^\infty \left[\frac{\int_t^\infty [1-G(x)dx]}{\alpha}\right]^k dt \le \frac{1}{\alpha^k}\frac{k\alpha^2}{2}\left(\gamma_s^2+1\right)\frac{2\alpha^{k-1}}{k(k+1)} = \alpha\frac{\gamma_s^2+1}{k+1}$, according again to the result of (Sathe, 1985) cited above. So

$$m_k \le \alpha\frac{\gamma_s^2+1}{k+1},\ k = 1,2,\ldots \tag{5}$$

Simple, although rather fastidious, computations allow the following rules to choose, the best upper bound to m_k:

a) $\rho\left(\gamma_s^2+1\right) > \frac{2}{3}$

a$_1$) $k < \frac{1}{4}\rho\left(\gamma_s^2+1\right) - \frac{1}{2}$

$$m_k \le \frac{1}{\lambda}$$

a₂) $\frac{1}{4}\rho\left(\gamma_s^2+1\right)-\frac{1}{2}\le k\le 2\rho\left(\gamma_s^2+1\right)-1$

$$m_k\le\alpha\sqrt{\frac{\gamma_s^2+1}{2\rho(2k+1)}}$$

a₃) $2\rho\left(\gamma_s^2+1\right)-1<k<4\rho\left(\gamma_s^2+1\right)-1$

$$m_k\le\min\left\{\alpha\sqrt{\frac{\gamma_s^2+1}{2\rho(2k+1)}};\alpha\frac{\gamma_s^2+1}{k+1}\right\}$$

a₄) $k\ge 4\rho\left(\gamma_s^2+1\right)-1$

$$m_k\le\alpha\frac{\gamma_s^2+1}{k+1}$$

b) $\frac{1}{2}<\rho\left(\gamma_s^2+1\right)\le\frac{2}{3}$

b₁) $k=1$

$$m_1\le\min\left\{\alpha\sqrt{\frac{\gamma_s^2+1}{6\rho}};\alpha\frac{\gamma_s^2+1}{2}\right\}$$

b₂) $k=2,3,\dots$

$$m_k\le\alpha\frac{\gamma_s^2+1}{k+1}$$

c) $\rho\left(\gamma_s^2+1\right)\le\frac{1}{2}$

$$m_k\le\alpha\frac{\gamma_s^2+1}{k+1},\ k=1,2,\dots$$

If the service time is exponential we have

$$m_k=\frac{\alpha}{k+\rho},\ k=1,2,\dots \qquad (6),$$

coincident to the one known to the M|M|∞ system.

If the service time is NBUE (New better than used in expectation), see for instance (Ross, 1983),

$$m_k\le\frac{\alpha}{k+\rho},\ k=1,2,\dots \qquad (7).$$

If the service time is NWUE (New worse than used in expectation), see again (Ross, 1983),

$$m_k \geq \frac{\alpha}{k+\rho},\ k = 1,2,\ldots \quad (8).$$

If the service time is DFR (Decreasing failure rate), see (Ross, 1983),

$$m_k \geq e^{k\left(\frac{1-\gamma_s^2}{2}\right)} \frac{\alpha}{k+\rho},\ k = 1,2,\ldots \quad (9).$$

If the service time is IMRL (Increasing mean residual life), see (Brown, 1981) and (Cox, 1962),

$$m_k \geq e^{k\left(1-\frac{2}{3}\frac{\alpha}{\mu_2^2}\mu_3\right)} \frac{\mu_2}{\mu_2\lambda + 2k\alpha},\ k = 1,2,\ldots \quad (10)$$

where μ_r is the r^{th} moment centered at the origin of $G(\cdot)$.

3. Concluding remarks

For, $k = 0$, whatever is the service time distribution, and for the M|M|∞ queueing system, whatever is k, the Markov Renewal Process mean sojourn time in state k is coincident with the ones known for the M|G|∞ queueing system. This allows us to hope that in the other cases it gives good approximations. We show results appliable to any service time distribution and for service time distributions important in reliability theory, NBUE, NWUE, DFR and IMRL.

4. Summary

Although, in generally, approximated we present some practical formulas for the mean sojourn time in state k, $k = 0,1,...$ for the M|G|∞ queue system. Its knowledge is of key importance for managing a group of servers in order to guarantee that the queueing systems act as an infinite server one.

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Manuel Ablerto M. Ferreira, Associate Professor

UNIDE/ISCTE, Avª das Forças Armadas, 1649-026 LISBOA, PORTUGAL, Telephone: 21 790 30 46/7, e-mail: vice.presidente@iscte.pt

INVENTORY MANAGEMENT IN SUPPLY CHAINS

PETR FIALA
UNIVERSITY OF ECONOMICS, PRAGUE, CZECH REPUBLIK

Abstract

Managing of supply chains is now seen as a very strong competitive advantage. One of the most costly aspects of supply chains is the management of inventory. In the inventory management are many inefficiencies. In the paper are presented some examples and approaches how a coordination of actions brings benefit for the whole supply chain.

1. Introduction

A supply chain is defined as a chain of organizations that are involved, through linkages, in the different processes and activities that produce value in the form of product and services in the hands of the ultimate customer. A structure of supply chains is composed from potential suppliers, producers, distributors, customers etc. The sets are interconnected by physical, financial, information and decisional flows. The analysis and design of the supply chains has been an active area of research (see [1] - [6]). Most supply chains are composed of independent units with individual preferences. Each unit will attempt to optimize his own preference. The competition degrades supply chain performance and the units can benefit from coordination. There are some possibilities to design some centralized or decentralized systems with different performance measures.

One of the most costly aspects of supply chains is the management of inventory. The importance of inventory management and the need for the coordination of inventory decisions has been evident for a long time. Order quantity is a very important factor in inventory management. In the paper we show some examples and approaches for optimal assessment of order quantity for the whole supply chain. The problems of double marginalization, applying the economic order quantity and risk pooling in supply chains are presented.

2. Double marginalization problem

Double marginalization is a well-known cause of supply chain inefficiency (see [6]) . Double margimalization problem occurs whenever the supply chain's profits are divided among two or more firms and at least one of the firms influences demand. Each firm only considers its own profit margin and does not consider the supply chain's margin.

We consider a supply chain with a supplier and a retailer that sells a product. The supplier produces each unit for a cost c and sells each unit to the retailer for a wholesale price w. The retailer chooses an order quantity q and sells q units at price $p(q)$, assuming that $p(q)$ is decreasing, concave and twice differentiable function.

Centralized solution assumes a single agent has complete information and controls the entire supply chain (this is referred as the firs-best solution) to maximize supply chain profit

$$z(q) = q\,(p(q) - c)\,.$$

Solution of the problem we denote q^0 .

Decentralized solution assumes the firms have incomplete information and make choices with the objective of maximizing their own profits. The retailer's profit and the supplier's profit are

$$z_r(q) = q\,(p(q) - w) \quad , \quad z_s(q) = q\,(w - c)$$

Solution of the problem we denote q^* .

If the centralized and decentralized solutions differ, investigate how to modify the firm's payoffs so that new decentralized solution corresponds to the centralized solution.

It can be shown that the retailer orders less than the supply chain optimal quantity ($q^0 > q^*$) whenever the supplier earns a positive profit and it holds

$$z(q^0) > z_r(q^*) + z_s(q^*)$$

Marginal cost pricing ($w = c$) is one solution to double marginalization problem, but the supplier earns a zero profit. A better solution is a profit sharing contract, where the supplier earns $v\,z(q)$ and the retailer earns $(1\text{-}v)\,z(q)$, for $0 \le v \le 1$. The wholesale price w is now irrelevant to each firm's profits and the supply chain earns the optimal profit.

3. Economic order quantity

It is assumed that the producer produces a product for which demand is relatively predictable and stable. The classic Economic Order Quantity (EOQ) model is a simple model that illustrates the trade-offs between ordering and holding costs. The question is how is applicable the model for supply chains.

We suppose that a producer produces a product for which demand is stable and the producer operates in an Economic Order Quantity type of environment. The problem arises because the order quantity that is optimal for the producer may not be optimal for the supply chain as a whole. One possibility of problem solving is focused on coordination of supply quantity between members of the supply chain. To illustrate a benefit of coordination we show a simple example.

4. Example

Suppose that a supply chain is composed of two members, a supplier and a producer. The producer produces D = 1000 units of a product per year at a constant rate. The producer purchases a component for the product from an upstream supplier. The ordering cost is S_P = 500 for a order and the holding cost of one component is H_P = 10 per year. Total cost for the producer is

$$TC_P = \frac{Q_P}{2} H_P + \frac{D}{Q_P} S_P \ .$$

The optimal order quantity for the producer is given by EOQ formula

$$Q_P = \sqrt{\frac{2DS_P}{H_P}} = 316 \text{ units.}$$

The supplier produces a bath of components with a production setup cost of S_S = 1000. The annual setup cost is a function of the producer order quantity

$$TC_S = \frac{D}{Q_P} S_S \ .$$

Total cost for the whole supply chain is

$$TC_C = \frac{Q_C}{2} H_P + \frac{D}{Q_C}(S_P + S_S)\,.$$

The optimal order quantity for the whole supply chain is given by EOQ formula

$$Q_C = \sqrt{\frac{2D(S_P + S_S)}{H_P}} = 548 \text{ units.}$$

We can compare the costs for optimal order quantity for the producer and the costs for optimal order quantity for the whole supply chain (see Tab. 1).

	Q_P = 316	Q_C = 548
TC_P	3162	3652
TC_S	3165	1825
TC_C	6327	5477

Tab. 1

The coordination of order quantity decreases total costs for the whole supply chain, but it is necessary to reallocate the costs between units of the supply chain.

5. Risk pooling

Risk pooling is an important concept in supply chain management (see [3]). In a supply chain is a variable demand for a product. We analyze connections between a supplier and retailers and can compare a decentralized distribution system with a specific warehouse for each retailer and centralized distribution system with a warehouse for all retailers. Risk pooling concept suggests that demand variability is reduced by aggregation of demand. It becomes more likely that high demand from one retailer will be offset by low demand from another. The reduction of variability allows to reduce safety stock and therefore reduce average inventory. The reallocation of inventory is not possible in a decentralized distribution system where different warehouses serve different retailers. Benefit from risk pooling increases by higher coefficient of demand variation and by more negative correlation of demand by different retailers.

The outputs are illustrated by reports. The screen of the Risk Pool Game see Fig. 1.

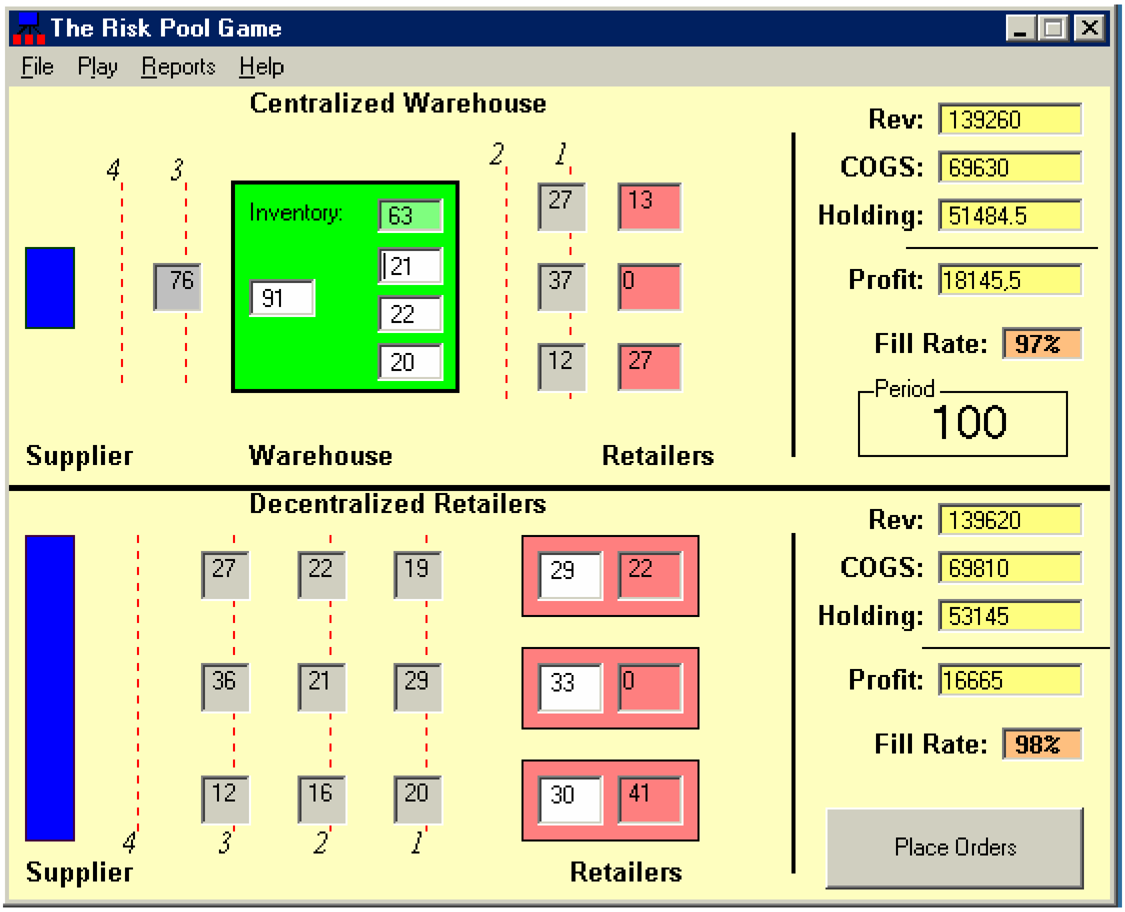

Fig. 1

There is a computerized version of the risk pool game (see [3]) to demonstrate effects of risk pooling concepts. The game proposes to compare a centralized system with a decentralized system by setting options:

- Initial inventories.

- Random demand parameters- mean, standard deviation, correlation.
- Inventory policy – safety stock policy, weeks of inventory policy.
- Costs- holding costs, revenue per item, cost per item.

6. Conclusion

Supply chain inventory management goes out the situation that supply chains are usually operated by independent units with individual preferences. There are many inefficiencies in supply chains. To be a supply chain more efficient as a whole it is necessary to apply coordination techniques to manipulate the behavior of one unit to the advantage of another. The Internet has affected inventory management most dramatically in the ability to be proactive and cooperative in the management of inventory systems. The paper presents some examples and approaches for coordination and cooperation activities in inventory management of supply chains.

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Doc.RNDr.Ing. Petr Fiala, CSc.

MBA, Department of Econometrics, University of Economics, W. Churchill Sq. 4, 130 67 Prague 3, Czech Republic, e-mail: pfiala@vse.cz

INTEGER PROGRAMMING IN AMPL

ZUZANA FÍGLOVÁ
UNIVERSITY OF ECONOMICS, PRAGUE, CZECH REPUBLIC

1 . Introduction

Many problems that arise in manufacturing and socio-economic systems, such as machine scheduling, vehicle routing, resource management, production planning, telecommunications network design, etc., can be modeled as integer programs. Integer programming (IP) usually involve optimization of a linear objective function subject to linear constraints, nonnegativity conditions and integer value conditions [1].

Problems containing integer variables fall into several classes: *pure IP problem* – a problem in which all variables are integer, *mixed IP problem* – a problem with some integer and some continuous variables, *0-1 linear IP* – a problem in which the integer variables are restricted to equal either zero or one, also called binary or Boolean variables. In this paper we present AMPL as a modeling language and system for formulating, solving and analyzing integer optimization problems.

2. AMPL

AMPL stands for "A Modeling Language for Mathematical Programming". It is a high level programming language that translates mathematical statements that describe a mathematical program into a format readable by most optimization software packages. The Student Edition version V1.6 consists of two implementations of the AMPL command environment and language processor – AMPL for DOS and AMPL Plus for Windows, together with the MINOS 5.5 and CPLEX 6.5.3 solvers. Mathematical programs generated by the Student Edition are limited to 300 variables and 300 constraints and it can be downloaded from www.ampl.com.

To start, AMPL needs a mathematical programming *model*, which describes variables, objectives and relationships without referring to specific data. It also needs an instance of the data, or a particular data set. The model and one (or more) data files are fed into the AMPL program. AMPL works like a *compiler*. It means, the model and input are put into an intermediate form, which can be read a *solver*. The solver actually finds an optimal solution to the problem by reading in the intermediate file produced by AMPL and applying an appropriate algorithm. The use of program we show on the lockbox problem as an example of 0/1 linear integer programming.

3. Lockbox problem

Lockbox problem is a subset of the fixed costs problem of determining the number of location of payment centers that minimize the sum of fixed operating costs and lost interest costs.

We define following model [2]:

$$\text{minimize} \qquad \sum_{i \in I} \sum_{j \in J} L_{ij} x_{ij} + \sum_{j \in J} c_j y_j$$

$$\text{subject to} \qquad \sum_{j \in J} x_{ij} = 1 \qquad \text{for all } i \in I,$$

$$\sum_{i \in I} x_{ij} \le |I| y_j \qquad \text{for all } j \in J,$$

$$x_{ij}, y_j \in \{0,1\} \qquad \text{for all } i \in I,\ j \in J,$$

where I is set of sources/regions (number = $|I|$), J is set of lockbox sites (number = $|J|$), c_{ij} is cost to open lockbox j, L_{ij} is average annual loss if source/region i is assigned to send its checks to lockbox j. Variables take values:

$$y_j = \begin{cases} 1 & \text{if lockbox } j \text{ is opened;} \\ 0 & \text{otherwise.} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{if source/region } i \text{ is assigned to send its check to lockbox } j; \\ 0 & \text{otherwise.} \end{cases}$$

In the model the first set of constraints means that every source/region must be assigned exactly one lockbox and the second means that if lockbox j is not open, no source/region is assigned.

3. 1 An example

Consider a US credit card company that receives payments from seven regions of the country with following daily amount of payments: Central \$65 000, Mid-Atlantic \$60 000, Midwest \$50 000, Northeast \$80 000, Northwest \$60 000, Southeast \$70 000 and Southwest \$60 000. A company earns 20% interest on these payments. We suppose opening lockboxes that are located in Los Angeles, Salt Lake City, Atlanta, New York, Austin and Springfield. The annual fixed cost of operating a lockbox is \$60 000.

The average duration (in days) from mailing to clearing and the losses due to lost interest in process on any given day for each possible assignment (for example, if the Central sends to Los Angeles, a yearly loss is 3*$65 000*20% = $39 000) is given in Table 1.

Table 1 Clearing Times/Lost Interest ($1000)

	Los Angeles		**Salt Lake City**		**Atlanta**		**New York**		**Austin**		**Springfield**	
Central	3	39	4	52	5	65	4	52	5	65	2	26
Mid-Atlantic	5	60	4	48	2	24	3	36	4	48	6	72
Midwest	4	40	3	30	5	50	6	60	4	40	7	70
Northeast	6	96	4	64	3	48	3	48	2	32	3	48
Northwest	2	24	2	24	7	84	5	60	5	60	3	36
Southeast	6	84	3	42	5	70	4	56	3	42	4	56
Southwest	2	24	3	36	2	24	8	96	6	72	3	36

A company wants to determine the lockbox configuration that minimizes the sum of lost interest and lockbox costs. For this problem we have 48 variables and 13 constraints.

3.2 Solving in AMPL

Figure 1 shows the entire data in AMPL syntax in data file. Comments can be added anywhere with "#" at the beginning.

Figure 1 An AMPL data file *lockbox.dat*

```
lockbox.dat
###  SETS  ###

set LOCKBOXES:= Los_Angeles Salt_Lake_City Atlanta New_York Austin Springfield;
set REGIONS:= Central Mid-Atlantic Midwest Northeast Northwest Southeast Southwest;

###  PARAMETERS  ###

param lost_interest:        Los_Angeles Salt_Lake_City Atlanta New_York Austin Springfield:=
                Central         39          52             65       52      65      26
                Mid-Atlantic    60          48             24       36      48      72
                Midwest         40          30             50       60      40      70
                Northeast       96          64             48       48      32      48
                Northwest       24          24             84       60      60      36
                Southeast       84          42             70       56      42      56
                Southwest       24          36             24       96      72      36;

param number_one:=1;
param lockbox_costs:=60;
param large_number:=100;
```

```
lockbox.mod
###  SETS  ###

set LOCKBOXES;
set REGIONS;

###  PARAMETERS  ###

param lost_interest {i in REGIONS, j in LOCKBOXES} >=0;
param number_one>0;
param lockbox_costs>0;
param large_number>0;

### VARIABLES  ###

var assignment {i in REGIONS, j in LOCKBOXES} binary;
var y {i in REGIONS} binary;

###  OBJECTIVE  ###

minimize total_costs: (sum {i in REGIONS, j in LOCKBOXES} lost_interest [i,j]*assignment[i,j]) +
sum {i in REGIONS}lockbox_costs*y[i];

###  CONSTRAINTS  ###

subject to constraintsI {j in LOCKBOXES}: sum {i in REGIONS} assignment[i,j]=number_one;
subject to constraintsII {i in REGIONS}: sum {j in LOCKBOXES} assignment[i,j]<=large_number*y[i];
```

Figure 2. An AMPL model file *lockbox.mod*

Figure 2 shows the entire model in AMPL syntax. This way of entering data and the model makes it easy to modify our example later.

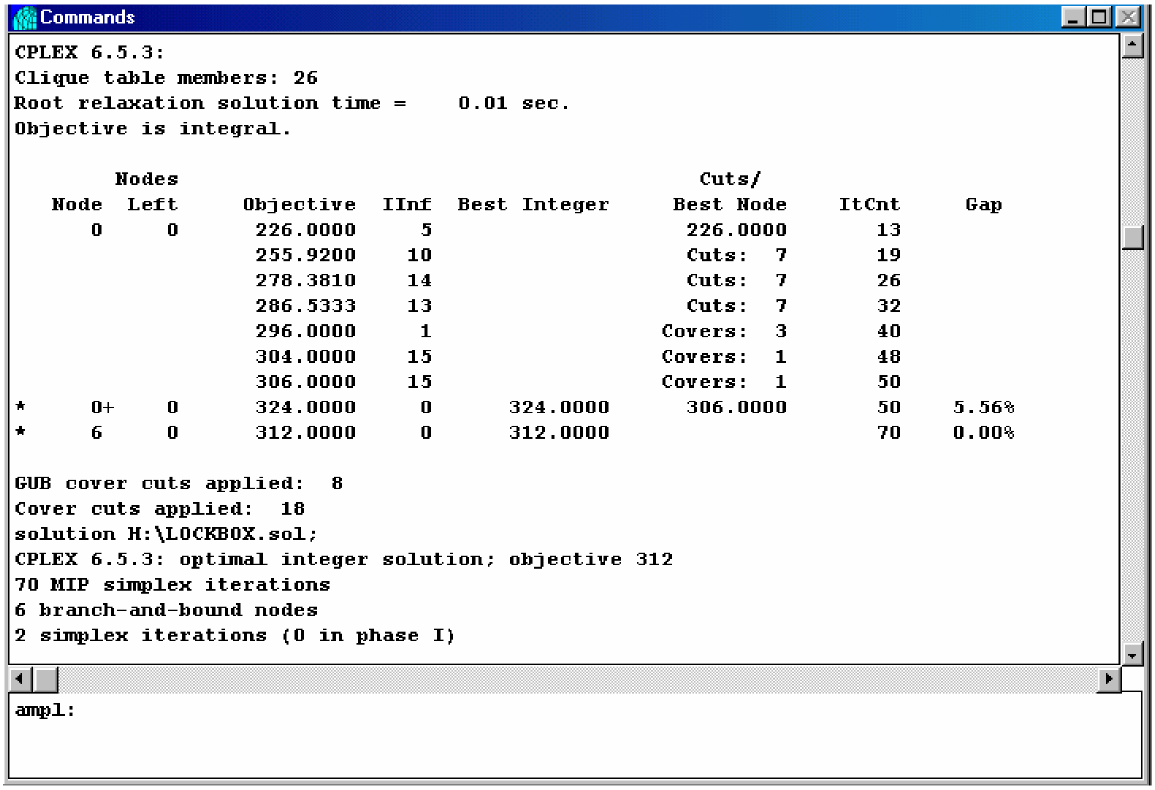
There are several ways to entering a model in AMPL. One of them is using the command windows with providing and enhanced form of the scrolling command line interface. We can also use text windows, which separate relationships between general form of the model and the problem of data values. An alternative way to supply data values is using data query window look like spreadsheet in MS Excel, for example.

After creating data and model file, we have to click on *Build Model*, *Build Data* and then *Solve Problem*. For solving problems that contain integer variables, CPLEX uses a branch-and-bound method. The optimizing algorithm maintains a hierarchy of related linear programming subproblems, referred to the search tree.

CPLEX allows to control a number of branching heuristics, for instance, the choice of the branching variable, the branch direction, the choice of the next node to be explored, and amount of backtracking, etc. For instance, we can control also the algorithm used to solve the root node as well as the algorithm used to solve subsequent nodes in the branch-and-bound tree. If you decide to use the barrier algorithm, you can also specify if the crossover (required to generate a vertex of the feasible polyhedron) uses primal or dual simplex.

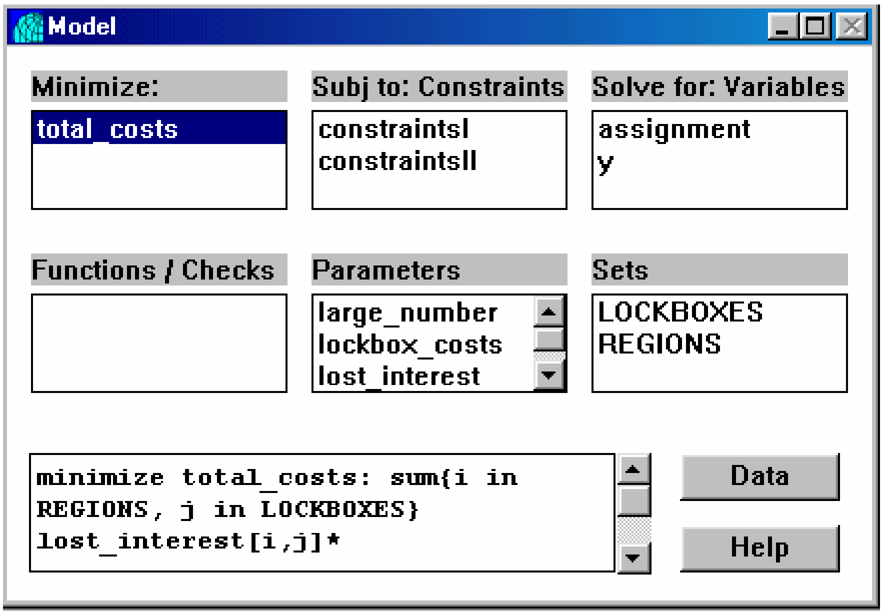
Figure 3 presents the steps of algorithm branch-and-bound method of our example. CPLEX takes 6 nodes, 70 MIP simplex iterations and 2 simplex iterations (0 in phase I) to find the optimal integer solution.

Figure 3 Iterations in *Command window*



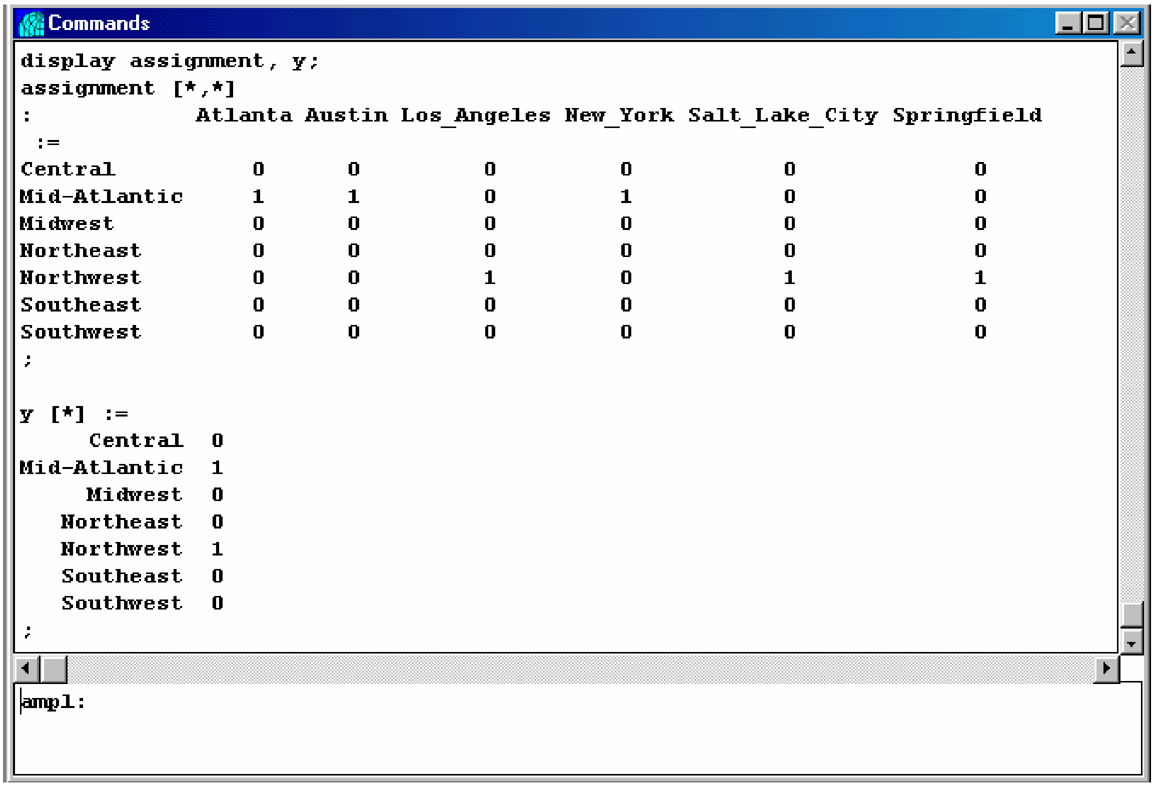
There are two possible ways to see other results. The first way displayed detailed optimal solution in window *Model* (Figure 4).

Figure 4 Detailed optimal solution in *Model window*



The second ways is using *Command window* (Figure 5), we choose displayed optimal values of variables *assignment* and *y* of our lockbox problem.

Figure 5 Optimal solution of variables assignment, y in *Command window*



```
Commands
display assignment, y;
assignment [*,*]
:             Atlanta Austin Los_Angeles New_York Salt_Lake_City Springfield
 :=
Central         0       0        0         0          0             0
Mid-Atlantic    1       1        0         1          0             0
Midwest         0       0        0         0          0             0
Northeast       0       0        0         0          0             0
Northwest       0       0        1         0          1             1
Southeast       0       0        0         0          0             0
Southwest       0       0        0         0          0             0
;

y [*] :=
     Central  0
Mid-Atlantic  1
     Midwest  0
   Northeast  0
   Northwest  1
   Southeast  0
   Southwest  0
;
ampl:
```

4. Conclusion

This paper is a brief report on the AMPL modeling language. It offers a number of features to facilitate development and testing of optimization models:

- a familiar GUI (graphical user interface),
- integrated text and mini-spreadsheet editing windows for creating and modifying AMPL models and data declarations,
- an SQL query capability, providing access through ODBC to virtually any relational database or external file,
- data can be selected, summarized, and loaded into AMPL Plus spreadsheet windows,
- a built-in model window which can be used to browse elements of the AMPL model and data and select the variables and constraints which will be included in the problem,
- a built-in solver window which provides progress information and user control while an AMPL-compatible solver is optimizing a model,

- an integrated commands window through which the Standard AMPL command-line interface may be used,
- an integrated project facility which collects model, data, query and command files, and automates the process of processing the model and solving the problem.

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Ing. Zuzana Fíglová

Department of Econometrics, University of Economics, W. Churchill Sq. 4, 130 67 Prague 3, Czech Republic, telephone: 00420-2-24095443, e-mail: figlova@vse.cz

INVESTMENT IN HUMANS, TECHNOLOGICAL DIFFUSION AND ECONOMIC GROWTH - AN OPTIMAL CONTROL INTERPRETATION

ORLANDO GOMES*[#]
*UNIDE AND ESCOLA SUPERIOR DE COMUNICAÇÃO SOCIAL, LISBOA, PORTUGAL

Abstract

In the mid 1960s, Nelson and Phelps (1966) proposed an economic growth model in which two factors played a central role on explaining how the physical output evolves over time. These two factors were human capital accumulation (investment in humans) and the dissemination of knowledge (technological diffusion).

In this paper we extend the Nelson-Phelps approach to economic growth by taking into consideration an optimal control problem. It is assumed that the economy has an objective function regarding technology goals, which are essentially two: (i) to expand the theoretical knowledge frontier and (ii) to reduce the gap between ready-to-use techniques and potentially available knowledge. By considering such an objective function, it is straightforward to build an intertemporal optimization setup concerning a two sector scenario. The first sector adapts already available technology to productive uses, while the second is an education sector. In this way, we can study the decisions of economic agents concerning the relation between technology and human capital accumulation under an intertemporal perspective.

1. Introduction

In most economic growth models human capital and technology alternatively arise as the main engines of growth. They acquire such crucial property when introduced into an aggregate production function alongside with physical capital and labor. Solow (1956) and Swan (1956) have demonstrated that under a neoclassical production function, these two former inputs are unable to produce, only by themselves, long run sustained growth. Uzawa (1965), Lucas (1988), Caballé and Santos (1993), Mulligan and Sala-i-Martin (1993), Xie (1994), Bond, Wang and Yip (1996) and Ladrón-de-Guevara, Ortigueira and Santos (1999), among others, have modeled human capital formation and they have included this type of input in the aggregate production function in order to support the concept of endogenous growth. Romer (1986, 1990), Aghion and Howitt (1992), Jones (1995), Evans, Honkapohja and Romer (1998) and Young (1998)

[#] I would like to acknowledge the helpful and dedicated comments and suggestions made by Professor Vivaldo Mendes (ISCTE). Any error that may subsist is obviously entirely my responsibility.

have chosen to put technology in the center of economic growth explanations, and a time dependent technological index arises in the production function as the vehicle to sustained growth.

The papers above mentioned, as well as the whole discussion around aggregate growth models, put the various inputs of production at a same level of analysis. This means that in both the physical capital/human capital approach and the physical capital/technology approach all the inputs that are relevant to growth have to appear as arguments in the final goods production function. As an early attempt to the interpretation of economic growth, Nelson and Phelps (1966) avoid such a straightforward view. For them, it makes sense to assume that human capital serves as a means to generate and spread technology and that technology is then usable in the physical goods production function. Recognizing that the Solow-Swan paradigm is valid when assuming a fixed level of technology, we may continue to use such a framework but should replace the constant level of technology by a time dependent variable, where the respective time path is determined by two factors: investment in humans and technological diffusion.

In this paper the Nelson-Phelps approach is revisited. We study growth giving particular attention to technology and human capital and attributing a subsidiary role to the capital accumulation constraint of the Solow/Swan model and to the consumption utility intertemporal maximization framework developed by Ramsey (1928), Cass (1964) and Koopmans (1964). Our main argument is that, since human capital and technology are the variables that determine the nature of growth, the basic growth results can be derived from the analysis of the factors that influence the time evolution of these variables. The Solow/Ramsey framework becomes accessory and may be used solely to justify that the main economic aggregates (*per capita* output, physical capital *per* labor unit, *per capita* consumption) follow a same long run growth rate as that of the technology variable.

Modeling human capital in a different way from those usually found in the literature on economic growth will allow for two new results. First, we will be able to define a human capital production function where decreasing returns are compatible with long run constant *per capita* economic growth, what implies that a counterfactual aspect of two sector growth models can be removed from such models: the linearity problem in aggregate capital as raised by Solow (1994). We do not have to assume any kind of artificial knife-edge linearity to encounter a constant long run growth rate. The second result will appear, at least at a first glance, a more awkward one. The way in which we model human capital will lead to a long run steady state where all relevant *per capita* variables (physical capital, output, consumption, technology) grow at a same rate while human capital will exhibit a zero growth rate in the long run solution. Growth models generally present human capital as growing alongside with the other economic variables. Here, we obtain the result that there is convergence of the human capital variable to a constant long run value. This may be supported by the idea that we may expand techniques and physical goods indefinitely but there is a limit to the expansion of human capabilities.

In what concerns the modeling of technology we follow the Nelson-Phelps approach by distinguishing two concepts of technology. First, there is a theoretical level of technology or a technological possibilities frontier; second, we assume the existence of a level of technology in practice which is a fraction of the former and can directly be applied to productive uses when embodied in an aggregate final goods production function. The treatment that will be given to technology choices implies the consideration of an optimal control problem in which the goals are simultaneously to amplify the frontier of technological knowledge and to approximate the level of ready-to-use technology to that frontier. This optimization problem is constrained precisely by the Nelson-Phelps motion equation that characterizes the relation between the two technology concepts.

The remainder of the paper is organized as follows. Section 2 develops the structure of the model, section 3 solves the model and refers to the steady state results and section 4 concludes.

2. A Two Sector Model of Optimal Technology Choices

Take technology and human capital as the engines for economic growth. The first variable, technology, may be decomposed into two parts. The technology possibilities frontier will be variable $T(t)$ and a ready-to-use in production technology is represented by $A(t)$. In every time moment $A(t) \leq T(t)$; both variables are assumed to be positive quantities for all $t \geq 0$. We will be concerned with understanding the temporal evolution of a gap variable $\phi(t) \equiv A(t)/T(t)$, $\phi(t) \in [0,1]$, that has a straightforward interpretation: the higher the value of $\phi(t)$ the smaller is the gap between the values of the technology variable representing knowledge immediately available to produce and of the technology variable representing the scientific state of the art. Another fundamental variable is the growth rate of the technology frontier: $\tau(t) \equiv \dot{T}(t)/T(t)$, $T(0) = T_0$ given. In our model this growth rate is assumed as a control variable for the representative agent; that is, in what concerns technology decisions, the economy is able to choose the rate at which scientific progress occurs. Nevertheless, obviously this must be a constrained decision process because in order to allocate more resources to research activities one faces an opportunity cost related to the resources that remain available for other economic activities. In particular, in our framework we assume that the choice in terms of basic technology progress is constrained by the necessity of using economic resources to apply technology to the goods production process. Therefore, a trade-off emerges between our first two endogenous variables: $\phi(t)$ and $\tau(t)$.

As far as technology choices are concerned, the basic economic goal is to maximize the intertemporal stream of $v[\phi(t), \tau(t)]$ functions, being a function v defined as follows.

Definition 1. Function v. The representative agent that makes technology choices faces a real valued objective function v: $\mathbb{R}^2_+ \mapsto \mathbb{R}$ that obeys the following properties:

i) Continuity, concavity and smoothness

ii) $$v_\phi . \frac{\phi(t)}{v} = \theta \in (0,1); \quad v_\tau . \frac{\tau(t)}{v} = \mu \in (0,1)$$

The second condition in definition 1 ensures that the utility of reducing the technology gap and the utility of increasing the pace of technological progress are both positive and diminishing for the representative agent. To simplify our treatment of the model we will work with a specific functional form of the above defined v function.

$$v[\phi(t), \tau(t)] = \phi(t)^{1/(1+\sigma)} . \tau(t)^{\sigma/(1+\sigma)} \quad (1)$$

The correspondence between parameters θ and μ in definition 1 and parameter σ in equation (1) is the following: $\sigma=\mu/\theta$ and $\mu=1-\theta$.

The trade-off between the two endogenous technology variables, $\phi(t)$ and $\tau(t)$, becomes explicit by considering the Nelson-Phelps technology constraint:

$$\dot{A}(t) = g[h(t)].[T(t) - A(t)],\ g(0) = 0,\ g' > 0, g'' < 0,\ A(0) = A_0 \text{ given} \quad (2)$$

Eq. (2) states that the rate of increase of the index $A(t)$ is a function of human capital [$h(t)$ is a human capital *per* unit of labor variable or a human capital efficiency index] and of the gap that exists between the two technology variables. First and second derivatives of g indicate that there are positive but diminishing returns of human capital in the production of technology. The gap term translates the idea that the level of technology in practice will evolve faster when there is a large gap between technology possibilities and the stock of knowledge instantly available to produce.

Recovering variable $\phi(t)$, from equation (2) we arrive at the final form of the first resource constraint of the optimal control problem:

$$\dot{\phi}(t) = g[h(t)].[1 - \phi(t)] - \tau(t).\phi(t), \quad \phi(0) = \phi_0 \quad (3)$$

To complete the presentation of the model it is necessary to define a rule for the evolution of the human capital variable. On this respect we follow the standard form in most growth models, i.e., human capital evolves in time through a production process that involves a production function, *f*, and a constant depreciation rate, δ.

$$\dot{h}(t) = f[h(t)] - \delta.h(t), \quad \delta > 0, \; h(0) = h_0 \text{ given} \qquad (4)$$

The human capital variable is, alongside with variable $\phi(t)$, a state variable of the intertemporal control problem. Equations (3) and (4) are the motion equations that constitute the resource constraints to which the optimization problem is subject to.

The analytical tractability of the model demands that we take explicit functional forms for functions *f[h(t)]* and *g[h(t)]*. Assuming that it is possible to choose at each moment in time the shares of human capital to allocate to each of the two economic sectors (technology and education sectors), we define a new variable *u(t)* that represents precisely the share of human capital allocated to the generation of technology. Obviously $u(t) \leq 1, \; \forall t$. The properties of function *g* were set forth in equation (2). Positive and diminishing returns of human capital in the production of technology imply a function with the following shape:

$$g[h(t)] = a.[u(t).h(t)]^{\eta}, \; a > 0, \eta \in (0,1) \qquad (5)$$

Function *f* may be defined in a similar way:

$$f[h(t)] = b.\{[1 - u(t)].h(t)\}^{\beta}, \quad b > 0 \qquad (6)$$

For equation (6) we find it convenient to impose $\beta \in (0,1)$ in order to obtain a long run balanced growth path. Thus, one assumes that the education sector exhibits diminishing returns in the accumulation of human knowledge. This point was referred to earlier in the introduction and should be stressed since it constitutes an innovation relatively to conventional growth models. In our model, where human capital contributes to the generation of physical goods solely in an indirect manner, we must assume diminishing returns in the accumulation of human capital in order to get a long term constant steady state growth. As a result of this assumption, the human capital *per* unit of labor variable will have a different behavior from those of the other inputs: its level will not grow in the long run in opposition to what happens to the several *per capita* variables, namely output, consumption and physical capital. Remembering that *h(t)* defines average individual skills, intuitively it is hard to support the concept that human capabilities may be improved further and further indefinitely at a constant rate. Technology and machines can be expanded without limit, individual skills cannot - this is a fundamental argument of our analysis.

The optimal control growth problem has now all the necessary ingredients. Definition 2 states the contours of the optimal solution.

Definition 2. Control problem optimal solution. An optimal solution is a set of paths {ϕ(t),h(t),τ(t),u(t)} that solve the maximization problem

$$\underset{\tau(t),u(t)}{Max}\int_0^{+\infty} v[\phi(t),\tau(t)].e^{-\rho.t}.dt$$

subject to constraints (3) and (4) and where functions v, f and g are defined respectively by (1), (5) and (6). All variables assume non negative values, initial values for state variables are given and shares ϕ(t) and u(t) remain always below unity. Furthermore, it is obvious from the problem that we take an infinite horizon and that future technology accomplishments are discounted at a constant discount rate, ρ>0.

3. Optimal Solution and Steady State

Let $p_\phi(t)$ and $p_h(t)$ be co-state variables. We synthesize our model's information into a current value Hamiltonian function:

$$\aleph[\phi(t),h(t),\tau(t),u(t),p_\phi(t),p_h(t)]\equiv \qquad (7)$$
$$\equiv v[\phi(t),\tau(t)]+\{a.[u(t).h(t)]^\eta.[1-\phi(t)]-\tau(t).\phi(t)\}.p_\phi(t)+$$
$$+(b.\{[1-u(t)].h(t)\}^\beta-\delta.h(t)).p_h(t)$$

Applying the Pontryagin's maximum principle, the first order optimality conditions can be computed:

$$\aleph_\tau=0\Rightarrow\frac{\sigma}{1+\sigma}.\left[\frac{\phi(t)}{\tau(t)}\right]^{1/1+\sigma}=p_\phi(t).\phi(t) \qquad (8)$$

$$\aleph_u=0\Rightarrow\eta.a.u(t)^{-(1-\eta)}.h(t)^\eta.[1-\phi(t)].p_\phi(t)= \qquad (9)$$
$$=\beta.b.[1-u(t)]^{-(1-\beta)}.h(t)^\beta.p_h(t)$$

$$\aleph_\phi=\rho.p_\phi(t)-\dot{p}_\phi(t)\Rightarrow \qquad (10)$$
$$\dot{p}_\phi(t)=\{\rho+a.[u(t).h(t)]^\eta+\tau(t)\}.p_\phi(t)-\frac{1}{1+\sigma}.\left[\frac{\tau(t)}{\phi(t)}\right]^{\sigma/1+\sigma}$$

$$\aleph_h=\rho.p_h(t)-\dot{p}_h(t)\Rightarrow\dot{p}_h(t)=\{\rho+\delta-\beta.b.[1-u(t)]^\beta.h(t)^{-(1-\beta)}\}.p_h(t)- \qquad (11)$$
$$-\eta.a.u(t)^\eta.h(t)^{-(1-\eta)}.[1-\phi(t)].p_\phi(t)$$

$$\aleph_{p_\phi}=\dot{\phi}(t)\Rightarrow\dot{\phi}(t)=a.[u(t).h(t)]^\eta.[1-\phi(t)]-\tau(t).\phi(t) \qquad (12)$$

$$\aleph_{p_h}=\dot{h}(t)\Rightarrow\dot{h}(t)=b.\{[1-u(t)].h(t)\}^\beta-\delta.h(t) \qquad (13)$$

$$\lim_{t \to +\infty} p_\phi(t).e^{-\rho.t}.\phi(t) = 0 \tag{14}$$

$$\lim_{t \to +\infty} p_h(t).e^{-\rho.t}.h(t) = 0 \tag{15}$$

Conditions (8)-(15) are sufficient conditions for optimality, given that the optimal Hamiltonian is concave in [$\phi(t),h(t)$]. The optimality conditions are the relations necessary to prove the following proposition.

Proposition 1. Existence and uniqueness of a balanced growth equilibrium. Under the condition $\beta+\rho/\delta<1$ there exists a unique balanced growth path or unique steady state four dimensional point that satisfies (8) to (15).

Proof: To prove the existence of a unique steady state point one has to solve the system $\left[\dot{\phi}(t)\ \dot{h}(t)\ \dot{\tau}(t)\ \dot{u}(t)\right] = \overline{0}$. The solution for this system consists on a set $\left\{\overline{\phi}, \overline{h}, \overline{\tau}, \overline{u}\right\}$ of constant values. To solve the system one is compelled to find equations of motion for the two control variables. Let us start with $\tau(t)$.

Differentiating (8) with respect to time, the following relation involving growth rates is obtained: [1]

$$\gamma_\tau = -\sigma.\gamma_\phi - (1+\sigma).\gamma_{p_\phi} \tag{16}$$

Replacing γ_{p_ϕ} and γ_ϕ in (16) by the corresponding expressions from (10) and (12), the motion equation for $\tau(t)$ comes as

$$\dot{\tau}(t) = \left\{\frac{1}{\sigma}.\tau(t) - (1+\sigma).\rho - a.[u(t).h(t)]^\eta.[1+\sigma/\phi(t)]\right\}.\tau(t) \tag{17}$$

The time evolution of $u(t)$ is derived from the following relation that is true under (9):

$$\gamma_u = \frac{1-u(t)}{(1-\eta)-(\beta-\eta).u(t)}.\left[\gamma_{p_\phi} - \gamma_{p_h} - \frac{\phi(t)}{1-\phi(t)}.\gamma_\phi + (\eta-\beta).\gamma_h\right] \tag{18}$$

Finally, we can arrive at the result

$$\dot{u}(t) = \frac{1-u(t)}{(1-\eta)-(\beta-\eta).u(t)}.\left\{\left[\frac{\phi(t)}{1-\phi(t)} - \frac{1-\sigma}{\sigma}\right].\tau(t) + \right. \tag{19}$$
$$\left. +[\eta+(\beta-\eta).u(t)].b.[(1-u(t)).h(t)]^{-(1-\beta)} - (1-\beta+\eta).\delta\right\}.u(t)$$

[1] The symbol γ represents the growth rate of the variable refered in index.

Equations (17) and (19), alongside with the two resource constraints, constitute the system from which we derive the steady state solution. The system has in fact a unique solution, which is

$$\begin{bmatrix} \bar{\phi} \\ \bar{h} \\ \bar{\tau} \\ \bar{u} \end{bmatrix} = \begin{bmatrix} \dfrac{(1-\sigma).\bar{g}}{\sigma.\rho+\bar{g}} \\ \left[\dfrac{b}{\delta}.(\beta+\rho/\delta)^{\beta}\right]^{1/(1-\beta)} \\ \dfrac{\sigma}{1-\sigma}.(\rho+\bar{g}) \\ 1-\beta-\rho/\delta \end{bmatrix} \quad (20)$$

with $\bar{g}=a.\left(\bar{u}.\bar{h}\right)^{\eta}$. If $\beta+\rho/\delta<1$ then we guarantee that $\bar{u}\in(0,1)$ and this is the only boundary condition that must be imposed in order to have (20) as a feasible four dimensional steady state point■

The steady state results deserve some comments. First, we notice that the share $\bar{u}$ depends upon parameters β, ρ and δ. The higher the elasticity parameter β and the discount rate, the lower is the value of the share of human capital allocated to technological development. The faster the depreciation of human capital, the more this form of capital is allocated to its own production relatively to a technological use. Second, the human capital efficiency index is indeed a constant amount on the balanced growth path. It depends on parameters b, δ, ρ and β. Third, we observe that $\bar{\phi}$ obeys the boundary condition $\bar{\phi}\leq 1$, because $\rho>0$ and $\bar{g}>0$. Fourth, the technology growth rate arises in the steady state depending on a multiplicity of factors, namely (i) the objective function parameter, (ii) the intertemporal discount rate, (iii) both sectors education functions parameters (a and b), (iv) the elasticity parameters (β and η) and (v) the depreciation rate.

For the steady state growth rate result,

$$\bar{\tau}=\frac{\sigma}{1-\sigma}.\left(\rho+a.\left\{\left[(1-\beta)-\rho/\delta\right].\left[(b/\delta).(\beta+\rho/\delta)^{\beta}\right]^{1/(1-\beta)}\right\}^{\eta}\right) \quad (21)$$

we stress that the partial derivatives $\partial\bar{\tau}/\partial\sigma>0$, $\partial\bar{\tau}/\partial a>0$, $\partial\bar{\tau}/\partial b>0$ and $\partial\bar{\tau}/\partial\eta>0$ have unquestionable signs. The same is not true for β, ρ and δ because when changes in these factors benefit the accumulation of human knowledge (h rises) they affect negativelly the transference of human capital to innovation purposes (u falls).

At this stage, after the long term technology growth rate has been found, we could transform the model by adding a physical capital accumulation constraint that should include a labor augmenting aggregate production function; which would lead to an economic growth framework. Due to lack of space, this framework will not be developed here but the results of its consideration can be stated in a brief manner. Under such a scenario the growth rate in (21) would be the steady state economic growth rate for the various *per capita* aggregates: physical capital, consumption and output. In such a model we have endogenous growth through technology choices, that is, long run constant *per capita* growth is obtained and is a function of several parameters of our analysis. These parameters concern the way in which the economy is able to create and diffuse technology and to invest in human capabilities. In fact, the growth rate obtained in this new framework is precisely the result of the particular way the economy handles the two true engines of growth: human capital and technology.

4. Final Remarks

This paper deals with the same issues as that of Nelson and Phelps (1966). We try to understand how physical capital accumulation plays a subsidiary role in terms of long run growth and the analysis focus on the two central engines of growth: investment in humans and technological diffusion. The way in which technological diffusion is modeled also relies on the Nelson-Phelps approach: the technology index to be included in an aggregate production function evolves over time according to a technological gap that relates to a reference value that may be understood as the scientific frontier or the state of the art in terms of knowledge capabilities of the economy.

The major modeling innovation in our paper consists of the assumption that the economy has the ability to control the growth of the knowledge frontier. This assumption makes sense if one considers a trade-off, arising from the technology resource constraint, between the *creation* of knowledge and the *application* of knowledge in material production. Therefore, the control of decisions about technology creation is conditioned by the economic non controllable rules of technology adoption. The way we have chosen to study simultaneously the behavior of human capital and technology variables leads to a two-sector optimal control problem, from which we have obtained some meaningful results:

(i) In the steady state, the state variables ("technology gap" and "human capital efficiency index") and the control variables ("growth rate" and "human capital share") they all display constant values.

(ii) The constant steady state result is possible only if one assumes decreasing returns in human capital accumulation in both the technology and the education sectors. In this way decreasing returns become compatible with sustained growth.

Finally, to translate our analysis to an economic growth setup it is just necessary to include a capital accumulation constraint and, eventually, an intertemporal consumption utility optimization framework. In this way, the rate that describes the evolution over time of the technology variables would simultaneously be the rate of economic growth, under a steady state perspective. Thus, in such a framework, it would be possible to talk about endogenous economic growth because the growth rate would be determined endogenously and because the several parameters that appear in its long run expression may be influenced by the economic agents decisions, including the policy measures that the government undertakes. Nevertheless, the endogenous nature of our growth model is different from the nature of conventional growth models. First, only technology and education decisions influence long run growth; factors as the consumption impatience, savings decisions or the rate of population growth are absent from the steady state growth rate expression. Second, despite the fact that the output, the physical capital stock and the technology all grow in the steady state, the human capital average efficiency does not, which implies that the model is somewhat pessimistic about human capabilities under our assumptions.

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Orlando Gomes

Escola Superior de Comunicação Social, Campus de Benfica do IPL, 1549-014 Lisboa, Portugal

e-mail: ogomes@escs.ipl.pt

COMPARING OF TIME SERIES IN ECONOMICS

MICHAL HOLČAPEK, ANNA MADRYOVÁ
VŠB-TU OSTRAVA, CZECH REPUBLIC

Abstract

In this paper we define a model of fuzzy quantifiers generalizing the classical quantifiers of predicate calculus of (fuzzy) logic. The new models of fuzzy quantifiers offer larger scale of applications than classical one. As an example we introduce an application of several models of fuzzy quantifier that represents the linguistic quantifier "for nearly all" and that is more suitable for studying of relation "to be better" of two time series of values of an economical index than the linguistic quantifier "for all".

1. Introduction

Some sequences of values are often a material for studying of some events or describing of some objects (e.g. firms, regions etc.) in economics. Sequences of values, which are dependent on some set of different times are called time series. In this paper we deal with this sort of sequences but described method (procedure) could be used for broader scale of sequences (e.g. values that express some level of satisfactions of given criteria). Besides studying a time series as unique object of our interest (e.g. by using of statistical methods) we can investigate some relations among more than one time series. For example, if we have two time series $\mathbf{x}=\{x_t|\ t\in T\}$ and $\mathbf{y}=\{y_t|\ t\in T\}$, where T denotes some set of times, then we can want to decide which one is "better". Before we propose a procedure of decision-making we analyze the mentioned situation. First, a decision on time series as some complex objects would be intuitively constructed by using of decisions on elements of time series in the same times. Further, a decision, if the value x_t of the first time series **x** is "better" than the value y_t of the second one **y**, is dependent on some model determining a truth value of the fact that "x_t is better than y_t". Hence, let **R** be a relation (e.g. expressed by using of natural language) then we can formulate a following general procedure:

1. To model truth values of the formulas φ_t:= "x_t is in the relation **R** with y_t" for each time $t\in T$.
2. To model (fuzzy) quantifier Q.
3. To model the following formula

$||\mathbf{x}$ is in the relation $\mathbf{R}$ with $\mathbf{y}||:= ||(Qt\in T)($ x_t is in the relation $\mathbf{R}$ with $y_t)||\ || =||(Qt\in T)(\varphi_t)||$,

where $||\ \ldots||$ denotes a truth value.

It is clear that greater truth value of the formula "**x** is in the relation **R** with **y**" supports a better decision that "**x** is in the relation **R** with **y**". This procedure uses some tools of classical and fuzzy logic (see [3,11]) that describe a human reasoning in natural sense. As the classical logic deals with two quantifiers only, namely general $\forall$ (for all) and existential $\exists$ (there exists), and it is not rich enough to express more complicated situations we propose a model of more general quantifier (see [4]) which is introduced in the section 3. In order to illustrate the mentioned procedure we use the relation "to be better" that is modelled in the next section. The section 4 is devoted to some application of this procedure in Economics.

2. Preliminaries

Throughout this paper the unit interval [0,1] is the set for truth values of formulas and membership degrees of fuzzy sets. The relation "to be better" is a kind of ordering. In order to model this relation we use a more general definition of ordering, namely T-E-ordering, where T (E) denotes t-norm (T-equivalence), that is introduced by U. Bodenhofer (see [2]). Note that it is a kind of fuzzy ordering (see [1]).

A *triangular norm* (t-norm for short) is a binary operation T on the unit interval [0,1], which is commutative, associative, non-decreasing and which has 1 as a neutral element. In the following example we introduce tree basic t-norms.

Example 1: Let $x,y\in[0,1]$ be an arbitrary elements. Then

Minimum t-norm: $T_{\mathbf{M}}(x,y)= x\wedge y$,

Product t-norm: $T_{\mathbf{P}}(x,y)= x\cdot y$,

Łukasiewicz t-norm: $T_{\mathbf{L}}(a,b)= 0\vee(a+b-1)$.

Let T be a t-norm, then a fuzzy relation E, i.e. $E: X\times X\rightarrow[0,1]$, is called T-*equivalence* if for all $x,y,z\in X$ we have

(i) $E(x,x)=1$ (reflexivity)

(ii) $E(x,y)=E(y,x)$ (symmetry)

(iii) $T(E(x,y), E(y,z))\leq E(x,z)$ (T-transitivity)

Note that a similarity of two objects is modeled as T-equivalences. If we want to construct some T-equivalence in practice, we can use some pseudo-metrics $d: X\times X\rightarrow[0,\infty]$. In order to show the correspondence between T-equivalences and pseudo-metrics, we introduce some additional properties for t-norms (see [1,5,6,11]). A t-norm is called *Archimedean* if and only if for each x, y $\in]0,1[$ there exists a natural number n such that $T(x,\ldots,x)=x_T^n<y$. It is clear to show that minimum t-norm is not Archimedean t-norm. An *additive generator* t of a continuous Archimedean t-norm T is a continuous, strictly decreasing mapping $t:[0,1]\rightarrow [0,\infty]$ with $t(1)=0$, such that is $T(x,y)= t^{(-1)}(t(x)+t(y))$ for all $x,y\in [0,1]$, where $t^{(-1)} = \mathrm{Inf}\{y\in [0,1] \mid t(y)<x\}$.

Theorem: *Let X be a universe and T is a continuous Archimedean t-norm with continuous additive generator t: $[0,1]\rightarrow[0,\infty]$. Then we have*

(i) *If d is a pseudo-metric on X, then the fuzzy relation E_d on X specified by $E_d(x,y)= t^{(-1)} o\, d(x,y)$ is a T-equivalence.*

(ii) *If E is a T-equivalence on X, then $d_E:X\times X\rightarrow[0,\infty]$ given by $d_E(x,y)=toE(x,y)$ is a pseudo-metric on X.*

Example 2: Let $d(x,y)=|x-y|$ be the common pseudo-metric on the set of real numbers. If $T_{\mathbf{P}}$ is product t-norm, then $E_{\mathbf{P},d}(x,y)=a^{-|x-y|}$, where $a>1$, is $T_{\mathbf{P}}$-equivalence. If $T_{\mathbf{L}}$ is Łukasiewicz t-norm, $E_{\mathbf{L},d}(x,y)=1-\min(|x-y|,1)$ is the $T_{\mathbf{L}}$-equivalence. Both T-equivalences are induced by the pseudo-metric $d(x,y)=|x-y|$.

The fuzzy relation $O:X\times X \rightarrow [0,1]$ is called *a fuzzy ordering* on X with respect to a t-norm T and T-equivalence E, shortly *T-E-ordering*, if for all x, y, z$\in$X we have

(i) $E(x,y) \leq O(x,y)$ (E-reflexivity),

(ii) $T(O(x,y),O(y,x)) \leq E(x,y)$ (T-E-antisymmetry),

(iii) $T(O(x,y),O(y,z)) \leq O(x,z)$ (T-transivity).

Example 3: Let E be a T-equivalence over set of real numbers, then a fuzzy relation O defined as follows: $O(x,y)=1$, if $x\leq y$, and $O(x,y)=E(x,y)$, if $x>y$, is T-E-ordering.

3. Fuzzy quantifiers

Generalized quantifiers have been studied in mathematical logic since fifties (pioneering work being done by A. Mostowski [9] and their study in connection with natural language has been set in motion by R. Montague [8]. The modelling of natural language quantifiers as fuzzy quantifiers being fuzzy numbers has been pointed out first by L.A. Zadeh in [12]. The modelling of classical quantifiers on the unit interval [0,1] is connected with operations *min* and *max* which are special examples of T-norms and S-conorms. T-quantifiers and S-quantifiers have been studied by R. Mesiar and H. Thiele in [7]. Linguistic quantifiers in fuzzy logic have been studied by V. Novák [10].

Most of definitions of generalized (fuzzy, cardinal, etc.) quantifiers are connected with a special mapping from a set of subsets (or fuzzy subsets) to a structure D of truth values of formulas, where subsets are chosen from a domain of a given formula. In the mentioned literature there is considered only finite domains of formulas and we also keep this restriction in our model of fuzzy quantifiers. Note, that the following definition of fuzzy quantifiers is similar to one of linguistic quantifiers in [10].

Definition: *Let X be a non-empty finite universe, T be a t-norm and* $\#Q: 2^X \rightarrow [0,1]$ *be a mapping such that*

(i) $\#Q(X)=1$ *and* $\#Q(\infty)=0$,

(ii) $\forall X_1, X_2 \in 2^X : X_1 \subseteq X_2 \Rightarrow \#Q(X_1) \leq \#Q(X_2)$,

(iii) $\forall X_1, X_2 \in 2^X : |X_1|=|X_2| \Rightarrow \#Q(X_1)= \#Q(X_2)$.

Then the truth value of a formula $(Qx \in X)(\varphi(x))$*, where Q is a fuzzy quantifier w.r.t. the mapping #Q, is defined by*

$$\|(Qx \in X)(\varphi(x))\| = \operatorname*{Sup}_{Y \in 2^X} \operatorname*{Inf}_{y \in Y} T(\#Q(Y), \|(\varphi(y))\|). \qquad (1)$$

According to this definition each fuzzy quantifier over a finite universe is determined by using of a mapping and the formula (1). It is easy to show that this model of general quantifier generalized classical quantifiers [see]. In the next part we use quantifier from the following example.

Example 4: Let X be a nonempty finite universe.

$$\forall Y \in 2^X : \# F_p(Y) = \left(\frac{|Y|}{|X|}\right)^p,$$

where p>0. This quantifier for p=1 is a special case of more general fuzzy quantifier defined by L.A.Zadeh in [10] to model language quantifiers as *most*, *a large fraction*, *much of*, etc.

It is obvious that $\|(\forall x \in X)(\varphi(x))\| \leq \|(Qx \in X)(\varphi(x))\| \leq \|(\exists x \in X)(\varphi(x))\|$ holds for all fuzzy quantifiers Q and formulas.

4. An application of comparison between two time series

In this section we show how to find, by using of the mentioned procedure in the introduction, truth values of formulas $\varphi_T(\mathbf{reg_i}, \mathbf{reg_j})$:="a region $\mathbf{reg_i}$ is better than a region $\mathbf{reg_j}$ in the Czech Republic with regards to the unemployment's index in the period T". The input data of our time series $\{reg_{-t} \mid t \in T\}$ are real values of unemployment's index in eight regions (Prague (**Pg**), Central-Bohemia (**Cb**), South-Bohemia (**Sb**), West-Bohemia (**Wb**), North-Bohemia (**Nb**), East-Bohemia (**Eb**), South-Moravia (**Sm**), North-Moravia (**Nm**)) of Czech Republic in 1995-1999 that are stated in the table 1.

	Pg	**Cb**	**Sb**	**Wb**	**Nb**	**Eb**	**Sm**	**Nm**
95	0,278	2,733	2,091	2,159	4,756	2,326	3,082	5,383
96	0,350	2,731	2,161	2,313	5,424	2,532	3,133	5,247
97	0,612	3,849	3,108	3,498	7,287	3,599	4,365	6,650
98	1,545	5,153	4,425	5,297	9,689	5,039	6,225	9,123
99	3,179	6,900	6,020	7,370	12,693	7,417	8,824	12,779

Table 1: Unemployment in years 1995-1999:

Let $\varphi_t(\mathbf{reg_i}, \mathbf{reg_j})$ be the similar to formula $\varphi_T(\mathbf{reg_i}, \mathbf{reg_j})$, where the part "in the period T" is replaced by "in the time t". If we use T_L , T_P-equivalences that are induced by the pseudo-metric d(x,y)=|x-y| (see example 2) for modelling of T-E-ordering $O_{\mathbf{L},d}$, $O_{\mathbf{P},d}$ (see example 3), respectively, then the truth values of formulas $\varphi_t(\mathbf{reg_i}, \mathbf{reg_j})$ can be expressed as follows

$$\|\varphi_t(\mathbf{reg_i}, \mathbf{reg_j})\|\_ := O\_{,d}(reg_{it}, reg_{jt}) = \begin{cases} 1, & \text{if } reg_{it} \le reg_{jt}, \\ E\_{,d}(reg_{it}, reg_{jt}), & \text{if } reg_{it} \ge reg_{jt}, \end{cases} \qquad (2)$$

where $- \in \{L,P\}$. In the case of $O_{\mathbf{P},d}$ we put a=1.1 (see example 2) to be larger numbers of $T_\mathbf{P}$-equivalence and so $T_\mathbf{P}$-E-ordering. Notice that the mentioned definition of truth values (2) corresponded to the natural imagination that a region is better than another one if it has smaller values of unemployment's index. In the table 2 we show a small part of the table of truth values of formulas $\varphi_t(\mathbf{reg_i},\mathbf{reg_j})$.

	$O_{\mathbf{P},d}$					$O_{\mathbf{L},d}$				
Formulas	**95**	**96**	**97**	**98**	**99**	**95**	**96**	**97**	**98**	**97**
φ_t**(Pg,Pg)**	1	1	1	1	1	1	1	1	1	1
…	…	…	…	…	…	…	…	…	…	…
φ_t**(Pg,Nm)**	1	1	1	1	1	1	1	1	1	1
φ_t**(Cb,Pg)**	0.79	0.8	0.73	0.71	0.7	0	0	0	0	0
…	…	…	…	…	…	…	…	…	…	…
φ_t**(Cb,Eb)**	0.96	0.98	0.98	0.99	1		0.8	0.75	0.89	1
φ_t**(Cb,Sm)**	1	1	1	1	1	1	1	1	1	1
…	…	…	…	…	…	…	…	…	…	…
φ_t**(Nm,Pg)**	0.62	0.63	0.56	0.49	0.4	0	0	0	0	0
φ_t**(Nm,Cb)**	0.78	0.79	0.77	0.69	0.57	0	0	0	0	0
…	…	…	…	…	…	…	…	…	…	…

Table 2: Several truth values of formulas $\varphi_t(\mathbf{reg_i},\mathbf{reg_j})$ interpreted by T-E-orderings.

Further, we use some linguistic quantifier "for nearly all" ($\forall'$ for short) that intuitively corresponds to our imagination about the relationship between a relation of two time series and relations of arguments of these time series. We consider two models of this quantifier "for nearly all" - #∀':= #F_p, where p=1,2 (see example 4). If we put t-norm T=Infimum, then we can calculate truth values of formulas $(\forall' t\in T)(\varphi_t(\mathbf{reg_i}, \mathbf{reg_j}))$, by using of the formula (1) and (2), as follows

$$\|(\forall' t \in T)(\varphi_t(\mathbf{reg_i}, \mathbf{reg_j}))\| = \operatorname*{Sup}_{Y\in 2^T} \operatorname*{Inf}_{y\in Y} (\#F_p(Y) \wedge O_{\_,d}(reg_{it}, reg_{jt})).$$

	$\varphi_T(\mathbf{reg_i},\mathbf{reg_j})$	Pg	Cb	Sb	Wb	Nb	Eb	Sm	Nm
$O_{\mathbf{P},d}$	**Pg**	1	1	1	1	1	1	1	1
	Cb	0.71	1	0.92	0.95	1	0.96	1	1
	Sb	0.76	1	1	1	1	1	1	1
	Wb	0.7	0.96	0.88	1	1	0.98	1	1
	Nb	0.53	0.65	0.61	0.66	1	0.64	0.72	0.94
	Eb	0.72	0.95	0.88	0.98	1	1	1	1
	Sm	0.64	0.83	0.8	0.87	1	0.87	1	1
	Nm	0.56	0.69	0.64	0.69	0.94	0.68	0.76	1
$O_{\mathbf{L},d}$	**Pg**	1	1	1	1	1	1	1	1
	Cb	0	1	0.36	0.6	1	0.75	1	1
	Sb	0	1	1	1	1	1	1	1
	Wb	0	0.8	0.6	1	1	0.8	1	1
	Nb	0	0	0	0	1	0	0	0.6
	Eb	0	0.8	0.51	0.8	1	1	1	1
	Sm	0	0.48	0.03	0.18	1	0.25	1	1
	Nm	0	0	0	0	0.8	0	0	1

Table 3: Truth values of formulas $(\forall' t\in T)(\varphi_t(\mathbf{reg_i}, \mathbf{reg_j}))$ for p=1

	$\varphi_T(\mathbf{reg_i},\mathbf{reg_j})$	Pg	Cb	Sb	Wb	Nb	Eb	Sm	Nm
$O_{\mathbf{P},d}$	**Pg**	1	1	1	1	1	1	1	1
	Cb	0.7	1	0.92	0.95	1	0.96	1	1
	Sb	0.76	1	1	1	1	1	1	1
	Wb	0.67	0.96	0.88	1	1	0.98	1	1
	Nb	0.46	0.64	0.61	0.64	1	0.64	0.69	0.94
	Eb	0.67	0.95	0.88	0.98	1	1	1	1
	Sm	0.64	0.83	0.77	0.87	1	0.87	1	1
	Nm	0.49	0.64	0.64	0.64	0.94	0.64	0.69	1
$O_{\mathbf{L},d}$	**Pg**	1	1	1	1	1	1	1	1
	Cb	0	1	0.27	0.58	1	0.64	1	1
	Sb	0	1	1	1	1	1	1	1
	Wb	0	0.64	0.36	1	1	0.74	1	1
	Nb	0	0	0	0	1	0	0	0.44
	Eb	0	0.64	0.39	0.78	1	1	1	1
	Sm	0	0.36	0.03	0.13	1	0.23	1	1
	Nm	0	0	0	0	0.64	0	0	1

Table 4: Truth values of formulas $(\forall' t\in T)(\varphi_t(\mathbf{reg_i}, \mathbf{reg_j}))$ for p=2

Tables 3 and 4 show the values of final formulas $\varphi_T(\mathbf{reg_i},\mathbf{reg_j})$ for p=1 and p=2. For parameter p=2 truth values are smaller. In general, the higher the values of parameter p are, the closer truth values of formulas are to those for quantifier "for all" (interpreted by operation infimum). Note that truth values are obtained by functions and procedures in the environment of Mathematica 4.1.

On the base of these tables can be constructed various orderings (to be better) of regions with regard to different indexes. This topic is very broad and rather complicated to present it in this paper.

5. Conclusion

In this paper, there is shown a simple model of fuzzy quantifiers that has been applied to an investigation of the relation "to be better" of two time series. This simple model could be easily extended by some modification of the definition of the mapping #Q. However, the modelling of evaluating formula (1) could be more complicated for more general fuzzy quantifiers in practice.

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Michal Holčapek, Anna Madryová

Department of Mathematical Methods in Economics, EkF VŠB-TU Ostrava, Sokolská 33, 701 21 Ostrava, e-mail: michal.holcapek@vsb.cz

SOCIAL ACCOUNTING MATRIX AS A TOOL OF EQUILIBRIUM ANALYSIS

JAROSLAV HUSÁR
EU BRATISLAVA, SLOVAKIA

1. Introduction

It is clear that a method of obtaining an overall picture of economic activity in an economy is an essential. This is the function of national income accounts. They are based on theory of an economic behaviour of economic agents. Within this theoretical superstructure we can analyze the functioning of a system. In last decades a new method of obtaining a picture on an economy was developed. It is a theory of social accounting matrix.[1] We present a completly new structure of social accounting matrix based on macroeconomic theory.

2. Social accounting matrix of Slovakia

Since each transaction between the sectors of the economy appears twice in each sector account in the system of national accounts, the form of the matrix is a convenient way to present an overall picture of the economy. Table 1 presents a general structure of social accounting matrix based on the macroeconomic variables listed later under the Table 1.

Table 1: Theoretical social accounting matrix

	Firms	House-holds	Govern ment	Nett foreign paymen ts	Net saving	Natio-nal income	Indirect busines s taxes	Depreci ation	Total
Firms		R_p	T_b		S_b	NI	T_i	D	GDP
Households	C		T_p		S_p				
Government	G	T_r		T_f	S_g				
Net export	X-M								
Net investm.	I_r			I_f					
Net nat. produ.	Y_n								
Ind. bus. taxes			T_i						
Deprecation	D								
Total	GDP								

Table 1 represents a payment, the left side of the sector account, while each column shows the corresponding receipts entered on the right side of another sector account. For example,

[1] This paper is the essence of my inaguration lecture in 1996.

consumption 251,89 is shown, reading across the Table 2 recording real Slovak data, as an expense of the household sector and also, reading down the table, as a business receipt.

Data are always incomplete and rarely consistent; different bodies of statistics are often collected by a variety of agencies which do not all make use of an agreed set of definitions and classifications; different methods of collecting give rise to sampling and other errors of many kinds. So the strategy is required in order to extract the best possible quantitative description of the economy from the statistical data available. We will have shown how to cope with this problem using the logic of our matrix.

The following notation is used to ease of presentation:

Gross domestic product – Y

Net national product – Y_n

National income – NI

Disposable income – Y_d

Consumption expenditure – C

Government expenditure – G

Net export – NX

Net foreign investment - I_f

Net domestic investment - I_d

Household receipts - R_p

Transfer payments (domestic) - T_d

Transfers to foreigners - T_f

Personal saving - S_p

Government saving - S_g

Business saving - S_b

Direct taxes (persons)- T_p

Direct taxes (business) – T_b

Indirect taxes - T_i

Depreciation

Using the real data for Slovak economy for the year 1994 we get this SAM:

Table 2: Social Accounting Matrix, Slovakia, 1994, bill. Sk

	Firms	House-holds	Govern ment	Net foreign paymen ts	Net saving	Natio-nal income	Indirect busines s taxes	Deprec ation	Total
Firms		285,20	27,47		14,03	326,70	36,23	35,07	398,00
Households	251,89		46,17		14,03				321,09
Government	79,48	26,88		1,75	1,75				109,86
Net export	4,09								4,09
Net investm.	27,47			2,34					29,81
Net nat. produ.	362,93								
Ind. bus. taxes			36,23						
Deprecation	35,07								
Total	398,00	312,08	109,87	4,09	29,81				

(Husár, June, 1995)

Now we are in a position to create several equations that algebraically describe the state of the economy. First of all, from column 1 of Table 1 and Table 2 we see that

$$GDP \equiv C + G + (X - M) + I_d + D \qquad (1)$$

$$398 \equiv 251{,}89 + 79{,}48 + 4{,}9 + 27{,}47 + 35{,}07.$$

Gross domestic product is the sum of the expenditures by all sectors in the economy. Again from the same column 1 we can write

$$Y_n \equiv GDP - D$$

or

$$Y_n \equiv C + G + (X - M) + I_d \qquad (2)$$

$$362{,}93 = 251{,}89 + 79{,}48 + 4{,}09 + 27{,}47$$

The net national product total represents the net addition to the flow of goods and services to individuals, either directly from business (domestic and foreign) or through the government., and to the capital stock. That is, net national product is gross domestic product less depreciation or, alternatively, the expenditures by the personal and government sectors and the net addition to the capital stock.[2]

The principle of the SAM is that the row sum must equal the column sum. By this principle from row 1 we see:

$$GDP \equiv R_p + T_b + S_b + T_i + D \qquad (3)$$

but from column 2 and row 2 we see that

[2] Both the change in inventory and net exports are given in net terms.

$$R_p + T_r \equiv C + T_p + S_p$$

or

$$R_p \equiv C + T_p + S_p - T_r \qquad (4)$$

and from column 3 and row 3,

$$T_b + T_p + T_i \equiv G + T_r + T_f + S_g$$

or

$$T_b + T_i \equiv G + T_r + T_f + S_g - T_p \qquad (5)$$

Now we made o good preparation for statement that follows. Our next ideas are an important information needed for macroeconomic analysis. Substituting (4) and (5) into (3), we have:

$$GDP \equiv C + T_p + S_p - T_r + G + T_r + T_f + S_g - T_p + S_b + D$$

or

$$GDP \equiv C + S_p + G + T_f + S_g + S_b + D \qquad (6)$$

Now we can equate the expression (1) and (6). We get:

$$(X - M) + I_d + D \equiv S_p {}^{+} S_g + T_f + S_b + D$$

Remembering that $(X - M) - T_f \equiv I_f$, we have

$$I_f + I_d + D \equiv S_p {}^{+} S_g + S_b + D \qquad (7)$$

2,34 + 27,47 + 35,07 ≡ 14,03 + 1,75 + 14,03 + 35,07.

This identity represents both sides of the gross saving and investment account as it is known from the system of national accounts. In equations (6) and (7) we got compleetly new picture on the GDP identity and identity for import. What is needed is a systematic and coherent framework whixh can accomodate the main bodies of data needed for macroeconomic description of the functioning of an economy.

Netting depreciation out of identity (7), we have

$$I_f + I_d \equiv S_p {}^{+} S_g + S_b$$

2,34 + 27,47 ≡ 14,03 + 1,75 + 14,03 and finally
29,81 ≡ 29,81.

This represents the equality of net saving and net investment.

The identity of saving and investment plays a large role in economic analysis and economic policy. The policy makers should insist on accepting this result of the SAM theory

and the macroeconomics. That part of final output not purchased by consumers and government has been called investment and must by definition be equal to that amount of receipts not paid out for final goods or services by persons, government, or firms (business).
National income may also be obtained by subtracting indirect taxes from Y.

$$Y - T_i \equiv NI$$

$$362{,}93 - 36{,}23 \equiv 326{,}70.$$

National income may olso be obtained by adding total personal income payments to direct business taxes and net saving. Accordingly, from row 1:

$$NI \equiv R_p + T_b + S_b$$

$$326{,}70 \equiv 285{,}20 + 27{,}47 + 14{,}03.$$

Further, from column 2 and row 2,

$$R_p + T_r \equiv T_p + S_p \qquad (8)$$

$$285{,}2 + 26{,}88 \equiv 251{,}89 + 46{,}17 + 14{,}03$$

The left side of (8) lists personal income (factor payments plus transfers to persons from government); the right side shows the way persons dispose of their income: consumption expenditures for goods and services, direct taxes, and saving.
If personal taxes are transposed in identity (8), the left side becomes personal disposable income. e. i., total income receipts of persons minus their tax payments.

$$R_p + T_r - T_p \equiv C + S_p \equiv Y_d$$

$$285{,}20 + 26{,}88 - 46{,}17 \equiv 251{,}89 + 14{,}03 \equiv 265{,}92$$

Disposable income, as shown by this identity, can be either spent (C) or saved (S_p). In some cases people can send some of their income to foreigners.
Disposable income can be derived from the net national product total by subtracting from Y that part which does not accrue to persons and adding government transfer payments to persons. Thus

$$Y_d \equiv Y - (T_i + T_p + T_b) + T_r - S_b$$

$$265{,}92 \equiv 362{,}93 - (36{,}23 + 46{,}17 + 27{,}47) + 26{,}88 - 14{,}03$$

Finally, let

$$T_i + T_p + T_b - T_r \equiv T$$

$$36{,}23 + 46{,}17 + 27{,}47 - 26{,}88 \equiv 82{,}99$$

where T is defined as net taxes, so that

$$Y \equiv Y - T - S_b$$

$$265{,}92 \equiv 362{,}93 - 82{,}99 - 14{,}03.$$

We would like to stress, moreover, that the government surplus can be calculated by subtracting government expenditures and transfers to foreigners from net taxes. Thus

$$S_g \equiv (G + T_f)$$

$$1{,}75 \equiv 82{,}99 - (79{,}48 + 1{,}75).$$

As we can see, we have highlighted several important identities in the Slovak economy. They result from the frame of the SAM, that is the construction based on macroeconomics and our attempt to create a new form of SAM. The entries in a matrix, as we have seen, can be presented in an equations form which help us to find new aspects of an economy functioning. Or better, if something is valid, what else must me valid too. Our SAM provides a data base for economic analysis, different way of looking at it.

In this explanation we have presented a fairly detailed picture of the national economy generally and the Slovak economy particularly. We showed how rhe transaction in the economy can be organize in matrix form. This form allows us to formulate a set of identities that help a policy maker to analyze the functioning of an economic system.

3. Conclusion

Our discussion indicated that the volume of output is determined by the expenditures for final goods and services (column 1), which, in turn, are related in part to the income received by the same units making expenditure decisions. Therefore, the GDP account is broken down by this method in terms of income receipts by economic units and expenditures by the same unit.

From Table 1 and 2 we can read of other important relationships using the requirement of the SAM that total of the first row should be equal to the total of first column. By making different substitutes one can get new information on an economy and so one can design a coherent economic policy.

Data are always incomplete and rarely consistent; different bodies of statistics are often collected by a variety of agencies which do not all make use of an agreed set of definitions and classifications; different methods of collecting give rise to sampling and other errors of many kinds. So the strategy is required in order to extract the best possible quantitative description of the economy from the statistical data available. We showed how to cope with this problem.

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prof. Ing. Jaroslav Husár, CSc., Mgr. ek.

Fakulta hospodárskej informatiky, Ekonomická univerzita, Dolnozemská 1b, Bratislava, e-mail: husar@euba.sk

COMPARISON OF ALTERNATIVE MODELS AND FORECASTS OF THE CZECH ECONOMY DEVELOPMENT[1]

ROMAN HUŠEK, MILAN VÁCLAVÍK, JIŘÍ UDATNÝ
UNIVERSITY OF ECONOMICS PRAGUE, CZECH REPUBLIC

Abstract

This study compares alternative estimates and forecasts of some output-oriented macroeconomic indicators of the Czech Republic. The analysed variables were obtained from an interdependent analytical-prognostic model of simultaneous equations which was specified in two ways. At first, the model was estimated from logarithmed values of the variables (see Hušek et al., 2002). Subsequently it was estimated from the year-on-year percentage growth rates.

1. Main Characteristics

The simultaneous model consists of 8 behavioural equations (EQ1 – EQ8) and 21 identities (ID1 – ID21). When using the natural logarithms of the variables, the model is addressed as LLM – **logarithmic level model**, for the percentage year-on-year growth rates we refer to ECM – **error correction model**. Parameter estimation was based upon 34 quarterly observations comprising the period from Q1/1994 to Q2/2002. Data were obtained from ČSÚ, ČNB, IMF and OECD; the estimates were performed by the two-stage least squares (2SLS) or ordinary least squares (OLS) methods. When needed to decrease the 1st order autocorrelation, Cochranne-Orcutt procedure was used. Equations EQ1 – EQ4 constitute a price block of deflators of the individual components of GDP and equations EQ5 – EQ8 form a real economy block of equations describing the GDP components expressed in constant prices of 1995.

The original specification was LLM which allowed an estimation and forecast of the levels of endogenous variables. LLM describes the long-term relations and interactions (Hušek et al., 2002). The ECM specification covers the short-term (dynamic) interaction of the differenced variables. One-year lagged residuals (multiplied by 100) from LLM – which may be taken for deviations from the long-term equilibrium – were used for the ECM specification as additional predetermined variables. The values of their estimated parameters allow us to assess the pace at which the relevant variables converge to its equilibrium. The

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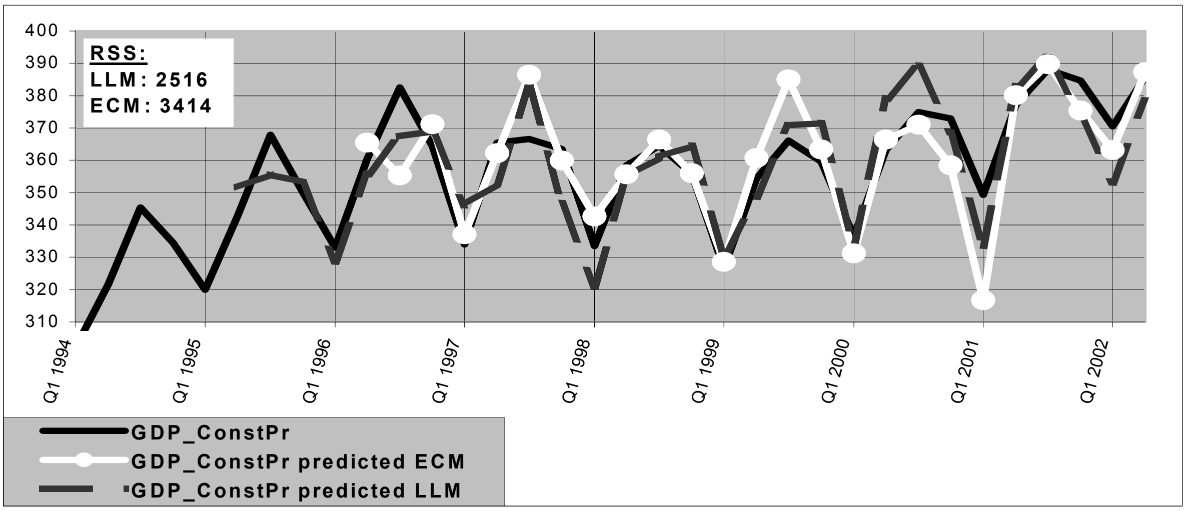
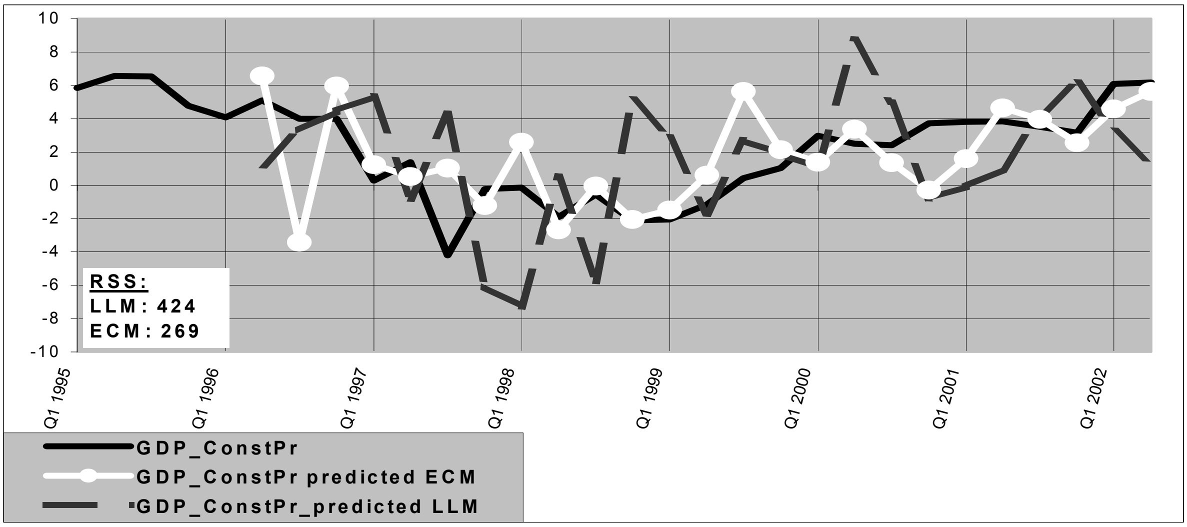
more negative they are, the faster the adaptation is (similar size of disequilibriums is assumed).

We have compared eg. the GDP in constant prices estimates from LLM with the estimates from ECM – after converting both back to level values. The data compliance measured by the root of mean square error (RSS) is better for LLM:



On the other hand, if we compare the estimate by LLM – after de-logging the values back to levels and subsequent transformation to year-on-year growth rates – with the same



equation estimated by ECM, than the RSS shows better data compliance for ECM:

2. Individual Behavioural Equations

EQ1 – Consumption Deflator (DEF_C)

		CPI_t	DEF_C_{t-1}	RES^{LLM}_{t-4}
LLM	coeff.	**0,798**	**0,130**	
	t-stat	*11,267*	*1,608*	
ECM	coeff.	**0,697**	**0,219**	**-0,617**
	t-stat	*7,296*	*2,107*	*-2,425*

	R^2_{adj}	SE	D H
LLM	0,999	0,739	1,039
ECM	0,985	0,787	1,092

The differences between the values of the coefficient estimates from LLM and ECM are relatively small which implies that this variable has got similar short-term and long-term dependencies. The value of the coefficient of residuals from LLM shows that convergence speed is rather small.

EQ2 – Gross Fixed Capital Formation Deflator (DEF_GFCF)

		CONST	DEF_M_t	$COPI_{t-1}$	DEF_GFCF_{t-1}	$D\_DEF\_GFCF_t$	$D\_DEF\_GFCF_{t-4}$ + + RES^{LLM}_{t-4}
LLM	coeff.	**0,016**	**0,116**	**0,155**	**0,474**	**-0,032**	
	t-stat	*4,287*	*1,606*	*2,520*	*3,481*	*-3,621*	
ECM	coeff.		**0,136**	**0,146**	**0,502**		**-0,536**
	t-stat		*2,766*	*1,904*	*3,039*		*-1,718*

	R^2_{adj}	SE	D H
LLM	0,966	1,063	0,716
ECM	0,886	1,222	-0,610

D_DEF_GFCF… equals to 1 on all first quarters starting with Q1/1999, otherwise this variable is equal to 0. Established due to extremely low values of the variable DEF_GFCF caused by changes in the structure of GFCF and the start of a new investment cycle.

EQ3 – Export of Goods and Services Deflator (DEF_X)

		PPI^{EU}_t	EER_t	DEF_X_{t-1}	RES_{t-1}	RES^{LLM}_{t-4}
LLM	coeff.	**0,707**	**0,413**	**0,208**	**0,948**	
	t-stat	*1,954*	*5,048*	*1,574*	*15,452*	
ECM	coeff.	**0,573**	**0,372**	**0,231**	**0,790**	**-0,320**
	t-stat	*1,833*	*5,845*	*1,805*	*6,054*	*-1,381*

	R^2_{adj}	SE	D H
LLM	0,989	1,190	-0,207
ECM	0,918	1,308	0,113

Positive value of the variable EER shows that the Czech economy is a foreign trade price-taker.

EQ4 – Import of Goods and Services Deflator (DEF_M)

At first, this equation was estimated with the predetermined variable PPI_EU being excluded from it – because of a strong colinearity between the PPI_EU variable and the

BRENT Oil price. Nevertheless, we have decided to prefer economic verification to the econometric one. The calibrated parameter of 0,5 of the PPI_EU variable is equal to the correlation coefficient between this variable and DEF_M. This again suggests that our economy is a price-taker.

		$CONST$	EER_t	P_BRENT_t	PPI^{EU}_t	DEF_M_{t-1}	RES_{t-1}	RES^{LLM}_{t-4}
LLM	coeff.	**-0,283**	**0,519**	**0,057**	**0,500**		**0,987**	
	t-stat	*-2,000*	*8,046*	*3,888*	*-*		*31,782*	
ECM	coeff.		**0,476**	**0,023**	**0,500**	**0,194**	**0,982**	**-0,788**
	t-stat		*10,871*	*2,754*	*-*	*2,761*	*12,275*	*-4,293*

	R^2_{adj}	SE	DW
LLM	0,930	0,960	1,672
ECM	0,967	0,987	1,694

EQ5 – Consumption (C_ConstPr)

The estimated parameters indicate that RDI has more effect on comsumption in the long run than in the short run. In the short run, consumption is influenced by its own persistency and consumers' habit – as may be observed from the value of the coefficient of the lagged endogenous variable, which is higher in ECM. Nevertheless, the consumption self persistence may be observed in the long run as well. The low value of the coefficient of RCH shows the unwillingness of the Czech households to cover their consumption from this source during the given period of time. On the other hand, from the high coefficient value of the RDI we may deduce the tendency to cover household consumption from this particular source, especially in the long run.

In order to increase the number of degrees of freedom of the following 4 equations and therefore in order to make the estimations more robust, we have used the composed variable SEAS. Theoretically, the value of its parameter should be equal to 1. The values of the individual seasonal variables Dx (for the x[th] quarter) are taken from the consumption equation where those seasonal variables were included individually.

		RDI_t	RCH_t	$C\_ConstPr_{t-1}$	$SEAS\_C_t$	D_C_t	D_C_{t-4} + + RES^{LLM}_{t-4}
LLM	coeff.	**0,473**	**0,046**	**0,515**	**0,986**	**0,027**	
	t-stat	*3,920*	*1,265*	*4,392*	*3,689*	*2,955*	
ECM	coeff.	**0,189**	**0,104**	**0,838**			**-1,245**
	t-stat	*1,441*	*1,923*	*7,049*			*-4,653*

	R^2_{adj}	SE	D H
LLM	1,000	1,743	-1,972
ECM	0,816	1,884	-0,908

D_C … equals 0 until Q4/1997 and equals 1 since Q1/1998. Represents the beginning of a new consumption cycle.

EQ6 – Gross Fixed Capital Formation (GFCF_ConstPr)

The coefficient of RGCP is higher in the long run which supports the hypothesis that corporate investments are influenced by long-term prosperity rather than by short-term (possibly random) profit fluctuations.

		$RFDIcum_t$	$RGCP_{t-1}$	RCC_{t-1}	$GFCF\_ConstPr_{t-4}$	$SEAS\_GFCF_t$	D_GFCF_t	RES^{LLM}_{t-4}
LLM	coeff.	**0,103**	**0,804**	**0,381**	**0,083**	**1,017**	**-0,058**	
	t-stat	*2,602*	*5,854*	*2,486*	*0,634*	*4,695*	*-1,824*	
ECM	coeff.	**0,041**	**0,342**	**0,210**	**0,274**			**-0,970**
	t-stat	*1,920*	*1,090*	*1,206*	*2,569*			*-3,530*

	R^2_{adj}	SE	DW
LLM	0,999	4,138	1,443
ECM	0,723	3,894	1,469

D_GFCF… equals 0 until Q4/1997 and equals 1 since Q1/1998. This is related to the change of GFCF structure and due to the start of a new investment cycle.

EQ7 – Export of Goods and Services (X_ConstPr)

This equation shows a strong dependency on the GDP_EU which is used to approximate the foreign demand for the Czech goods. This relation is strong in the short run, as once a company establishes itself on a foreign market, its demand-dependency on a foreign economic cycle decreases.

The P_X variable represents the price elasticity of export. This variable is given as the ratio of local (domestic) and foreign price level, both expressed in CZK.

		$GDP^{EU}\_ConstPr_t$	P_X_t	$X\_ConstPr_{t-1}$	$SEAS\_X_t$	RES^{LLM}_{t-4}
LLM	coeff.	**1,881**	**-0,719**	**0,420**	**1,025**	
	t-stat	*10,545*	*-4,246*	*7,492*	*9,582*	
ECM	coeff.	**2,429**	**-0,792**	**0,357**		**-1,378**
	t-stat	*5,623*	*-5,796*	*4,044*		*-5,014*

	R^2_{adj}	SE	D H
LLM	1,000	2,494	-0,084
ECM	0,948	2,830	0,413

EQ8 – Import of Goods and Services (M_ConstPr)

Imports are highly dependent on local demand (so-called 'domestic absorbtion': C, GCF and X). In this context, we talk about import intensity of consumption etc.

The coefficient of the P_M variable represents the price elasticity of import. This variable is given as the ratio of foreign and local price level, both expressed in CZK.

The coefficient of residuals from LLM shows that convergence to the equilibrium is quick.

If we compare the coefficients of export and import elasticity we realize that import is much less elastic than export. It supports the hypothesis of large dependency of the Czech economy on imports of raw materials and semi-finished goods.

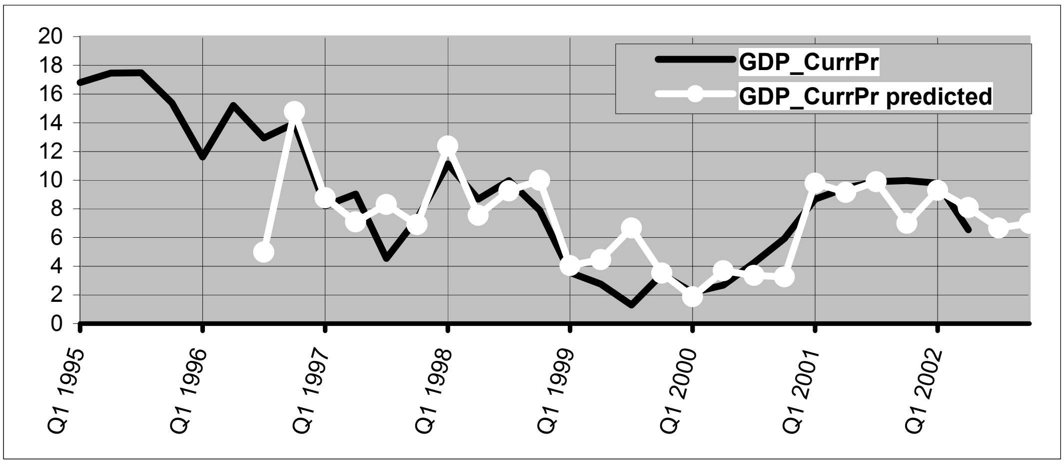
		$C\_ConstPr_t$	$GCF\_ConstPr_t$	$X\_ConstPr_t$	P_M_t	$SEAS\_M_t$	RES_{t-1}	RES^{LLM}_{t-4}
LLM	coeff.	**0,484**	**0,184**	**0,801**	**-0,187**	**0,978**		
	t-stat	*11,042*	*4,022*	*31,652*	*-2,028*	*7,675*		
ECM	coeff.	**0,506**	**0,255**	**0,793**	**-0,152**		**-0,563**	**-1,601**
	t-stat	*10,533*	*11,099*	*53,470*	*-4,520*		*-3,268*	*-8,186*

	R^2_{adj}	SE	DW
LLM	1,000	1,439	2,560
ECM	0,993	1,047	2,114

3. ECM-Based Forecasts

We shall step forward to the ex-ante forecasting of the endogenous variables for the period from Q3/2002 to Q4/2002. Because LLM is logarithmed and is of larger scale and the available time series are too short, we have not used the reduced form of the equations. Instead, the forecasts were calculated iteratively from the structure form of the model (Hušek et al., 2002). We shall also present outcome of the forecasts performed using discrete deterministic simulations, both static and dynamic. The next table contains ECM-based forecasts of year-on-year growth rates of all endogenous variables. All relevant exogenous variables were forecasted (set) by expert method. The next figure shows both observed and forecasted values of the GDP in current prices growth rate – as this variable can be considered as the main output of our model.

	Constant Prices		Deflator		Current Prices	
	Q3/2002	Q4/2002	Q3/2002	Q4/2002	Q3/2002	Q4/2002
C	3,3	3,7	1,2	2,0	4,5	5,8
GFCF	3,8	3,3	-0,2	0,3	3,6	3,6
X	4,1	4,8	-6,0	-4,5	-2,1	0,1
M	4,0	4,6	-8,0	-6,0	-4,3	-1,7
GDP	2,2	2,7	4,3	4,1	6,6	6,9



4) List of Variables and Identities

R...	**in Constant Prices of 1995 (Real)**	FDIcum	Cumulated Foreign Direct Investments
..._ConstPr	**in Constant Prices of 1995 (Real)**	G	Government Expenditures
..._CurrPr	**in Current Prices (Nominal)**	GCF	Gross Capital Formation
C	Consumption	GDP	Gross Domestic Product
CC	Credits to Corporates	GDP EU	Gross Domestic Product of European Union
CoPI	Construction Price Index	GFCF	Gross Fixed Capital Formation
CPI	Consumer Price Index	CH	Credits to Households
CSI	Change of Stocks & Inventories	M	Import of Goods & Services
CZKDEM,CZKUSD	Nominal Exchange Rate CZK/DEM, CZK/USD	P_BRENT	Price of Brent Oil
D_C	Dummy Variable for C_ConstPr	P_M	Price Ratios for Import of Goods & Services
D_DEF_GFCF	Dummy Variable for DEF_GFCF	P_X	Price Ratios for Export of Goods & Services
D_GFCF_ConstPr	Dummy Variable for GFCF_ConstPr	PPI	Producers Price Index
DEF_C	Consumption Deflator	PPI EU	Producers Price Index of European Union
DEF_GDP	GDP Deflator	RES	Residuals from the same equation (Cochr.-Orc.)
DEF_GFCF	GFCF Deflator	RES LLM	Residuals from LLM
DEF_M	Import of Goods & Services Deflator	SEAS	Aggergate Seasonal Dummy Variable
DEF_X	Export of Goods & Services Deflator	WS	Wages & Salaries
DI	Disposable Income	X	Export of Goods & Services
EER	Nominal Effective Exchange Rate		

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Roman Hušek, Milan Václavík, Jiří Udatný

Department of Econometrics, University of Economics; Winston Churchill Sq. 4; 130 67 Prague 3, e-mail: husek@vse.cz, milan_vaclavik@centrum.cz, udatny@centrum.cz

SYNCHRONISE OF THE DEVELOPMENT OF MONEY STOCK, INFLATION RATE AND INTEREST RATE THE THEORETICAL APPROACH

IVANIČOVÁ ZLATICA, RUBLÍKOVÁ EVA
ECONOMIC UNIVERSITY, BRATISLAVA, SLOVAKIA

There were present a simple model that is consistent with relations between money, inflation rate and interest rate. From this model it is possible to understand that how changes in the money stock affect interest rates depends not only on what is happening to money today, but also on what is expected to happen to money in the future (impact of inflation). According to the model, if the money stock is changed today and future money growth rates are not expected to change (inflation rate is zero), than interest rates move in the opposite direction as the money stock, which is liquidity effect view. But if the money stock, which is changed today and future money growth rates (inflation rate is not zero), the interest rate is moving in that direction too, which is the Fischer equation view. The model was formulated in many studies (for example [1],[2],[3]).

The model represents an exchange economy without production. There are describing the relation between good, bond and money market. All agents in economy receive an identical endowment of goods $\boldsymbol{Y}$ at the beginning of each period and each of them has identical preferences. The assumptions of this model are: goods are disappear at the end of the period if they are not consumed before then, the agents are not unable to consume only their own quantity of goods, they must shop for goods from other agents; agents cannot carry their own goods around to barter with other agents (good are very hard to transport) – this assumption provides a role for fiat money; each agent as a household is a seller and a shopper; in each period the seller stays home to sell the household's goods to other agents for money; the shopper uses the receipts from the previous period's good sales to buy good from other agents; shoppers spend all their money in each period; shoppers can use a random fraction ν of their current period sales receipts for theirs current period purchases; there exists traders and nontraders, traders (fraction of households $0 < \lambda \leq 1$) operate on the bond market before going to the good market.

On the basis of following symbols

Y_t - endowment of goods at the beginning of each period, t=1,2,...

λ - proportion of the traders, $0 < \lambda \leq 1$,

C^T_t - consumption of traders in period t,

C^N_t - consumption of nontraders in period t,

is possible to formulate the resource constraint

$$Y = \lambda\, C^T_t + (1-\lambda)\, C^N_t \qquad (1)$$

what means that the holseholds‘ total consumption must be equal their total endowment.

The fraction of the households (λ) - traders, visit a bond market before going to the goods market. In the bond market, money is exchanged for government bonds, meaning that traders are on the other side of all open market operations engaged in by the monetary authority. As a result, traders absorb all changes in the per capita money supply, that occur through open market operations in time period **t**. If the change in the money supply in period **t** is

$$M_t - M_{t-1} = \mu_t M_{t-1},$$

where M_t, M_{t-1} - stock of money supply in time period t and t−1,

μ_t - money supply growth rate in time period t,

than each trader gets $\mu_t M_{t-1}/\lambda$ units of fiat money in the period **t** bond market. Since this new money is spent in the goods market, the budget constraint of traders is

$$P_t C^T_t = (1-\nu_{t-1})P_{t-1}Y + \nu_t P_t Y + \mu_t M_{t-1}/\lambda \qquad (2)$$

where P_t, P_{t-1} - price level in time period t and t−1,

ν_t, ν_{t-1} - log of velocity of money.

Nontraders face a budget constraint of the form

$$P_t C^N_t = \nu_t P_t Y + (1-\nu_{t-1})P_{t-1}Y. \qquad (3)$$

This constraint states that the nominal expenditures on consumption in the current period must be equal the fraction of receipts from selling the endowment that can be spent in the current period plus the unspent fraction of receipts from selling the endowment in the previous period.

Substituting equation (2) and (3) into (1) yields

$$P_t Y = (1-\nu_{t-1})P_{t-1}Y + \nu_t P_t Y + \mu_t M_{t-1}. \qquad (4)$$

Since the total number of units of fiat money earned into period ***t*** is

$$M_{t-1} = (1-\nu_{t-1})P_{t-1}Y, \qquad (5)$$

equation (4) is a version of the quantity theory of money. Specifically, (4) can be rewritten as the growth rate version of that theory: the rate of inflation

$$\pi_t = (P_t / P_{t-1}) - 1 \qquad (6)$$

is equal the rate of money supply growth μ_t plus the rate of velocity growth $\nu_t - \nu_{t-1}$, i.e.

$$\pi_t = \mu_t + \nu_t - \nu_{t-1}. \tag{7}$$

Solving equation (2) with the help of equations (5), (6) and (7) and on the assumption that the velocity ν is constant yields

$$C_t^T = Y\left[1 + \left(\mu_t / \lambda\right)\right] / \left(1 + \mu_t\right) \tag{8}$$

As long as not all agents are traders, the consumption of traders increases with the rate of growth of the money supply.

On the basis of equilibrium in the bond market and marginal condition for pricing assets is possible build equation

$$\left(1 + r_t\right)^{-1}\left[U'\left(C_t^T\right) / P_t\right] = \left(1 + r^o\right)^{-1} E_t\left[U'\left(C_{t+1}^T\right) / P_{t+1}\right] \tag{9}$$

Assume that bonds issued in period t are promises to one unit of fiat money in period $t+1$, that r_t is the nominal rate of interest on those bonds in period t, that $E_t(.)$ is an expectation conditional on history in period t and earlier, r^0 is the agent's subjective rate of time preference, and U' is marginal utility. Then the left side of (9) is the marginal utility of the goods that agents have to give up in order to buy a bond in period t. The right side of (9) is the discounted expected marginal utility of the goods that will be received in period $t+1$. The marginal utilities are evaluated at the consumption of traders, because only traders can participate in the bond market.

The useful approximation to (9), see [4], is

$$r_t = r^0 + E_t\left(\mu_{t+1}\right) + \Phi\left(E_t \mu_{t+1} - \mu_t\right) + E_t \nu_{t+1} - \nu_t \tag{10}$$

where Φ is a risk correction factor. The equation for the determination of the interest rate (10) is consistent with both views of the relationship between money and interest rates.

Assume again, that there exist some nontraders ($\lambda < 1$) and that velocity is constant ($\nu_t = \nu$). Farther assume that in the long run, money growth fluctuates randomly around some mean growth rate μ,

$$\mu_t = \mu + \varepsilon_t \tag{11}$$

where ε_t is a noise error term that can be interpreted as a transient charge in the money shock in period ***t*** which does not change the expected future rates of money growth. Substituting (11) into (10) yields

$$r_t = r^0 + \mu - \Phi\varepsilon_t. \qquad (12)$$

Consistent with the liquidity effect view, (12) shows that money growth rate shocks lead to changes in the interest rate in the opposite dimension. Consistent with the Fischer equation view, (12) shows that changes in the mean (or long-run) rate of growth of the money supply lead to changes in the nominal interest rate in the same direction.

The relationship between money and interest rates implicitly assumes that central bank states its monetary policy in terms of money supply growth. Today most central banks state their policy in terms of interest rates. Do money and interest rates have the same relationship when central bank use interest rate rules rather than money supply rules ? Yes.
Incorporating an interest rate policy rule into model can see this. A simple interest rate rule that approximates the way in which many central banks currently seem to operate is

$$r_t = r^0 + \pi + \theta(\pi_t - \pi) \qquad (13)$$

with $\theta > 0$. According to this policy a central bank raises the nominal interest rate above its target $r^0+\pi$ whenever current inflation is above the target rate of π and lowers the nominal interest rate whenever inflation is below that target rate. Following the policy rule (13) the central bank responds by raising the nominal interest rate which is achieved by reducing the current rate of money growth. Thus, under this policy, a central bank fights inflation by doing what is traditionally thought of a monetary tightening-reducing the money supply and raising interest rates.

How central banks should translate their interest rate targets into changes in the money supply is under consideration. Economic theory offers two views about its. One, the liquidity effect view, is that increasing interest rates requires a decrease in the money theory. The other view, the Fischer equation view, is that increasing interest rates requite an increase in the rate of growth of the money supply. We examined the empirical evidence on the Slovak data and found that it is consistent with both views.

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Ivaničová Zlatica, Rublíková Eva
KOVE, FHI, EU BRATISLAVA, Dolnozemská 1/b, 852 35 BRATISLAVA, tel.: 00421267295830, ivanic@euba.sk, rublik@euba.sk

SUPER EFFICIENCY DATA DATA ENVELOPMENT ANALYSIS MODELS[1]

JOSEF JABLONSKÝ

UNIVERSITY OF ECONOMICS PRAGUE, CZECH REPUBLIC

Abstract

Data Envelopment Analysis (DEA) offers several models for comparison the relative efficiency of decision making units (DMU) described by multiple inputs and outputs. In most DEA models the best DMUs reach the efficient score 100%. Typically the number of units with 100% efficiency can be high (depending on the number of units and/or inputs and outputs used in the model). The necessity to classify the efficient units leads to the several definition of super efficiency. The efficient units in super efficient models can reach the efficient score higher than 100%. In this way they can be simply classified. The paper presents two super efficiency models and discusses their results on a numerical example.

1. Basic DEA models

Data envelopment analysis (DEA) is a tool for measuring the relative efficiency and comparison of decision making units (DMU). The DMUs are usually characterised by several inputs that are spent for production of several outputs. Let us consider set E of n decision making units E = {DMU_1, DMU_2, …, DMU_n}. Each of the units produces *r* outputs and for their production spent *m* inputs. Let us denote $X_j = \{x_{ij}, i=1,2,\ldots,m\}$ the set of inputs and $Y_j = \{y_{ij}, i=1,2,\ldots,r\}$ the set of outputs for the DMU_j. Then X is the (m,n) matrix of inputs and Y the (r,n) matrix of outputs.

The basic principle of the DEA in evaluation of efficiency of the DMU_q, $q \in \{1,2.,,,.n\}$ consists in looking for a virtual unit with inputs and outputs defined as the weighted sum of inputs and outputs of the other units in the decision set - $X\lambda$ a $Y\lambda$, where $\lambda=(\lambda_1, \lambda_2,\ldots, \lambda_n)$, $\lambda>0$ is the vector of weights of the DMUs. The virtual unit should be better (or at least not worse) than the analysed unit DMU_q. The problem of looking for a virtual unit can generally be formulated as standard linear programming problem:

minimise $\quad z = \theta$,

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subject to $\quad Y\lambda \geq Y_q,$ (1)

$X\lambda \leq \theta X_q,$

$\lambda \geq 0.$

The DMU_q is to be considered as efficient if the virtual unit is identical with analysed unit (does not exist the virtual unit with better inputs and outputs). In this case $Y\lambda = Y_q$, $X\lambda = X_q$ and minimum value of $z = \theta = 1$. Otherwise the DMU_q is not efficient and minimum value of $\theta<1$ can be interpreted as the need of proportional reduction of inputs in order to reach the efficient frontier. The presented model is so called input oriented model because its objective is to find reduction of inputs in order to reach the efficiency. Analogously can be formulated output oriented model.

Model (1) presents just the basic philosophy of the DEA models. The input oriented form of the Charnes, Cooper, Rhodes model (CCR-I) is formulated as follows:

minimise $\quad z = \theta - \varepsilon(e^T s^+ + e^T s^-),$

subject to $\quad Y\lambda - s^+ = Y_q,$ (2)

$X\lambda + s^- = \theta X_q,$

$\lambda, s^+, s^- \geq 0,$

where $e^T = (1,1,\ldots,1)$ and ε is a infinitesimal constant (usually 10^{-6} or 10^{-8}). Presented formulations (1) and (2) are very close each other. The variables s^+, s^- are just slack variables expressing the difference between virtual inputs/outputs and appropriate inputs/outputs of the DMU_q. Obviously, the virtual inputs/outputs can be computed with optimal values of variables of the model (2) as follows:

$$X'_q = X_q\theta^* - s^-,$$

$$Y'_q = Y_q + s^+.$$

2. Super efficiency models

The efficiency score in the standard CCR input oriented model is limited to unity (100%). Nevertheless, the number of efficient units identified by the CCR model and reaching the maximum efficiency score 100% can be relatively high and especially in problems with a small number of decision units the efficient set can contain almost all the units. In such cases it is very important to have a tool for a diversification and classification of efficient units. That is why several DEA models for classification of efficient units were formulated. In these models the efficient scores of inefficient units remain lower than 100% but the efficient score for efficient units can be higher than 100%. Then, the efficient score can be taken as a basis for a complete ranking of efficient units. The DEA models that relax the condition for unity efficiency are called super efficiency models.

The first super efficiency DEA model was formulated by Andersen and Petersen in [1]. Its input oriented formulation (3) is very close to the standard input oriented formulation of CCR model (2). In this model the weight λ_q of the evaluated unit DMU_q is equated to zero. This cannot influence the efficient score of the inefficient units but the efficient score of the efficient units is not limited by unity in this case. The input oriented formulation of the Andersen and Petersen model is as follows:

minimise $z = \theta$,

subject to
$$\sum_{j=1,\neq q}^{n} x_{ij}\lambda_j + s_i^- = \theta x_{iq}, \quad i = 1,2,...,m, \quad (3)$$
$$\sum_{j=1,\neq q}^{n} y_{ij}\lambda_j - s_i^+ = y_{iq}, \quad i = 1,2,...,r,$$
$$\lambda, s^+, s^- \geq 0.$$

Tone in [6] proposes a slack based measure of efficiency (SBM model) that is basis for his formulation of the super efficiency model presented in [7]. The SBM model is formulated as follows:

minimise
$$\rho = \frac{1 - \frac{1}{m}\sum_{i=1}^{m} s_i^- / x_{iq}}{1 + \frac{1}{r}\sum_{i=1}^{r} s_i^+ / y_{iq}},$$

subject to
$$\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = x_{iq}, \quad i = 1,2,...,m, \quad (4)$$
$$\sum_{j=1}^{n} y_{ij}\lambda_j - s_i^+ = y_{iq}, \quad i = 1,2,...,r,$$
$$\lambda, s^+, s^- \geq 0.$$

The formulation shows that the SBM model is non-radial and deals directly with all the slacks variables. The model returns efficiency score between 0 and 1 and is equal to 1 if and only if the DMU_q is on the efficient frontier without any slacks. It is possible to prove that the efficient score of the SBM model is always lower or equal than the efficient score of the appropriate CCR input oriented model. The formulation of the model (4) with fractional objective function can be simply transformed into a standard problem with linear objective function.

The super efficiency SBM model removes the evaluated unit DMU_q from the set of units (like Andersen and Petersen model) and looks for a DMU* with inputs x* and outputs y* being SBM (and CCR) efficient after this removal. It is clear that all the inputs of the unit DMU* have to be higher or equal than the inputs of the unit DMU_q and all the outputs will be lower or equal comparing to outputs of DMU_q. The super efficiency is measured as a distance of the inputs/outputs of both the units. As a distance measure in the mathematical formulation of the super SBM model below the index δ is used:

minimise $$\delta = \frac{\frac{1}{m}\sum_{i=1}^{m} x_i^* / x_{iq}}{\frac{1}{r}\sum_{i=1}^{r} y_i^* / y_{iq}}, \tag{5}$$

subject to
$$\begin{aligned}
&\sum_{j=1,\neq q}^{n} x_{ij}\lambda_j + s_i^- = x_{iq}, && i = 1,2,...,m,\\
&\sum_{j=1,\neq q}^{n} y_{ij}\lambda_j - s_i^+ = y_{iq}, && i = 1,2,...,r,\\
&x_i^* \geq x_{iq}, && i = 1,2,...,m,\\
&y_i^* \leq y_{iq}, && i = 1,2,...,r,\\
&\lambda, s^+, s^-, \ y_i^* \geq 0.
\end{aligned} \tag{6}$$

The numerator in the ratio (5) can be interpreted as a distance of both the units in the input space and an average reduction rate of inputs of DMU* to inputs of DMU_q. The same holds for the output space in the denominator of the ratio (5). The input oriented formulation of the super SBM model is given as follows:

minimise $$\delta = \frac{1}{m}\sum_{i=1}^{m} x_i^* / x_{iq}, \tag{7}$$

subject to
$$\begin{aligned}
&\sum_{j=1,\neq q}^{n} x_{ij}\lambda_j + s_i^- = x_{iq}, && i = 1,2,...,m,\\
&\sum_{j=1,\neq q}^{n} y_{ij}\lambda_j + s_i^- = y_{iq}, && i = 1,2,...,r,\\
&x_i^* \geq x_{iq}, && i = 1,2,...,m,\\
&y_i^* = y_{iq}, && i = 1,2,...,r,\\
&\lambda, s^+, s^- \geq 0.
\end{aligned} \tag{8}$$

The formulation of the previous model is derived from the model (5)-(6) by setting the denumerator equal to 1, i.e. $y_i^* = y_{iq}$.

The super SBM model (7)-(8) gives optimal objective values greater or equal 1. The optimal efficient score is greater than 1 for efficient DMUs – higher value is assigned to more efficient units. All the SBM inefficient units reach in the super SBM model optimal score 1. It means that this model cannot be used for classification of inefficient units. The SBM models have to be used in two steps. The first step is applied to the entire set of units in order to identify efficient units and classify inefficient units. The second step is the computation of the super efficiency scores by means of the model (7) and (8).

3. A numerical example

The basic features of the super efficiency models will be illustrated on the numerical example. This example is based on a real data set originating from the survey among the Central European firms. Table 1 contains a data set of 12 firms of the meat processing industry from Czech Republic (CZ), Germany (GW) and Hungary (H). Each firm is described by 6 characteristics: turnover, market share, fixed costs, # of workers, floor space and investments (all the financial characteristics are in millions of Euro). The first and second characteristics are taken as outputs of the model, the remaining ones are inputs.

	Turnover	**Market share**	**Fixed costs**	**# of workers**	**Floor space**	**Invest.**
CZ1	1.486	40	0.8	39	2500	0.01
CZ2	3.143	5	0.023	30	960	0.057
CZ3	3.871	70	0.949	73	800	0.266
CZ4	2.857	5	1	52	500	0.057
GW1	7.669	40	6.136	26	2500	0.102
GW2	3.835	5	1.023	18	1000	0.102
GW3	30.678	40	4.001	100	2700	0.511
GW4	5.466	15	2.572	65	2900	0.378
H1	1.753	10	0.253	20	1300	0.078
H2	1.851	10	0.07	19	400	0.148
H3	3.409	10	0.031	27	920	0.249
H4	1.648	12	0.105	27	4601	0.23

Tabulka 1 – *The data set.*

Table 2 contains the efficiency scores computed by:

- standard CCR input oriented model (2),
- SBM model (4),
- input oriented Andersen and Petersen model (3) and
- input oriented super SBM model (7)-(8).

	CCR	SBM	Andersen Petersen	Super SBM
CZ1	1.000	1.000	10.200	4.505
CZ2	1.000	1.000	3.588	1.818
CZ3	1.000	1.000	3.634	2.095
CZ4	0.798	0.502	0.798	xxx
GW1	1.000	1.000	1.997	1.288
GW2	0.694	0.537	0.694	xxx
GW3	1.000	1.000	2.643	1.945
GW4	0.371	0.294	0.371	xxx
H1	0.803	0.644	0.803	xxx
H2	1.000	1.000	1.234	1.115
H3	1.000	1.000	1.733	1.309
H4	0.829	0.612	0.829	xxx

Tabulka 2 – *Results of DEA models*.

On the example of the DMU_1 we will explain the difference between both the super efficiency models. The optimal efficiency score for DMU_1 in the Andersen and Petersen model is 10.2. This value is the absolute expansion rate for input values of this unit, i.e. if all the inputs of the projected unit are expanded 10.2 times the unit still becomes CCR efficient and the projected point has the following reduced inputs x* = (8.16, 397.8, 25500, 0.102) with the identical outputs. The projected inputs for the point on the efficient frontier result directly from the optimisation model (7)-(8). For DMU_1 they are x* = (0.8, 40.99, 2500, 0.1497). The optimal value of δ* = 4.505 is the average distance of the vector of projected inputs and the vector of inputs x^1 = (0.8, 39, 2500, 0.01), i.e.

$$\frac{0.8/0.8 + 40.99/39 + 2500/2500 + 0.1497/0.01}{4} = 4.505.$$

It is clear that the projected point given by the super SBM model is much closer (in sense of the selected metric) to the efficient frontier. It can be taken as advantage of the super SBM model because it leads always to lower super efficiency scores comparing to the Andersen and Petersen model. These lower super efficiency scores can be better explained and interpreted. The ranking of units by both the super efficiency models cannot be identical. The results in Table 2 show some rank reversals of efficient units in Andersen and Petersen and super SBM model.

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Josef Jablonský

Department of Econometrics, Faculty of Informatics and Statistics, University of Economics Prague, W. Churchill Sq. 4, 130 67 Praha 3, e-mail: jablon@vse.cz, www: nb.vse.cz/~jablon

DESIGN AND ANALYSIS OF MANY-TO-MANY DISTRIBUTION SYSTEM

JAROSLAV JANÁČEK
UNIVERSITY OF ŽILINA, SLOVAKIA

1. Introduction

A transportation system, which has approximately the same number of primary sources as number of customers, seems to be a marginal case of a distribution system. This case includes such instances as National Postal Network [2] or cargo railway system [5], which provides transport of carriages between railway stations. In these cases, demands of customers form a matrix of yearly flows from sources to places of destination. We denote this matrix as $\boldsymbol{B} = \{b_{sj}\}$, for $s \in S$ and $j \in J$, where S is a set of sources and J denotes set of customers. The fact that unit cost of transportation is smaller when bigger bulks of items are transported, approves concentration of flows between different pairs of source and customer to stronger flows at least on a part of their way. This flow concentration needs terminals, in which transhipment of transported items is performed and bigger bulks are formed or, on the other side, where bulks are split into smaller groups designated to different customers.

On the contraries to the classical distribution systems, in which big bulks leave primary source, another situation emerges in the many-to-many distribution systems. Primary sources send relative small bulks of items and it is useful to concentrate them to bigger bulks in the terminals located near the sources and then to send these bigger bulks to remote terminals and to split them there (see Fig. 1).

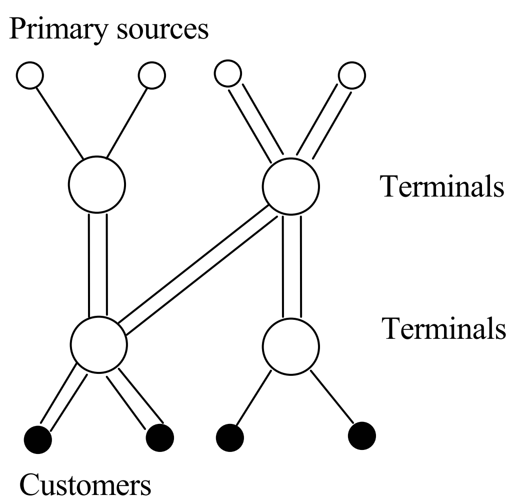

Fig. 1

We restrict ourselves here to the distribution system, in which a customer-source is assigned to only one terminal and an exchange of the consignments between the customer-source and other primary sources or customers is done via this assigned terminal, as it is shown in Fig. 1. Furthermore, we consider the general case, in which any source is a customer simultaneously, what is the case of post offices, railway stations and so on. We do not make any difference between a primary source and a customer hereafter and we introduce the set *J'* of customer-sources, for which matrix ***B*** gives by coefficients b_{sj} the yearly volume of the consignments, which are sent from the object *s* to the object *j* and it gives by coefficients b_{js} the total yearly volume sent from the object *j* to the object *s*. In the next section we try to model a symmetrical many-to-many distribution system with unique assignment of customers to terminals.

2. Model of many-to-many distribution system

Let us consider a case with a linear cost estimation function with unit cost e_0 for transport of one item along unit distance on the way from a primary source to a terminal or from a terminal to a customer. Next, let us consider unit cost e_1 for transport of one item along unit distance on the way between terminals. Furthermore we denote a set of possible terminal locations by symbol *I*, where each place $i \in I$ is associated with yearly fixed charges f_i for building and performance of the terminal *i* and with unit cost g_i for transhipment of one unit in the terminal. In accordance to the previous definition, we denote *J'* the set of objects which mutually send consignments with the yearly total amounts b_{sj} from $s \in J'$ to $j \in J'$. Symbol d_{ij} denotes the distance between objects *i* and *j*. Our goal is to assign each sending or receipting object to exactly one terminal so that the total yearly cost of the designed system be minimal. If we denote by $y_i \in \{0, 1\}$ for $i \in I$ the bivalent variable, which corresponds to the decision if a terminal will or will not be built at place *i* and if we introduce the variable $z_{ij} \in \{0, 1\}$ for $i \in I$ a $j \in J'$, which says if object *j* will or will not be assigned to place *i*, than we can establish following mathematical programming model for the problem.

$$\text{Minimise} \sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{j \in J'} (e_0 d_{ij} + g_i)(\sum_{s \in J'} b_{js} + \sum_{s \in J'} b_{sj}) z_{ij} +$$

$$+ \sum_{i \in I} \sum_{k \in I} e_1 d_{ik} \sum_{j \in J'} \sum_{s \in J'} b_{sj} z_{ij} z_{ks} \quad . \tag{1}$$

$$\text{Subject to} \quad \sum_{i \in I} z_{ij} = 1 \qquad \text{for } j \in J', \tag{2}$$

$$z_{ij} \le y_i \qquad \text{for } i \in I, j \in J', \tag{3}$$

$$y_i \in \{0, 1\} \qquad \text{for } i \in I, \tag{4}$$

$$z_{ij} \in \{0, 1\} \qquad \text{for } i \in I, j \in J'. \tag{5}$$

The model belongs to discrete quadratic programmes due the third term of *(1)*. Such problems of the usual size are not yet exactly solvable due time consumption of relevant methods. If we define

$c_{ij} = (e_0 d_{ij} + g_i)(\sum_{s\in J'} b_{js} + \sum_{s\in J'} b_{sj})$ and $m_{ijks} = e_1 d_{ik} b_{sj}$, then it is possible to solve the problem

$$\textit{minimise} \sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J'} c_{ij} z_{ij} + \sum_{i\in I}\sum_{k\in I}\sum_{j\in J'}\sum_{s\in J'} m_{ijks} z_{ij} z_{ks} \qquad (6)$$

subject to *(2) - (5),* using a suitable heuristic.

Another approach is based on a linearization of objective function *(1)*, what brings exactly solvable problem with less precise objective function.

3. Linearization of the model

A possibility, how to perform linearization of objective function *(1)*, consists in an estimation of relation between distances d_{is} and d_{ik} in the form $d_{ik} = \alpha d_{is}$. When similar system works and supplies set of his customers via terminals located at places from set I_0 and when assignment of customers to terminals is given by decisions $\underline{z}_{ij}$, then it is possible to estimate α by expression

$$\alpha = \frac{\sum_{s\in J'}\sum_{k\in I_0}\sum_{i\in I_0}(\sum_{j\in J'} b_{sj}\underline{z}_{ij}\underline{z}_{ks})d_{ik}/d_{is}}{\sum_{s\in J'}\sum_{j\in J'} b_{sj}}.$$

Having replaced the distance d_{ik} by the estimation αd_{is} in the third term of *(1)*, we obtain a surrogate objective function in the form:

$$\sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J'}(e_0 d_{ij} + g_i)(\sum_{s\in J'} b_{js} + \sum_{s\in J'} b_{sj})z_{ij} + \sum_{i\in I}\sum_{k\in I}\sum_{j\in J'}\sum_{s\in J'} e_1 \alpha d_{is} b_{sj} z_{ij} z_{ks} =$$

$$= \sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J'}(e_0 d_{ij} + g_i)(\sum_{s\in J'} b_{js} + \sum_{s\in J'} b_{sj})z_{ij} + \sum_{i\in I}\sum_{j\in J'}\sum_{s\in J'} e_1 \alpha d_{is} b_{sj} z_{ij} \sum_{k\in I} z_{ks} =$$

$$= \sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J'}(e_0 d_{ij} + g_i)(\sum_{s\in J'} b_{js} + \sum_{s\in J'} b_{sj})z_{ij} + \sum_{i\in I}\sum_{j\in J'} e_1 \alpha (\sum_{s\in J'} d_{is} b_{sj}) z_{ij} =$$

$$\sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J'} \underline{c}_{ij} z_{ij} \quad , \qquad (7)$$

where $\underline{c}_{ij} = \sum_{s\in J'} [(e_0 d_{ij} + g_i)(b_{js} + b_{sj}) + e_1 \alpha (d_{is} b_{sj})]$.

The resulting problem, minimize *(7)* subject to *(2) - (5)*, is the well known uncapacitated location problem [2], [5].

Another way of linearization of the quadratic term in *(6)* consists in introducing bivalent variables $x_{ijks} \in \{0, 1\}$ for $i, k \in I, j, s \in J'$ and substituting them for the product of z_{ij} and z_{ks} to the third term of *(6)*. In addition, it is necessary to extend the system of constraints *(2) - (5)* by binding constraints, which ensure that x_{ijks} takes value *1* if and only if both variables z_{ij} and z_{ks} take value

1. These linear constraints can be established by the following way:

$$x_{ijks} \le z_{ij} \qquad for\ i,\ k \in I,\ j,\ s \in J', \qquad (8)$$

$$x_{ijks} \le z_{ks} \qquad for\ i,\ k \in I,\ j,\ s \in J', \qquad (9)$$

$$z_{ij} + z_{ks} - 1 \le x_{ijks} \quad for\ i,\ k \in I,\ j,\ s \in J'. \qquad (10)$$

4. Solving techniques

As mentioned above, some of metaheuristics can be employed to obtain a good solution of the problem *(6)*, *(2) - (5)* for a practical use. Especially genetic algorithm seems to be very suitable for the problems, solution of which is given by *0 – 1* variables (genes), where no strong constraints are imposed on them. This assumption unfortunately does not fully hold in this case because there is no possibility of easy determination of optimal values of z_{ij} if values of y_i are known on the contrary to the classical uncapacitated location programme, where structure of an optimal solution is unambiguously determined by values of y_i. Including of variables z_{ij} to „chromosome" causes that constraints *(2) - (5)* would have to be checked whenever crossover of the chromosomes is performed. This brings considerable time loss of the metaheuristic.

Another way, how to handle this trouble with the quadratic term, is to replace optimal determination of z_{ij} by an approximate computational process. In this case a result of the algorithm would be burdened with impreciseness, which could be greater than the impreciseness caused by the linearization in *(7)*.

Regarding the substitution of x_{ijks}, the problem *(6)*, *(2) - (5)*, *(8) - (10)* is though integer linear problem, but there is no specific efficient algorithm and that is why a general method for integer linear programming problems would have to be employed under associated risk.

To solve problem *(7)*, *(2) - (5)* there exist several implementations [2], [5] of branch and method, which make use of fast and relative precise computation of lower bounds provided by the dual ascent and dual adjustment procedures [2] to obtain an optimal solution of the problem.

5. Conclusion

We have designed two ways of linearization of the many-to-many distribution system design problem. The exact linearization lead to integer linear programme *(6)*, *(2) - (5)*, *(8) - (10)* after the substitution to *(6)* had been done. The second linearization using proportional coefficient α has yield model *(7)*, *(2) - (5)*. Having considered the size of common practical problems of this type [3], [6], we find that the cardinality of the location set is *100* concerning the order and the cardinality of the set of customers equals to *1000*. Thus in the former case we obtain problem containing 10^{10} zero-one variables and in the latter case the problem involves 10^5 zero-one variables. Furthermore the latter case the problem has the integrality property as regards to variables z_{ij}.

While it is impossible to obtain an optimal solution of problem *(6)*, *(2) - (5)*, *(8) - (10)* by a general method of integer linear programming due time consumption of the method, the latter problem is solvable in the time of several tens seconds [1], [5], thanks to the specific structure of the system of constraints. This property enables repetition of the computational process for different values of α and obtaining a more precise result, what was exploit in the decision support system [6].

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Prof. RNDr. Jaroslav Janáček CSc.

Katedra dopravných sietí, Fakulta riadenia a informatiky, Žilinská univerzita v Žiline, 01026 Žilina, tel. 0415134226, e-mail: jardo@frdsa.fri.utc.sk

A Remark on Stability in Multiobjective Stochastic Programming Problems

Vlasta Kaňková

Institute of Information Theory and Automation AS CR,
E-mails: kankova@utia.cas.cz

Abstract. The paper deals with multiobjective optimization problems in which the objective functions are in the form of mathematical expectation of functions depending on a random element and a constraints set depends on a probability measure. The aim of the paper is to present some results on a stability (considered w.r.t. the probabilty measures space) of the above mentioned type of the problems. To prove these assertions the results on the stability of the stochastic one–objective programming are employed.

Keywords: Multiobjective stochastic programming, stability, Kolmogorov metric, Wasserstein metric, efficient solution, strongly efficient solution.

AMS classification: 90 C 15, 90 C 29
JCL classification: C 44, J 64

1 Introduction

It happens rather often that it is reasonable to evaluate an economic activity simultaneously by several "utility" functions (see e.g. [8]). If, moreover, there exist a random element and a "parameter" (not determined completely but whose value must only fulfil some conditions), then a multiobjective optimization problem with a random factor usually corresponds (from the mathematical point of view) to such situation. However, since mostly, the value of the parameter must be determined without knowing the random element realization, some deterministic optimization problem must be lastly assigned to the original economic problem. It can depend (generally) on the random element only through the "underlying" probabilty measure. Moreover, this problem can be one–objective as well as multiobjetive. In this paper, we shall deal with the multiobjective case.

To introduce the multiobjective stochastic programming problem in a general form; let $(\Omega, \mathcal{S}, P)$ be a probability space, $\xi(= \xi(\omega) = [\xi_1(\omega), \ldots, \xi_s(\omega)])$ be an s–dimensional random vector defined on $(\Omega, \mathcal{S}, P)$, $F(:= F(z), z \in R^s)$ be the distribution function of ξ, P_F denote the probability measure corresponding to F, $Z_F \subset R^s$ denote the support of P_F. Let, furthermore, $g_i(x, z)$, $i = 1, \ldots, l$, $l \geq 1$ be real–valued functions defined on $R^n \times R^s$, $X_F \subset R^n$ be a nonempty set depending (generally) on the distribution function F. The symbol R^n, $n \geq 1$ is reserved for the n–dimensional Euclidean space.

A rather general multiobjective stochastic programming problem can be introduced in the form.

Find

$$\min \mathsf{E}_F g_i(x, \xi), \quad i = 1, 2, \ldots, l \quad \text{subject to} \quad x \in X_F. \tag{1}$$

(E_F denotes the operator of mathematical expectation corresponding to F.)

To solve the problem (1) the complete information on F is necessary. However in applications very often at least one of the following cases happens:

- F must be replaced by its statistical estimate,
- F must be (for numerical difficulties) replaced by some simpler one,
- the actual distribution function is a little modified F.

Consequently, it is suitable (or even necessary) to investigate the stability of the problem (1) w.r.t. the probability measures space as well as the corresponding statistical estimates. In the literature, a great attention was paid to these problems in one–objective case; we can recall e.g. papers [1], [6], [11], [12], [13]. However, the works dealing with the multiobjective case have already began to appear also; see e.g. [4], [9], [14], [15]. The aim of the paper is to deal with the stability of the multiobjective problem (1). We generalize by this the results of [9]. Moreover, we present the quantitative stability results employing the Kolmogorov and the Wasserstein metrics.

2 Some Definitions and Auxiliary Assertions

To study the stability of the multiobjective problem (1), we first recall the sets that correspond to the "optimal value" and the "optimal solution" in one–objective case. Moreover, we recall some assertions from the deterministic multiobjective optimization theory. To this end, let a multiobjective deterministic optimization problem be in the form.

Find

$$\min f_i(x), \; i = 1, \ldots, r \quad \text{subject to } x \in \mathcal{K}. \tag{2}$$

$f_i(x)$, $i = 1, \ldots, r$ are real–valued functions defined on R^n, $\mathcal{K} \subset R^n$ is a nonempty set.

Definition 1. The vector x^* is an efficient solution of the problem (2) if and only if there exists no $x \in \mathcal{K}$ such that $f_i(x) \le f_i(x^*)$ for $i = 1, \ldots, r$ and such that for at least one i_0 one has $f_{i_0}(x) < f_{i_0}(x^*)$.

Definition 2. The vector x^* is a properly efficient solution of the multiobjective optimization problem (2) if and only if it is efficient and if there exists a scalar $\bar{M} > 0$ such that for each i and each $x \in \mathcal{K}$ satisfying $f_i(x) < f_i(x^*)$ there exists at least one j such that $f_j(x^*) < f_j(x)$ and

$$\frac{f_i(x^*) - f_i(x)}{f_j(x) - f_j(x^*)} \le \bar{M}. \tag{3}$$

Proposition 1. [3] Let $\mathcal{K} \subset R^n$ be a convex set and let $f_i(\cdot)$, $i = 1, \ldots, r$ be convex functions on $\mathcal{K}$. Then x^0 is a properly efficient solution of the problem (2) if and only if x^0 is optimal in

$$\min_{x \in \mathcal{K}} \sum_{i=1}^{r} \lambda_i f_i(x) \quad \text{for some } \lambda_1, \ldots, \lambda_r > 0; \quad \sum_{i=1}^{r} \lambda_i = 1.$$

Remark Employing Proposition 1 we can apply the stabilty results achieved for one–objective case to the multiobjective problems.

Definition 3. If $\mathcal{K}^{'}, \mathcal{K}^{''} \subset R^n$ are two non–empty sets, then the Hausdorff distance of these sets $\Delta[\mathcal{K}^{'}, \mathcal{K}^{''}] := \Delta_n[\mathcal{K}^{'}, \mathcal{K}^{''}]$ is defined by

$$\begin{aligned} \Delta_n[\mathcal{K}^{'}, \mathcal{K}^{''}] &= \max[\delta_n(\mathcal{K}^{'}, \mathcal{K}^{''}), \delta_n(\mathcal{K}^{''}, \mathcal{K}^{'})], \\ \delta_n(\mathcal{K}^{'}, \mathcal{K}^{''}) &= \sup_{x^{'} \in \mathcal{K}^{'}} \inf_{x^{''} \in \mathcal{K}^{''}} \|x^{'} - x^{''}\|. \end{aligned}$$

($\| \cdot \| := \| \cdot \|_n$ denotes the Euclidean norm in R^n.)

Definition 4. Let $\bar{h}(x)$ be a real–valued function defined on a convex set $\mathcal{K} \subset R^n$. $\bar{h}(x)$ is a strongly convex function with a parameter $\rho > 0$ if

$$\bar{h}(\lambda x^1 + (1-\lambda)x^2) \le \lambda \bar{h}(x^1) + (1-\lambda)\bar{h}(x^2) - \lambda(1-\lambda)\rho\|x^1 - x^2\|^2$$

for every $x^1, x^2 \in \mathcal{K}$, $\lambda \in \langle 0, 1 \rangle$.

Proposition 2. [5] Let $\mathcal{K} \subset R^n$ be a nonempty, compact, convex set. Let, moreover, $\bar{h}(x)$ be a strongly convex with a parameter $\rho > 0$, continuous, real–valued function defined on $\mathcal{K}$. If $x^0 = \arg\min_{x \in \mathcal{K}} \bar{h}(x)$, then

$$\|x - x^0\|^2 \le \frac{2}{\rho} |\bar{h}(x) - \bar{h}(x^0)| \quad \text{for every } x \in \mathcal{K}.$$

3 Problem Analysis

To investigate the stability of the problem (1) we define the sets $\mathcal{G}_F$, $\bar{\mathcal{X}}_F$, $\bar{\mathcal{G}}_F$.

$$\begin{aligned} \mathcal{G}_F &= \{y \in R^l : y_j = \mathsf{E}_F g_j(x, \xi), \ j = 1, \ldots, l \quad \text{for some } x \in X_F; \\ & \qquad y = (y_1, \ldots, y_l)\}, \\ \bar{\mathcal{X}}_F &= \{x \in X_F : x \text{ is a properly efficient point of the problem (1)}\}, \\ \bar{\mathcal{G}}_F &= \{y \in R^l : y_j = \mathsf{E}_F g_j(x, \xi), \ j = 1, \ldots, l \quad \text{for some } x \in \bar{\mathcal{X}}_F\}. \end{aligned} \tag{4}$$

If we replace in (4) F by another s–dimensional distribution function G, then we obtain the sets $\mathcal{G}_G$, $\bar{\mathcal{X}}_G$ and $\bar{\mathcal{G}}_G$ instead of the original $\mathcal{G}_F$, $\bar{\mathcal{X}}_F$ and $\bar{\mathcal{G}}_F$.

Let $\mathcal{P}(R^s)$ denote the set of all (Borel) probability measures in R^s. To investigate the stability of the problem (1) we can first state the assumptions under which for a $\delta > 0$ there exist "regions" $\mathcal{F}_1(\delta)$, $\mathcal{F}_2(\delta)$, $\mathcal{F}_3(\delta) \subset \mathcal{P}(R^s)$ such that

$$\begin{aligned} G \in \mathcal{F}_1(\delta) &\Longrightarrow \Delta(\mathcal{G}_F, \mathcal{G}_G) \le \delta, \\ G \in \mathcal{F}_2(\delta) &\Longrightarrow \Delta(\bar{\mathcal{X}}_F, \bar{\mathcal{X}}_G) \le \delta, \\ G \in \mathcal{F}_3(\delta) &\Longrightarrow \Delta(\bar{\mathcal{G}}_F, \bar{\mathcal{G}}_G) \le \delta. \end{aligned} \tag{5}$$

Furthermore, we shall state the assumptions under which there exist functions m_1, m_2 and m_3 defined on R^1 such that

$$\begin{aligned} \Delta(\mathcal{G}_F,\, \mathcal{G}_G) &\leq m_1(d_s(P_F,\, P_G)), \\ \Delta(\bar{\mathcal{X}}_F,\, \bar{\mathcal{X}}_G) &\leq m_2(d_s(P_F,\, P_G)), \\ \Delta(\bar{\mathcal{G}}_F,\, \bar{\mathcal{G}}_G) &\leq m_3(d_s(P_F,\, P_G)), \end{aligned} \tag{6}$$

for distribution functions G that are "near" to F. d_s is a "suitable" metric in the probability measures space; say e.g. Kolmogorov or Wasserstein.

It is known that the Kolmogorov metric $d_K(P_F,\, P_G)$ is defined by

$$d_K(P_F,\, P_G) := d_K(F,\, G) = \sup_{z \in R^s} |F(z) - G(z)|. \tag{7}$$

To define the Wasserstein metric $d_{W_1}(F,\, G) := d_{W_1}(P_F,\, P_G)$ let

$$\mathcal{M}_1(R^s) \;=\; \{\nu \in \mathcal{P}(R^s) : \int\limits_{R^s} \|z\| \nu(dz) < \infty\}.$$

If $\mathcal{D}(\nu,\, \mu)$ denotes the set of those measures in $\mathcal{P}(R^s \times R^s)$ whose marginal measures are ν and μ, then

$$d_{W_1}(\nu,\, \mu) \;= \inf\{ \int\limits_{R^s \times R^s} \|z - \bar{z}\| \kappa(dz \times d\bar{z}) : \kappa \in \mathcal{D}(\nu,\, \mu)\}, \quad \nu, \mu \in \mathcal{M}_1(R^s).$$

4 Main Results

To present the stability results we introduce the systems of the assumptions. To this end let $\delta > 0$ and $Z_F(\delta)$ denote the δ–neighbourhood of the set Z_F.

A.1 There exists $X \subset R^n$ such that $X_F \subset X$ and, moreover,

- a. $g_i(x,\, z)$, $i = 1, \ldots, l$ are uniformly continuous functions on $X \times R^s$,
- b. for every $x \in X$, $g_i(x,\, z)$, $i = 1, \ldots, l$ are Lipschitz functions of $z \in Z_F(s\delta)$ with the Lipschitz constants not depending on $x \in X$,

A.2 there exists $X \subset R^n$ such that $X_F \subset X$ and, moreover,

- a. $g_i(x,\, z)$, $i = 1, \ldots, l$ are Lipschitz functions on $X \times Z_F(s\delta)$ (with the Lipschitz constants L_i),
- b. X is a convex set and simultaneously $g_i(x,\, z)$, $i = 1, \ldots, l$ are strongly convex (with a parameter $\rho > 0$) functions on X,
- c. X_F is a nonempty, convex, compact set,

A.3

- a. P_F is absolutely continous w.r.t. the Lebesgue measure in R^s. We denote by the symbol h the probability density corresponding to P_F,
- b. $Z_F = \prod\limits_{j=1}^{s} \langle c_j,\, c_j^{'} \rangle$, $c_j,\, c_j^{'} > 0$, $c_j < c_j^{'}$, $j = 1, 2, \ldots, s$ and, moreover, there exists a constant $\vartheta > 0$ such that $h(z) \geq \vartheta$ for every $z \in Z_F$.

Theorem 1. Let $\delta > 0$, X_F be a nonempty compact set, $P_F \in \mathcal{P}(R^s)$. If

1. the assumptions A.2a is fulfilled,
2. for every $x \in X$ there exist finite $\mathsf{E}_{F\xi}\, g_i(x,\, \xi)$, $i = 1, \ldots, l$,

then there exist $\bar{\mathcal{F}}_{\mathcal{G}_F}(\delta) \subset \mathcal{P}(R^s)$ and constants $\delta_{\mathcal{G}_F}$, $C_{\mathcal{G}_F} > 0$ such that for an arbitrary $P_G \in \mathcal{P}(R^s)$ with finite $\mathsf{E}_G\, g_i(x,\, \xi)$, $i = 1, \ldots, l$ the following implication holds

$$G \in \bar{\mathcal{F}}_{\mathcal{G}_F}(\delta), \quad \Delta[X_F,\, X_G] \le \delta_{\mathcal{G}_F}, \qquad X_G \subset X \text{ a nonempty, compact, set} \implies \Delta[\mathcal{G}_F,\, \mathcal{G}_G] \le C_{\mathcal{G}_F}\delta. \tag{8}$$

If, moreover, the assumptions A.2b, A.2c are fulfilled, then there exist $\bar{\mathcal{F}}_{\bar{\mathcal{X}}_F}(\delta)$, $\bar{\mathcal{F}}_{\bar{\mathcal{G}}_F}(\delta) \subset \mathcal{P}(R^s)$ and constants $\delta_{\bar{\mathcal{X}}_F}$, $\delta_{\bar{\mathcal{G}}_F}$, $C_{\bar{\mathcal{X}}_F}$, $C_{\bar{\mathcal{G}}_F}$, $K_{\bar{\mathcal{G}}_F} > 0$ such that

$$G \in \bar{\mathcal{F}}_{\bar{\mathcal{X}}_F}(\delta), \quad \Delta[X_F,\, X_G] \le \delta_{\bar{\mathcal{X}}_F}, \qquad X_G \subset X \text{ a nonempty, compact, convex set} \implies \Delta[\bar{\mathcal{X}}_F,\, \bar{\mathcal{X}}_G]^2 \le C_{\bar{\mathcal{X}}_F}\delta, \tag{9}$$

$$G \in \bar{\mathcal{F}}_{\bar{\mathcal{G}}_F}(\delta), \quad \Delta[X_F,\, X_G] \le \delta_{\bar{\mathcal{G}}_F}, \qquad X_G \subset X \text{ a nonempty, compact, convex set} \implies \Delta[\bar{\mathcal{G}}_F,\, \bar{\mathcal{G}}_F] \le C_{\bar{\mathcal{G}}_F}\sqrt{\delta}(K_{\bar{\mathcal{G}}_F} + \sqrt{\delta}). \tag{10}$$

Remark. It follows from the proof of Thorem 1 that $\mathcal{F}_1(\delta)$, $\mathcal{F}_2(\delta)$, $\mathcal{F}_3(\delta)$ (the relation (5)) depend on F, the "multifunction" X_F and the Lipschitz constants of g_i, $i = 1, \ldots, l$.

Employing the Kolmogorov metric we can furthermore obtain.

Theorem 2. Let X_F be a nonempty compact set, $P_F \in \mathcal{P}(R^s)$. If

1. the assumptions A.2a, A.3 are fulfilled,
2. $G \in \mathcal{P}(R^s)$ is an arbitrary s–dimensional distribution function such that

$$Z_G \subset Z_F(\delta') \quad \text{for } \delta' = \left(\frac{2d_K(P_F,\, P_G)}{\vartheta}\right)^{\frac{1}{s}} \le \min_j(c_j' - c_j),$$

then there exist constants $\delta'_{\mathcal{G}_F}$, $C'_{\mathcal{G}_F} > 0$ such that the following implication holds

$$\Delta[X_F,\, X_G] \le \delta'_{\mathcal{G}_F} d_K(P_F,\, P_G), \qquad X_G \subset X \text{ a nonempty, compact set} \implies \Delta[\mathcal{G}_F,\, \mathcal{G}_G] \le C'_{\mathcal{G}_F} d_K(P_F,\, P_G).$$

If, moreover, the assumptions A.2b, A.2c are fulfilled, then there exist constants $\delta'_{\bar{\mathcal{X}}_F}$, $\delta'_{\bar{\mathcal{G}}_F}$, $C'_{\bar{\mathcal{X}}_F}$, $C'_{\bar{\mathcal{G}}_F}$, $K'_{\bar{\mathcal{G}}_F} > 0$ such that

$$\Delta[X_F,\, X_G] \le \delta'_{\bar{\mathcal{X}}_F} d_K(P_F,\, P_G), \qquad X_G \subset X \text{ a nonempty, convex, compact set} \implies \Delta[\bar{\mathcal{X}}_F,\, \bar{\mathcal{X}}_G]^2 \le C'_{\bar{\mathcal{X}}_F} d_K(P_F,\, P_G),$$

$$\Delta[X_F,\, X_G] \le \delta'_{\bar{\mathcal{G}}_F} d_K(P_F,\, P_G), \qquad X_G \subset X \text{ a nonempty, convex, compact set} \quad \Longrightarrow$$
$$\Delta[\bar{\mathcal{G}}_F,\, \bar{\mathcal{G}}_F] \le C'_{\bar{\mathcal{G}}_F} \sqrt{d_K(P_F,\, P_G)}(K'_{\bar{\mathcal{G}}_F} + \sqrt{d_K(P_F,\, P_G)}).$$

It can be suitable to employ the Wasserstein metric in the special case.

Theorem 3. Let $P_F \in \mathcal{M}_1(R^s)$, the system of the assumptions A.1 be fulfilled. If

1. There exists a nonempty, compact $X \subset R^n$ such that $X_F = X$ indepedently on $P_F \in \mathcal{P}(R^s)$,
2. G is an arbitrary s–dimensional distribution function such that $P_G \in \mathcal{M}_1(R^s)$,

then there exists a constant $C''_{\mathcal{G}_F} > 0$ such that

$$\Delta[\mathcal{G}_F,\, \mathcal{G}_G] \le C''_{\mathcal{G}_F} d_{W_1}(P_F,\, P_G). \tag{11}$$

If, moreover, the system of the assumptions A.2 is fulfilled, then there exists a constants $C''_{\bar{\mathcal{X}}_F},\, C''_{\bar{\mathcal{G}}_F},\, K''_{\bar{\mathcal{G}}_F} > 0$ such that

$$\begin{aligned} \Delta[\bar{\mathcal{X}}_F,\, \bar{\mathcal{X}}_G]^2 &\le C''_{\bar{\mathcal{X}}_F}(d_{W_1}(P_F,\, P_G)), \\ \Delta[\bar{\mathcal{G}}_F,\, \bar{\mathcal{G}}_F] &\le C''_{\bar{\mathcal{G}}_F}\sqrt{d_{W_1}(P_F,\, P_G)}(K''_{\bar{\mathcal{G}}_F} + \sqrt{d_{W_1}(P_F,\, P_G)}). \end{aligned} \tag{12}$$

5 Conclusion

Some new stability results concerning the stability of the multiobjective stochastic programming problems are introduced in the paper. It is over possibilities of this paper to present the corresponding proofs in details; the complete proofs can be found in [10]. At the end of the paper, we try at least to sketch these proofs. To prove the stability assertions (8) the known results for one–objective problems (introduced in [7]) are employed. To prove the relation (9), mainly the assertions of Proposition 1, Proposition 2 and the results of [7] are employed. The relation (10) follows from the relation (9) and the Lipschitz property of the objective funtions.

The proof of Theorem 2 is very similar to the proof of Theorem 1. However, instead of the general assertions on the stability of one objective problems, the results of the quantitative stability with the Kolmogorov metric [7]) are employed. The assertions of Theorem 3 follow from the results of [2]; a little generalized results of [11] (for details see [15]).

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RNDr. Vlasta Kaňková, CSc.,
Department of Econometrics, Institute of Information Theory and Automation of AS ČR, Pod vodárenskou věží 4, 182 08 Praha 8, Czech Republic.

The Role of Inflation Rate on the Dynamics of an Extended Kaldor Model[1]

Jan Kodera, Karel Sladký, Miloslav Vošvrda
Department of Econometrics
Institute of Information Theory and Automation
Academy of Sciences of the Czech Republic

Abstract. This contribution is devoted to the dynamics of an extended Kaldor model. Attention is focused primarily on the influence of inflation rate on the limiting behaviour, stability and robustness of the model. Both the analytical approach and computer modelling are discussed. Two illustrative examples are supplied.

JEL Classification: C061, E012

1 Introduction – the Static IS-LM Model

The IS-LM model is a macroeconomic model describing conditions for the equilibrium between the commodity market and the money market.

In a neo-Keynesian model, the commodity market is described by the so-called IS Model (Investment–Saving Model), i.e. by means of the functions of savings S and investments I. The savings $S(Y,R)$ are assumed to be an increasing function of the real product Y and the nominal interest rate R. Similarly, the investments $I(Y,R)$ are again an increasing function of the real product Y, but a decreasing function of the nominal interest rate R. Considering a static IS model, we are looking for an equilibrium point, say $(Y^\star, R^\star)$ given by the equation

$$S(Y,R) = I(Y,R), \tag{1.1}$$

i.e. for the equilibrium point we have $S(Y^\star, R^\star) = I(Y^\star, R^\star)$.

Similarly, the money market is described by the so-called LM Model (Liquidity–Money Model); in particular, the nominal supply of money M^s must be equal to the nominal demand for money L. In a Keynesian model we assume that the nominal demand for money depends only on the real product Y and the nominal interest rate R. The equilibrium in the money market is then given by

$$M^s = L(Y,R), \tag{1.2}$$

i.e. for the equilibrium point $(Y^\star, R^\star)$ we have $M^s = L(Y^\star, R^\star)$.

Unfortunately, the static model yields no information about the development of the considered macroeconomic system over time, e.g. no information is available about the speed of adjustment to the equilibrium point, and only the nominal inflation is taken into account.

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In this note we shall study a more sophisticated dynamical model. To describe the output dynamics of a macroeconomic system we shall employ the traditional form of the Kaldor dynamic model (for details see e.g. [?] or [?]). The dynamics of the monetary market will be described by the Fisherian equation of demand for money, Tobin's equation for adaptive expectation of inflation and the price adjustment equation. Two numerical examples illustrate the differencies in the behaviour of the real product which is either the function of the price or if the price is an endogeneous variable.

2 The Output Dynamics

We denote (in continuous time $t \geq 0$) by $Y(t)$, $K(t)$, $S(\cdot,\cdot)$ and $I(\cdot,\cdot,\cdot)$ the real product, capital stock, savings and real investments of the considered economy respectively. Recall that in a neo-Keynesian approach we assume that the nominal interest $R(t)$ depends also on the the expected inflation $\pi(t)$, hence $R(t) = r(t) + \pi(t)$ where $r(t)$ is the real interest rate. The output dynamics is then assumed to be given by the following differential equation

$$\frac{\mathrm{d}Y(t)}{\mathrm{d}t} = \alpha \left\{ I\left(\frac{Y(t)}{K(t)}, Y(t), r(t)\right) - S(Y(t), r(t)) \right\} \tag{2.1}$$

where the positive constant α signifies the speed of adaptation.

We assume that the capital formation can be described by the following differential equation

$$\frac{\mathrm{d}K(t)}{\mathrm{d}t} = I\left(\frac{Y(t)}{K(t)}, Y(t), r(t)\right) - \beta[K(t)]^{\gamma} \tag{2.2}$$

where the numbers β, $\gamma \in (0,1]$. The last term on the RHS of (2.2) is called the consumption function $D(t)$, i.e. $D(t) = \beta[K(t)]^{\gamma}$.

As usual, we introduce the logarithmic scale for production $Y(t)$ and the capital stock $K(t)$, in particular, let

$$y(t) = \ln Y(t) \Longleftrightarrow Y(t) = e^{y(t)} \Longrightarrow \frac{\mathrm{d}y(t)}{\mathrm{d}t} = \frac{1}{Y(t)} \cdot \frac{\mathrm{d}Y(t)}{\mathrm{d}t}$$
$$k(t) = \ln K(t) \Longleftrightarrow K(t) = e^{k(t)} \Longrightarrow \frac{\mathrm{d}k(t)}{\mathrm{d}t} = \frac{1}{K(t)} \cdot \frac{\mathrm{d}K(t)}{\mathrm{d}t}$$

Introducing the so-called propensity to invest $i(\cdot,\cdot,\cdot) = I(\cdot,\cdot,\cdot)/Y(\cdot)$ and the propensity to save $s(\cdot,\cdot) = S(\cdot,\cdot)/Y(\cdot)$ from (2.1) we get

$$\begin{aligned} \frac{\mathrm{d}y(t)}{\mathrm{d}t} &= \alpha \cdot \left\{ i\left(e^{y(t)-k(t)}, e^{y(t)}, r(t)\right) - s(e^{y(t)}, r(t)) \right\} \\ &\stackrel{\text{def}}{=} \alpha \cdot \{\imath(y(t)-k(t), y(t), r(t)) - \bar{s}(y(t), r(t))\} \end{aligned} \tag{2.3}$$

and from (2.2) we conclude that

$$\begin{aligned} \frac{\mathrm{d}k(t)}{\mathrm{d}t} &= i\left(e^{y(t)-k(t)}, e^{y(t)}, r(t)\right) \cdot e^{y(t)-k(t)} - \beta e^{(\gamma-1)k(t)} \\ &\stackrel{\text{def}}{=} \imath\left(y(t)-k(t), y(t), r(t)\right) \cdot e^{y(t)-k(t)} - \beta e^{(\gamma-1)k(t)} \end{aligned} \tag{2.4}$$

Moreover, from (2.3), (2.4) we can express dependence of $y(t)$ on $k(t)$. After some algebra we obtain

$$\left(\frac{\mathrm{d}k(t)}{\mathrm{d}t}+\beta e^{(\gamma-1)k(t)}\right)e^{k(t)}=\left(\frac{1}{\alpha}\cdot\frac{\mathrm{d}y(t)}{\mathrm{d}t}+s(e^{y(t)},r(t))\right)\cdot e^{y(t)} \tag{2.5}$$

In what follows we shall assume that the investments can be well approximated by the following nonlinear function:

$$I\left(\frac{Y(t)}{K(t)},Y(t),r(t)\right)=\frac{1}{1+r(t)}\cdot J\left(\frac{Y(t)}{K(t)}\right)\cdot Y(t) \tag{2.6}$$

where $J\left(Y(t)/K(t)\right)$ is the so-called investment ratio.
Since $Y(t)=e^{y(t)},\ K(t)=e^{k(t)}$, if $\varepsilon(t)=y(t)-k(t)$

$$J\left(\frac{Y(t)}{K(t)}\right)=J\left(e^{y(t)-k(t)}\right)=\jmath(y(t)-k(t))=\jmath(\varepsilon(t)) \tag{2.7}$$

Hereafter we assume that

$$\jmath(\varepsilon(t))=\lambda\cdot\hat{\jmath}(\varepsilon(t)) \tag{2.8}$$

where $\lambda>0$ is a constant number and $\hat{\jmath}(\varepsilon)$ is a logistic function of ε.

It is well-known that the logistic function takes on the form

$$\hat{\jmath}(\varepsilon)=\frac{a}{b+ce^{-a\varepsilon}}$$

where the parameters $a,b>0$ and c is an arbitrary real number.
Obviously, $\hat{\jmath}(\varepsilon)$ is an increasing function for all $\varepsilon>0$ that is convex in the interval $(0,\frac{1}{a}\ln\frac{c}{b})$ and concave in $(\frac{1}{a}\ln\frac{c}{b},\infty)$ (hence $\tilde{\varepsilon}=\frac{1}{a}\ln\frac{c}{b}$ is the only inflex point of $\hat{\jmath}(\varepsilon)$ and $\hat{\jmath}(0)=\frac{a}{b+c},\ \lim\limits_{\varepsilon\to\infty}\hat{\jmath}(\varepsilon)=\frac{a}{b}$.) Recall that $\hat{\jmath}(\varepsilon)$ is a solution of the differential equation

$$\frac{\mathrm{d}\hat{\jmath}(\varepsilon)}{\mathrm{d}\varepsilon}=\hat{\jmath}(\varepsilon)\cdot[a-b\hat{\jmath}(\varepsilon)]$$

with the initial condition $\hat{\jmath}(0)=\hat{\jmath}^0=\frac{a}{b+c}\Longrightarrow c=a(\hat{\jmath}^0)^{-1}-b$. So the investment ratio can be written as

$$\jmath(\varepsilon(t))=\lambda\cdot\frac{a}{b+[a(\hat{\jmath}^0)^{-1}-b]e^{-a\varepsilon(t)}}=\lambda\cdot\frac{a\hat{\jmath}^0}{b\hat{\jmath}^0+[a-b\hat{\jmath}^0]e^{-a\varepsilon(t)}} \tag{2.9}$$

Hence the propensity to invest takes on the following form

$$i\left(\frac{Y(t)}{K(t)},Y(t),r(t)\right)=\frac{1}{1+r(t)}\cdot J\left(\frac{Y(t)}{K(t)}\right)=\frac{1}{1+r(t)}\cdot\jmath(y(t)-k(t))\stackrel{\text{def}}{=}\bar{\jmath}(y(t)-k(t)) \tag{2.10}$$

and, moreover, if $\frac{1}{1+r(t)}\doteq 1$ then even

$$i\left(\frac{Y(t)}{K(t)},Y(t),r(t)\right)=\jmath(y(t)-k(t))$$

We assume that also savings $S(Y(t), r(t))$ can be well approximated by the following expression

$$S(Y(t), r(t)) = Y(t) \cdot [s_0 + s_1 \cdot y(t)) + s_2 \cdot r(t)] \quad \text{with} \quad s_0 < 0, \text{ and } s_1, s_2 > 0 \tag{2.11}$$

Hence the propensity to save $s(\cdot,\cdot) = S(\cdot,\cdot)/Y(\cdot)$ can be written as

$$s(Y(t), r(t)) \stackrel{\text{def}}{=} \bar{s}(y(t), r(t)) = s_0 + s_1 \cdot y(t) + s_2 \cdot r(t) \tag{2.12}$$

Hence under condition (2.10) equation (2.3) can be written as

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = \alpha \cdot \left\{ \frac{1}{1+r(t)} \cdot \jmath\,(y(t) - k(t)) - [s_0 + s_1 \cdot y(t) + s_2 \cdot r(t)] \right\} \tag{2.13}$$

and similarly from (2.4) we get

$$\frac{\mathrm{d}k(t)}{\mathrm{d}t} = \frac{1}{1+r(t)} \cdot \jmath\,(y(t) - k(t))\, e^{y(t)-k(t)} - \beta \cdot e^{(\gamma-1)k(t)} \tag{2.14}$$

Observe that if $r(t) \equiv r$ then (2.13), (2.14) yield for a given initial conditions y^0, k^0 unique solutions $y(t)$, $k(t)$.

Moreover, we can express dependence of $y(t)$ on $k(t)$ more explicitly. In particular, inserting from (2.12) into (2.5) we conclude that

$$\left(\frac{1}{\alpha} \cdot \frac{\mathrm{d}y(t)}{\mathrm{d}t} + s_0 + s_1 y(t) + s_2 r(t)\right) \cdot e^{y(t)} = \left(\frac{\mathrm{d}k(t)}{\mathrm{d}t} + \beta e^{(\gamma-1)k(t)}\right) \cdot e^{k(t)} \tag{2.15}$$

and if $r(t) \equiv 0$ the above expression can be simplified to

$$\left(\frac{1}{\alpha} \cdot \frac{\mathrm{d}y(t)}{\mathrm{d}t} + s_0 + s_1 y(t)\right) \cdot e^{y(t)} = \left(\frac{\mathrm{d}k(t)}{\mathrm{d}t} + \beta e^{(\gamma-1)k(t)}\right) \cdot e^{k(t)}$$

Finally observe that for $r(t) \equiv 0$ (2.13), (2.14) are identical with equations (1) and (4) in the paper [**?**], in case that

$$I\left(\frac{Y(t)}{K(t)}, Y(t), r(t)\right) = \frac{1}{1+r(t)} \cdot \bar{I}(Y(t)) \quad \text{then} \quad \jmath\,(y(t) - k(t); y(t)) \stackrel{\text{def}}{=} \jmath\,(y(t)). \tag{2.16}$$

This form was employed in the papers Sladký, Kodera, Vošvrda [**?**], [**?**].

3 Money Market

As to the money market, let (in continuous time $t \geq 0$) $L(Y(t), R(t))$ denote the demand for money and let M^s be the supply of money. The dynamics of the money market (i.e. the dynamic LM model) is given by the following differential equation:

$$\frac{\mathrm{d}r(t)}{\mathrm{d}t} = \zeta\,\{\ell(y(t), R(t)) - m^s\} = \zeta\,\{\ell(y(t), r(t) + \pi(t)) - m^s\} \tag{3.1}$$

where $\ell(y(t), R(t)) = \ln(L(Y(t), R(t))$, $m^s = \ln M^s$, and $\zeta > 0$ signifies the speed of adjustment.

Observe that that the real interest rate $r(t)$ increases if the demand for money $L(\cdot, \cdot)$ in (3.1) is greater than the supply of money M^s (or iff $\ell(\cdot, \cdot) > m^s$), and decreases iff $L(\cdot, \cdot) < M^s$ (or iff $\ell(\cdot, \cdot) < m$) which correspond with the laws of money market. Obviously, the equilibrium is given by $M^s = L(Y^\star, R^\star)$ for some $Y(t) \equiv Y^\star$, $R(t) \equiv R^\star$ or equivalently by $m^s = \ell(y^\star, R^\star)$ (cf. (1.2)).

The demand for money is described by the traditional Keynesian demand-for-money function being in the following form

$$\ell(y, r) = \ell_0 + \ell_1 y - \ell_2 R = \ell_0 + \ell_1 y - \ell_2 (r + \pi), \tag{3.2}$$

where the parameters $\ell_i > 0$, $i = 0, 1, 2$ are known.

Similarly, according to Tobin [?], for $\pi(t)$ the following equation is valid

$$\frac{\mathrm{d}\pi(t)}{\mathrm{d}t} = \omega \left[\frac{\mathrm{d}p(t)}{\mathrm{d}t} - \pi(t) \right] \tag{3.3}$$

where ω is the coefficient of adaptation and $p(t) = \ln P(t)$; $P(t)$ is the price level.

For what follows we need to express $p(t)$. To this end in the papers [?], [?] the authors employed the following relation obtained using the Cobb–Douglas production function on condition of the full employment and an additional assumption the real wage corresponds to the productivity of labor

$$\frac{\mathrm{d}p(t)}{\mathrm{d}t} = \mu \frac{\mathrm{d}y(t)}{\mathrm{d}t} \tag{3.4}$$

Inserting from (3.3) into (3.4) we get

$$\frac{\mathrm{d}\pi(t)}{\mathrm{d}t} = \omega \left[\mu \frac{\mathrm{d}y(t)}{\mathrm{d}t} - \pi(t) \right] \tag{3.5}$$

In the papers [?], [?] the dynamics of the complete macroeconomic system was described by relations (2.13), (3.1) and (3.5), however, the authors assumed that $k(t) \equiv 0$, $\bar{s}(y, r) = s_0 + s_1 y + s_2 r$ (hence the term $s_2 r$ is nonzero, in numerical examples, its magnitude was comparable with that of the term $s_1 y$).

On the contrary in the paper [?] the author assumed that $\bar{s}(y, r) = s_0 + s_1 y$ and the dynamics of output $y(t)$ is then fully described by the solution of equations (2.13) and (2.14) where $r(t) \equiv 0$. In addition in [?] the dynamics of the money market is given quite independently of the output $y(t)$. It is assumed the Fisherian model, where the money demand at time t, denoted $M^d(t)$, is given by

$$M^d(t) = \frac{1}{V(\pi(t))} P(t) Y(t) \tag{3.6}$$

$V(\pi)$ is the velocity of circulation of money.

Taking logarithms we get from (3.6)

$$m^d(t) = -v(\pi(t)) + p(t) + y(t) \tag{3.7}$$

where $m^d(t) = \ln M^d(t)$, $p(t) = \ln P(t)$, $y(t) = \ln Y(t)$, $v(\pi) = \ln V(\pi) = v_0 + \kappa\theta(\pi)$, $\theta(\pi)$ is a logistic function.

Along with the adaptive relation of expected inflation (3.3) also the following equation for adaptation of $p(t)$

$$\frac{\mathrm{d}p(t)}{\mathrm{d}t} = \sigma \left[m^s - m^d(t)\right] \tag{3.8}$$

where $m^s = \ln M^s$ is considered,

On inserting from (3.6) into (**??**) we get

$$\frac{\mathrm{d}p(t)}{\mathrm{d}t} = \sigma \left[m^s - y(t) - p(t) + v(\pi)\right] \tag{3.9}$$

and on inserting (**??**), (3.3) takes on the following form

$$\frac{\mathrm{d}\pi(t)}{\mathrm{d}t} = \omega \left\{\sigma \left[m^s - y(t) - p(t) + v(\pi)\right]\right\} \tag{3.10}$$

Supposing that we know the output function $y(t)$ we are able to find the functions $p(t)$ and $\pi(t)$ on solving (**??**), (**??**).

4 Illustrative Examples

We now compare dynamic behavior of two complete macroeconomic systems that were represented either:

— by the three equations system where the output $y(t)$ is governed by (2.13) ($I(\cdot)$ is simplified according to (2.16)), the dynamics of interest rate is given by (3.1) (where $\ell(\cdot)$ is approximated by (3.2)) and the expected inflation follows (3.5), see [**?**], [**?**], or

— by the four equations system where the output and capital formation is given by (2.13), (2.14) (for $\jmath$ we substitute from (2.7), (2.8)) and the expected inflation along with the price level is given by (**??**), (**??**). We assume that $\alpha = 35$, $\beta = 0.1$, $\gamma = 1$, $s_0 = 0.16$, $s_1 = 0.05$, $s_2 = 0.016$, $a = 1$, $b = 1$, $\lambda = 0.25$, $r(t) \equiv r = 0.08$ in (2.13), (2.14) and $\sigma = 0.6$, $\omega = 0.8$, $v_0 = 1$, $\kappa = 15$, $m^s = 0.535$ in (**??**), (**??**). The initial conditions are $\hat{\jmath}^0 = 0.1$ (implying $y^0 = 1$), $k^0 = 0.38$, $\pi^0 = 1.1$, $p^0 = 1$. In addition, for the three equation system we assume that $\ell_0 = 0.25$, $\ell_1 = 0.12$, $\ell_2 = 0.66$, $\zeta = 1$.

The obtained numerical results can be found in Figures 1 and 2.

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Fig. 1. Dynamic behaviour of the complete macroeconomic system (the three equations system) for $m^s = 0.535$.

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Prof. Ing. RNDr. Jan Kodera, CSc., Ing. Karel Sladký, CSc., Doc. Ing. Miloslav Vošvrda, CSc., Department of Econometrics, Institute of Information Theory and Automation of AS ČR, Pod vodárenskou věží 4, 182 08 Praha 8, Czech Republic.
e-mails: kodera@vse.cz, sladky@utia.cas.cz, vosvrda@utia.cas.cz,

THE SCHEDULING AIRCRAFT LANDINGS PROBLEM

VÁCLAV KOŘENÁŘ, JAN FÁBRY, JAN PELIKÁN, STANISLAV MĚLNÍČEK
UNIVERSITY OF ECONOMICS PRAGUE, CZECH REPUBLIK

Abstract

The problem studied in this paper is an air traffic problem on an airport runway. The goal is to find an aircraft landing sequence that meets the time window condition for the particular aircraft and at the same time the separation conditions between pairs of landing aircrafts, what is necessary for the security of landings. The integer programming formulations and the relationship to the traveling salesman problem with cumulative costs are shown. Also computational results with testing data sets are presented.

1. The scheduling aircraft landings problem

The problem is one of deciding the landing time for the set of planes, which are in the radar horizon of an air traffic controller at an airport, which involves the decision of sequencing of planes. There are two basic conditions for this time: the landing time of each plane has to lie within a predetermined time window and the landing times should follow separation conditions.

The lower bound of the time window for a particular plane depends on the distance of the plane from the airport and the speed of the plane. The upper bound of the time window depends on the amount of fuel. An economic speed for each plane determines the preferred landing time, so called target landing time.

The second main important constraint is the separation time between two planes. Each plane generates an air turbulence that can be dangerous for successive planes. The intensity of the turbulence depends on the type and weight of the plane. Because of the turbulence problem it must be specified certain time distance (separation time) between planes. There are two separation conditions:

- complete separation conditions, if we have to ensure separation to all previous landing planes,
- successive separation conditions, which ensure only separation to directly previous landing plane.

It can be proved that if the triangular condition for separation times is satisfied the successive separation conditions ensure the complete separation conditions. Otherwise the successive separation conditions are weaker than the complete separation conditions.

The goal is either to maximize the number of planes scheduled to land in certain time period or to minimize the average landing time of all planes or to minimize the total cost of deviation of real (scheduled) landing times from appropriate target landing times.

The problem can be formulated for one or more runways, for plane landings only, for plane takeoffs only, or for both landings and takeoffs.

2. Mathematical model of the aircraft landing problem with complete separation

In [1] the following mixed integer zero-one formulation of the problem is presented.

Notation:

P the number of planes

E_i the earliest landing time for plane i (i=1,2,...,P)

L_i the latest landing time for plane i (i=1,2,...,P)

T_i the target (preferred) landing time for plane i (i=1,2,...,P)

S_{ij} the required separation time between plane i landing and plane j landing, where plane i lands before plane j

$g_i > 0$ the penalty cost per time unit for landing before the target time T_i for plane i (i=1,2,...,P)

$h_i > 0$ the penalty cost per time unit for landing after the target time T_i for plane i (i=1,2,...,P)

It is supposed that $E_i \le T_i \le L_i,\ i = 1, 2, ..., P$ and $S_{ij} \ge 0$ for all i, j.

The variables in the model are:

t_i the landing time for plane i (i=1,2,...,P)

t_i^+ how soon plane i lands before target time T_i (i=1,2,...,P)

t_i^- how soon plane i lands after target time T_i (i=1,2,...,P)

Let

x_{ij}=1 if plane i lands before (not necessary directly) plane j, $i, j = 1, 2, ..., P,\ i \ne j$

x_{ij}=0 otherwise.

Mathematical model of the aircraft landing problem with the complete separation:

$$\min \sum_{i=1}^{P} g_i t_i^+ + h_i t_i^- \quad (1)$$

subject to

$$x_{ij} + x_{ji} = 1,\ i,j = 1,2,...,P,\ i \neq j \quad (2)$$

$$t_i + S_{ij} - (L_i + S_{ij} - E_j)x_{ji} \leq t_j,\ i,j = 1,2,...,P,\ i \neq j \quad (3)$$

$$E_i \leq t_i \leq L_i,\ i = 1,2,...,P \quad (4)$$

$$t_i + t_i^+ - t_i^- = T_i,\ i = 1,2,...,P \quad (5)$$

$$t_i, t_i^+, t_i^- \geq 0,\ i = 1,2,...P,\ x_{ij} \in \{0,1\},\ i,j = 1,2,...,P, i \neq j \quad (6)$$

Equation (2) means that either plane *i* lands before plane *j* or plane *j* lands before plane *i*. The landing time t_j should be greater than t_i with a difference, which is the separation time S_{ij} (inequality (3)). The landing time t_i should lie within the time window <E_i, L_i> (inequality (4)). Equation (5) defines time differences t_i^+ and t_i^-, which are the differences of t_i from the target time T_i. The variables t_i^+ and t_i^- are not defined by (5) uniquely, nevertheless the uniqueness of t_i^+ and t_i^- is guaranteed by the fact that g_i>0 and h_i>0 in the objective function (1).

An alternative objective function is the minimal average landing time $(1/P)\sum_{i=1}^{P} t_i$.

Comment

For a given sequence of planes the determination of the optimal landing times is a linear programming problem. We can obtain this model by putting all variables x_{ij} to the appropriate values into the model (1)-(6).

3. The heuristic method

Because of NP hardness of the ALP (Aircraft Landing Problem) heuristic methods were proposed for the problem. One of them [3] is the greedy approach based on priorities numbers p^k_j, in which *k*-th plane is picked to be landed next according to the lowest priority number p^k_j . The priority numbers are calculated as

$$p_j^k = \delta T_j + \varepsilon EE_j^k + \alpha_j ,$$

where δ, ε are arbitrary positive weights and α_j is a perturbation of the priority.

EE_j^k is defined as the earliest time in which the plane j can land given by the previous sequence of planes, that is, if the partial sequence $s_1, s_2, \ldots, s_{k-1}$ of planes has been constructed already, then

$$EE_j^k = \max\{E_j, \max_{i<k} \{EE_{s_i}^i + S_{s_i j}\}\}.$$

The next plane to land is $s_k = \underset{j \notin \{s_1, s_2, \ldots, s_{k-1}\}}{\arg\min} \; p_j^k$.

The earliest possible landing time for plane s_k is $EE_{s_k}^k$.

This heuristic will not necessary find a feasible landing sequence (it is possible that $EE_{s_k}^k > L_{s_k}$ for some k).

4. Mathematical model of the aircraft landing problem with successive separation

In this section we will solve the problem in which only successive separation is enforced. If the triangular inequality $S_{ik} \le S_{ij} + S_{jk}$ for all $i \ne j \ne k$ holds the successive separation is sufficient to ensure complete separation.

The aircraft landing problem with successive separation can be viewed as an open traveling salesman problem with time windows, where nodes in this problem are the planes. The objective function is cumulative, so the special formulation, called traveling salesman problem with cumulative costs (or the deliveryman problem) should be used [4].

The following formulation of the aircraft landing problem with successive separation is proposed.

Let x_{ij}=1 if plane i lands directly before plane j, $i, j = 1, 2, \ldots P$, $i \ne j$,

x_{ij}=0 otherwise.

$$\min \sum_{i=1}^{P} g_i t_i^+ + h_i t_i^- \tag{7}$$

subject to

$$\sum_{i=0}^{P} x_{ij} = 1, \; j = 0,1,..,P \tag{8}$$

$$\sum_{j=0}^{P} x_{ij} = 1, \;\; i = 0,1,..,P \tag{9}$$

$$\begin{aligned} & t_i + S_{ij} - (L_i + S_{ij} - E_j)(1 - x_{ij}) \le t_j, \; i, j = 0,1,\ldots,P, \; i \ne j, \; j > 0 \\ & t_0 = 0 \end{aligned} \tag{10}$$

$$E_i \le t_i \le L_i,\ i = 1,2,...,P \quad (11)$$

$$t_i + t_i^+ - t_i^- = T_i,\ i = 1,2,...,P \quad (12)$$

$$t_i, t_i^+, t_i^- \ge 0,\ i = 1,2,...,P,\ x_{ij} \in \{0,1\},\ i, j = 1,2,...,P,\ i \ne j \quad (13)$$

Equations (8) and (9) ensure that only one plane precedes and only one plane follows each plane. Plane number zero is artificial, so that S_{0i}=S_{i0}=0 for all *i*.

5. Aircraft landing problem for multiple runways

There are usually two or more runways on great international airports. The aircraft landing problem solves the question on which runway the plane will land and at which landing time. There are two different groups of separation times:

S_{ij} separation time for planes *i* and *j* landing on the same runway,

s_{ij} separation time for planes *i* and *j* landing on different runways.

It is assumed that $0 \le s_{ij} \le S_{ij}$.

Let z_{ij}=1 if planes *i* and *j* land on the same runway, $i, j = 1,2,...P,\ i \ne j$,

z_{ij}=0 otherwise.

The mathematical model (1)-(6) has to be modified so that equation (3) for determination of landing times should be replaced by

$$t_i + S_{ij} z_{ij} + s_{ij}(1 - z_{ij}) - (L_i + S_{ij} - E_j) x_{ji} \le t_j,\ i, j = 1,2,...P,\ i \ne j. \quad (14)$$

6. Computational results

Many computational results of experiments using the models shown above were published in [1].

The problem and models can be tested on the data sets provided by the OR problem library maintained by Beasley (http://mscmga.ms.ic.ac.uk/info.html). There are 8 problems in this data set. All of them were solved and the results were compared. The LINGO ver. 7 was used and run on Pentium II. The results are presented in Table 1, where the original computational times from [1] are written in brackets (if differs significantly).

Table 1. Numerical experiments

No. data set	No. planes	No. runways	Cost Heuristic m.	Opt. cost LINGO	Runtime (sec.)
1	10	1	1210	700	1
		2	120	90	1
		3	0	0	1
2	15	1	2030	1480	6
		2	210	210	3
		3	0	0	1
3	20	1	2870	820	4
		2	60	60	2
		3	0	0	2
4	20	1	4480	2520	483 (220)
		2	680	640	2754 (1920)
		3	130	130	75 (2299)
		4	0	0	2
5	20	1	7120	3100	1379 (922)
		2	1220	650	X (11510)
		3	240	170	1539 (1655)
		4	0	0	3
6	30	1	24442	24442	2 (33)
		2	882	554	2482 (1568)
		3	0	0	3
7	44	1	3974	1550	37 (11)
		2	0	0	5
8	50	1	4390	1950	77 (112)
		2	260	135	301 (3451)
		3	0	0	9

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Václav Kořenář, Jan Fábry, Jan Pelikán, Stanislav Mělníček

Department of Econometrics, University of Economics Prague, Nám. W. Churchilla 4, 130 67 Praha 3, Czech Republic, e-mail: korenar@vse.cz, fabry@vse.cz, pelikan@vse.cz, melnicek@vse.cz

OPTIMIZATION METHODS AND BULLWHIP EFFECT

MARTINA KUNCOVÁ
UNIVERSITY OF ECONOMICS, PRAGUE, CZECH REPUBLIK

1. Introduction[1]

In this article I would like to make a connection between methods of the operational research and bullwhip effect, which is usually mentioned when talking about supply chain management. Supply chain management (SCM) is nowadays seen as one of the most important areas that should be developed. Supply chain itself can be characterized as a net that consists of suppliers, manufacturing centres, warehouses, distribution centres, and retail outlets, as well as raw materials, work-in-process inventory, and finished products that flow between the facilities. The main effort is dedicated to the interconnection of all subjects in the supply chain and to the optimization of all the flows that exist among these subjects. But because of the fact that firms are used to compete and not to cooperate, they usually don't want to share any information about their products, inventory, costs or profit. So it is not easy to manage the whole supply chain the way that is acceptable for everyone. I've tried to describe some examples connected with inventory optimisation and show, how the effort of minimization of the enterprise inventory cost can cause the bullwhip effect.

2. Bullwhip Effect

Whenever we read any book or an article about the supply chain or the supply chain management, we probably find there something about the bullwhip effect (also called demand amplification effect or whiplash). Forester was the first man who described this effect but the experts of Procter & Gamble gave it its name (after the way the amplitude of a whip increases down its length) and also its publicity. It's well known, that during the monitoring of the customers demand and retailers and distributors orders of the diaper product "Pampers" they found out the amplification of order variability, though the demand was almost fixed. Recently the interest in the exploration and measurement of the bullwhip effect has grown up because of the fact that the amplification of orders influences the distributor's and the manufacturer's costs, inventory, reliability and other important business processes. By reason that nowadays the main effort is dedicated to the coordination and communication among customers, suppliers, distributors and manufacturers (it means it is dedicated to the development of the supply chain management), some specialists of this branch have started to look for the reasons for the bullwhip effect, for the methods of its measurement and also for the methods of its elimination.

2.1 Four Causes of the Bullwhip Effect

- **Demand forecast**. It's not assumed that the retailer knows the exact form of the customer demand process. Instead, he uses historical data and some forecasting techniques to estimate the demand. The supplier doesn't know the retailer's data and so he supposes the retailer's order to be the real demand. Due to this, the forecast could be very different and that's why the orders can vary.
- **Order batching**. In most cases (see below) the cost (or inventory) policy is the main why for ordering in batches. But due to this, the next link of the supply chain has to have higher inventory to avoid the depletion of inventory.
- **Price fluctuation**. Customers are driven to buy in larger quantities by attractive offers on quantity discounts or price discounts. If their behaviour is rational, they buy more when the price is down or less when the price is up. However this strategy doesn't reflect their true needs, and so this may give birth to the bullwhip effect.
- **Rationing and shortage gaming**. This cause might be similar to the price fluctuation. It occurs when demand exceeds supply or when the customers think it may happen. Then they start to exaggerate their real needs to be sure that the existing demand will be satisfied. The demand amplification effect will grow up even further if customers are allowed to cancel their orders when their real demand is fulfilled.

It's possible to consider more reasons, for example the information distortion, lead times or the maximization of the profit (or the minimization of the costs) that can influence each of the causes mentioned above.

Every retailer usually knows a lot of information – about customers, demand, prices, discounts, inventory, etc. – but it doesn't want to share this information with another company even though it might be its supplier. Then the supplier and the manufacturer don't know much about the real situation on the market, and therefore they can hardly follow the customers' needs. This effect is called the information distortion.

Lead times aren't conventionally mentioned separately as the main cause of the bullwhip effect, although this factor is included in the bullwhip effect formula. It's clear that the longer the lead times are, the larger the demand and the safety stock must be (to avoid the inventory shortage), and that's why the larger the demand amplification could be.

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The most often aim of every company is to maximize their profit or to minimize their costs irrespective of their suppliers or customers. If they try to fulfil this stated goal, they have to realize some of the actions mentioned above (discounts, order batching, etc.) and by this means they contribute to the demand amplification effect.

2.2 Problems With Measuring of the Bullwhip Effect

According to other similar articles, the bullwhip effect is usually measured as the ratio of variances Var(q*) / Var(Q), where Var(q*) represents variance of (optimal) order and Var(Q) is variance of demand faced by the enterprise. Although this formula is used very often, I've found out that it is dependent on the type of data aggregation.

I have obtained the real data about the intensity of the sales and orders of one type of summer tyres[2] from the tyre repair and service shop. Chart 1 show the real demand, order and inventory per month.

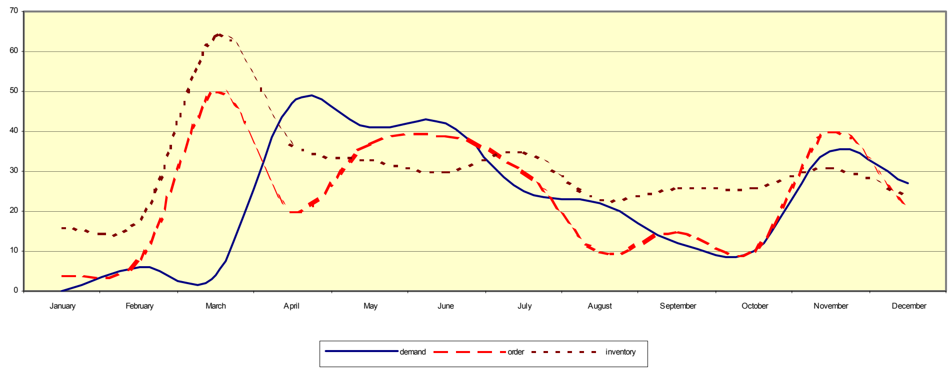

Chart 1 – Monthly demand, order and inventory

Demand is between 0 and 47 units (tyres) per month, but the firm orders every month (as you can see later, sometimes even every week or twice a week). The variance of the sales is 241 and the variance of the orders is 210, so there is no demand amplification effect because the quotient of these variances is 0.87.

As I know, the customers don't buy the tyres once a month but conventionally they want to buy it every week or every day and so does the retailer. When you look at the Chart 2, you can see that the inventory level is much higher than the demand level (they can afford it because of the fact that the inventory holding costs are very small). Although the orders are close to the demand, the bullwhip effect is suddenly 1.34.

[2] Brilantes 160/70/R13 OR60 – usually used for the Czech cars Skoda Fabia and Skoda Felicia

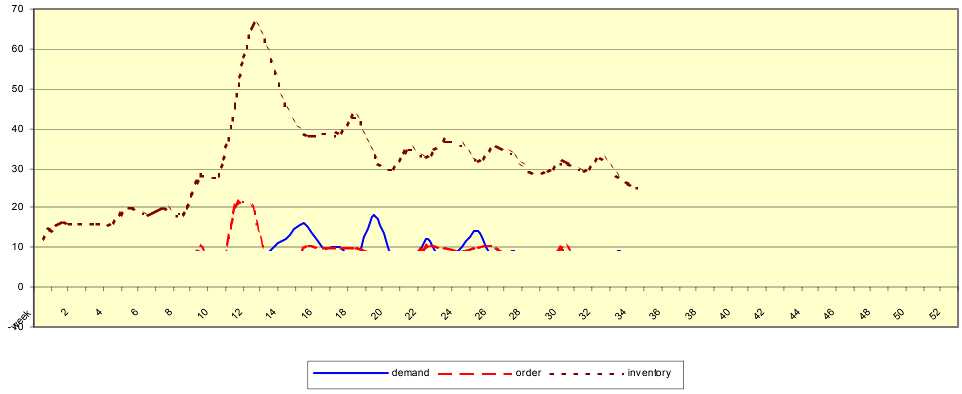

Chart 2 – Demand, order and inventory per week

Finally I've investigated the daily demand and order (and then also the same but without weekends because the shop is closed in those days). When I've taken into account all work days of the year 2001, the bullwhip effect has been 3.27. The question is whether the days with zero demand and zero order should be involved or not. If they aren't involved, the bullwhip effect is surprisingly 5.84. As you can see in Chart 3, the bullwhip effect also depends on the period of data aggregation.

Another possibility, how to count this effect, is the ratio of coefficients of variation of order and demand, where coefficient of variation is equal to standard deviation divided by average demand (order). This formula is much worse than the previous one, especially when the standard deviations of demand and order are nearly equal, because the ratio of average orders (demands) doesn't tell us nearly anything about the oscillations.

Because of these difficulties with measuring, I've tried to count the mean costs of every member of the supply chain (in the next example) and define the bullwhip effect (with regard to costs) as a percentage growth of costs.

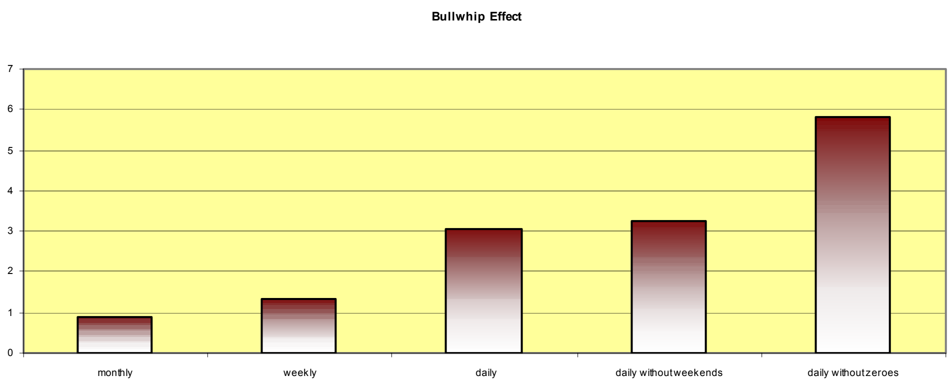

Chart 3 – The bullwhip effect when using various periods for data aggregation

3. Stochastic Models of Inventory and the Bullwhip Effect

It is supposed that demand is a random variable (usually a discrete random variable) that can be approximated with a normal distribution with known mean and standard deviation. If we want to calculate the optimal order quantity (to minimize the inventory costs), we can use EOQ formula as follows (I assume the inventory shortage doesn't occur, so this model doesn't reflect the inventory shortage costs):

$$\text{EOQ formula:} \qquad q^* = \sqrt{\frac{2\mu_Q c_2}{c_1}}$$

where: q^* = the optimal order quantity (EOQ) for minimization of the inventory costs

μ_Q = the mean demand in units per one period

c_1 = the inventory carrying costs (per one unit per one period)

c_2 = the costs per order

Since we don't know the actual demand, it is necessary to build up a safety stock (w) that should compensate the variations of demand. The safety stock level depends on the desired percents product availability (α = the probability that the inventory shortage won't occur). This desired service level is a function of the normal loss curve, which provides the area in a right tail of a normal distribution. If the factor that corresponds with α is z and we know the lead times (which are constant), then we can calculate the safety stock level and also the reorder point (the inventory level showing the necessity to order) as:

$$w = z * \sigma_Q * \sqrt{L} \qquad\qquad s = L * \mu_Q + w$$

where: w = safety stock in units, z = the z factor that corresponds with service level α

L = lead time, s = reorder point in units, μ_Q = the mean demand

σ_Q = the standard deviation of demand

Now let's say that the customers demand oscillates randomly between 20 and 30 units per month (mean is 25 units and standard deviation is 1,67). The fixed costs are 1,5 crowns per month and variable costs are 120 crowns (for all members of the supply chain), the lead times are 2 weeks (0.43478 per month) and the service level is 99%. Then the optimal characteristics for the retailer are in the Table 1. In this case the bullwhip effect is 316 because the oscillations of demand are very small compared to the oscillations of the orders.

Since the demand is random, the supplier can't find out when the retailer really orders (which week or day). Because the retailer's order is now its demand, which the supplier faces, and it is not normally distributed, let's suppose that the supplier has 3 possible strategies (A, B, C). All characteristics are in Table 1.

Table 1 – Main characteristics of retailer and supplier

	Retailer	Supplier A	Supplier B	Supplier C
μ_Q (units)	25	21	63	31,5
σ_Q (units)	1,67	30	0,67	10,5
q* (units)	63,24	57,96	100,6	71
t* (months)	2,53	2,76	1,6 (3-5)	2,25
w (units)	3	47,59	1,06	16,66
s (units)	14,63	57,56	32	32
S (units)	79	116	133	103
$\mu_{N\ (crowns/month)}$	99,37	158,33	152,19	131,48

As you can see in all examples shown above, when any subject of the supply chain uses the EOQ method for order quantity calculation (to minimize its inventory costs), it always orders in batches after some period and so it should enhance the bullwhip effect. But when I've tried to simulate this small supply chain (customer-retailer-supplier) and than count the effect, I've found out that the point of view (or the measuring method) is the critical factor. When you look into the table above, you can see that the lower the safety stocks are, the higher the optimal order must be, and vice versa. But the most used formula for counting the demand amplification effect doesn't care about the safety stocks although they are very important when you are interested in costs and inventory levels. So the bullwhip effect counted as the ratio of variances differs from the ratio of mean costs (see Table 2).

Table 2 – Comparison of the classical BE and the growth of mean costs

	Retailer vers. Supplier A	Retailer vers. Supplier B	Retailer vers. Supplier C
Ratio of variances (BE)	0,959	2,168	1,07
Ratio of mean costs	1,59	1,53	1,32

In the first case there is no bullwhip effect, because the supplier orders nearly with the same frequency and nearly the same amount as the retailer does, but it doesn't reflect that the supplier must hold high level of safety stock which increase its inventory holding and carrying costs. So it is possible to say that the bullwhip effect is hidden in inventory level or in safety stocks.

4. Conclusion

Supply chain management (SCM) is nowadays seen as one of the most important areas that should be developed because the present trend aims at cooperation and managing the whole chain. The bullwhip effect is one of the main problems inside any supply chain because it has bad impact on those enterprises, which are not closed to the end customer. I've tried to show how it may come into existence, for example when using the EOQ policy (to minimize the inventory holding and carrying costs), which leads to ordering in batches, and how it is possible to measure it. I've found out that the usual method for its measuring shows only the growth of amplification of orders (if occurs), but it doesn't tell us anything about the impact on the inventory level or inventory costs. If each member of the supply chain wants to minimize its inventory costs, the bullwhip effect always appears. As I've described above, sometimes it doesn't appear as the demand amplification effect, but it is hidden in the growth of the inventory level and the inventory costs. Unfortunately, it is nearly impossible to avoid it, because any company must count with uncertainty.

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Ing.Martina Kuncová
Department of Econometrics, University of Economics, Winston Churchill sq. 3, 130 67 Prague 3, Czech republic, tel. +420 224 095 449, kuncovam@vse.cz

ECONOMIC TIME SERIES MODELLING BY APPLICATION OF KALMAN FILTERING PROCEDURES

MARČEK DUŠAN
UNIVERSITY OF ZILINA, SLOVAK REPUBLIC

Abstract

The paper shows the construction and application state-space model with the associated Kalman recursions for the estimates of the trend and seasonal component for an additive model with constant magnitude of the seasonal swing over time. We try also to estimate the variance of the model from the given time and savings deposits.

1. Introduction

The amount of the time and saving deposits of housholds is a very important variable which reflects the savings of housholds which can be used by commercial banks for long-term credits. In this text, we will use the variable savings deposits - y_t as the time series variable. The period of time or frequency for which the data were collected is quarter.

This paper presents an application of state-space model and the Kalman recursions for time and saving deposits forecasting. In Section 2 we will analyse the quarterly data presented in [2] and build a state-space representation for this time series. In Section 3 Kalman rekursions are performed to determine the trend and seasonal component and calculate their best linear mean square predictors.

2. State-Space Representation of a Discrete Time Series Model

In this section we present an example of the state-space model specification. The collection of 24 valid reliable savings and deposits data are available (see [2]) and are listed in Tab. 1 and shown in Fig. 1. We would like to develop a time series model for this process so that a predictor for its trend and seasonal component can be developed.

Tab. 1

The quarterly actual values for time and savings deposits of housholds for the 6 years 1961-1966.
The data set is listed by rows.

524	365	317	309	563	364	344	365	513	373	361	366
562	393	382	389	625	433	438	417	614	481	466	455

The graphical presentation of the time series $\{ y_t \}$ exhibits an upward trend suggesting that the qrowth is linear over the time. There is a definable drop in third quarter (summer) and a rise to a pick during the first quarter (winter). We assume that we have an additive model. From this data, we can express the underlying model in the version with the local linear trend and constant seasonality by the following model equations system

$$y_t \; = x_{t,1} + x_{t,2} \tag{1}$$

$$x_{t,1} = x_{t-1,1} + v_t \tag{2}$$

$$x_{t,2} = \sum_{j=1}^{s-1} x_{t-j,2} \tag{3}$$

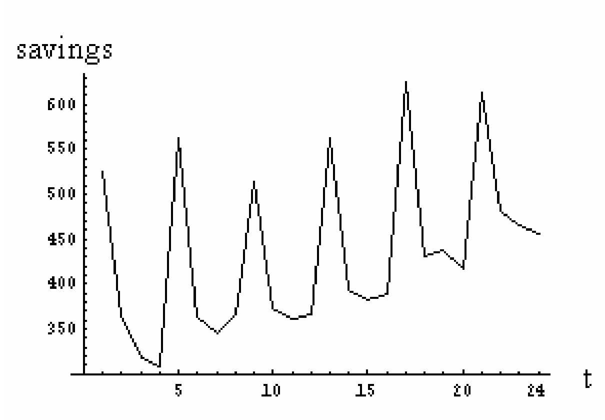


Fig. 1. The quarterly actual values for time and savings deposits of housholds for the 6 years 1961-1966.

where v_t is the white noise variable with zero mean and σ_v^2 variance. The first equation is a transition equation. By third equation is modeled seasonal component suggesting that the seasonal component is the some any one season each year. The equations (1) and (2) are also the behavioral equations, the equations (3) are identities.

The equations system (1) to (3) can be represented in linear state-space form, i.e., the discrete series $\{ y_t \}$ satisfies the equations of the form

$$\mathbf{Y}_t = \mathbf{G}_t \mathbf{X}_t + \mathbf{W}_t \tag{4}$$

$$\mathbf{X}_t = \mathbf{F}_t \mathbf{X}_{t-1} + \mathbf{V}_t \tag{5}$$

where X_t is the $v \times 1$ vector of inputs (state) or explonatory variable (in our case X_t=[$x_{t,1}$, $x_{t,2}$, $x_{t-1,2}$, $x_{t-2,2}$]′), v is number of inputs, Y_t is the $w \times 1$ vector of observed (measured) or explained variables (in our case Y_t= $\{ y_t \}$), w is the number of outputs. $\mathbf{W_t}, \mathbf{V}_t$ are the $v \times 1$

or $w \times 1$ vectors of independent Gaussian white noise variables with zero mean and covariance matrices by $E(\mathbf{V}_t\mathbf{V}_t')$ =Q_t and $E(\mathbf{W}_t\mathbf{W}_t')$ =R_t respectively.

The equation (4) defines a sequence of observations. The equation (5) is interpreted as describing the evolution of the state X_t of a system at time *t* in terms of a known sequence of $v \times v$ matrix F_t. F, G are known matrices or vector at time *t*. Next, we assume that the coefficient matrices are independent of time.

3 . Application of the Kalman Recursions, Simulation Results

Assuming that the coefficients of the matrices **F**, **G** and the initial estimates of the unobservable state vector *X* are known, one may to obtain an estimate of the state vector $X_{t|t}$ based on the information of Y_t available at time *t*, i.e., based on the information of Y up to Y_t. Denote the current period by *s*. We wish to estimate the state vector *X* at period *t* based on the information at time *s*. Let $\hat{X}_{t|s}$ represents the estimate of X_t based on the information up to and including period *s* and $P_{t|s}$ represents its mean square error matrix. Kalman rekursions find out the best linear estimate of the state cector X_t based on the observations $Y_1, Y_2, ..., Y_s$. Then the appriopriate set of Kalman recursions for the estimate $X_{t|s}$ in terms of :

s=t is defined as filtering recursion,
s>t is defined as th smoothing recursion, and
s<t is defined as the prediction recursion.

The appropriate set of equations which constitute the above Kalman rekursions can be found in [1].

The Kalman recursion results are given in the Fig. 2 - Fig. 7. The Kalman filter gives the estimate of the state variable $x_{t,1}$ and $x_{t,2}$. The estimated values of trend ($x_{t,1}$) and seasonal component ($x_{t,2}$) given $\{y_1, y_2, ..., y_s\}$ are ploted as a function of *t* in Fig. 2.

Similarly, the smoothed estimates of trend and seasonal component are ploted in Fig. 3. The plot of the predicted trend and seasonal component for the next 4 points is shown in Fig. 4.

As metioned earlier the first *v* values of data were used to calculate the starting values of X and P. To estimate the variance of the random component v_t the log likelohood function of the state-space representation of the discrete time series model (1) to (3) was used in the following form

$$\ln \ell(\sigma_v^2; Y_1, Y_2, \ldots, Y_N) = -\frac{1}{2}\sum_{t=v+1}^{N} \ln\left|\Sigma_{t|t-1}\right| - \frac{1}{2}(Y_t - \hat{Y}_{t|t-1})' \Sigma_{t|t-1}^{-1}(Y_t - \hat{Y}_{t|t-1}) + const \qquad (6)$$

vhere $\Sigma_{t|t-1}^{-1}$ is the inverse of the error matrix $\Sigma_{t|t-1} = E[(Y_t - \hat{Y}_{t|t-1})(Y_t - \hat{Y}_{t|t-1})']$. The equation (6) gives the estimated variance σ_v^2 = 701.9.

Now assuming that the seasonal component of the state variable $x_{t,2}$ in equation (1) is multiplicative - the magnitude of the seasonal swing is proportional (a ratio) to the trend $x_{t,1}$. We can define the version of the structural model with multiplicative seasonal movements over time with respect to the model (1) to (3) by

$$y_t \ = x_{t,1} + x_{t,2} / x_{t,1} \qquad (7)$$

$$x_{t,1} = x_{t-1,1} + v_t \qquad (8)$$

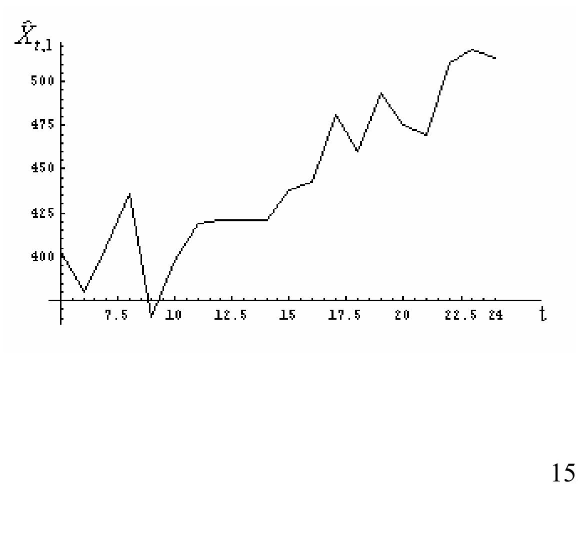
$$x_{t,2} / x_{t,1} = -\sum_{j=1}^{s-1} x_{t-j,2} / x_{t-j,1} \qquad (9)$$

Similarly the equations (7) to (9) can be represented in linear state-space form (4) and (5) with

$\mathbf{X}_t = [\, x_{t,1},\ x_{t,2}/x_{t,1},\ x_{t-1,2} / x_{t-1,1},\ x_{t-2,2} / x_{t-2,1}]'$;

$\mathbf{X}_{t-1} = [\, x_{t-1,1},\ x_{t-1,2}/x_{t-1,1},\ x_{t-2,2} / x_{t-2,1},\ x_{t-3,2} / x_{t-3,1}]'$

The Kalman recursions give the some estimates of the state values (trend and seasonal component) as in the case of the model with additive seasonal movements over time. This is because the trend $x_{t,1}$, seasonal component $x_{t,2}$ and the estimations of $X_{t,1}$, $X_{t,2}$ given y_1, $y_2 \ldots, y_N$ is given by the Kalman filter. These estimations obtaining by Kalman filtering are dependent only on the initial values of X_t, P_t and the known matrices **F**, **Q**, **G**, R only and in this case both the information set y_1, $y_2 \ldots, y_N$ and the matrices **F**, **Q**, **G**, R are the some.



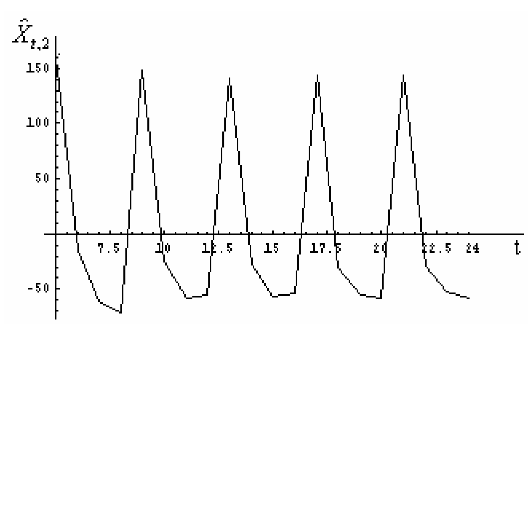


Fig. 2. The Kalman filter estimates of trend $\hat{x}_{t,1}$ and seasonal $\hat{x}_{t,2}$

4. Conclusion

Esentially there are two available techniques associated with modelling time series data containing trend, seasonal and error components. The first one is the decomposition method, the second one is the state-space representation with the associated Kalman recursions. In this paper we have presented the methods for the estimate of trend and seasonal components based on Kalman recursions. We have estimated the variance parameter of the state-space form model from the given data using the likelihood function. The information are obtained from Time Series Pack procedure Kalman Filter, Kalman Smoothing, Kalman Prediction FindMinimum-LogLikelihood [3] applied to the data of Tab 1.

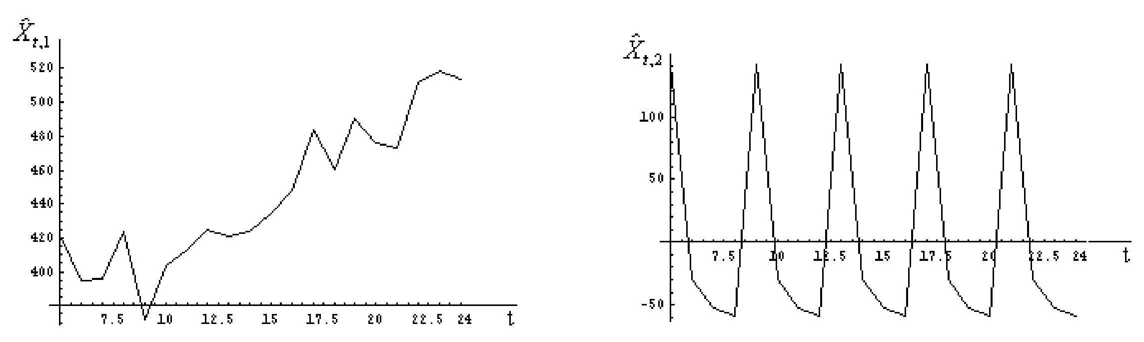


Fig. 3. The smoothed estimates of trend $\hat{x}_{t,1}$ and seasonal $\hat{x}_{t,2}$

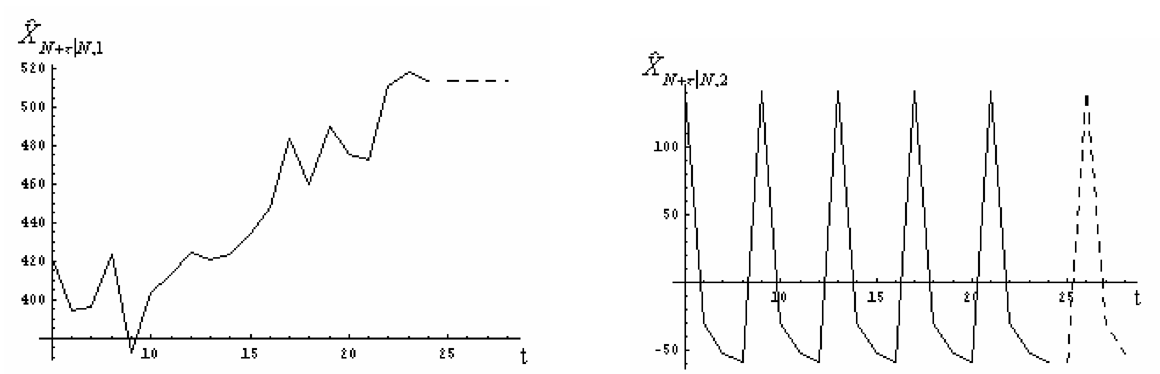


Fig. 4. The predicted values of the trend $\hat{x}_{t,1}$ and seasonal $\hat{x}_{t,2}$ for the next 4 period savings

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Dušan Marček

Faculty of Management Science and Informatics, University of Zilina, Moyzesova 20, 010 26 Žilina, Slovakia, marcek@fria.utc.sk

STATISTICAL INFORMATION SYSTEMS FOR LOCAL GOVERNMENT SUPPORT

CRISTINA MARTELLI
UNIVERSITY OF FLORENCE, ITALY

Abstract

In a complex society (Le Moigne, 1997) political decisions and government acts need a very rich informational framework; this issue is particularly important for local administrations, who are requested to answer to society needs improving their services level (Gibert and Andrault, 1984; Martini and Cais, 1999).

Administrative archives, especially when managed in an automatic context, may represent a rich opportunity and a strategic tool to accomplish these goals. In Italy public administrations are actually interested by a very deep improvement and innovation process: recent laws have drawn a more systemic model in public administration organization, and in this new context is now possible to develop Statistical Information Systems (S.I.S) oriented to monitoring, auditing, management control and government decision supporting (Stame,1998). This paper presents the main designing topics that have been assumed during some join projects involving the Florence University Statistics Department "G.Parenti" and the Municipality and the Province of Florence.

1. Statistical information systems

A decision supporting context must rely on a statistical information system designed to answer to informational requirements. In recent time the distinction between informational and operational systems has been specified in the data warehousing perspective (Kimball, 1996; Inmon, 1995 e 1996) and many tools and products have been proposed. Usually the concept of datawarehouse is not wide enough for local administration statistical information systems: the extension of their pertinence, the role of rules, laws and bureaucracy itself make the issue more difficult than in firms.

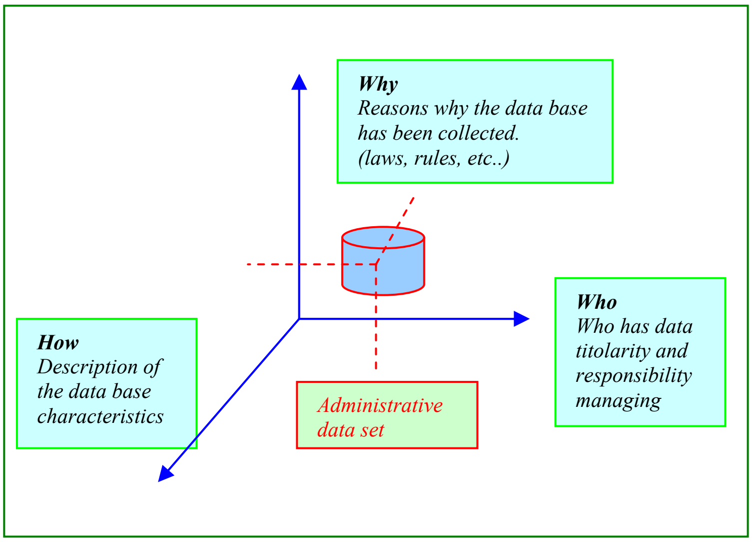
A statistical information system has some important peculiarities: the observed entities are frequently very complex with significant interrelationships, and the role of time is usually more critical than in operational one. Time series are always requested and very often archival data are not updated (in the sense that the new data substitutes the old one) but preserved for posterity. Retrieval of data is often “volumetric , that is, based on two or more conjunctive range searches.

The S.I.S projecting activity always starts from the required indicators (Buzzigoli, 2002) and from the models that must be estimated; afterwards the necessary data must be produced or detected in the administrative data bases and mapped into the new statistical information system. Whenever informational gaps are detected new *ad hoc* sources must be developed. Therefore two are the fundamental goals that must be performed when building a local administrations oriented statistical information system: a clear image of the required indicators and a very accurate knowledge of the administrative data sets.

2. Administrative data for statistical use in local administrations: from operational data to statistical sources

2.1. The metadata survey

One of the most important issue in datawarehouse methodology is that informative and operational systems must be kept apart: the difference between the two contexts in terms of users, technology and requirements suggests not to overlap the two functionalities. Starting from the administrative and operational framework a new information system must be developed: in order to that, a clear picture of the administrative data sets must be obtained. A metadata data base is the first instrument to develop and every archive has to be described in terms of three main axes (Picture 1).



Picture 1 – The specification axes of administrative archive

This metadata data base will be precious, for instance, whenever arises the need of monitoring government decision consequences, because it will help to find out all the archives related with the topic of interest, along with their technical features.

Sometimes administration have good information about their data, but more often the detail level at which the archives are described is not enough for developing a metadata database: in this case a survey must be performed .

2.2. *The questionnaire model*

The conceptual model of the metadata survey is made by several actors: in particular in Picture 1 is shown the interdependent role of organization, laws and storing technology.

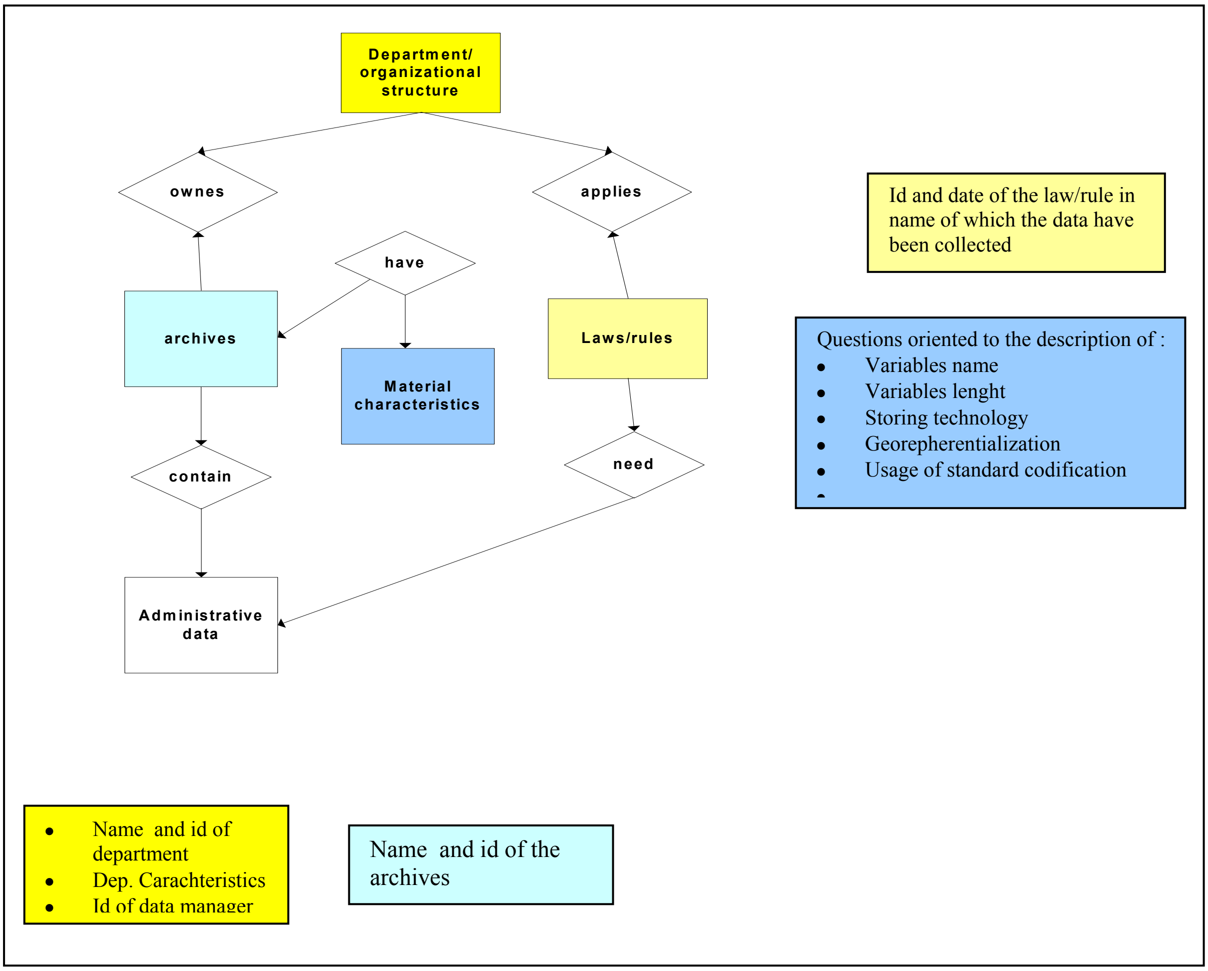
In Picture 2 the correspondence between conceptual model and survey questions is shown. Collecting such information and organizing them in a relational data base logically organized in coherence with this conceptual model, it will be possible to be informed on situations like: "Where can I find data to study the consequences of the application of a certain rule/law?" or "Can I link together two or more administrative data sets (on the basis of the adoption of the same standard codes)?"

It must be stressed that such a survey is not interested in data collection, but only in metadata gathering. In other words, in this first step of S.I.S construction, we are not interested in having the availability of administrative data but only in drawing a sort of "map" of them. Is very frequent, for instance to discover that there could be a rich quantity of potential information that is wasted because of different coding politics that don't allow any linkage, or that the usage of traditional storing and managing systems in practice forbid the statistical usage of the data. The metadata survey recently made for the Province of Florence, for instance (Risolo, Rogai,1998), has shown how, in some departments, usage of informatics for data archiving is low and data coding protocols not homogeneous; to check how the data archives situation is evolving, a second wave of the metadata survey is about to be distributed among the Province's different departments.

2.3. *The linkage between the logistic and organizational system and the metadata database.*

Through the "who" axe (Picture 1) it is possible to set a very useful linkage between the metadata database and the local administration organizational structure.

In a recent work, all the indicators already produced by the Municipality of Florence departments have been linked with a data set describing their characteristics, in terms of objective, resources, responsibilities, and so forth.



Picture 2: metadata survey conceptual model

When the Comune di Firenze metadata survey has been designed, a particular attention has been paid to information linking between administrative archives and organizational data.

The first wave of the Municipality of Florence metadata survey is actually on going: when it will be terminated it will be possible to answer to questions like: given the responsibilities of a certain structure which are the more appropriate indicators to monitor its activities? The indicators that they calculating at the moment are performing enough? Where could we find the best data to monitor and valuate their activity?

2.4. Role of the relational model in a statistical information system

A statistical information system is not oriented to managing and, in this sense, the non redundancy issue, that is usually compulsory in operational data bases, can be partially released. Many authors (Kimball, 1996) underline that in informational data bases the relational approach is too difficult to adopt, and that a multidimensional data modelling achieves a more friendly approach to data querying and retrieving.

It is well known that the four hardest thing for a data base to do are: 1) Join tables, 2) aggregate data, 3) Sort data, 4) Scan large volume of data. By the way these are the four most often requested operations in a decision support system. In order to get around these four operations data base vendors suggest that the data base:1) use a denormalized or dimensional model (to avoid the joins) 2) use summary tables (avoid aggregation) 3) store data in sorted order (avoid sorting) 4) rely heavily on indexes (avoid scanning large volumes of base table data). All of these operations require that data base designer to "know" what queries will be asked. Once you "know" all the questions, you can create a dimensional model that will provide high performance for these queries. Another popular suggestion is to start to break the data up into "distributed data bases" to combat the data volume issues.

The biggest problem with dimensional models is that they are designed on the base of factors that are already well known. They are not adaptive to new relations that may be discovered and in fact discourage the discovery of such relationships. An E/R approach to system designing allows for true exploratory analysis such data mining. Dimensional modelling technique does not allow the user to expand their understanding of data relationships, change the way they look at the problem , nor does it support growth into more advanced analysis and emerging data types. A statistical information system must be equipped with a relational layer, which is the result of the Entity/Relationship conceptual modelling of the problematic context. Only the existence of this layer guarantees about the realization of a real systemic description of the reality we want to explore. This part of the system will be more appropriate to explore and recognize new data relationships. Contextually with this S.I.S component, a dimensional model especially tailored to feed routine statistics may be realized: a data mart approach could be a good answer to management control needs, but it doesn't fit the requirements of decision and valuation supporting systems for whom an entity relational context is necessary.

2.5. The problem of the user information needs modelling

An important issue when developing a statistical information system in support to local administrations is to succeed in giving a reliable technical translation of users informational needs. In particular, it will be very important to draw the boundaries (legal, bureaucratic, organizational, logistic) that limit and define the process we want to evaluate. In this sense is very important to use high level modelling graphical languages like, for instance, Uml, *Universal Modelling Language* (Whitman, Huff and Presley, 1997) and IDEF0 (IEEE, 1998) to sketch, describe and organize (M.E.A.N.S., 1993; Retzgui, 1998) the users information.

Users know the substance of the problems, but usually they are not able to organize a source for finding solutions and answers. Technical professionals have the so said "know how" but lack in "know why" and "know what". Conceptual languages may represent a bridge between these two worlds because they are enough formal to be considered a convenient input for technical projecting, but they are also very simple and expressive, so that even end users are competent for understanding and controlling whether their inputs have been correctly understood.

Formal linguistic analysis of laws texts and users informal descriptions and expressions may be adopted: process functionalities may be detected in verbal forms, while informative actors, classes and entities may be identified in substantives.

3. Conclusions

In this paper a methodology for projecting and developing a statistical information system in support to local government administrations has been presented. At the base of a statistical information system we must put an exact comprehension of the processes to evaluate: the adoption of formal conceptual modelling languages may represent a solution to the task. A particular attention has been dedicated to the problem of building the statistical data sources necessary to evaluate the indicators requested by the users. Administrative data sets and archives are, in this sense, an important opportunity provided that an exact description of their characteristics and their usage limits is drawn. A metadata survey is the first step toward this objective: particular consideration must be paid to the problem of the conceptualisation of the survey itself, in order to get a full linkage with the public administration's organizational system.

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Cristina Martelli

Dipartimento di Statistica “G. Parenti”, Viale Morgagni 59, 50134 Italy, Tel.: +39 055 4237262, Fax: +39 055 4223560, e-mail: martelli@ds.unifi.it

MODELLING MARKET INTEGRATION BY COINTEGRATION ANALYSIS

RUI MENEZES
ISCTE, LISBOA, PORTUGAL

1. Introduction

This paper investigates the extent to which Portuguese codfish imports from Norway and Iceland compete in the same market, both for wet and dried salted codfish. In this case the goods are said to be substitutes and there is market integration (Perry, 1989). In integrated markets the prices of goods tend to uniformity after allowing for transportation costs, presentation differences, etc (Stigler, 1969). This notion of market applies to markets for identical goods separated by distance. It relates the evolution of prices in the long run. However, prices are allowed to deviate from each other in the short run. On the basis of this type of relation one can test for correlation, causality, and proportionality, leading to the empirical definition of market integration (Ravallion, 1986).

The methodology used in this paper is based on cointegration techniques. This methodology is well established in the literature and has been applied to studies of the market for seafood products (see, e.g., Asche *et al.*, 2001b; Menezes *et al.*, 2002).

The relevant relations in our case refer to the average price series of dried and wet salted cod imports from Norway and Iceland. To this end, we use monthly price series covering the period from January 1988 to December 1999. The data used here were obtained from the National Statistics Institute of Portugal.

The paper will proceed with a short description of the Portuguese cod market, followed by a presentation of the market integration methodology and the empirical results obtained. Some final remarks will be also presented.

2. The Portuguese Cod Market

The fish and seafood is since long an important component of the diet of the Portuguese population, with an average annual gross consumption *per capita* over 60 kg (lwe). For this reason Portugal holds the first place in the ranking of fish and seafood consumption among the EU countries and the second place within Europe. A substantial part of this consumption is dried salted cod, which is either imported directly as a processed product, or produced in the Portuguese drying and salting industry using other forms of imported cod.

The foundation of dried salted cod consumption in Portugal dates back to at least the end of the 15th century, when the Portuguese discoveries were at their peak. The cod, caught in the banks of Terra Nova, was submitted to a double process of preservation: onboard salting and inland drying. Both processes were known and utilized since the ancient times. The salting of the scaled cod was a rather cheap and expedited process to store onboard the cod caught in Terra Nova. The drying inland allowed the storage of cod for long times with relatively few demands. Over the last years, however, the onboard salting has been gradually substituted by freezing.

The cod salting and drying industry is the only significant one in the sector of salting, drying, and smoking in Portugal, and it was until recently an integrated industrial activity, very dependent on catches made by the national long distance fishing fleet. For several reasons, however, the fishing activity of the Portuguese fleet collapsed in the mid 1980s, and these units had then to rely almost exclusively on imported raw material (Dias *et al.*, 2001).

Portugal is the world's largest importer of dried salted cod, which outside of Portugal is produced virtually only in Norway. In addition, Portugal is a big importer of salted cod, which is dried in Portugal before consumption. This cod comes from all the larger harvesting nations, especially from Norway and Iceland, and small quantities from other places. In the 1990s, imports of frozen cod took market share from salted cod as the Portuguese industry thaws, salts and dries this input. Unfortunately, however, the quality of the data available for Portuguese imports of frozen cod is not good enough to be used in an empirical analysis of cod price relationships.

There is evidence that there is a highly competitive global market for frozen cod (Gordon and Hannesson, 1996). Given the large number of suppliers it is also likely that this is the case for the market for wet salted cod. Moreover, although dried salted cod is produced only in two countries, there are many small companies in the industry in both countries giving little scope for market power (Guillotreau and Le Grel, 2001). Market integration analysis and investigation of substitution relationships between different product forms of cod also indicate that the different cod markets are highly related (Gordon *et al.*, 1993; Asche *et al.*, 2001a). This is as expected given that the budget share of the raw fish is high for all product forms (Toft and Bjørndal, 1996).

3. Modelling Market Integration

The markets for a group of products are integrated if the underlying prices move proportionally to each other along time, that is, if the Law of One Price holds. The LOP, described by Cassel (1918) and other prominent economists, can be incorporated as a special case of the following deterministic relationship between prices:

$$p_{1t} = \theta(p_{2t})^{\Lambda} \tag{1}$$

where p_{1t} and p_{2t} are respectively the prices of goods 1 and 2. The parameter Λ gives the degree of nonlinearity in this relationship. If $\Lambda = 0$, there is no relationship between the variables. If $\Lambda = 1$, the relationship between the variables is proportional where θ is the coefficient of proportionality. If $\Lambda \neq 1$ the prices are not proportional.

The parameter θ measures the inter-temporal relationship between prices and is a constant. It has a direct economic interpretation only if $\Lambda = 1$, so that the relationship is linear. This is the case where the LOP holds. For $\theta = 1$, the two prices are alike. For $\theta \neq 1$, the prices move proportionally but differ one to each other due to different transportation costs, etc. The former case is called the strict version of the LOP; the latter is the weak version of the LOP (Asche *et al.*, 1999).

Taking logarithms on both sides of equation (1) leads to

$$\ln p_{1t} = \Theta + \Lambda \ln p_{2t} \tag{2}$$

where $\Theta = \ln \theta$ is a constant. Equation (2) can be empirically estimated using a basic regression model relating the two prices:

$$\ln p_{1t} = \Theta + \Lambda \ln p_{2t} + \Xi_t \tag{3}$$

where Ξ_t is a random disturbance. The parameter Λ allows for testing different hypotheses about the price transmission. In particular, the price transmission elasticity will be one if $\Lambda = 1$.

Equation (3) can be extended to the case where p_{1t} depends not only on p_{2t} but also on lagged values of both variables, allowing the distinction between instantaneous and long run market integration (Ravallion, 1986; Slade, 1986; Goodwin *et al.*, 1990).

Conventional inferences for equation (3) are only valid if the price series p_{1t} and p_{2t} are stationary, i.e. integrated of order zero. If this is not the case, one can face a problem of spurious regression, where the usual t and F tests on regression parameters may be misleading (Granger and Newbold, 1974). To test the order of integration of the price series one can use an Augmented Dickey-Fuller test (ADF). The ADF test is based on the following regression (Dickey and Fuller, 1979; 1981):

$$\Delta \ln p_{kt} = \mu_0 + \mu_1 T + (\rho - 1)\ln p_{k,t-1} + \sum_{i=1}^{p} \gamma_i \Delta \ln p_{k,t-i} + \varepsilon_t \qquad (4)$$

where the hypothesis to be tested is that ρ is equal to 1, using to this end the critical values of MacKinnon (1991). The number of lags in the model is chosen so that the residuals are white noise. The null hypothesis of the test is that there is a unit root. A significant test statistic rejects the null hypothesis, and thus indicates stationarity ($\rho < 1$).

The usual procedure in these tests is to test the null hypothesis that $\rho = 1$ for the variables in levels and in first differences. If the null hypothesis is not rejected in the former case, but is rejected in the latter, then the variable p_k is said to be integrated of first order, *I*(1). If the null hypothesis is only rejected in second differences the variable p_k is integrated of second order, and so on.[1]

When the series are non-stationary, the appropriate tool to infer causal long-run relationships in the data is cointegration analysis. The cointegration approach may be represented as follows.[2] Consider two price series p_{1t} and p_{2t}. Each series is by itself non-stationary and is required to be differenced once to produce a stationary series. In general, a linear combination of non-stationary data series will also be non-stationary. In this case there is no long-run relationship between the series. However, when the data series form a long-run relationship they will move together over time, and a linear combination of the series will produce a residual series ζ_t which is stationary:

$$\zeta_t = \ln p_{1t} - \beta \ln p_{2t} \qquad (5).$$

In this case the prices p_{1t} and p_{2t} are said to be cointegrated, and the vector $(1, \beta)$ is the cointegrating vector (Engle and Granger, 1987). This is straightforward to extend to the multivariate case, which is usually denoted by vector autoregression (VAR).

Two different tests for cointegration are commonly used in the literature: the Engle-Granger test (Engle and Granger, 1987) and the Johansen test (Johansen, 1988; 1991). We

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[1] Despite their popularity due to their simplicity, the ADF tests are not free of problems. Several authors have analysed the problem of size and power of ADF tests of unit roots. Blough (1992), for example, shows that some unit root processes display a finite sample behaviour closer to a (stationary) white noise process than to a (non-stationary) random walk. On the contrary, some trend-stationary processes behave more like random walks in finite samples. There is, therefore, a problem of trade-off between size and power of the tests due to the use of critical values based on the asymptotic Dickey-Fuller distribution. Alternative tests have been suggested in the literature, but unfortunately they are not yet standard output of the econometric software.

[2] See e.g. Hendry and Juselius (2000) for a more thorough discussion about modelling of nonstationary data series and cointegration.

will here use the latter, as this is the most powerful test (Gonzalo, 1994) and allows parametric tests on the long-run parameters.

The Johansen method is based on an unrestricted VAR in the levels of the relevant variables to the analysis. To illustrate the process, consider the following vector equation:

$$\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1} + \cdots + \mathbf{A}_p\mathbf{y}_{t-p} + \boldsymbol{\mu} + \mathbf{u}_t \qquad (6)$$

where $\mathbf{y}_t$ represents a k-vector of nonstationary endogenous variables, the $\mathbf{A}_i$ represent $k \times k$ matrices of parameters and $\mathbf{u}_t$ represents a vector of *iid* residuals with mean zero and contemporaneous covariance matrix $\boldsymbol{\Omega}$. The matrix $\boldsymbol{\Omega}$ is definite positive, so that the residuals are not serially correlated although they can be contemporaneously correlated.

The Johansen method consists basically in studying the cointegrating rank (r) of the VAR system. To this end, the system represented by equation (6) can be written in a vector error correction form (VECM):

$$\Delta\mathbf{y}_t = \boldsymbol{\Pi}\mathbf{y}_{t-1} + \sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i\Delta\mathbf{y}_{t-i} + \boldsymbol{\mu} + \mathbf{u}_t \qquad (7)$$

where $\boldsymbol{\Pi} = \sum_{i=1}^{p}\mathbf{A}_i - \mathbf{I}$ and $\boldsymbol{\Gamma}_i = -\sum_{j=i+1}^{p}\mathbf{A}_j$. This specification of the system contains information on the short and long run adjustment parameters of the model through the estimates of $\hat{\boldsymbol{\Gamma}}_i$ and $\hat{\boldsymbol{\Pi}}$, respectively. If $\mathbf{y}_t$ is a vector of $I(1)$ variables then $\Delta\mathbf{y}_t$ and $\boldsymbol{\Gamma}_i\Delta\mathbf{y}_{t-i}$ are $I(0)$ and $\boldsymbol{\Pi}\mathbf{y}_{t-1}$ is a linear combination of $I(1)$ variables, being itself $I(0)$, given the assumptions made for the disturbances. The matrix $\boldsymbol{\Pi}$ can be factorized as $\boldsymbol{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$, where $\boldsymbol{\alpha}$ represents the adjustment speed to disequilibrium and $\boldsymbol{\beta}$ is the matrix of long run coefficients, that is, the cointegrating vectors. This is so when there are r cointegrating vectors, where $0 < r < k$. The cointegrating vectors denote the error correction mechanism in the VAR system. Once the number of cointegrating relations is determined and the matrices $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$ are estimated, the VAR is estimated incorporating these cointegrating relations.

When the cointegrating rank r is equal to the number of endogenous variables in the system, k, the variables in levels are stationary and the usual methods for estimating the VAR are applied. When $r = 0$ we have $\boldsymbol{\Pi} = \mathbf{0}$. In this case, there is no cointegrating relation between the variables in the system and a VAR in first differences without long run elements should be used.

Notice that determining the cointegrating rank amounts, in practice, to determining how many cointegrating vectors exist in $\boldsymbol{\beta}$ or, equivalently, how many columns are null in the matrix $\boldsymbol{\alpha}$. This is equivalent to determining the number of linearly independent rows/columns that exist in matrix $\boldsymbol{\Pi}$. Johansen proposed two tests for testing the hypothesis that the cointegrating rank is at most *r* (less than *k*): the trace statistic and the max statistic. In the former test, the alternative hypothesis is that the rank is *k*, and in the latter, the alternative hypothesis is that the rank is $r + 1$.

Johansen's method allows for a wide range of hypotheses testing involving the coefficients of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ using likelihood ratio tests (Johansen and Juselius, 1990). Among them, one can perform tests of proportionality between variables, and tests of weak exogeneity. In a bivariate context, if the variables are cointegrated, $\boldsymbol{\Pi}$ has rank equal to 1, so that $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are (2×1) vectors. In this case, testing the proportionality of the variables is equivalent to testing whether $\boldsymbol{\beta} = (1, -1)$. Testing for weak exogeneity, on the other hand, is equivalent to testing whether, for instance, the *i*th row of $\boldsymbol{\alpha}$ is zero. In this case, the *i*th endogenous variable is said to be weakly exogenous with respect to the parameters $\boldsymbol{\beta}$.

4. Empirical Results

The results of the ADF tests for the logarithm transformations of each price series are shown in Table 1. Here the number of lags was chosen so as to select the highest one with a significant final coefficient (Hendry and Doornik, 2001). The resulting residuals follow a white noise process. The results are presented for the variables in levels and in first differences.

For all the variables, the test for the variables in levels does not allow us to reject the null hypothesis that there is a unit root, independently of the inclusion of a deterministic trend in the ADF regression. However, the null hypothesis of a unit root is significantly rejected by the tests for the variables in first differences, with or without the inclusion of a deterministic trend. This means that the price variables are integrated of first order and cointegration procedures are required to perform tests on these series. It is apparent that the main trends in the prices are similar, but that there is substantial short-run variation.

Table 1
Augmented Dickey-Fuller Tests

Product	**Statistic**		**Stat with trend**		**Lags**
Levels:					
Wet salted import Iceland	-1.740		-3.216		3
Wet salted import Norway	-0.971		-1.506		2
Dried salted import Norway	-1.476		-1.896		1
First Differences:					
Wet salted import Iceland	-12.470	**	-17.080	**	0
Wet salted import Norway	-13.630	**	-13.640	**	1
Dried salted import Norway	-17.070	**	-17.080	**	0

** significant at the 1% level
Critical value is -3.478 at the 1% level (test statistic)
Critical value is -4.026 at the 1% level (test statistic with trend)

To investigate now the exact structure of the relationships between the price series, we estimated the long-run relationships with Johansen's method as described earlier. In Table 2 we present the test statistics for cointegration (panel a) and for proportionality and weak exogeneity (panel b). The VAR model was specified with two lags in the system. As expected, all prices are cointegrated so that they form long-run relationships. The results of the cointegration tests are consistent with the idea that cod prices for the different presentations/states are part of an integrated price system for this species, and do not represent separate or independent prices. Furthermore, prices are found to be (strictly) proportional in all cases except for the relation between wet salted cod imported from Iceland and Norway. If the joint variation of prices is reflected in terms of proportionality between two or more prices, then the data is compatible with the LOP. For the proportional relationships the elasticity of price transmission is one between all levels. Finally, the tests for weak exogeneity indicate that the dried and wet salted cod imported from Norway is exogenous (leading price) when compared with the price of cod imported from Iceland. Neither price is exogenous in the pairwise relation between dried and wet salted cod imported from Norway.

Table 2
Bivariate Johansen Tests (a)

Product: Price 1	Product: Price 2	Rank = *p* (1)	Eigenvalue (2)	Trace (3)		Max (4)	
Wet salted Iceland	Dried salted Norway	p==0	0.2617	44.9211	**	43.0890	**
		p<=1	0.0128	1.8321		1.8321	
Wet salted Iceland	Wet salted Norway	p==0	0.2601	45.6806	**	42.7668	**
		p<=1	0.0203	2.9138		2.9138	
Wet salted Norway	Dried salted Norway	p==0	0.2076	33.9959	**	33.0361	**
		p<=1	0.0067	0.9597		0.9597	

* significant at 5%; ** significant at 1%. Critical values are 15.41 and 3.76 at the 5% level and 20.04 and 6.65 at the 1% level for the Trace test and 14.07 and 3.76 at the 5% and 18.63 and 6.65 at the 1% level for the Maximum Eigenvalue test.

Table 2
Bivariate Johansen Tests (b)

Product: Price 1	Product: Price 2	LOP (5)		Exogeneity Price 1: Statistic (6)		Price 1: *p*-value	Exogeneity Price 2: Statistic (7)		Price 2: *p*-value
Wet salted Iceland	Dried salted Norway	2.9683		36.1047	**	0.0000	1.9447		0.1632
Wet salted Iceland	Wet salted Norway	3.8705	*	34.0439	**	0.0000	0.5736		0.4488
Wet salted Norway	Dried salted Norway	0.0017		13.2293	**	0.0003	12.0753	**	0.0005

* significant at 5%; ** significant at 1%. Critical values are 15.41 and 3.76 at the 5% level and 20.04 and 6.65 at the 1% level for the Trace test and 14.07 and 3.76 at the 5% and 18.63 and 6.65 at the 1% level for the Maximum Eigenvalue test.

5. Final Remarks

To conclude, this study has shown that the Portuguese cod market suffered profound structural transformations over the last decades. Although the national salting and drying cod industry continues to be the main supplier of retailers, the raw material used in this industry, formerly caught by the national fishing fleet of long distance, is now primarily imported. Norway represents a very important share of the Portuguese cod import market.

This study allows us to conclude that the cod market is strongly integrated where most prices are transmitted proportionally and almost instantaneously over the value chain. The cod prices are endogenous within a single origin but Norway leads the market for wet salted cod when confronted with Iceland. This seems to reflect the importance of the Norwegian market in terms of price determination.

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Rui Menezes

ISCTE, Av das Forças Armadas, 1649-026, Lisboa, Portugal, e-mail: rui.menezes@iscte.pt

MONEY MARKET – AN EFFICIENT FRONTIER ANALYSIS

MLYNAROVIČ VLADIMÍR, RICHARD KOLÁRIK
UNIVERSITY OF ECONOMICS IN BRATISLAVA- SYMPATIA FINANCIE, O.C.P., a.s., SLOVAKIA

Questions about profitability of using exchange rates changes at various kinds of financial investments are a subject of a frequent analysis and speculations as well. At present the question is alive mainly in connection with conjectures about possible exchange rate losses at the conversion of dollars returns from the sale of Slovak Gas Industry into Slovak currency. But the goal of the paper is not to analyse this special case. We concentrate our attention on historical analysis and optimisation of portfolio of selected international money market tools.

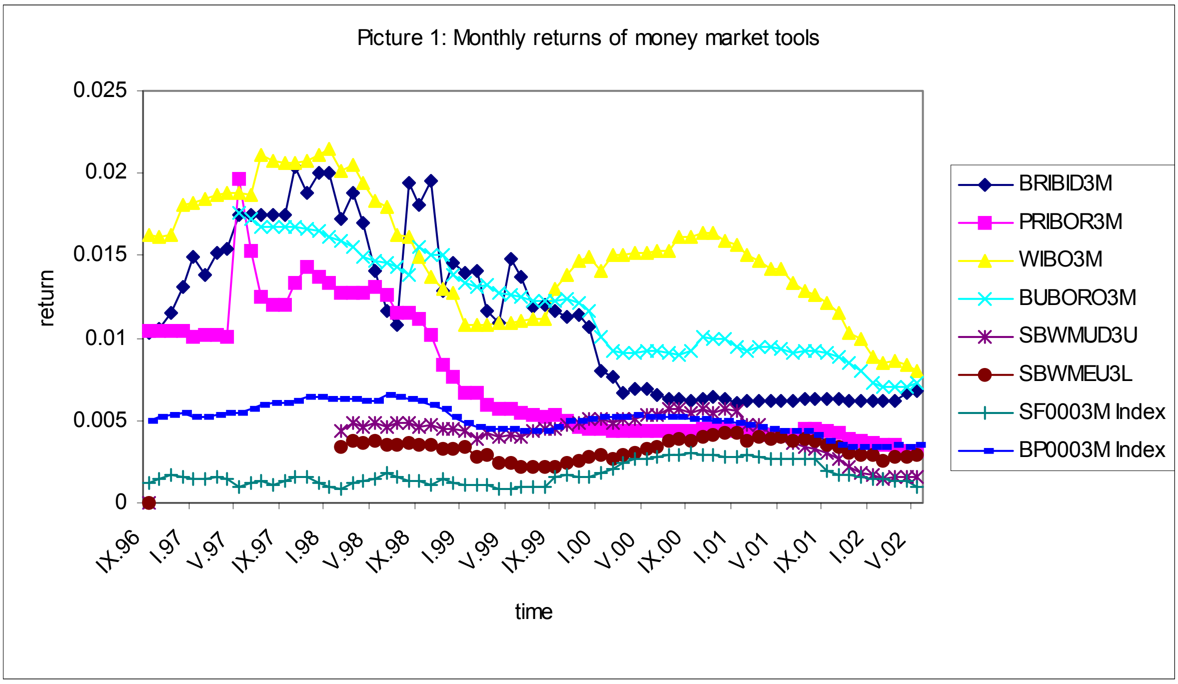
In general, for an international investor, fluctuations in asset prices must be converted from the local currency into the currency in which portfolio performance is evaluated. Exchange rates changes are therefore critical for measuring and comparing the returns from different countries.

In the paper the special money market that consists of three months tools of Slovak (BRIBID3M), Czech (PRIBOR3M), Polish (WIBOR3M), Hungarian (BUBOR3M), Switzerland (SF003M) and British (BP003M) money markets together with a one dollar (SBWMUD3U) and eur (SDWMEU3L) three months tool of money market, is created. We look for an answer to the question what kind of currencies was efficient for investors to hold from historical point of view. Monthly returns for period from September 1996 to May 2002 that are illustrated on Picture 1 were used at the analysis. Together with them the data about monthly exchange rates of Slovak currency with corresponding currencies of assumed money market tools were used as well[1].

Characteristics of the money market tools are presented in Table 1. The Table 2 describes returns of assumed money market tools after the conversion into the Slovak currency where the corresponding monthly exchange rates were taken into account. It is evident that higher returns in SKK go together with higher risk that is measured with standard deviation of returns. We would also like to note that the whole analysis is simplified in this

[1] The source: Bloomberg

way that transaction costs connected with conversions of returns into Slovak currency are not taken into account.



Picture 1: Monthly returns of money market tools

Pictures 2 – 4 illustrate monthly developments of returns in original currency and after their conversion into the Slovak currency for three selected tools of created money market. In the Table 3 we have the correlation coefficients that describe tendencies of mutual movements in returns of tools after the conversion into SKK.

Table 1: Characteristics of the money market tools

Currency	SKK	CZK	PLN	HUF	USD	EUR	CHF	GBP
Tool	BRIBID3M	PRIBOR3M	WIBO3M	BUBORO3M	SBWMUD3U	SBWMEU3L	SF0003M Inde	BP0003M Index
Average return per month	1.133%	0.752%	1.503%	1.184%	0.422%	0.326%	0.172%	0.506%
Standard deviation	0.48%	0.39%	0.36%	0.32%	0.12%	0.06%	0.07%	0.09%
Maximum return	2.035%	1.963%	2.145%	1.763%	0.575%	0.428%	0.299%	0.656%
Month	31.10.1997	30.5.1997	30.1.1998	30.5.1997	31.8.2000	29.12.2000	29.9.2000	30.6.1998
Minimum return	0.611%	0.310%	0.800%	0.700%	0.145%	0.220%	0.083%	0.335%
Month	31.1.2001	31.5.2002	31.5.2002	29.3.2002	28.2.2002	30.9.1999	30.4.1999	30.11.2001
Average return, p.a.	14.48%	9.41%	19.61%	15.18%	5.18%	3.99%	2.09%	6.24%
Standard deviation, p.a.	1.67%	1.36%	1.24%	1.09%	0.42%	0.20%	0.23%	0.31%

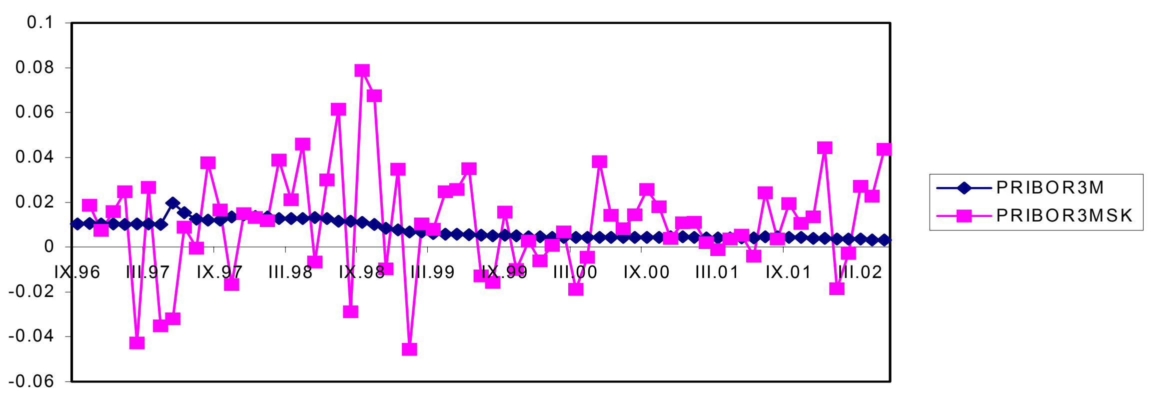
Table 2: Characteristics of the money market tools after conversions into SKK

Currency	SKK	SKK	SKK	SKK	SKK	SKK	SKK	SKK
Tool	BRIBID3M	PRIBOR3MSK	WIBO3MSK	BUBORO3MSK	SBWMUD3USK	SBWMEU3LSK	SF0003MSK	BP0003MSK
Average return per month	1.133%	1.108%	1.620%	1.169%	0.986%	0.588%	0.467%	1.045%
Standard deviation	0.482%	2.359%	2.807%	1.689%	2.564%	1.547%	1.839%	2.326%
Maximum return	2.035%	7.864%	6.483%	5.068%	6.400%	3.962%	4.615%	6.562%
Month	31.10.97	30.09.98	30.09.98	31.05.01	26.02.99	31.08.98	30.10.98	31.03.99
Minimum return	0.611%	-4.574%	-6.287%	-1.907%	-4.990%	-1.785%	-3.583%	-5.280%
Month	31.01.01	29.01.99	31.08.98	30.09.99	29.12.00	31.08.99	31.01.97	30.11.00
Average return, p.a.	14.480%	14.144%	21.276%	14.966%	12.499%	7.289%	5.750%	13.280%
Standard deviation, p.a.	1.670%	8.170%	9.725%	5.850%	8.883%	5.359%	6.370%	8.056%

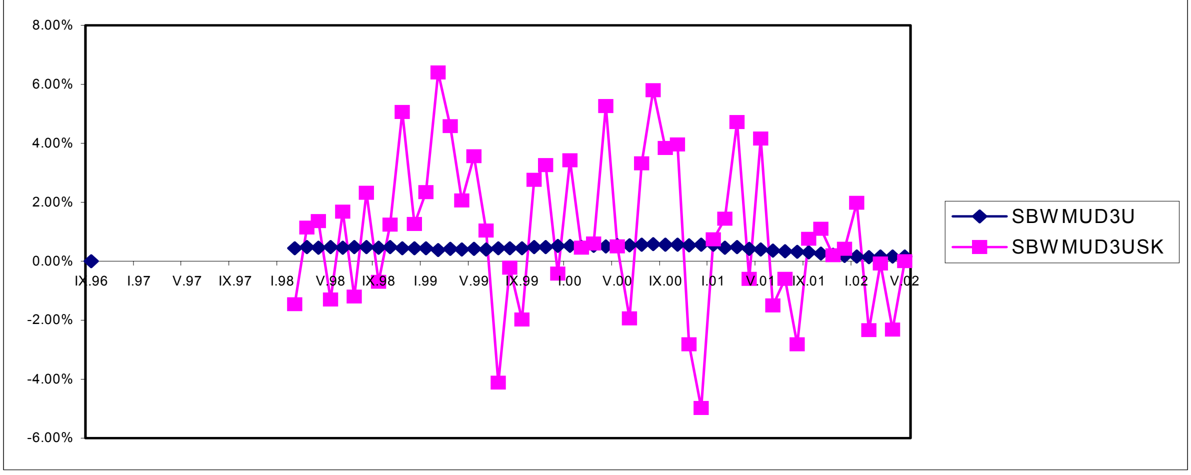
Table 3: The matrix of correlation coefficients

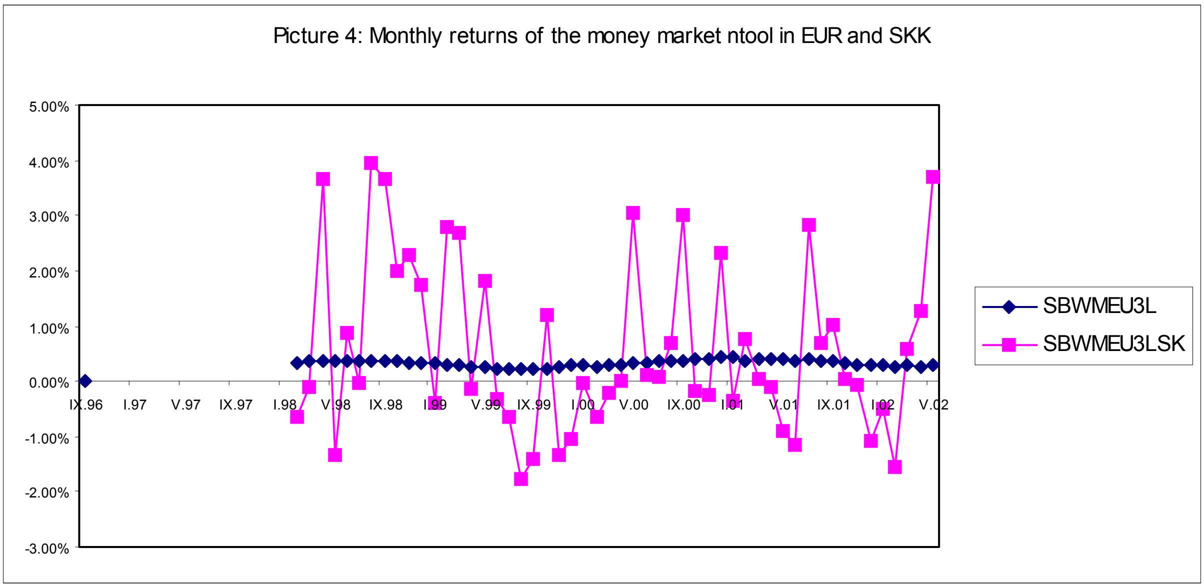
	BRIBID3M	PRIBOR3MSK	WIBO3MSK	BUBORO3MSK	SBWMUD3USK	SBWMEU3LSK	SF00003MSK	BP0003MSK
BRIBID3M	1.0000	0.0507	-0.0170	0.0117	0.0767	0.2352	0.0763	0.1654
PRIBOR3MSK	0.0507	1.0000	0.4481	0.3820	-0.1199	0.3716	0.2216	-0.1716
WIBO3MSK	-0.0170	0.4481	1.0000	0.4407	0.3106	0.1078	-0.0065	0.1356
BUBORO3MSK	0.0117	0.3820	0.4407	1.0000	0.3369	0.5805	0.3592	0.2104
SBWMUD3USK	0.0767	-0.1199	0.3106	0.3369	1.0000	0.2074	0.1524	0.6849
SBWMEU3LSK	0.2352	0.3716	0.1078	0.5805	0.2074	1.0000	0.7628	0.3413
SF00003MSK	0.0763	0.2216	-0.0065	0.3592	0.1524	0.7628	1.0000	0.1583
BP0003MSK	0.1654	-0.1716	0.1356	0.2104	0.6849	0.3413	0.1583	1.0000

Picture 2: Monthly returns of the Czech maney market in CZK and SKK



Picture 3: Monthly returns of the dollar money market tool in USD and SKK





1. A frontier of investment opportunities set with short sales allowed

At now on the described market of assets the frontier of investment opportunities is constructed in such a way to consists of the assets portfolios that ensure required (expected) returns at as low risk as possible. The part of the frontier, from the point that corresponds to the portfolio with global minimum risk, as it is known from a modern portfolio theory, creates the set of efficient portfolios. From the technical point of view one need to solve a series of quadratic programming problems that are known as Markowitz portfolio selection problems. When the short sales are allowed it means that investor can also borrow tools (assets). From the formal point of view it means that the proportion of such an asset in the portfolio is expressed with negative weight or with negative percentage proportion. For an effective realisation of solving process the Excel solver with a special created VBA procedure was used. Resulted optimal portfolios are described in Table 4 and the frontier they generated is illustrated in Picture 5. The problems are formulated and solved for monthly returns, but each row of Table 4 contains in the first two columns data about portfolio return and risk per annual. In the following columns we have the percentage structure of the corresponding portfolios. The Table 5 describes three selected efficient portfolios together with the selected probability characteristics.

Table 4: Efficient portfolios with short sales allowed

2	Výnos p.a.	Riziko p.a.	BRIBID3M	PRIBOR3MSK	WIBO3MSK	BUBORO3MSK	SBWMUD3USK	SBWMEU3LSK	SF00003MSK	BP0003MSK
3	11.350%	2.3215%	77.470%	0.268%	-3.097%	-10.658%	8.631%	22.257%	13.126%	-7.996%
4	11.616%	2.2219%	78.634%	0.400%	-2.803%	-9.099%	7.865%	19.270%	12.907%	-7.174%
5	11.882%	2.1267%	79.777%	0.555%	-2.528%	-7.532%	7.119%	16.280%	12.687%	-6.358%
6	12.149%	2.0357%	80.925%	0.710%	-2.252%	-5.956%	6.369%	13.276%	12.466%	-5.539%
7	12.416%	1.9503%	82.068%	0.865%	-1.977%	-4.388%	5.623%	10.286%	12.245%	-4.723%
8	12.683%	1.8712%	83.205%	1.019%	-1.704%	-2.828%	4.881%	7.312%	12.026%	-3.912%
9	12.952%	1.7980%	84.353%	1.175%	-1.428%	-1.252%	4.131%	4.307%	11.805%	-3.092%
0	13.221%	1.7326%	85.496%	1.330%	-1.153%	0.315%	3.385%	1.318%	11.585%	-2.276%
1	13.488%	1.6759%	86.633%	1.484%	-0.880%	1.875%	2.643%	-1.657%	11.366%	-1.465%
2	13.760%	1.6280%	87.782%	1.639%	-0.603%	3.451%	1.893%	-4.661%	11.145%	-0.645%
3	14.030%	1.5904%	88.924%	1.794%	-0.329%	5.019%	1.147%	-7.651%	10.924%	0.170%
4	14.301%	1.5636%	90.067%	1.949%	-0.054%	6.587%	0.401%	-10.640%	10.704%	0.986%
5	14.573%	1.5482%	91.210%	2.104%	0.221%	8.154%	-0.345%	-13.630%	10.484%	1.802%
6	14.845%	1.5445%	92.352%	2.259%	0.495%	9.722%	-1.091%	-16.619%	10.264%	2.617%
7	15.117%	1.5527%	93.495%	2.414%	0.770%	11.290%	-1.837%	-19.609%	10.044%	3.433%
8	15.389%	1.5723%	94.632%	2.568%	1.044%	12.850%	-2.580%	-22.583%	9.825%	4.244%
9	15.663%	1.6032%	95.775%	2.723%	1.318%	14.418%	-3.326%	-25.573%	9.605%	5.060%
0	15.938%	1.6447%	96.918%	2.878%	1.593%	15.986%	-4.072%	-28.562%	9.384%	5.876%
1	16.214%	1.6962%	98.066%	3.033%	1.869%	17.561%	-4.822%	-31.567%	9.163%	6.695%
2	16.490%	1.7565%	99.209%	3.188%	2.144%	19.129%	-5.568%	-34.556%	8.943%	7.511%
3	16.767%	1.8248%	100.352%	3.343%	2.418%	20.697%	-6.314%	-37.545%	8.723%	8.326%
4	17.044%	1.9002%	101.494%	3.498%	2.693%	22.265%	-7.060%	-40.535%	8.503%	9.142%
5	17.321%	1.9821%	102.637%	3.653%	2.968%	23.833%	-7.806%	-43.524%	8.282%	9.958%
6	17.598%	2.0691%	103.774%	3.807%	3.241%	25.393%	-8.548%	-46.499%	8.063%	10.769%
7	17.878%	2.1619%	104.922%	3.963%	3.517%	26.968%	-9.298%	-49.503%	7.842%	11.589%
8	18.157%	2.2586%	106.065%	4.117%	3.792%	28.536%	-10.044%	-52.493%	7.622%	12.404%
9	18.436%	2.3585%	107.202%	4.272%	4.065%	30.096%	-10.786%	-55.468%	7.403%	13.216%

Table 5: Selected efficient portfolios and their probabilistic characteristics

Portfolio		I	II	III
Expected return, p.a.		14.845%	16.767%	18.436%
Risk measured with standard deviation, p.a.		1.545%	1.825%	2.359%
Minimum return (5% level of confidence)		12.304%	13.765%	14.556%
Probability of nonpositive return		0.000%	0.000%	0.000%
Probablity for return less than 10%		0.085%	0.010%	0.017%
Probability for retur higher than 20%		0.042%	3.820%	25.361%
Money market	BRIBID3M	93.495%	100.352%	107.202%
	PRIBOR3MSK	2.414%	3.343%	4.272%
	WIBO3MSK	0.770%	2.418%	4.065%
	BUBORO3MSK	11.290%	20.697%	30.096%
	SBWMUD3USK	-1.837%	-6.314%	-10.786%
	SBWMEU3LSK	-19.609%	-37.545%	-55.468%
	SF00003MSK	10.044%	8.723%	7.403%
	BP0003MSK	3.433%	8.326%	13.216%

Picture 5: Money market portfolios frontier with short sales allowed

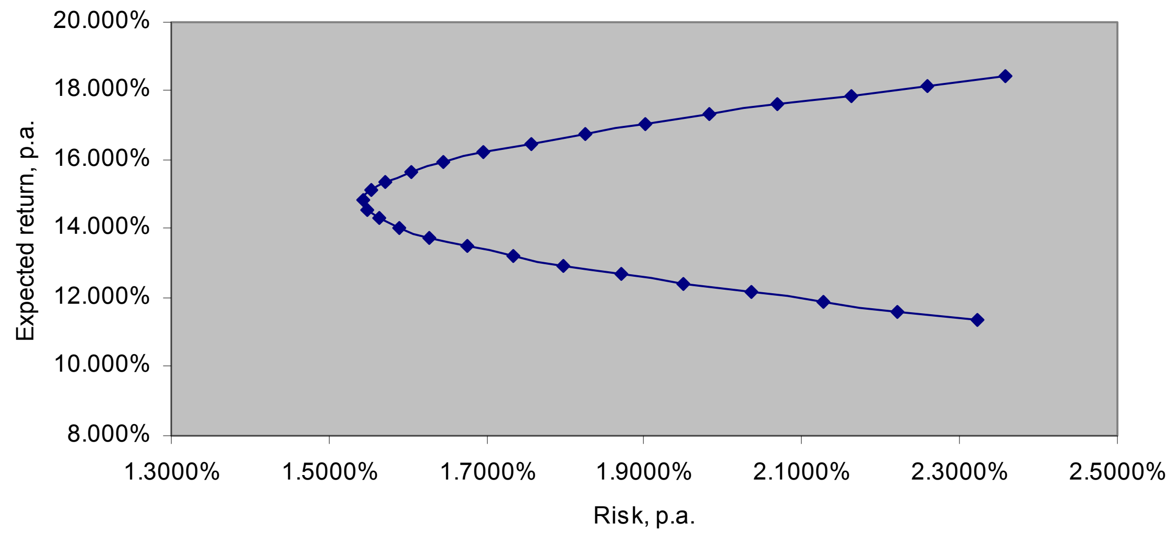


Table 6: Efficient portfolios with short sales not allowed

Výnos p.a.	Riziko p.a.	BRIBID3M	PRIBOR3MSK	WIBO3MSK	BUBORO3MSK	SBWMUD3USK	SBWMEU3LSK	SF00003MSK	BP0003MSK
11.350%	2.4946%	61.641%	0.000%	0.000%	0.000%	0.779%	15.011%	22.570%	0.000%
11.616%	2.3596%	64.938%	0.000%	0.000%	0.000%	0.877%	13.046%	21.139%	0.000%
11.882%	2.2319%	68.218%	0.000%	0.000%	0.000%	0.976%	11.091%	19.715%	0.000%
12.148%	2.1121%	71.498%	0.000%	0.000%	0.000%	1.074%	9.137%	18.291%	0.000%
12.415%	2.0014%	74.779%	0.000%	0.000%	0.000%	1.172%	7.182%	16.867%	0.000%
12.683%	1.9016%	78.059%	0.000%	0.000%	0.000%	1.271%	5.228%	15.443%	0.000%
12.951%	1.8144%	81.250%	0.080%	0.000%	0.000%	1.383%	3.281%	14.006%	0.000%
13.219%	1.7404%	83.787%	0.836%	0.000%	0.000%	1.604%	0.886%	12.888%	0.000%
13.488%	1.6805%	85.702%	1.080%	0.000%	1.061%	1.469%	0.000%	10.688%	0.000%
13.758%	1.6366%	87.343%	1.054%	0.073%	2.485%	1.159%	0.000%	7.886%	0.000%
14.029%	1.6090%	88.793%	0.687%	0.781%	3.504%	0.655%	0.000%	5.581%	0.000%
14.300%	1.5977%	90.261%	0.316%	1.496%	4.501%	0.143%	0.000%	3.283%	0.000%
14.571%	1.6032%	91.587%	0.115%	2.102%	5.385%	0.000%	0.000%	0.810%	0.000%
14.843%	1.6421%	90.864%	0.000%	5.199%	3.937%	0.000%	0.000%	0.000%	0.000%
15.116%	1.7596%	89.147%	0.000%	9.493%	1.360%	0.000%	0.000%	0.000%	0.000%
15.389%	1.9460%	86.302%	0.000%	13.698%	0.000%	0.000%	0.000%	0.000%	0.000%
15.665%	2.1925%	82.177%	0.000%	17.823%	0.000%	0.000%	0.000%	0.000%	0.000%
15.939%	2.4805%	78.072%	0.000%	21.928%	0.000%	0.000%	0.000%	0.000%	0.000%
16.214%	2.7980%	73.967%	0.000%	26.033%	0.000%	0.000%	0.000%	0.000%	0.000%
16.489%	3.1343%	69.883%	0.000%	30.117%	0.000%	0.000%	0.000%	0.000%	0.000%
16.767%	3.4886%	65.758%	0.000%	34.242%	0.000%	0.000%	0.000%	0.000%	0.000%
17.044%	3.8518%	61.653%	0.000%	38.347%	0.000%	0.000%	0.000%	0.000%	0.000%
17.321%	4.2228%	57.548%	0.000%	42.452%	0.000%	0.000%	0.000%	0.000%	0.000%
17.599%	4.5997%	53.443%	0.000%	46.557%	0.000%	0.000%	0.000%	0.000%	0.000%
17.878%	4.9813%	49.339%	0.000%	50.661%	0.000%	0.000%	0.000%	0.000%	0.000%
18.156%	5.3645%	45.254%	0.000%	54.746%	0.000%	0.000%	0.000%	0.000%	0.000%
18.437%	5.7545%	41.129%	0.000%	58.871%	0.000%	0.000%	0.000%	0.000%	0.000%

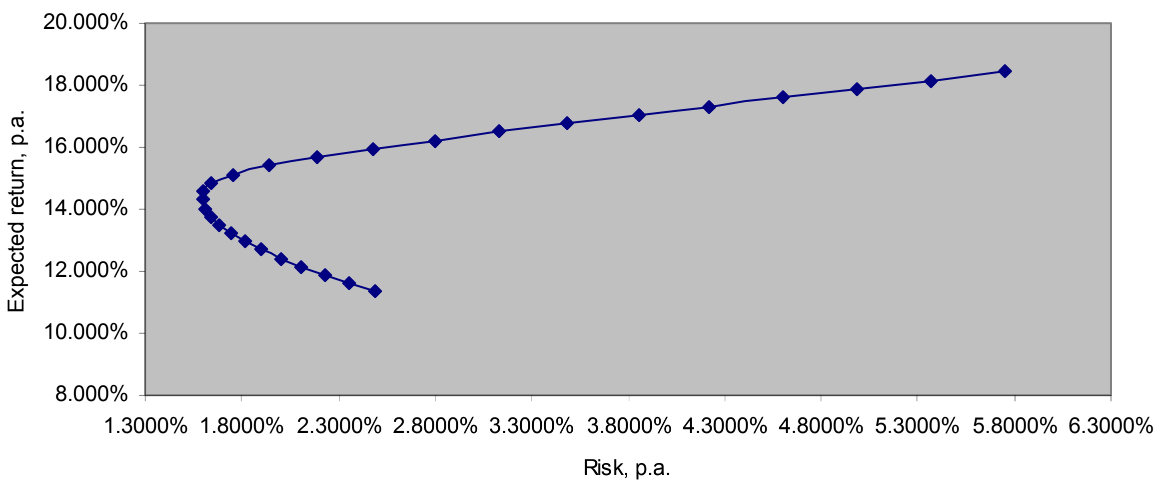
2. A frontier of investment opportunities set without short sales

At now we will approximate a frontier of investment opportunity set that contains such portfolios of assets which ensure to achieve required expected return with as low risk as possible. The difference in comparison with approach described in the previous part is that weights of assets must be nonnegative. Optimal portfolios are described in the Table 6 that has the same structure as Table 4 and corresponding frontier is illustrated at the Picture 6. The Table 7 contains three selected efficient portfolios together with their probabilistic characteristics.

Table 7: Selected efficient portfolios and their probabilistic characteristics

Portfolio		I	II	III
Expected return, p.a.		14.300%	16.489%	18.437%
Risk, p.a.		1.598%	3.134%	5.755%
Minimum return (5% level of confidence)		11.672%	11.333%	8.972%
Probability of nonpositive return		0.000%	0.000%	0.068%
Probability for return less than 10%		0.356%	1.921%	7.130%
Probabilty for return higher than 20%		0.018%	13.131%	39.298%
Money market	BRIBID3M	90.261%	65.758%	41.129%
	PRIBOR3MSK	0.316%	0.000%	0.000%
	WIBO3MSK	1.496%	34.242%	58.871%
	BUBORO3MSK	4.501%	0.000%	0.000%
	SBWMUD3USK	0.143%	0.000%	0.000%
	SBWMEU3LSK	0.000%	0.000%	0.000%
	SF00003MSK	3.283%	0.000%	0.000%
	BP0003MSK	0.000%	0.000%	0.000%

Picture 6: Money market portfolio frontier without short sales



3. Appendix

Analyses described are based on solving quadratic programming problems that look for such portfolios where defined portfolio expected return E_P is achieved with as low risk as possible measured with variance of portfolio returns. The problem can be written in the form

$$\min \quad \mathbf{w}^T \mathbf{C} \mathbf{w}$$

subject to: $\mathbf{E}^T\mathbf{w} = E_P$

$\mathbf{e}^T\mathbf{w} = 1$

$\mathbf{w}^l \le \mathbf{w} \le \mathbf{w}^u$

where: $\mathbf{C}$ - covvariance matrix $n \times n$, where n is number of assets,

$\mathbf{w}$ **-** vector of assets weights,

$\mathbf{E}$ - vector of assets expected returns ,

E_P - portfolio expected return,

$\mathbf{w}^l$ - vector of lower bounds on weights,

$\mathbf{w}^u$ - vector of upper bounds of weights,

$\mathbf{e}$ - vector which all elements equals 1.

The described analysis required to solve series of optimisation problems and for an effective realisation in Excel we have created user functions for computation of portfolio expected return (*PortfolioReturn*) and portfolio variance of returns (*PortfolioVariance*) and VBA procedure (*MM_frontier*) that automatizes formulation and solving of a sequence problems with solver. The VBA procedure and user function have the following structure:

a) procedure for approximation of investment opportunity set frontier

```
Sub MM_frontier()
' ' Start: Ctrl+Shift+A
' Contents of excel ranges:
'       ovciel' – cell where expected return is automatically put down
'       ov_ciel'dh – cell with lower bound for expected return
'       váhy – range where optimal weighs are automatically put down
'       váhy_dh – range with lower bounds on weights
'       váhy_hh – range with upper bounds on weights
'       suma – the cell that contain the formula for sum of range „váhy“
'       ov – cell that ask for function PortfolioReturn
'       so – cell that ask for function PortfolioVariance
'       ad – cell that define step change for an increase of return
'       ciel' – beginning of the range where required returns are archived
'       výnos – beginning of the range where computed returns are archived
'       riziko – beginning of the range where computed risk are archived
'       optváhy – beginning of the range where optimal weights are archived
```

```
Range("ovciel’").Value = Range("ov_cieľdh").Value
SolverReset
Call SolverAdd(Range("váhy"), 3, Range("váhy_dh"))
Call SolverAdd(Range("váhy"), 1, Range("váhy_hh"))
Call SolverAdd(Range("suma"), 2, 1)
Call SolverAdd(Range("ov"), 2, Range("ovciel’"))
Call SolverOk(Range("so"), 2, 0, Range("váhy"))
n = (Range("ov_cieľhh").Value - Range("ov_cieľdh").Value) / Range("ad").Value
ciel’ = Range("ovciel’").Value
ad = Range("ad").Value
For i = 0 To n
Call SolverSolve(True)
Range("ovciel’").Copy
Range("ciel’").Offset(i, 0).PasteSpecial Paste:=xlValues
Application.CutCopyMode = False
Range("ov").Copy
Range("výnos").Offset(i, 0).PasteSpecial Paste:=xlValues
Application.CutCopyMode = False
Range("so").Copy
Range("riziko").Offset(i, 0).PasteSpecial Paste:=xlValues
Application.CutCopyMode = False
Range("váhy").Copy
Range("optváhy").Offset(i, 0).PasteSpecial Paste:=xlValues
Application.CutCopyMode = False
ciel’ = ciel’ + ad
Range("ovciel’").Value = ciel’
Call SolverChange(Range("ov"), 2, Range("ovciel’"))
Next i
SolverFinish
End Sub
```

b) user function for variance computation

```
Function PortfolioVariance(wtsvec, vcvmat)
''        wtsvec – range of the weights
```

```
'          vcvmat – range with covariance matrix
'   Dim v1 As Variant
    If wtsvec.Columns.Count > wtsvec.Rows.Count Then
      wtsvec = Application.Transpose(wtsvec)
    End If
    v1 = Application.MMult(vcvmat, wtsvec)
    PortfolioVariance = Application.SumProduct(v1, wtsvec)
End Function
```

c) user function for portfolio return computation

```
Function PortfolioReturn(retvec, wtsvec)
''         wtsvec - range with weigths of assets
'          retvec – range with expected returns of assets
'If Application.Count(retvec) = Application.Count(wtsvec) Then
   If retvec.Columns.Count > retvec.Rows.Count Then
     retvec = Application.Transpose(retvec)
   End If
   If wtsvec.Columns.Count > wtsvec.Rows.Count Then
     wtsvec = Application.Transpose(wtsvec)
   End If
   PortfolioReturn = Application.SumProduct(retvec, wtsvec)
Else
   PortfolioReturn = -1
End If
End Function
```

References

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Mlynarovič Vladimír, doc., ing., CSc.

Ekonomická univerzita Bratislava, e-mail: vladimir.mlynarovic@sympatia.sk

Kolárik Richard, ing.

Sympatia Financie, o.c.p., a.s., e-mail: richard.kolarik@sympatia.sk

GAMES IN CODE FORM VERSUS GAMES IN EXTENSIVE FORM

CRISTINA PEIXOTO, MANUEL ALBERTO M. FERREIRA
UNIDE/ISCTE, PORTUGAL

1.Introduction

The idea of a new game representation occurred when we analysed a game with 3 players in the normal form. It seemed that a representation that gave us a global picture of the game would be the ideal. As a consequence of this idea, we were tried a game representation that agglutinated the information of extensive form and of normal form. We imagined the codified representation and we verified that this representation has great advantages when the number of players is greater or equal than 3, when the game is an imperfect information one and mainly when it is sequential. We present the code form comparing that representation with the extensive representation.

2.Extensive

The extensive form contains all the information about a game, defining all the moves and when they are performed, what each player knows when he moves, which moves are available to him and where each move leads.

Definition *1* - A game can be described using a tree. This is called the Extensive Form of the game.

The set of players includes the agents taking part in the game. However, in many games there is a place for chance. More concretely, it is necessary to consider the chance that has an uncertainty on some relevant fact. To represent these possibilities we introduce a fictional player: the nature. In terminal nodes do not exist payoff for the nature, and every time a node is allocated to nature a probability distribution over the branches that follow needs to be specified.

Definition *2* - Information set is a collection of nodes $\{m_1,...,m_k\}$, such that: The same player i is to move at each of these nodes; The same moves are available at each of these nodes.

Definition *3* - A strategy of a player i is a list of contingent plans; that is a description of which action he takes at each information set where he is to move.

After some considerations about extensive form we present the aim of this work: a new game representation – the code form. We will introduce this new representation step by step. That is we construct the model on the basis of games with different features and comparing it with the extensive form.

3.Perfect Information

In this section we analyse the dynamic games with perfect information. We have a game of perfect information if when a player has to move he knows everything that happened in the game up to that point: he knows all other players' moves up to that point.

Let there be a game with 3 players with perfect information where there are two strategies for each player: $"D"$ and $"E"$.

The roles of the game are:

- If any player chooses $"E"$ the payoff vector is $(0,0,0)$; If only one player chooses $"D"$, this player obtains a payoff 1 and each of others obtain $-\frac{1}{2}$; If only two players choose $"D"$, then each of them obtain $\frac{1}{2}$ and the other player who chooses $"E"$ obtains -1; If all players choose $"D"$ everyone obtains 0; Order of play is: first player 1, second player 2 and third player 3.

Extensive Form

In figure 1 we model the above situation as an extensive form game.

The labeled nodes, $m_1,...,m_{15}$, represent the various states of the game. In this figure the node labeled 1, m_1, indicate that player 1 makes the first move. The two lines branching from this node indicate that, at this node, player 1 has two choices, $"D"$ and $"E"$. Each of these lines leads to nodes labeled 2, nodes m_2 and m_3, which indicates that after player 1's action it is player 2 turn to play. There are two lines branching from each of the nodes labeled 2. This indicates that in each of the nodes player 2 has two choices, $"D"$ and $"E"$. Each of these actions leads to nodes labeled 3 (nodes m_4, m_5, m_4, and m_7). Hence at these nodes it is player 3 turn to move. The nodes $m_8,...,m_{15}$, are terminal nodes of the game. These nodes represent payoffs of the game.

To find the game equilibrium we use the backward induction method. "The concept of backward induction corresponds to the assumption, that is common knowledge, that each player will act rationally at each node were he moves, even if his rationality would imply that such a node will not be reached".

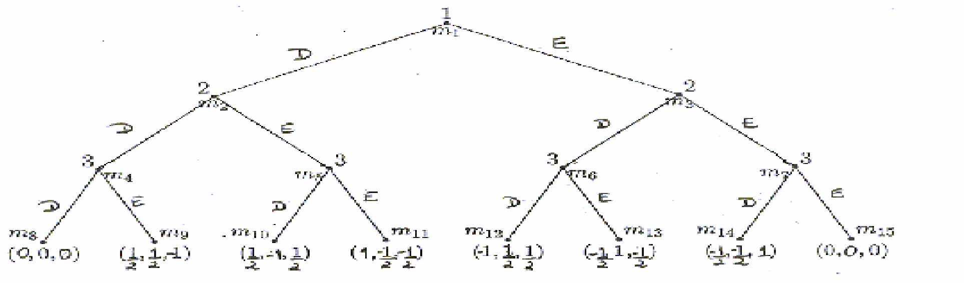

Figure 1 - Perfect information, three players – Extensive Form

In our game we have: in each of the final action nodes of this game player 3 has to take an action. Player 3 chooses an action that maximizes his payoff. Since agent 3 does not have any other node at which he must take an action we conclude that his strategy found by backward induction is $s_3^* = ("D","D","D","D")$. Assigning the payoff vector associated with this move that gives this player the highest payoff to the node at hand, we delete all the moves stamming from this node so that we have a shorter game, where our node is a terminal node. We repeat the process to player 2. In all the final action nodes of this game agent 2 must choose an action that maximizes his payoff. Hence the strategy of agent 2 obtained through backward induction is $s_2^* = ("D","D")$. The reduced game obtained by replacing the action nodes of agent 2 with a terminal node and associating it with the payoff profile resulting from the optimal action of agent 2 leads us to another reduced game. In this final reduced game we see that agent 1 optimal action is to choose $"D"$. Hence the strategy of agent 1 obtained through backward induction is $s_1^* = ("D")$. As a result we have one strategy profile that is obtained through backward induction: (s_1^*, s_2^*, s_3^*). Hence the play induced by these strategy profiles is: first agent 1 chooses $"D"$ then agent 2 chooses $"D"$ and finally agent 3 chooses $"D"$. As a result the game solution corresponds to the payoffs vector $(0,0,0)$.

Code Form

Figure 2 shows the code form representing the above game.

Code form game idea seats in the game estimated linear reading. We build a table that contains the whole game information. How to organize the information?

Reading from left to the right, the first table column indicates the move number. If the players move simultaneously, this column is filled with the code $"S"$. The following columns mention each one of the players. Last column indicates the payoffs vector in accordance with the strategies chosen by the players. This column identifies the player number and the strategy chosen for it. The order of the players is arbitrary when the moves are simultaneous. If there is a blank field it means that information is the same than in the above field.

1	(1,“D”)			
2		(2,”D”)		
3			(3,”D”)	$(0,0,0)$
			(3,”E”)	$\left(\frac{1}{2},\frac{1}{2},-1\right)$
2		(2,”E”)		
3			(3,”D”)	$\left(\frac{1}{2},-1,\frac{1}{2}\right)$
			(3,”E”)	$\left(1,-\frac{1}{2},-\frac{1}{2}\right)$
1	(1,“E”)			
2		(2,”D”)		
3			(3,”D”)	$\left(-1,\frac{1}{2},\frac{1}{2}\right)$
			(3,”E”)	$\left(-\frac{1}{2},1,-\frac{1}{2}\right)$
2		(2,”E”)		
3			(3,”D”)	$\left(-\frac{1}{2},-\frac{1}{2},1\right)$
			(3,”E”)	$(0,0,0)$

Figure 2 - Perfect information, three players – Code Form

The table is built in accordance with the following algorithm:

Let Γ be a given game with n players. For each $i \in N = \{1,2,\ldots,n\}$, S_i is the set of all strategies that are available to player i.

1. Let j be the starting move of the game
2. Assume player i is the player who has taken an action at j.
3. From S_i select one of the possible actions for player i. Let a_i be this action.
4. If j is a terminal move, select the player number, the action and the payoff vector associated with this move. Change the line and repeat the above steps after step 3. Otherwise, change the line and the column and repeat the above steps, from step 1, with j replaced by j'.
5. If the player who has taken an action at the move j' has no more actions to take select the player who took an action immediately before. Select his column, and repeat the above steps, from step 1, replacing j with the respective move.

6. The process is completed when the player who has taken an action at move j has no more actions to take.

To find the game equilibrium we use the best payoff method. This method has the same philosophy that backward induction method. Mechanically, it is computed as follows:

1. Let i be the player who has taken an action at the terminal move.
2. Consider any move that comes just before terminal move. Select the payoff sets associated with each move.
3. In each of those sets select the best payoff for player i.
4. Delete the column for player i so that we have a shorter game where player i''s move is a terminal move and where player i' is the player who has move just before player i.
5. Repeat the process until the first player is reached.
6. Select the strategies profile induced by the payoff vector obtained.

In our game we have:

Player 3 is the player who takes an action in the last move. He has 4 payoffs sets:

$$\left\{(0,0,0),\left(\frac{1}{2},\frac{1}{2},-1\right)\right\},\ \left\{\left(\frac{1}{2},-1,\frac{1}{2}\right),\left(1,-\frac{1}{2},-\frac{1}{2}\right)\right\},\ \left\{\left(-1,\frac{1}{2},\frac{1}{2}\right),\left(-\frac{1}{2},1,-\frac{1}{2}\right)\right\} \text{ and } \left\{\left(-\frac{1}{2},-\frac{1}{2},1\right),(0,0,0)\right\}.$$

In each of those sets select the best payoff for player 3. They are:

$$(0,0,0),\ \left(\frac{1}{2},-1,\frac{1}{2}\right),\ \left(-1,\frac{1}{2},\frac{1}{2}\right) \text{ and } \left(-\frac{1}{2},-\frac{1}{2},1\right).$$

Now consider the shorter games and repeat the process for player 2. Player 2 is the player who takes an action in the last move for this shorter game. He has 2 payoffs sets:

$$\left\{(0,0,0),\left(\frac{1}{2},-1,\frac{1}{2}\right)\right\} \text{ e } \left\{\left(-1,\frac{1}{2},\frac{1}{2}\right),\left(-\frac{1}{2},-\frac{1}{2},1\right)\right\}.$$

In each of those sets select the best payoff for player 2. They are: $(0,0,0)$ and $\left(-1,\frac{1}{2},\frac{1}{2}\right)$.

Now consider the shortest games and repeat the process for player 1. Player 1 is the player who takes an action in the last move for this shortest game. He has one set of payoffs:

$$\left\{(0,0,0),\left(-1,\frac{1}{2},\frac{1}{2}\right)\right\}.$$

In this set we select the best payoff for player 1. This payoff is $(0,0,0)$. The solution for this game results from all players choosing strategy $"D"$.

4. Imperfect Information

We have a game of imperfect information if when a player has to move he does not know all the choices that other players have made.

Let there be a game with 3 players with imperfect information where there are two strategies for each player: $"a"$ e $"b"$.

Suppose: Player 1 prefers $"a"$ to $"b"$; Player 2 prefers $"b"$ to $"a"$; Player 3 prefers $"a"$ to $"b"$.

The roles of the game are:

- Player 1 chooses and communicates to the other players his decision. Then the other players choose simultaneously. The option with the highest number of votes is chosen.

Extensive Form

Figure 3 gives the above described situation extensive form representation.

The dashed line connecting nodes m_4 and m_5 indicates that player 3 does not know whether the game is at a state represented with m_4 or m_5. That is, player 3 does not know if player 2 chooses $"a"$ or $"b"$. The nodes m_4 and m_5 are not connected to any of the nodes m_4 and m_7 (at which player 3 most vote), because player 3 knows the choice of player 1 when it is his turn to take an action. Similarly, the dashed line connecting the nodes m_4 and m_7 indicates that player 3 does not know whether the game is at a state represented with node m_4 or m_7. We will say that the nodes connected with dashed lines belong to the same information set of a player. When it is a player's turn to play, the player knows the information set that contains the node representing the state of the game, but does not know which of the nodes in the information set it is (unless there is only one node in the set). Player 1 has only one information set: $\{m_1\}$. Player 2 has two information sets: $\{m_2\}$ and $\{m_3\}$. Player 3 has two information sets: $\{m_4,m_5\}$ and $\{m_6,m_7\}$. Thus when it is player 2 turn to move he knows if the game is at a state represented by m_2 or at a state represented by m_3. When it is player 3 turn to move player 3 knows if the game is at a state represent by a node in the set $\{m_4,m_5\}$ or $\{m_6,m_7\}$, but he does not know whether the state is m_4, m_5, m_4, or m_7.

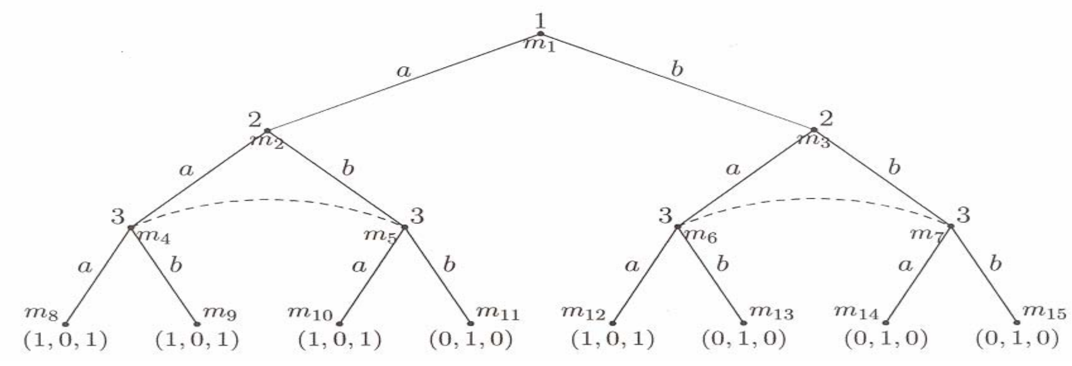

Figure 3 - Imperfect information, three players – Extensive Form

To find game equilibrium we cannot apply the backward induction method because we cannot apply it beyond perfect information games with a finite horizon. Subgame perfect equilibrium is a generalization of the backward induction equilibrium to extensive form games with imperfect information. Our game has three subgames, namely Γ_{m1}, Γ_{m2} e Γ_{m3}. Γ_{m1} is the game Γ and is called the trivial subgame of Γ. The other two subgames Γ_{m2} and Γ_{m3} (shown in Figure 4) are called nontrivial subgames.

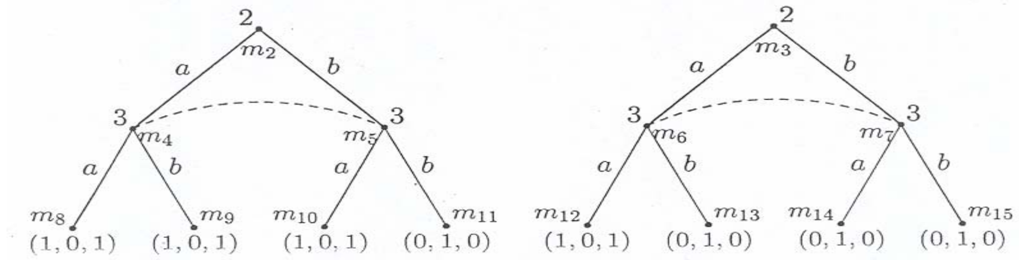

Figure 4 - Subgames – Extensive Form

Definition 4 - A strategic profile s^* in an extensive form game Γ is a subgame perfect equilibrium (SPE) if the restriction of s^* to any subgame of Γ is a subgame equilibrium.

To find the subgame perfect solutions of a game Γ we start with a subgame of Γ which does not contain any other subgames. We find the solutions of this subgame. Then we truncate the original game Γ to a game Γ' by replacing the subgame with a terminal node and associating with it the payoff profile resulting from the play of one of the solutions of the subgame. Then we repeat the same steps with Γ replaced by Γ' until all the subgames are eliminated. Analysing subgames J_{m2} e J_{m3} we verify that $(0,1,0)$ corresponds to J_{m3} subgame perfect equilibrium and $(1,0,1)$ corresponds to J_{m2} subgame perfect equilibrium. Thus the perfect equilibrium in reduced subgame corresponds to payoff vector $(1,0,1)$.

Code Form

The figure 5 represents the game in code form.

1	(1,"a")			
2 - S		(2,"a",1,3)	(3,"a",1,2)	(1,0,1)
			(3,"b",1,2)	(1,0,1)
		(2,"b",1,3)	(3,"a",1,2)	(1,0,1)
			(3,"b",1,2)	(0,1,0)
1	(1,"b")			
2 - S		(2,"a",1,3)	(3,"a",1,2)	(1,0,1)
			(3,"b",1,2)	(0,1,0)
		(2,"b",1,3)	(3,"a",1,2)	(0,1,0)
			(3,"b",1,2)	(0,1,0)

Figure 5 - Imperfect information, three players – Code Form

When we construct this representation we verify that one more code is necessary to indicate that there is imperfect information. In the column of the player who has imperfect information we add code $"I"$ and the player number to whom the imperfect information refers. This code only exists for moves with imperfect information.

The best payoff algorithm used to find the solution, referred above, most likely works for games of this type. Nevertheless, as it has not been completely tested, the SPE was used to find the payoff - $(1,0,1)$.

5. Conclusion

- Although both representations provide a global view of the gameCode form does this more clearly
- Furthermore, code form provides more information about the game than extensive form
- For example, using code form simultaneous moves can be readily observed
- The solution in code form can be computed more linearly
- Because code form solution has not been thoroughly tested, the SPE can be used to find the actual solution
- The solution of a game in the codified form is perfect in subgames
- Considering that the importance of a representation rests on its ease of interpretation, ease of solving a game and truthful information about the game, code form representation is the only one, which fulfils all these requirements.

6. Summary

The aim of this paper is to present and discuss a new representation form games with more than 2 players. This formalization centres the consequences of different strategies without suppressing the meticulousness of the extensive form.

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Maria Cristina Peixoto A. S. R. Matos

Second Triennium Assistant, Campus Politécnico, 3504-510 Viseu, PORTUGAL, Telephone – 232480592, (cristinapeixoto@mat.estv.ipv.pt

Manuel Alberto M. Ferreira

Associated Professor, Av. das Forças Armadas, 1649-026 Lisboa, PORTUGAL, Telephone – 217903046/7, (vice.presidente@iscte.pt)

SOLVING TSP VIA SQL QUERIES

ŠTEFAN PEŠKO, JURAJ HURTÍK
UNIVERSITY OF ŹILINA, SLOVAKIA

Abstract

The possibility of using SQL language for solving the travelling salesman problem is considered. Difficulties and advantages of the approximate and the exact SQL algorithms based on the method of partial sums are discussed. Experiences with small instances are presented.

1. Introduction

The travelling salesman problem (TSP) can be stated clearly and exactly in simple terms: Given an n x n distance matrix $C = (c_{ij})$, find a cyclic permutation $\pi = (\pi(1), \pi(2), \ldots, \pi(n))$ of the set $N = \{1,2,\ldots,n\}$ that minimize the function

$$c(\pi) = \sum_{i \in N} c_{i,\pi(i)} \qquad (1)$$

We note XX the set of integer numbers and suppose without loss of generality that $c_{ij} \in XX$. The set of all cyclic permutation of set N we note $\Pi(n)$.

The TSP is known to be **NP** hard problem. For more information refer to Lawler, Lenstra, Rinnooy Kan and Shomoys [5].

2. Method of partial sums

The theorem of the partial sum which was first proved end applied in Martinec [6], can be used for the enumeration bounds in the branch and bound method used by Peško [2] and Hurtík [1] for solving some **NP-**hard combinatorial problems. We use following simplificated formulation.

Theorem Let $\{d_i\}_{i=1}^{n}$ is finite sequence of real numbers satisfying condition

$$\sum_{i=1}^{n} d_i > 0 \qquad (2)$$

Then there exists index k such that $1 \leq k \leq n$ and

$$\forall_j : k \leq j \leq n \quad \sum_{i=k}^{j} d_i > 0 \qquad (3)$$

$$\forall_j : 1 \le j < k \quad \sum_{i=k}^{n} d_i + \sum_{i=1}^{j} d_i > 0 \qquad (4)$$

Proof. in Peško

As an application of the theorem for solving the minimization problem

$$\pi^* = \arg\min \{c(\pi), \pi \in \prod(N)\},$$

consider two distinct feasible solutions $\pi,\ \phi \in \prod(N)$. We set

$$a_i = c_{i,\phi(i)}, b_i = c_{i,\pi(i)}, d_i = a_i - b_i, \quad \text{for } i \in N$$

The solution ϕ is known and we find a solution π so that

$$c(\phi) - c(\pi) = \sum_{i\in N} a_i - \sum_{i\in N} b_i = \sum_{i\in N} (a_i - b_i) = \sum_{i\in N} d_i > 0$$

If we do not find better π then ϕ is optimal solution. Computer experiments shown that is useful set $a_i = c(\phi)/n$ (i.e. mean weight). Then is not necessarily to known concrete x. It suffices if we know the value of its goal function or some upper bound. In this case it is not necessari to find index k from Theorem. In text SQL implementation we use this strategy.

3. SQL algorithm

The method of the partial sums for the min-sum problem we can simple implement in SQL language as the repeated call of a command SELECT. The key structure in (simple) edge weighted digraph G = (V, E, c) where V = N, $H \subset V \times V, c: H \to \mathbf{R}$ is q/path

$$\mathbf{P}_q = \upsilon_1 \to \upsilon_2 \to \upsilon_3 \to \ldots \to \upsilon_q, (\upsilon_1, \upsilon_2), (\upsilon_2, \upsilon_3), \ldots, (\upsilon_{q-1}, \upsilon_q) \in E$$

weighted by number

$$c(\mathbf{P}_q) = \sum_{r=1}^{q-1} c(\upsilon_r, \upsilon_{r+1}).$$

For every Hamiltonian n-path $\mathbf{P_n} = \upsilon_1 \to \ldots \to \upsilon_n$ we assign Hamiltonian n-cycle $\mathbf{C}_n = \upsilon_1 \to \ldots \to \upsilon_n \to \upsilon_1$.

For SQL queries we use two tables only. The table Edge with three columns:

- Edge.u is first vertex of edge (u, υ),
- Edge.v is last vertex of edge (u, υ),
- Edge.c is cost of edge (u, υ)

and the table Solution with two columns:

- Solution.path is head of q-path,
- Solution.sum is head of partial sum

We can now describe the algorithm which generates all optimal solutions for nonoptimal start $\text{mean} = \sum_{i=1}^{n} a_i / n$ cost (mean weight of the edge in the solution).

```
procedure MinSum (mean)
    for υ ∈ V do (* Initialization*|
        Solution . path [υ]=υ, Solution . sum [υ]= 0
    for q = 1 to n-1 do
        SELECT DISTINCT (*Better q-path*)
            Solution.path → Edge. v AS Solution.path
            Solution.sum + Edge.c AS Solution.sum
            FROM Solution, Edge
            WHERE
                Edge.v ∈ Solution.path AND
                Edge.u = LastVertex (Solution.path) AND
                Solution.sum + Edge.c < mean*q
            TO FILE Solution
        SELECT DISTINCT (*Better n-cycle*)
        Solution.path → Edge.v AS Solution.path
        Solution.sum + Edge.c AS Solution.sum
        FROM Solution, Edge
        WHERE
            Edge.u = LastVertex (Solution.path) AND
            Edge.v = FirstVertex (Solution.path) AND
            Solution.sum + Edge.c < mean*n
            TO FILE Solution
```

For bigger instances it is possible that table Solutions will grows drastically. Smaller table we can generate if change step of the initialization code with

```
    (*Initialization*)
        Solution.path [1] = υ1, Solution.sum [1] = 0
```

Note, that after this initialization we have first Solution table with one row only but in the exact initialization we have the table with n rows.

Then we need to find an index k ($1 \leq k \leq n$) i.e. $\upsilon_1 = k$. When for some k we find a better solution, it is not optimal solution necessarily. But we have better approximation solution with new record cost.

4. Experiences with small instances

Open question is how efficient is presented SQL approach. We made some experiments with instances of small size only. We have not total computer times for solving of the instances. We utilised that SQL enumeration can be interrupted.

We found the optimal solution for instance with n = 26 and 1.05-heuristic solution for instances [7] berlin52 (n=52) and eil76 (n=76). The instances were Euclidean.

The probabilistic analysis of both exact and heuristic method may be very interesting. It would be useful to study the SQL approach for other **NP**-hard min-sum combinatorial problems. the new scheduling version of the TSP called the fastidious traveling salesman problem formulated by Palúch [3] has many innovative applications in transport and can be easy solved via SQL queries for small istances.

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RNDr.Štefan Peško,CSc., University of Žilina, Fakulty of Management Science and Informatics, Dept. of Mathematical Methods, 010 26 Žilina, Slovakia, e-mail.: pesko@frcatel.fri.utc.sk

Ing.Juraj Hortík, University of Žilina, Faculty of Management Science and Informatics, Dept. of Transportation Networks, 010 26 Žilina, Slovakia, e-mail: hurtik@frcatel.fri.utc.sk

AN M/M/N QUEUE MODEL WITH PARTIAL RETURN OF UNSATISFACTORILY SERVED CUSTOMERS

JÚLIUS REBO, ONDREJ BARTL
UNIVERSITY OF ŽILINA, SLOVAKIA

1. Introduction

Analysis of Markov queues usually reposes no-failure operation mode of customers in service point. A theoretic approach to reality claims contemplations respecting impacts of individual servers for the service quality. Thus, some models, as in [3], assume failures (physical harm) of servers that remove any given number of repairmen. Supposing under some models also physical failures of servers, we can entertain incidence of low quality service free of their breakdown. A simple example to observe impacts of failures service to define a probability of successful service of customer and measures of effectiveness derived with it, e.g. in [2], [4].

The paper deal in modelling systems of service supposing such like channels occupancy that any of served customers come to a resolution to return for new completion of service. So we shall establish it in the next part.

2. Modelling of Unsatisfactory

We shall suppose the customers income to the system respecting Poisson distribution with a rate $\lambda > 0$ and with probabilities $\pi_r = \frac{(\lambda t)^r}{r!} e^{-\lambda t}$ to enter exactly *r* customers during time *t*. We shall be assuming too that incoming customers are ordered in common queue respecting their arrival time to the waiting line. Let have $n \geq 1$ service points with the same exponentially distributed rates of service with a density probability function $f(t) = \overline{\mu} e^{-\overline{\mu} t}$, $\overline{\mu} > 0$.

If we are imagining that per time unit is a part of customers passing through the one service point served in a good quality and a part in bad quality, we can divide a total rate of service $\overline{\mu}$ for each service point into the rate of good quality and the rate of bad quality, denoting by $\mu > 0$ res. $\nu > 0$. Moreover, we shall assume the both descriptions of service quality are independent to each other i.e. the quality of being service of customer is not depend on the quality of service of an antecedent customer. So define the total rate of service (jointly a good and a bad quality) as the exponentially distributed value with a parameter $\overline{\mu} = \mu + \nu$. (1)

We claim so as, each of served customers, accordingly customers with bad service quality, after completions go out from the system. Whereas the customer's service is closed but the customer can feel dissatisfaction with it, we shall call that as an unsatisfactory service. Unsatisfactory served customers either leave the system and they not require next service or make a decision at random for returning into the system and realising a new full service.

Technical point of view leads us to the assumption that enter customers from outside as a customers turning back exploit only one entry point through pass in singles. Denote as $p > 0$ a probability of unsatisfactory served customer's return into the system (1-p be a probability of unsatisfactory served customer who is made a decision to leave the system). We know, every service point will unsatisfactory serve ν customers per time unit and so number of returns into the system will form Poisson flow with intensity $p\nu$ customers per unit of time.

Joint work independence of service points leads to the fact that *k* service points will produce Poisson flow unsatisfactory served customers with intensity $kp\nu, k = 1,...,n$.

We can express the return intensity of customers σ from the one service point by the stand-alone expression as $\sigma = p\nu$ or beyond the satisfactory service rate as $\sigma = pb\mu, 0 \le b < 1$. Return intensity produces by *k* service points will express as $k\sigma, k = 1,...,n$.

3. M/M/n Model with an Infinite Queue

Consider now the queue with $n \ge 1$ service points and in aggregate $m \ge 1$ places in the system and a waiting line, which holds forgoing assumptions of probability distribution of entering customers and their services. To common solving point will take a general birth-death process.

If we consider the states of the system as a number of customers in the system we can express transition probabilities during arbitrary small time $dt > 0$ as:

From *k* to *k+1*: $(\lambda + k\sigma)dt, k = 0,1,...,n-1$, $\quad (\lambda + n\sigma)dt, k = n, n+1, n+2,...$

From *k* to *k-1*: $k\bar{\mu}dt = k(\mu + \nu), k = 1,2,...,n$, $\quad n\bar{\mu}dt = n(\mu + \nu), k = n+1, n+2,...$

From *k* to *k*: $1 - \lambda dt, k = 0$, $1 - (\lambda + k\sigma + k\bar{\mu})dt, k = 1,2,...,n-1$

$1 - (\lambda + n\sigma + n\bar{\mu})dt, k = n, n+1, n+2,...$

The expressions of states probabilities aided the general birth-death process after substituting $\alpha_k = \frac{\lambda + k\sigma}{\mu + \nu}, \rho = \frac{\lambda + n\sigma}{n(\mu + \nu)}$ yields:

$$p_k = \frac{\prod_{i=0}^{k-1}(\lambda + i\sigma)}{k!(\mu + \nu)^k} p_0 = \frac{\prod_{i=0}^{k-1}\alpha_i}{k!} p_0, k = 1,2,...,n,$$

$$p_k = \frac{(\lambda + n\sigma)^{k-n} \prod_{i=0}^{n-1} (\lambda + i\sigma)}{n^{k-n} . n! (\mu + \nu)^k} p_0 = \frac{\prod_{i=0}^{n-1} \alpha_i}{n!} \rho^{k-n} p_0, k = n+1, n+2, ...$$

$$p_0 = \left[1 + \sum_{k=1}^{n} \frac{\prod_{i=0}^{k-1} \alpha_i}{k!} + \frac{n^n}{n!} \frac{\prod_{i=0}^{n-1} (\lambda + i\sigma)}{(\lambda + n\sigma)^n} \sum_{k=n+1}^{\infty} \rho^k \right]^{-1} = \left[1 + \sum_{k=1}^{n-1} \frac{1}{k!} \prod_{i=0}^{k-1} \alpha_i + \frac{1}{n!} \frac{1}{1-\rho} \prod_{i=0}^{n-1} \alpha_i \right]^{-1}. \quad (2)$$

A convergence condition of infinite series with quotient ρ within expression for p_0 gives necessary and sufficient condition of the system stability in the form $\rho = \frac{\lambda + n\sigma}{n(\mu + \nu)} < 1$. (3)

Measurements of system effectiveness will be study in the 4th section.

3.1 Single Server Model

Simple modifications of the above model for a number of service points $n = 1$ allow to us to derive states probabilities when $\alpha_i = \rho = \frac{\lambda + \sigma}{\mu + \nu} < 1$, $\alpha_0 = \frac{\lambda}{\mu + \nu}$.

Then we get: $p_0 = \left[1 + \alpha_0 + \sum_{k=2}^{\infty} \alpha_0 \rho^{k-1} \right]^{-1} = \frac{1-\rho}{1-\rho+\alpha_0}$,

$$p_k = \alpha_0 \rho^{k-1} p_0 = \frac{\alpha_0 (1-\rho) \rho^{k-1}}{1-\rho+\alpha_0}, k = 1,2,.... \quad (4)$$

4. The Systgem Measurements

Many measures of effectiveness for the above prescribed model with unsatisfactory served customers can derived by similar techniques than model wherein is considered only successful service of customers (called basic model), e.g. [1].

The mean value of busy channels of service we shall derive likewise for the basic model

$$E(S) = \sum_{k=0}^{n} kP\{S = k\} = \sum_{k=0}^{n-1} kp_k + n\sum_{k=n}^{\infty} p_k = \sum_{k=1}^{n-1} k \frac{1}{k!} \prod_{i=0}^{k-1} \alpha_i p_0 + n\sum_{k=n}^{\infty} \rho^{k-n} p_n = \sum_{k=1}^{n-1} \frac{1}{(k-1)!} \prod_{i=0}^{k-1} \alpha_i p_0 + np_n \frac{1}{1-\rho}.$$

Considering a steady-state mode of operation are busy channels proportional divided to service points occupied by satisfactory or unsatisfactory services. Let us $E(S_s)$, $E(S_u)$ denote the mean values of busy service points with satisfactory res. unsatisfactory served customers then we can express

$$E(S_s) = \sum_{k=1}^{n-1} k \frac{\mu}{\mu + \nu} p_k + n\sum_{k=n}^{\infty} \frac{\mu}{\mu + \nu} p_k = \frac{\mu}{\mu + \nu} \left[\sum_{k=1}^{n-1} \frac{1}{k!} \prod_{i=0}^{k-1} \alpha_i p_0 + n\sum_{k=n}^{\infty} \rho^{k-n} p_n \right] = \frac{\mu}{\mu + \nu} E(S),$$

$$E(S_u) = \frac{\nu}{\mu + \nu} E(S) \text{ and } E(S) = E(S_u) + E(S_n).$$

Since customers who enter from outside of the system or return for new service generate the mean value of waiting customers for service in queue $E(L)$ without reference to their next quality of service, we have done

$$E(L)=\sum_{l=0}^{\infty} lP\{L=l\}=\sum_{l=n}^{\infty}(l-n)p_l=\sum_{l=n+1}^{\infty}(l-n)\rho^{l-n}p_n=p_n\sum_{l=n+1}^{\infty}(l-n)\rho^{l-n}=\frac{\rho}{(1-\rho)^2}p_n=\frac{1}{n!}\prod_{i=0}^{n-1}\alpha_i\frac{\rho}{(1-\rho)^2}p_0 .$$

Let T_L denote a random variable of time spent in the waiting line per one customer and let $W_L(t)=P\{T_L\le t\}$ is a distribution function of random variable T_L. For $t=0$ we get

$$W_L(0)=P\{T_L\le 0\}=P\{T_L=0\}=P\{n-1 \textit{ or less in the system}\}=$$
$$=\sum_{k=0}^{n-1}p_k=p_0\left(1+\sum_{k=1}^{n-1}\frac{1}{k!}\prod_{i=0}^{k-1}\alpha_i\right).$$

From (2) for p_0 yields

$$\frac{1}{p_0}=1+\sum_{k=1}^{n-1}\frac{1}{k!}\prod_{i=0}^{k-1}\alpha_i+\frac{1}{n!(1-\rho)}\prod_{i=0}^{n-1}\alpha_i \Rightarrow \sum_{k=1}^{n-1}\frac{1}{k!}\prod_{i=0}^{k-1}\alpha_i=\frac{1}{p_0}-\left(1+\frac{1}{n!(1-\rho)}\prod_{i=0}^{n-1}\alpha_i\right),$$

substituting this to the previous expression and setting we have

$$W_L(0)=1-\frac{p_0}{n!(1-\rho)}\prod_{i=0}^{n-1}\alpha_i . \tag{5}$$

For $T_L>0$, then,

$$W_L(t)=P\{T_L\le t\}=\sum_{k=n}^{\infty}\left[P\{k-n+1 \textit{ completions in} \le t/ \textit{ arrival found } k \textit{ in system}\}p_k\right]+W_L(0).$$

Now when $k\ge n$, the system average output is Poisson with a mean $n(\mu+\nu)$, so that the time between successive completions is exponential with mean $\frac{1}{n(\mu+\nu)}$, and the distribution of the time for $k-n+1$ completions is Erlang type $(k-n+1)$. Thus we can write

$$W_L(t)=p_0\sum_{k=n}^{\infty}\frac{1}{n!}\prod_{i=0}^{n-1}\alpha_i\rho^{k-n}\int_0^{\infty}\frac{n(\mu+\nu)[n(\mu+\nu)x]^{k-n}}{(k-n)!}e^{-n(\mu+\nu)x}dx+W_L(0)=$$
$$=\frac{p_0}{n!}\prod_{i=0}^{n-1}\alpha_i\int_0^{\infty}n(\mu+\nu)e^{-n(\mu+\nu)x}\sum_{k=n}^{\infty}\frac{[n\rho(\mu+\nu)x]^{k-n}}{(k-n)!}dx+W_L(0)=\frac{p_0}{n!}\prod_{i=0}^{n-1}\alpha_i\int_0^{\infty}n(\mu+\nu)e^{-n(\mu+\nu)x}e^{(\lambda+n\sigma)}dx+W_L(0)=$$
$$=\frac{p_0}{(n-1)!}\prod_{i=0}^{n-1}\alpha_i\frac{(\mu+\nu)\left(1-e^{-[n(\mu+\nu)-(\lambda+n\sigma)t]}\right)}{n(\mu+\nu)-(\lambda+n\sigma)}+W_L(0)=\frac{p_0}{n!}\prod_{i=0}^{n-1}\alpha_i\frac{1-e^{-n(\mu+\nu)(1-\rho)t}}{1-\rho}+W_L(0). \tag{6}$$

Summarizing (5) and (6) we get

$$W_L(t)=\begin{cases}1-\dfrac{p_0}{n!(1-\rho)}\displaystyle\prod_{i=0}^{n-1}\alpha_i, & t=0\\ \dfrac{p_0}{n!}\displaystyle\prod_{i=0}^{n-1}\alpha_i\dfrac{1-e^{-n(\mu+\nu)(1-\rho)t}}{1-\rho}+W_L(0), & t>0\end{cases} \tag{7}$$

To obtain the expected waiting-time measure in queue W_L we must find the mean of random variable of T_L according to

$$W_L=E(T_L)=\int_0^{\infty}tdW(t)=\int_0^{\infty}t\left[\frac{1}{n!}\prod_{i=0}^{n-1}\alpha_i\frac{p_0}{1-\rho}\left(n(\mu+\nu)(1-\rho)e^{-n(\mu+\nu)(1-\rho)t}\right)\right]dt=$$
$$=\frac{p_0}{n!(1-\rho)}\prod_{i=0}^{n-1}\alpha_i\int_0^{\infty}n(\mu+\nu)(1-\rho)te^{-n(\mu+\nu)(1-\rho)t}dt=\frac{\prod_{i=0}^{n-1}\alpha_i}{n!}\frac{p_0}{n(\mu+\nu)(1-\rho)^2}. \tag{8}$$

A following table includes different samples of several parameters of the system and shows their impact to the mean-values (by previous) references. Specifically is considered the system $M/M/3/\infty$ for incoming rate $\lambda = 2c/h$, the total intensity of service $\overline{\mu} = \mu + \nu = 2.2c/h$ and values of unsatisfactory service rates ν in the table are represented together with the return probability *p*.

$M/M/3/\infty$	$\nu = 0$	$\nu = 0.2, p = 0.1$	$\nu = 0.2, p = 0.5$
ρ	0.303	0.312	0.348
p_0	0.3997	0.3978	0.3899
$E(S)$	0.909	0.917	0.952
$E(L)$	0.031	0.034	0.046
W_L	0.0156	0.0164	0.0201
$E(S_s)/E(S_u)$	0.909/0	0.834/0.083	0.866/0.087
	$\nu = 0.5, p = 0$	$\nu = 0.5, p = 0.1$	$\nu = 0.5, p = 0.5$
ρ	0.303	0.326	0.417
p_0	0.3997	0.3949	0.3737
$E(S)$	0.909	0.930	1.026
$E(L)$	0.031	0.038	0.081
W_L	0.0156	0.0177	0.0293
$E(S_s)/E(S_u)$	0.702/0.207	0.719/0.211	0.792/0.233

5. Conclusions

It is clear to see, from the previous consecution, characteristic mean values of the system are substantially influencing by the individual values of the service and the return of customers to the system under the unsatisfactory service. We see, if we are setting $\nu = 0$ i.d. suppose the service with non-availability, the total rate of service $\overline{\mu}$ will equal to the rate of good service $\mu = \overline{\mu}$ and we shall done results for a so-called basic model as in [1]. If we are disposing $\nu \neq 0$, $p = 0$ we can actually model specific case of systems with unsatisfactorily served customers without returning as in [5], when total rates of service for all service points are equal.

If we are respecting no return with unsatisfactorily served customers into the system it will more easily deal only a satisfactory service and use a successful rate of service for evaluations of the system effectiveness. From this point of view will be sufficient to define a probability of successful service (good service quality) and the probability $1-p$ declares that the service will no pass (ever or successful).

If we are denoting μ as the total rate of service and $\overline{\mu}$ as a rate of successful service, then we will have $\overline{\mu} = \mu - (1-p)\mu = p\mu$. That is the way to the models with the successful service as in [2].

Likewise considering, but for different rates of service in every service point, enables to us to define the successful service of *i*th service point as $\overline{\mu}_i = \mu_i - (1-p)\mu_i = p\mu_i$, according to $[4]$. Furthermore we have a possibility to seak an optimal rate of service for each service point respecting operating costs of the system.

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RNDr. Július Rebo, PhD.

DP Fakulty riadenia a informatiky, Žilinská univerzita, Ul. Bakalárov 2 971, 01 Prievidza e-mail: rebo@utcpd.sk

Ing. Ondrej Bartl, PhD.

Fakulta riadenia a informatiky, Žilinská univerzita, Veľký diel, 010 26 Žilina, e-mail: bartl@kst.utc.sk

IMPLEMENTATION OF MULTI-PERIODICAL OPTIMIZING MODELS AND GOAL PROGRAMMING FOR THE IDENTIFICATION OF OPTIMAL INVESTMENT STRATEGY

JOZEF REPISKÝ, ARTAN QINETI, PAVEL CIAIAN
SLOVAK AGRICULTURE UNIVERISTY NITRA, SLOVAKIA

Abstract

The development strategy of the majority of firms is given by their decisions in the area of asset investments, which then determines the future growth of the firm. The optimal investment strategy of breeding rabbits is possible to be identified by using multi-periodic optimisation model. The model is constructed in spreadsheet program Excel and solved with configuration utility solver. The strategic goal is represented by an objective function, which maximises the assets of the enterprise. The value of the assets is represented by the indicator net present value of the cash flows obtained during the lifetime of the investment project. There are several sets of constraints in the model, such as balancing the financial resources. Credit restriction and the liquidity of the enterprise have to be ensured for each period – i.e. a positive cash flow and sufficient financial resources have to be provided to cover capital investment and operating costs. The results of the model give the optimal size of the investment during each particular period and in the same time they give the possibility to select an optimal way of financing the investment.

The compromising solution is obtained by applying goal programming. The two goals between which the compromise is made are: 1. a minimum level of net present value obtained during the lifetime of the investment and 2. not overdrawing the maximum credit available.

1. Introduction

Investment decision making is an impartial component of strategic decision making process. Its implication becomes apparent after a certain period of time and from the short-term perspective they can have negative financial effects. A remarkable improvement in this process, especially for the purposes of the investment projects construction and evaluation, is the construction of different models. Supporting instruments of the decision process can be taken into account like for example scenarios, optimisation and simulation models or models of decision analysis and those of complex evaluation of variants. Implementation of multi-periodical optimising models followed by goal programming application is a convenient procedure for the identification of the optimal strategy of broiler rabbits breeding. Rabbits breeding is not constrained by climatic conditions, what classifies it among attractive and technologically not very complicated investment activities.

In the project it is projected the breeding hall K-90 reconstruction, adjusted for manure hole and concentration as well as ventilation shafts. In this way, there are provided isolated breeding capacities with ideal climatic conditions - temperature, relative humidity, air ventilation, light intensity etc. Cage modulation enables for a flexible increase in the number of breeding rabbits. The core input production parameters are based on such assumptions that, breeding material can be in the reproduction process no longer than three years, with number of fertilisation of 11 per year and number of rabbits 8 per case. The four-line hybrid is projected to be fed, up to a weight of 2,58 kg with average daily weight increases of 0,03 kg and a normative consumption of granulated mixture of 4 kg per kg of added weight.

Economic parameters of the model are derived from the current prices of investment costs (module for female-rabbits with a capacity of 10 s at a price of 10 440 Sk, for male-rabbits with a capacity of 10 heads at a price of 11 268 Sk and for young rabbits with a capacity of 72 heads at a price of 7 668 Sk), from the prices of production inputs (breeding material – 680 Sk per head, foodstuff – 650 Sk, respectively 660 Sk per 100 kg, different types of medicaments etc.), as well as from output prices (meet – 62 Sk per kg, manure – 15 Sk per 100 kg). The borrowing cost of the loan interest rates is estimated at 15%.

2. The construction of the multi-periodic optimisation model

The multi-periodic optimisation model applied in this paper consists of the following parts:

- Costs estimation: investment costs, variable costs and fixed costs.
- Production estimation
- Amortisation
- Investment capital requirements, credit, the schedule for credit repayment
- Projected profit-and-loss statement
- Projected cash flow statement
- Projected balance sheet statement

The strategic goal is represented by an objective function, which maximises the entrepreneur's assets. Specifically, it is represented by the net present value (NPV) of all the financial flows incurred during the lifetime of the investment project.

$$NPV = \sum_{t=1}^{n} \frac{CF_t}{(1+i)^t} - I_0$$

where: NPV is the net present value

CF_t is the net flow of revenues in each year during the duration of the investment

t is the time period

n is the duration of the investment

I_0 are initial investment costs

An acceptable investment project, for any entrepreneurial firm, is the project which gives a positive value of expected net present value of future cash flows. This is because all the expenses incurred during the lifetime of the investment project first of all have to cover the incurred costs, secondly, have to provide a normal level of profits (represented by the discount rate) to investor and also should increase the cash flows, which will be reflected in the higher market valuation of the firm. A more detailed description of the net present value methodology can found in Lumby (1996), McLaney (1994) and Repiský (2000).

The key variable in the model is the category female rabbits, from which the numbers of male rabbits and the numbers of rabbits meant for meat production are determined. The values of the other variables are obtained from the production, accounting and other relationships that are relevant in rabbit breeding.

The model parameters are the following:

p^t_1 is the change of the numbers of female rabbits in year t (for t = 1,2 … n)

u^t_q $q \in N_3$ is the set of the methods available for credit repayment

f^t_p $p \in N_4$ is the set of methods available to finance the investment

O^t_{p1} $p1 \in N_5$ is the set of the methods applicable for the amortisation of the purchased cages

b^t_{p2} $p2 \in N_6$ is the set of all the possibilities available to acquire the necessary building

DSU^t is the period during which the credit borrowed at time t has to be repaid.

The model contains 42 functional constraints mainly representing the production limitations and balancing relationship that take into consideration accounting, financial, taxing and other legal requirements. In this paper only the constraints that transform the model from its balancing structure to its optimising structure is provided. A full version of the model can be found in Repiský (2000). These constrains are as follows:

1. The constraint that ensures a positive value of the cumulative cash flows

$KCF^t \geq 0$ where KCF is cumulative cash flow in year t

2. The constraints that balances the coverage of investment and operational capital

$$ZK^t - KN^t - PK^t \geq 0 \qquad for\ t = 1$$

$$ZK^t + KCF^{t-1} - KN^t - PK^t \geq 0 \qquad for\ t = 2,3,...n$$

where:	ZK^t	are sources for financing the investment capital and operational costs
	PK^t	are operational costs
	KN^t	is investment capital
	n	is the duration of the investment

3. The constraint limiting the number of purchased cages

$$\sum_{s \in N_1} k_s^t - CK \leq 0 \qquad \text{for } t = 1$$

$$\sum_{s \in N_1} k_s^t + \sum_{S \in N_1} \sum_{u=1}^{t} k_s^u - CK \leq 0 \qquad \text{for } t = 2,3,...n$$

where:	k_s^t	is the number of cages needed for category s
	$s \in N_1$	is the set of rabbit categories
	CK	is the maximum number of cages, which is given by the capacity of the building

4. The constraint limiting the credit

$$PU^t - LU^t \leq 0$$

where:	PU^t	is the total borrowed credit
	LU^t[22]	is the maximum amount of credit

The objective function maximises the assets of the entrepreneurial firm obtained during the duration of the investment project (12 years). The assets are represented by the NPV.

$$\max \sum_{t=1}^{n} \frac{(CF^t - VZ^t + ZC)}{(1 + DS)^t} \qquad t = 1,2,...n$$

where:	n	is the duration of the investment project
	CF^t[2]	is the cash flow obtained in each year of the investment project duration
	VZ^t	is own capital invested
	DS	is discount rate
	ZC	is the market value of the assets at the end of lifetime of the investment

The exogenous variable, the numbers of female rabbits given in a year with the possibility of full capacity usage for five years, and the exogenous variable, the methods of credit repayment (either with a constant yearly payment of the principal or with a constant yearly payment of the annuity), determine the values of all of the variables in the model.

Table 1 gives the result for the simulations when the purchase of the building is considered, the credit of 1 500 000 Sk is borrowed in the first year of the project and with zero credit allowed to be borrowed afterwards. The simulation were calculated using the configuration utility solver provided under the spreadsheet program Excel

Table 1. The increase of the number of rabbits in each category for the scenario when the credit is available only in the first year (1.5 mill. SK).

Year	1	2	3	4	5
Female rabbits (heads)	95	72	108	213	91
Male rabbits (heads)	14	11	15	30	13
Rabbits for meat (heads)	8313	6293	9540	18736	8038

Table 2 Projected cash flow statement

Year	1	2	3	4
Revenues from sold rabbits	1329741	2336263	3862378	6859518
Revenues from sold manure	14210	33288	55033	97739
Credit	1500000	0	0	0
TOTAL revenues	2843951	2369551	3917411	6957257
Costs of purchasing the building	815675			
Costs of the stall modification	30000			
Costs of constructing the manure hole	140000			
Costs of purchasing technology	317688	240468	364604	716046
Costs of purchasing reproduction herd	70175	53117	80537	228342
Material costs	499593	1119268	1850406	3286290
Wages	95155	167181	276388	490860
Cost of energy	30000	30000	30000	30000
Other costs	1250	1250	1250	1250
Maintenance costs	3176	5581	9227	16388
Principal payments	135541	155872	179253	206140
Interest payments	225000	204669	181288	154401
Income tax	95876	194135	383869	723722
TOTAL costs	2459129	2171541	3356822	5853439
CF	384822	198010	560589	1103818
Cumulative CF	384822	582832	1143421	2247239
NPV	3595180			

For this scenario, the most of the financial recourses borrowed are used for purchasing the building, which is necessary to place the cages in it. The remaining financial resources are able to cover the costs of purchase and cost of breeding only for 95 female rabbits and for corresponding numbers of the other rabbit categories as given in table 1. The credit is considered to be repaid during the 7 years period and in each year a constant payment of annuity is incurred. The associated costs with the rabbit population enlargement (investment costs and variable costs) effectuated in the next period are covered from the own resources (or cash flow) totalling 384 822 Sk (table 2).

The cash flow obtained from the investment allows to increase the numbers of rabbit population in the remaining years of the project, such that the capacity is fully used. Table 2 shows that the NPV is positive equalling to 3 595 180 Sk, which indicates that the project is acceptable and might be considered as a relatively good investment opportunity. Ceteris paribus, when the building is rented instead of bought, the NPV increases to 5 067 590 Sk. Thus, from the economic point of view this second alternative, that of renting the building, is preferable. In this case the credit and the cash flow obtained from the investment make it possible to reach the full capacity use already in the third year of he project.

3. The application of the goal programming

The problem that looks for a compromising solution for two goals taken in consideration- a specified level of NPV and a specified level of credit - is possible to be solved using goal programming (see Hillier, F.S. – Lieberman, G.J. (1990)).

Objective function is given as follows:

$$\min g(y) = \sum_{k=1}^{n} p_k \, y_k^+ + \sum_{k=1}^{n} r_k \, y_k^-$$

The set of constrains are those applied for the multi-periodic programming model plus additional ones specified as follows:

$$\vec{c}_k \vec{x} - y_k^+ + y_k^- = h_k$$

where: y_k^+, y_k^- - is the value representing the over-fulfilment and non-fulfilment of the goal, respectively

h_k - is the targeted value of the *k* criteria function

p_k, r_k -are the penalisation rates for non-fulfilment of the targeted goal

$\vec{c}$ - is the vector valuation of the k goal function

$\vec{x}$ - is the vector of the endogenous variables

One solution includes two goals: the NPV at least at 6 millions Sk and a loan in the first year a maximum of 2 millions Sk and in the following years 500 000 Sk (table 3), while following penalising rates are used: the rate 7 per unit of not reaching the first goal and the rate 6 per unit of not reaching loan goals. This solution fails to reach the first goal by an amount of 21 821 Sk and cross the borrowing limit in the first year by 146 130 Sk. In the second year the amount of 500 000 is borrowed but in the following years borrowing is not necessary. The housing capacity is used completely after the third year and financing is provided not only from loans but as well as from own resources in the form of cumulative cash flow in the given period. The objective function that was to minimise the weighted sum of residuals of each criterions from targeted values reached the minimum value of 1 030 124 from the total amount of penalising points.

Table 3 Numbers of animals – goals: I - NPV 6 millions Sk, II - loans: 1-st year 2 millions and following years 500 000 Sk

YEAR	1	2	3	4	5
Female (heads)	214	303	62		
Male (heads)	31	43	9		
Young (heads)	18782	26597	5541		

In the case where not crossing the borrowing constraints is considered as a priority goal (the penalisation for not reaching the goal consists in multiplying the prohibitive rate for example by 1000 Sk) we get such a solution where priority goal loan would be overcome in the first year by 111 970 Sk while other priority are satisfied (borrowing in the following years) at the expense of secondary goal i.e. NPV is lower by 88 120 Sk. The number of female rabbits increases in the first year to 208 heads, in the second 295 heads and in the third 76 heads. From this number is derived (dependent upon) the number of male rabbits and the number of rabbits for meat production as well as the required number of modules to be purchased for each category.

In the last solution variant the following goals were set: the NPV 6 millions Sk and the borrowing limits in the first year 1,5 millions Sk and in the following years 500 000 Sk (solution outputs are presented in table 4). The NPV in this variant is 4 873 980 Sk that means goal is missed by 1 126 020 Sk. Also financial injection is needed: 1 536 928 Sk in the first year(that would be provided by a loan - the goal value is overcome by 36 928 Sk); 926 981 Sk in the second year and 1 636 313 Sk in the third year. Financial requirements for the second and third year will be provided by loans at 500 000 Sk each and the rest should be accumulated by own financial resources (i.e. cash flow).

Table 4 Number of animals – goals: I - NPV 6 millions Sk, II – loans: 1-st year 1,5 millions Sk and other year 500 000 Sk

YEAR	1	2	3	4	5
Male (heads)	101	173	305		
Female (heads)	15	25	43		
Meat prod.	8911	15186	26823		

4. Conclusion

The implementation of multi-periodical optimising models is a convenient procedure for the identification of the optimal strategy of broiler rabbits breeding. The following application of goal programming provides solution variants. The goals of the model include the value of the property owned by the company that is given in the form of the Net Present Value (NPV) and the limitation of borrowing on the behalf of the company. The multi-periodical optimising model allows for the definition of different factors as structural variables of the model set for optimisation. Similarly, the application of goal programming allows for the definition of different goals that numerical quantification is possible and it helps finding a compromise solution. It depends on the concrete conditions of the decision maker to define which parameters are explicitly given (constraints), which parameters are set to be optimised (structural variables of the model) and for which ones is possible to find a compromise solution (numerically defined goals of goals programming application).

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Repiský Jozef

Department of Statistics and Operations Research, Faculty of management a agriculture, Slovak Agricultural University Nitra, 949 76 Nitra, e-mail: Jozef.Repisky@uniag.sk

Artan Qineti

Faculty of Applied Biological Sciences , Catholic University of Leuven, 3000 Leuven, Belgium, e-mail: Artan.Qineti@agr.kuleuven.ac.be

Pavel Ciaian

Faculty of Economics and Applied Economics, Catholic University of Leuven, 3000 Leuven, Belgium, e-mail: Pavel.Ciaian@econ.kuleuven.ac.be

DATA ENVELOPMENT ANALYSIS (DEA) AND ITS PRACTICAL APPLICATIONS

JIŘÍ ROZKOVEC
UNIVERSITY OF ECONOMICS, PRAGUE, CZECH REPUBLIC

Abstract

This paper disserts on DEA - Data Envelopment Analysis, especially CCR model (by Charnes, Coopers, Rhodes) and its applications for comparison of efficiency of Czech pension funds.

1. Definitions

DEA is concerned with evaluations of performance of organisations such as business firms, government agencies, hospitals, educational institutions etc. In general they are called Decission Making Units (DMU). The elementary term of DEA is efficiency. It is usually stated in the form of a ration like the following,

Efficiency = Inputs/Outputs

Let us apply this formula to the following single example. Suppose there are 8 stores, which we label A to H, in Table 1 there are numbers of employees and sales (measured in 100.000 CZK):

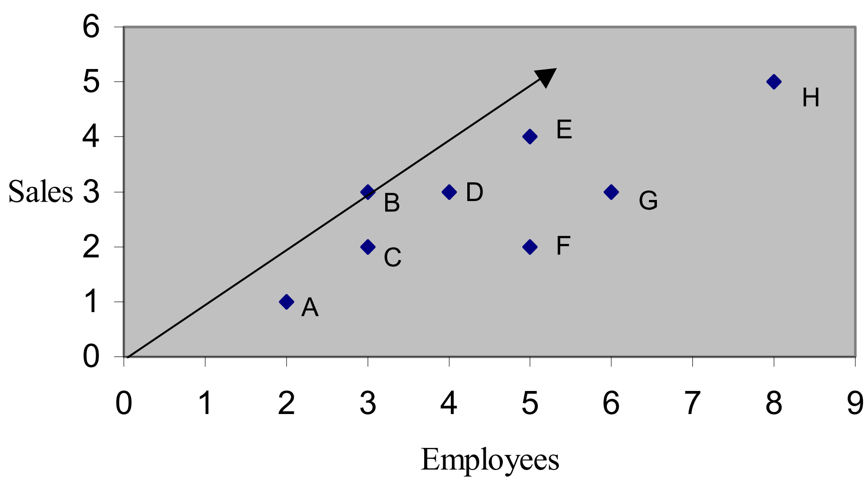
Table 1. – Single Input and Single Output Case

Store	A	B	C	D	E	F	G	H
Employees	2	3	3	4	5	5	6	8
Sales 100.000 CZK	1	3	2	3	4	2	3	5
Sales / Employees	0,500	1,000	0,667	0,750	0,800	0,400	0,500	0,625

The bottom line in Table 1 shows the sales per employee – measure of “productivity”. The highest value we find for store B and, by this measure, we may identify B as the most efficient branch and F as least efficiency.

Let us represent these data as in Figure 1 by plotting “number of employess” on the horizontal and “sales” on the vertical axis. From a geometric point of view, the values form the bottom line of the table mean the slopes of the lines connecting each point (A – H) to the origin and the highest value, of course, is attained by the line from the origin through B. This line is called the “efficient frontier”. Notice that this frontier touches at least one point and all points are therefore on or below this line. The name Data Envelopment Analysis, as used in DEA, comes from this property because in mathematical parlance, such as a frontier is said to "envelop" these points.

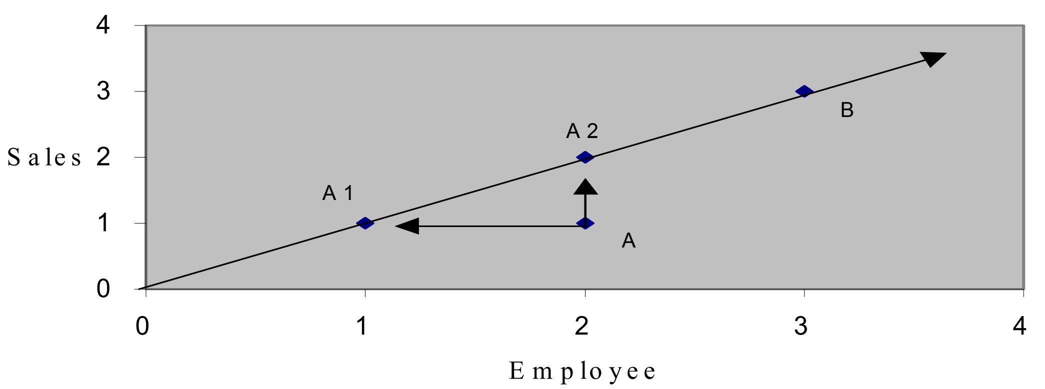
Figure 1 - Comparison of Branch Stores



2. How to Make the Inefficient Stores Efficient

Shortly, in several ways. We have to move the inefficient stores up to the efiicient frontier. In this simple case it is achieved by reducing the input (number of employees) to A1 with coordinates [1, 1] on the efficient frontier. Another is achieved by raising the output (sales in 100.000 CZK) up to A2=[2, 2].

Figure 2 - Improvement of Store A



3. Relative efficiency, CCR-model

This way of calculation of the efficiency is depend on the units of measure. For instance, if sales were stated in 10.000 CZK, the ratio for F would change from 2/5 =0.4 to 20/5=4.0. Furthermore, searched DMUs could have various inputs and outputs, so it is impossible to used the first simple formula for efficiency.

Therefore we define relative efficiency, supposed by M. J. Farell. Let us have:

- **N** searched DMUs
- every DMU has **m** inputs and **s** outputs

The inputs and outputs forms matrix X, Y respectively. Every column represents one DMU in the both matrices. There are m rows and N columns in the matrix **X and** s rows and N columns in Y respectively. Let $\mathbf{x_j}$ be the j[th] column of **X**, $\mathbf{y_j}$ be the j[th] column of **Y**. Then the relative efficiency θ_j for j[th] DMU is defined like a ratio of the outputs and inputs formed by "output weights" and "input weights":

$$\theta_j = (\mathbf{u}.\, \mathbf{y_j})/(\mathbf{v}.\mathbf{x_j}) \qquad \text{for } j=1,\dots, N$$

where $\mathbf{u} = (u_1, u_2,.., u_m)$ is a vector of the output weights and $\mathbf{v} = (v_1, v_2,.., v_s)$ the output weights respectively. Now the problem is how to set the weights because they are common for all the evaluated DMUs. Its solution supposed Charnes, Cooper a Rhodes (1978): every DMU can set its weights to make the best for itself. Such relative efficiency we can compare with the others relative efficiencies. The relative efficiency we obtain by a solution of following linear programming problem:

$$\begin{aligned} &\text{MAX} && \theta = (\mathbf{u}.\, \mathbf{y_0})/(\mathbf{v}.\mathbf{x_0}) \\ &\text{Subject to} && (\mathbf{u}.\, \mathbf{y_j})/(\mathbf{v}.\mathbf{x_j}) <= 1 \qquad \text{for } j = 1,\dots, N. \\ & && \mathbf{u}>=\mathbf{0},\ \mathbf{v}>=\mathbf{0} \end{aligned}$$

if $\mathbf{x_0}$, $\mathbf{y_0}$ are inputs and outputs respectively for the DMU just being searched, where **0** means null vector. No efficiency is greater than 1. The described model is called **CCR-model**.

For the comparison of the DMUs at first we solve N LP problems mentioned above and then compare obtained relative efficiencies. It is easy to transfer this problem into following one:

$$\begin{aligned} &\text{MAX} && \theta = \mathbf{u}.\, \mathbf{y_0} \\ &\text{Subject to} && \mathbf{v}.\mathbf{x_j} = 1 \qquad \text{for} \quad j = 1,\dots, N \\ & && \mathbf{u}.\, \mathbf{y_j} <= \mathbf{v}.\mathbf{x_j} \quad \text{for} \quad j=1,\dots, N \end{aligned}$$

u>=0, v>=0

4. Czech Pension Funds

We focused on Czech pension funds and their performance, especially the interest credited to their clients' accounts between 1995-2000, see Table 2. The interest was calculated by a methodology of the Czech Pension Funds Association and this is the input. As the outputs we used various data from funds' accountings: the volume of assets, of financial property or sales and revenues etc financial investment per client and like. We evaluated only the year 2000.

In Table 3 there are the efficiencies calculated within the particular LP tasks. As the most efficient we may identify ČSOB PF and PF ČP and Spořitelní PF.

However, two questions appear at this moment: whether we chose the data which are able to describe fund's performance fair and whether the interests published by the funds were not by an accounting trick. Unfortunately, it is not a problem of DEA.

5. Conclusion

CCR-model presented in this paper is not the only one DEA model. Also BCC-model (*Banker, Charnes, Cooper* – 1984) and additive model (*Charnes – 1985*) could be used for the comparison of the pension funds. However these models are not involved in this paper and their results will not be published here yet.

INTEREST CREDITED TO CLIENTS' ACCOUNTS BY PF IN THE CZECH REPUBLIC (in percents, calculated by methodology of the APFCR)

Table 2.

Name	1995	1996	1997	1998	1999	2000
Allianz PF formerly Allianz – Živnobanka PF	**-**	**-**	**8,90**	**9,10**	**6,00**	**3,80**
Commercial Union PF formerly Všeobecný vzájemný PF	**10,10**	**10,20**	**10,00**	**9,30**	**5,00**	**2,90**
ČSOB PF formerly PF spokojenosti Českých přístavů	**0,00**	**16,40**	**8,00**	**10,90**	**7,70**	**5,62**
Generali PF formerly Generali-Creditanstalt PF	**10,30**	**10,61**	**14,60**	**11,40**	**5,30**	**3,60**
Hornický PF Ostrava	**0,00**	**3,47**	**7,84**	**7,70**	**4,41**	**2,04**
ING penzijní fond	**12,80**	**12,10**	**11,00**	**8,50**	**6,00**	**4,40**

PF České pojišťovny	**10,30**	**9,20**	**9,60**	**9,72**	**6,60**	**4,50**
PF Komerční banky	**9,44**	**8,36**	**9,10**	**9,50**	**7,20**	**4,89**
Spořitelní PF	**-**	**-**	**-**	**-**	**-**	**4,20**
Vojenský otevřený PF	**9,45**	**10,00**	**10,03**	**9,70**	**6,70**	**4,10**
WINTERTHUR PF	**12,80**	**11,45**	**11,20**	**10,10**	**6,50**	**4,10**
Zemský PF	**11,80**	**7,00**	**7,00**	**7,00**	**7,00**	**5,01**

Table 3.

Name	**Efficiency**
Allianz-Živnobanka PF	0.698
ČSOB PF	**1.000**
Generali PF	0.741
ING PF	0.744
P F České pojišťovny	0,986
PF Komerční banky	0,648
Spořitelní PF	0,951
Vojenský otevřený PF	0,671
Všeobecný vzájemný PF	0,702
WINTERTHUR PF	0,837
Zemský PF	0,952

6. Summary

Concept of the work Models of the Data Envelopment Analysis (DEA) and their practical use

Recent years have seen a great variety of applications of DEA (Data Envelopment Analysis) for use in evaluating the performances of many different kinds of entities in many different activities in many different contexts in many different countries. One reason is that DEA has opened up possibilities for use in cases which have been resistant to other approaches because of the complex nature of the raletions between the multiple inputs and multiple outputs involved in many of these activities. The Czech pension funds just appear as such entities. Using DEA we try to compare their efficiecy by means of the data from their accountings. As an output we use the annual yield attained by the funds, as inputs is possible to take the volume of assets, of financial property or sales and revenues etc. At first we introduce the DEA method in general, the CCR-model as well. Afterwards we state the values of annual gains of selected pension funds for illustration, because the evaluation is more complex and has not been finished yet.

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Jiří Rozkovec

Vysoká škola ekonomická v Praze, Nám. W. Churchilla 4, 130 67 PRAHA 3, Česká republika, e-mail: rozkovec@centrum.cz

NONLINEAR MODELS FOR MACROECONOMIC DATA

EVA RUBLÍKOVÁ, MANUELA MAGALHAES HILL
ECONOMIC UNIVERSITY, BRATISLAVA, SLOVAKIA
INSTITUTO SUPERIOR DE CIENCIAS DO TRABALHO E DA EMPRESA, LISBON

1. Introduction

The eim of the paper is description and application of nonlinear models used to model univariate time series with time varying variance. The basic model allowing non-constant conditional variances is the autoregressive conditional heteroscedastic process or ARCH(q) process proposed first by Engle in 1982. This model was enhanced by Bollerslev in 1986 to the Generalised ARCH(p,q) or GARCH(p,q) process. Will will look for an appropriate ARCH or GARCH model to describe the development of mothly series of exchange rate of Slovak crone to USD dollar from January 1994 till December 2001 together with annualy series of Import to Portugal from 1947 till 1985.

2. Modelling Volatility

Box-Jenkins methodology of ARIMA models is concentrated on forecasting time series as conditional means, so the point forecast of a time series at time t, y_t, given information on the series up to time t – 1 is $E(y_t|y_{t-1}, y_{t-2},...)$, implicitly assuming, that the conditional variance remain constant..

But there are many financial time series in which this assuption is not valid. Especially exchange rates exhibit changes in variance over time, which are usually serially correlated, with groups of highly volatile observations occurring together, so one might prefer to forecast variances that are affected by past information.

ARCH(q) model

The more general approach was proposed by Engle (1982) which allows the variance to depend upon the available information set. Assuming conditional normality, a general specification of the evolution of y_t would be

$$y_t|Y_{t-1} \sim N(g_t, h_t) \qquad (1)$$

where $Y_{t-1} = \{y_{t-s}, s \geq 1\}$, and where both g_t and h_t are functions of the past observations of the variable Y_{t-1}. So in general g_t could be expressed by appropriate AR(p) model, but h_t is defined as follows

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + ... + \alpha_q \varepsilon_{t-q}^2, \; \varepsilon_t = y_t - g_t. \qquad (2)$$

Equations (1) and (2) together are known as an autoregressive conditional heteroscedasticity – ARCH(q) regression model.

The modeling of financial time series is very often done by the combination of AR(1) model with ARCH(1) errors, so we show their properties to see the usefulness of this combination. Models AR(1) with ARCH(1) errors are as follows:

$$y_t = \phi_1 y_{t-1} + \varepsilon_t$$

$$E(\varepsilon_t | E_{t-1}) = 0$$

$$\mathrm{var}(\varepsilon_t | E_{t-1}) = h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2, \qquad (3)$$

where

$|\phi_1| < 1$, so that y_t is stationary,

$\alpha_0 > 0$ and $\alpha_1 \geq 0$, so that h_t is positive.

The errors ε_t although serially uncorrelated through the white noise assumption, are not independent since they are related through their second moments. If ε_t is conditionally normal, the fourth unconditional moment of ε_t will exceed $3\sigma^4$ so that the marginal distribution of ε_t exhibits fatter tails than the normal. A finite fourth moment is ensured if $3\alpha_1^2 < 1$.

From (3) it follows that conditional mean and conditional variance of the model AR(1) for y_t with ARCH(1) model for errors are

$$E(y_t | Y_{t-1}) = \phi_1 y_{t-1}$$

$$\mathrm{var}(y_t | Y_{t-1}) = h_t = \alpha_0 + \alpha_1 (y_{t-1} - \phi_1 y_{t-2})^2,$$

so that both conditional mean and conditional variance of the one-step ahead forecast depend on the available information set. In particular, the conditional variance is increased by large "surprises" in y_t or vice versa.

Computation of conditional variance is useful only for short time horizon h, because if the horizon of forecast is large, the conditional variance is well approximated by

$$\operatorname{var}(y_{t+h} | Y_t) = \sigma^2 \sum_{i=0}^{h-1} \phi_1^{2i} ,$$

where the unconditional variance of ε_t is $\operatorname{var}(\varepsilon_t) = \sigma^2 = \frac{\alpha_0}{1-\alpha_1}$. For ARCH(q) process the unconditional variance is $\operatorname{var}(\varepsilon_t) = \sigma^2 = \frac{\alpha_0}{1-\sum_{i=1}^{q} \alpha_i}$, if $\alpha_0 > 0$, $\alpha_i \geq 0$ for i = 1, 2, ..., q and $\sum_{i=1}^{q} \alpha_i < 1$.

GARCH(p,q) model

The standard AR(p) –ARCH(q) model can be extended in various ways. We will speak about Bollersev extension (1986) of the model (3) generalizing h_t to

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^{2} + \sum_{i=1}^{p} \beta_i h_{t-i} , \qquad (4)$$

where

$\beta_i \geq 0.$

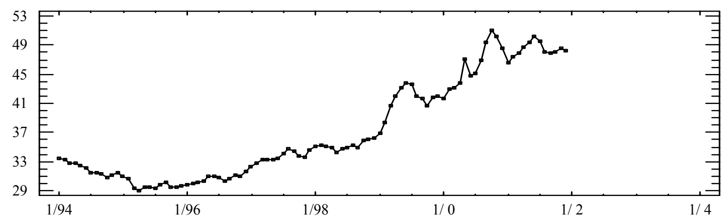
This function for h_t, which allows lagged conditional variances to enter, is termed as Generalized ARCH(p,q) process or GARCH (p,q) process.[1] ,[2], [3].

3. Application

ARCH and GARCH models have became very popular especially for financial time series because they enable to estimate the changing variance at a particular point in time. We shall discuss empirical results of application ARCH or GARCH model for monthly exchange rate of SKK/USD during the period from January 1994 to December 2001, which yields 96 observations.

From Figure 1 we can see strong evidence of nonstationarity, which is close to random walk.

Figure 1 Monthly spot exchange rate SKK/USD, January 1994 –December 2001



We will use the Dickey and Fuller test to make sure that the time series is random walk. Null hypotheses about y_t = SKK/USD$_t$ which follows random walk means that $y_t = \phi_1 y_{t-1} + \varepsilon_t$ only if $\phi_1 = 1$. So we can test hypothesis

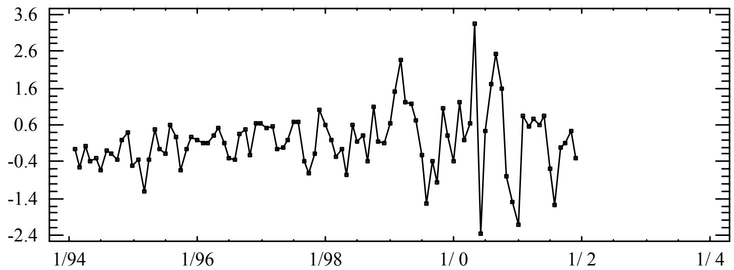
$$H_0: \phi_1 = 1 \qquad H_{1:}\ \phi_1 \langle 1. \tag{5}$$

The asymptotic distribution of the t statistic $t_{\hat{\gamma}} = \frac{\hat{\phi}_1 - 1}{se_\gamma}$ under Ho is known as the Dickey Fuller distribution, which is dependent on the sample size and the form of the regression.

In our series the null hypothesis cannot be rejected because $t_{\hat{\gamma}}$ =1,813 < c, where c is one of the negative values –1,95 and –2,60 for 5 % and 1 % significance level respectively, if the sample size is 100. [2] Since we cannot reject null hypotheses, we assume, that series of SKK/USD is generated by random walk.

From Figure 2, there is evident change in variability of the first differences or errors (residuals) of the series. The reason is, that on October 2, 1998 National Bank of Slovakia changed regime from the fixed exhange rate to the floating one.

Figure 2 Monthly changes of exchange rate SKK/USD



The ARCH effect was tested by Lagrange Multiplier test, for which gives LM = 27,3 greater than $\chi^2(3)$ not only on 5 % level but on 1 % level, as well.

We would like to find out the best AR(p)–ARCH(q) model for the growth rates of exchange rate rather than for the first absolute differences of SKK/USD. So we will use the series w_t = ln (SKK/USD)$_t$ – ln(SKK/USD)$_{t-1}$. The estimation of the AR(1) model for w_t is of the form

$$\underset{}{(1-B)w_t} = \underset{(0,1)}{0{,}258}\,w_{t-1}$$

, with standard error in parentheses. The MAPE during the analysed period from January 1994 till december 2001 is 1,5 %.

From the sample autocorrelation and sample partial autocorrelation function of squared residuals of the AR(1) model which are on Figure 4a) and Figure 4b) respectively, it is evident, that there is not simple ARCH effect, because we can use tentative models ARMA(3,1) or ARMA(2,2) to fit squared residuals. In this situation ARMA(p,q) model is called GARCH(p,q) model.

Figure 4a) SACF for squared residuals

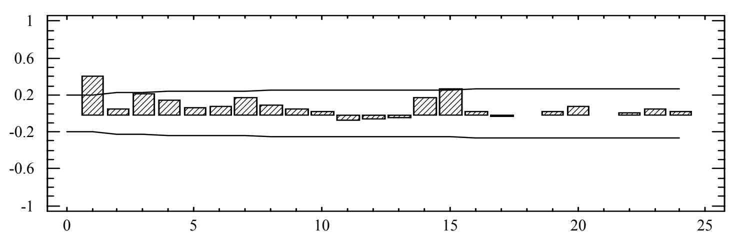
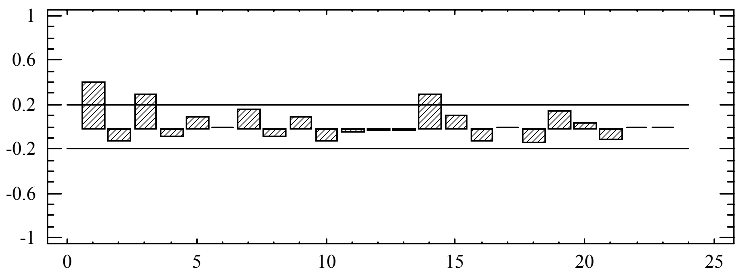


Figure 4b) SACF for squared residuals



After estimation of tentative models ARMA(3,1) and ARMA(2,2) we have found that the ARMA(2,2) or GARCH(2,2) model fit conditional variance the best, and estimation of the equation h $_t$ with P-values in parentheses is as follows:

$$h_t = 0{,}589E - 5 + 0519\varepsilon_{t-1}{}^2 + 0{,}33\varepsilon_{t-2}{}^2 + 0{,}663h_{t-2} \atop \qquad\qquad (0{,}08) \qquad (0{,}02) \qquad (0{,}002)$$.

Both models AR(1) with GARCH(2,2) for errors are good approximation of the development of the growth rate for exchange rate SKK/USD for the given period.

Conclusion

When we fit the growth rate of SKK/USD by means of the model ARIMA(1,1,0) together with GARCH(2,2) for squared residuals of ARIMA(1,1,0) model, than we gain possibility to get forecasts not only for level of the series but also for variance as well, conditionally on the past observations.

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Eva Rublíková, Department of Statistics, FHI EÚ Bratislava, Slovakia, e-mail: rublik@euba.sk Manuela Magalhaes Hill
ISCTE, Avenida Forças Armandas, 1940-026 Lisboa, Portugal, e-mail: hill@iscte.pt

POSSIBILITIES OF ARCH MODEL USING FOR ANALYSIS OF THE CZECH AND SLOVAK EQUITY MARKET

PETR SEĎA

VŠB-TU OSTRAVA, CZECH REPUBLIK

Abstract

In this paper an application of ARCH models is shown. These models are specifically designed to model and forecast conditional variances. The variance of the dependent variable is modeled as a function of past values of squared variances and squared residuals. This article examines possibilities of ARCH model using for analysis time series of Czech and Slovak stock return data. The appropriate types of ARCH models, estimation of the ARCH model parameters are also described in a simple manner.

1. Introduction

There is accordingly a strong need for modeling and forecasting volatility. In this paper, I introduce probably the most extensively applied family of volatility models in finance, Autoregressive conditional heteroskedasticity (ARCH) models. It is well known that financial time series exhibit changes in variance over the time and unconditional distribution of many financial prices and return series exhibit fatter tails then normal distribution. ARCH models can capture this characteristic under the assumption of conditional normality.

At the root of understanding ARCH is the distinction between the conditional and unconditional volatility. The first ARCH model, introduced by Engle (1982), was later generalized by Bollerslev (1986) and many variations on the basic GARCH model have been introduced in the last ten years.

The idea of ARCH is to add a second equation to the standard regression model, an equation that models the conditional variance. The first equation in the ARCH model is the conditional mean equation. This can be anything, but because the focus of ARCH is on the conditional variance equation it is usual to have a very simple conditional mean equation, such as

$$r_t = constant + \varepsilon_t , \qquad (1)$$

where r_t denotes for example daily rate of returns and ε_t denotes error.

In normal ARCH models we assume that ε_t is conditionally normally distributed with conditional variance σ_t^2[1]. However, in high frequency data there may still be insufficient leptokurtosis in normal ARCH to capture the full extent of kurtosis the data, and in this case a *t*–distribution could be assumed or a ARCH model defined on a mixture of normals.

The ARCH specification

Very many types of GARCH have been proposed in the academic literature, but only a few of these have found good practical applications. The generalization of Engle`s ARCH (p) model by Bollerslev (1986) adds *q* autoregressive terms to moving averages of squared unexpected returns. It takes form:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + ... + \alpha_p \varepsilon_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + ... + \beta_q \sigma_{t-q}^2, \qquad (2)$$

$$\omega \rangle 0,\ \alpha_1, ..., \alpha_p, \quad \beta_1, ..., \beta_q \geq 0. \qquad (3)$$

The parsimonious GARCH (1, 1) model, which is just one lagged error square and one autoregressive term, is most commonly used:

$$\sigma_t^2 = \omega + \alpha\, \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \qquad (4)$$

$$\omega \rangle 0, \quad \alpha, \beta \geq 0. \qquad (5)$$

The method used to estimate GARCH model parameters is maximum likelihood method estimation, which is a powerful and general statistic procedure, widely used because it always produces consistent estimates. The idea is to choose parameter estimates to maximize the likelihood of the data under an assumption about the shape of the distribution of the data generation process. The log–likelihood function is then maximized.

Data description

We used the Prague Stock Exchange Index PX 50 and Bratislava Stock Exchange Index SAX in time period from December 2, 1994 to December 21, 2001, which provides a total of 1600 observations to model volatility by appropriate ARCH model. The PX 50 is a weighted index containing 50 the most liquid titles with weights changing according to the market capitalization. At the present time, the actual number of the basic issues is variable[2]. The SAX is a capital-weighted index that compares the market capitalization of a selected set of shares with the market capitalization of the same set of shares as of a given reference day.

[1] The unconditional returns distributions will then be leptokurtic–that is, have the fatter tails than the normal–because the changing conditional variance allows for more outliers or unusually observations.

[2] However, in accordance with the Principles of Updating the Base of the Index PX 50, which were approved in December 2001, the base cannot consist of more than 50 issues.

The SAX Index is also a weighted index containing 10 the most liquid titles. We model the level of equity markets daily market returns of indices as:

$$r_t = 100\,.\,ln\,\frac{Index_t}{Index_{t\text{-}!}} \qquad (6)$$

where t denotes days.

The conditional volatility as a GARCH (p, q) process is written like equation (2). The following Figure (1) and Figure (2) show the distribution of daily returns and descriptive statistics of PX 50 and SAX indices.

Figure 1 : *PX 50 daily returns change histogram and descriptive statistics*

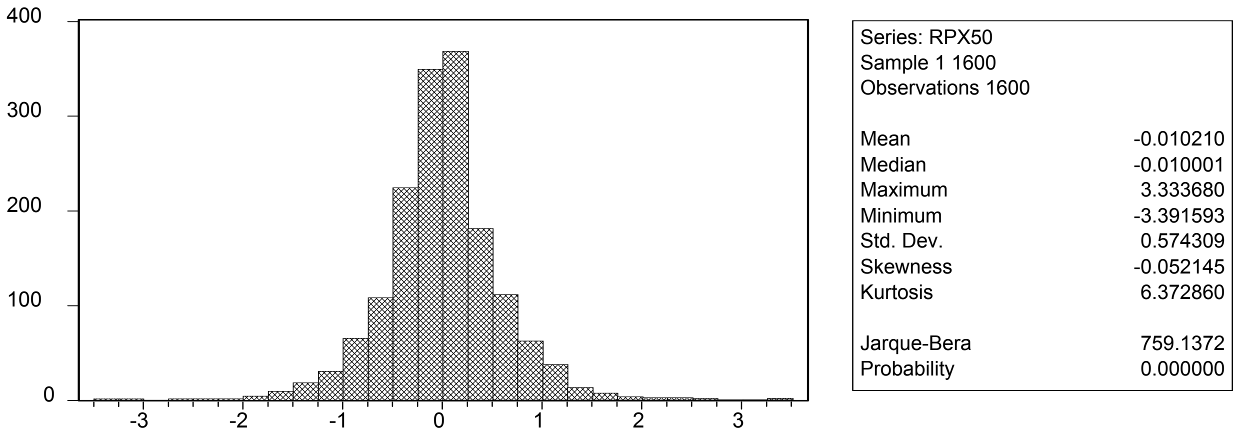
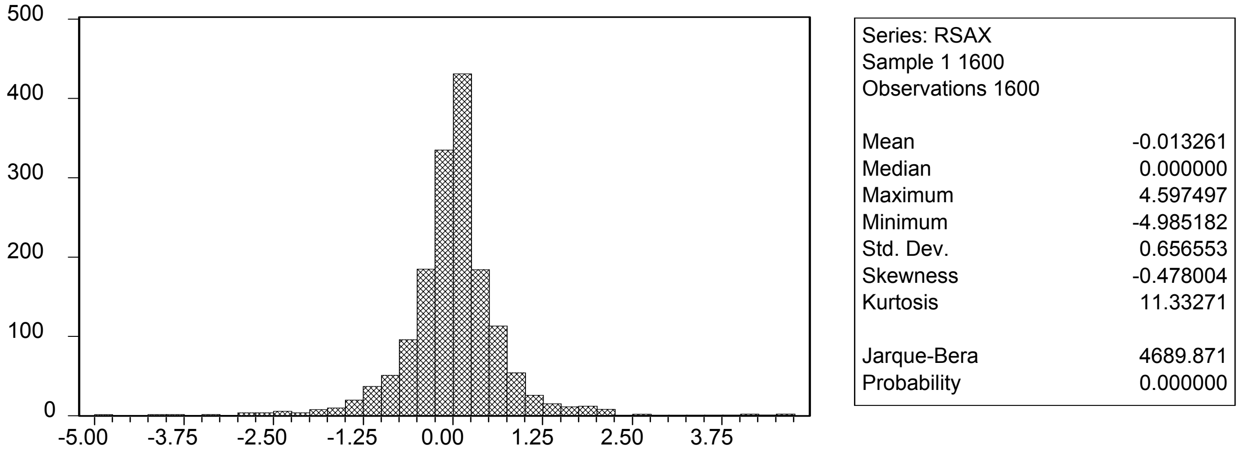


Figure 2 : *SAX daily returns change histogram and descriptive statistics*



Unit root test applied on the returns of PX 50 and SAX rejected the hypothesis of a unit root and thus the hypothesis of non stationary of the series. On the basis of estimated values we can reject the hypothesis of non stationary of the PX 50 and SAX returns[3].

[3] Results are available on request.

The rejection of the hypothesis of non stationary of stock market returns does not imply that returns are uncorrelated. We have therefore computed the values of the autocorrelation function for 15 lagged returns of PX 50 and SAX.

Table (1) shows in the third column that there is statistically significant autocorrelation between r_t and one day lagged variable r_t, surprisingly between r_t and six and also eight and thirteen days lagged variable r_t of PX 50. In case of the Slovak equity market, presented by partial autocorrelation in seventh column, there is statistically significant autocorrelation between r_t and two days lagged variable r_t, surprisingly between r_t and eight and also eleven days lagged variable r_t.

Table 1 : *Autocorrelation analysis of PX 50 and SAX daily returns*

	PX 50				**SAX**			
Lags	Autoc.	Part.aut.	Ljung-Box Q	Probab.	Autoc.	Part. aut.	Ljung-Box Q	Probab.
1	0,150	**0,150***	36,296	0,000	-0,033	-0,033	1,797	0,180
2	0,038	0,016	38,650	0,000	-0,066	**-0,067***	8,810	0,012
3	0,034	0,027	40,529	0,000	0,004	-0,001	8,837	0,032
4	0,015	0,006	40,906	0,000	0,048	0,044	12,479	0,014
5	0,010	0,005	41,053	0,000	0,004	0,008	12,509	0,028
6	-0,061	**-0,066***	46,995	0,000	0,031	0,037	14,030	0,029
7	-0,022	-0,005	47,778	0,000	-0,030	-0,027	15,500	0,030
8	0,065	**0,073***	54,489	0,000	0,072	**0,073***	23,826	0,002
9	0,040	0,025	57,039	0,000	0,002	0,003	23,834	0,005
10	0,024	0,013	57,939	0,000	-0,046	-0,040	27,310	0,002
11	0,034	0,025	59,752	0,000	0,059	**0,059***	32,856	0,001
12	0,063	0,048	66,071	0,000	0,050	0,042	36,941	0,000
13	0,081	**0,060***	76,587	0,000	-0,036	-0,025	39,021	0,000
14	0,043	0,027	79,599	0,000	-0,007	-0,006	39,104	0,000
15	-0,006	-0,018	79,6630	0,000	0,007	0,001	39,176	0,001

*significant at 1 % level except two days lagged variable r_t of SAX (significant at 5 % level)

Application of ARCH technique

In this chapter we examine and apply ARCH technique to develop reasonably accurate time series models of daily returns on the Czech and Slovak equity markets. On the basis of autocorrelation functions we have chosen the basic mean equation (1), including one, six, eight and thirteen days lagged variable r_t in case of the Czech equity market and two, eight and eleven days lagged variable r_t in case of the Slovak equity market.

The comparison of different models suggested that GARCH (1, 2) model fits the PX 50 returns in the best way. The estimated parameters of the GARCH processes are shown in Table (2).

Table 2: *GARCH (1, 2) model for Czech equity market daily returns*

	Coefficient	Std. Error	z-Statistic	Probability
Mean Equation				
$R_{PX50(-1)}$	0,2308	0,0291	7,9212	0,0000
$R_{PX50(-6)}$	-0,0490	0,0276	-1,7743	0,0760
$R_{PX50(-8)}$	0,0745	0,0254	2,9367	0,0033
$R_{PX50(-13)}$	0,0830	0,0232	3,5748	0,0004
Variance Equation				
C	0,0101	0,0016	6,2135	0,0000
ARCH(1)	0,2254	0,0265	8,4949	0,0000
GARCH(1)	0,2949	0,0763	3,8649	0,0001
GARCH(2)	0,4741	0,0649	7,3034	0,0000
R-squared	0,0269	Mean dependent var		-0,0110
Adjusted R-squared	0,0226	S,D, dependent var		0,5742
S.E. of regression	0,5677	Akaike info criterion		1,4804
Sum squared resid	508,8922	Schwarz criterion		1,5074
Log likelihood	-1166,6780	Durbin-Watson stat		2,1882

In case of the Slovak equity market the results are similar. The comparison of different models suggested again that GARCH (1, 2) model fits the SAX returns in the best way. The estimated parameters of the GARCH processes are shown in Table (3).

Table 3: *GARCH (1, 2) model for Slovak equity market daily returns*

	Coefficient	Std. Error	z-Statistic	Probability
Mean Equation				
$R_{SAX(-2)}$	-0,0552	0,0282	-1,9579	0,0502
$R_{SAX(-8)}$	0,0879	0,0262	3,3494	0,0008
$R_{SAX(-11)}$	0,0951	0,0259	3,6715	0,0002
Variance Equation				
C	0,0169	0,0021	8,0012	0,0000
ARCH(1)	0,0417	0,0068	6,1604	0,0000
GARCH(1)	1,4905	0,0515	28,9234	0,0000
GARCH(2)	-0,5714	0,0438	-13,0386	0,0000
R-squared	0,0111	Mean dependent var		-0,0159
Adjusted R-squared	0,0073	S,D, dependent var		0,6478
S.E. of regression	0,6454	Akaike info criterion		1,8872
Sum squared resid	658,9579	Schwarz criterion		1,9108
Log likelihood	-1492,3700	Durbin-Watson stat		1,9864

The results shows that all estimated coefficients are significant at 1 % level except six days lagged variable r_t of PX 50 (significant at 10 % level) and two days lagged variable r_t of SAX (also significant at 10 % level) therefore GARCH (1, 2) model may be used to generate the fitted values of time series and to successively generate one - step - ahead forecast variances.

Some diagnostic tests are available for checking the adequacy of the model. When the model is adequately specified for the level and volatility, the standardized residual, ε_t/σ_t, and the standardized residual square, $(\varepsilon_t/\sigma_t)^2$, should be independent and identically distributed and thus, not exhibit any time series properties such as autocorrelation[4]. Results of all diagnostic tests are available on request.

2. Conclusion

Although many versions of GARCH models are relatively easy to estimate with widely reported success, they are still some limitations. ARCH models have became very popular because they enable to estimate the variance of financial time series at a particular point of time. We have demonstrated analysis and modeling of time series of Czech and Slovak equity daily returns for length of 1600 observations. We also have informatively focused on the problem of estimates of variance forecasts using appropriate type of ARCH processes.

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Petr Seďa

Department of Mathematical Methods in Economics, Faculty of Economics VŠB-TU Ostrava

Sokolská tř. 33, Ostrava 1, 701 21, The Czech Republic, e-mail: petr.seda@vsb.cz

[4] Normalized residuals should be also tested by Lagrange Multiplier test. The results indicate that after application of GARCH (1, 2) model the residuals are homoskedastic.

Calculating the Variance in Markov Reward Chains with a Small Interest Rate[1]

Milan Sitař, Karel Sladký
Institute of Information Theory and Automation
Academy of Sciences of the Czech Republic
Pod vodárenskou věží 4, CZ 182 08 Praha 8, Czech Republic

Abstract. We consider a discrete time Markov reward process with finite state space and assume that the rewards associated with the transitions are random variables with known probability distributions and finite first and second moments. Formulas for expected value and variance of the cumulative (random) reward are obtained for finite horizon case and infinite horizon models with discounting. Employing the Laurent expansion techniques we obtain explicit formulas for the variance of the long run discounted reward in the terms of undiscounted models if the discount factor tends to unity (or equivalently if the interest rate tends to zero).

1 Introduction

In this note, we consider a discrete time Markov reward process with finite state space. We assume that the rewards associated with the transitions, instead of known constants, are random variables with known probability distributions with finite mean and variance. Dynamic programming recursions for Markov decision chains with random rewards accrued to one-stage transitions were studied in Benito [?] where recurrence formulas for the mean and variance of the cumulative rewards are established, along with infinite horizon model for the case with discounting. Recall that the usual criteria examined in the literature on Markov reward processes, e.g. total expected discounted reward or mean reward per transition, may be quite insufficient to characterize the problem from the point of the decision maker (see e.g. the classical work of Markowitz [?] on mean variance selection rule for the portfolio selection problem).

Under assumption that one-stage rewards associated with transitions are deterministic, higher moments and variance of cumulative rewards in finite state space models were studied in Jaquette [?], [?], Mandl [?], and Sobel [?] mostly in the framework of Markovian decision processes (in the last paper also a formula for distribution function of cumulative reward was given), and in the review paper by White [?]. In the above papers second moments (or variance) of total expected or mean reward is usually considered only in the class of stationary mean or discounted optimal policies to select in the class of discounted (or mean) optimal policies decisions guaranteeing lower variance (equivalent to the lower second moment) of the cumulative reward.

In this article we are interested in the properties of expected value and variance of the cumulative reward earned in the subsequent transitions of the Markov chain. For the

[1]This research was supported by the Grant Agency of the Czech Republic under Grants 402/01/0539 and 402/02/1015.

infinite discounted cost case, formulas for expected value and variance of the cumulative (random) reward are obtained. Employing the Laurent expansion techniques we obtain explicit formulas for the variance of the long run discounted reward in the terms of undiscounted models if the discount factor β tends to unity, or equivalently if the interest rate ρ tends to zero (recall that $\beta = \frac{1}{1+\rho}$). This approach was initiated by Blackwell in his seminal paper [?] and followed by many other authors (see e.g. [?], [?]).

2 Markov Chains with Rewards

Let $X = \{X_n, n = 0, 1, \ldots\}$ be a (homogeneous) Markov chain with finite state space $\mathcal{S} = \{1, 2, \ldots, S\}$, initial distribution $\boldsymbol{p}(0)$ and the transition probability matrix $\boldsymbol{P} = [p_{ij}]_{i,j=1}^{S}$ (obviously, $\sum_{j=1}^{S} p_{ij} = 1$, $p_{ij} \geq 0$). Then the probability distribution $\boldsymbol{p}(n)$ after n transitions is given by $\boldsymbol{p}(n) = \boldsymbol{p}(0)\boldsymbol{P}^n$; $\boldsymbol{P}^n$ is the n-th power of $\boldsymbol{P}$ (its ij-th element equals $p_{ij}^{(n)}$ is the probability of reaching state j from state i in the n next transitions). For convenience we set $\boldsymbol{P}^0 = \boldsymbol{I}$ (the unit matrix), $\boldsymbol{e}$ is reserved for a unit vector. Notice that without loosing generality we can suppose that the transitions occur at discrete time points $t = 0, 1, \ldots$.

It is well known from Markov chain theory (cf. e.g. [?]) that the following matrices exist:

$$\begin{cases} \boldsymbol{P}^* = \lim\limits_{n\to\infty} \frac{1}{n} \sum\limits_{k=0}^{n-1} \boldsymbol{P}^k & \text{(the Cesaro steady state limit)} \\ \boldsymbol{Z} = (\boldsymbol{I} - \boldsymbol{P} + \boldsymbol{P}^*)^{-1} & \text{(the fundamental matrix of } \boldsymbol{P}\text{)} \end{cases} \tag{1}$$

It can be shown (cf. [?]) that $\boldsymbol{Z}\,\boldsymbol{e} = \boldsymbol{e}$, $\boldsymbol{Z}\boldsymbol{P}^* = \boldsymbol{P}^*\boldsymbol{Z} = \boldsymbol{P}^*$, and that $\boldsymbol{P}\boldsymbol{Z} = \boldsymbol{Z} - \boldsymbol{I} + \boldsymbol{P}^*$. Moreover, if $\boldsymbol{P}$ has one recurrent class then the rows of $\boldsymbol{P}^*$ are identical and equal to the stationary probability distribution (row) vector $\boldsymbol{p}^* = [p_1^*, \ldots, p_S^*]$ (i.e. the row of $\boldsymbol{P}^*$) of the considered discrete-time Markov chain. Moreover, if $\boldsymbol{P}$ is also aperiodic (i.e. if one is the only eigenvalue of $\boldsymbol{P}$ with the modulus equal to one and hence $\lim\limits_{n\to\infty} \boldsymbol{P}^n = \boldsymbol{P}^*$) then

$$p_j^* = \lim_{n\to\infty} \mathsf{P}\{X_n = j | X(0) = i\} \quad \text{for every} \quad i, j \in \mathcal{S}. \tag{2}$$

and the convergence in (??) is geometric.

In contrast with the classical Markov reward models we shall assume that if state $j \in \mathcal{S}$ is reached from state $i \in \mathcal{S}$ an immediate *random* reward ξ_{ij} is earned. Let $F_{ij}(\cdot)$ be the probability distribution function of ξ_{ij} with (finite) expectation r_{ij} (i.e. the first moment of ξ_{ij}) and the (finite) second moment s_{ij}, i.e. $r_{ij} = \int_{\infty}^{\infty} \xi \,\mathrm{d}F_{ij}(\xi) < \infty$, $s_{ij} = \int_{-\infty}^{\infty} \xi^2 \,\mathrm{d}F_{ij}(\xi) < \infty$ and the resulting one-stage variance $[\sigma_{ij}]^2 = \int_{-\infty}^{\infty} [\xi - r_{ij}]^2 \mathrm{d}F_{ij}(\xi) = s_{ij} - [r_{ij}]^2 < \infty$. Observe that $F_{ij}(\xi)$ represents the conditional probability that the reward earned will be non-greater than ξ on condition that the chain in state i will next enter state j.

Obviously, the expected reward in state i is equal to $r_i = \sum_{j=1}^{S} p_{ij} r_{ij}$, and the second moment of the (random) reward earned in state i is equal to $s_i = \sum_{j=1}^{S} p_{ij} s_{ij} = \sum_{j=1}^{S} p_{ij} \{[r_{ij}]^2 + [\sigma_{ij}]^2\}$. The symbol $\boldsymbol{r}$, resp. $\boldsymbol{s}$ denotes the $S \times 1$ vector whose i-th element equals r_i, resp. s_i, and $\boldsymbol{R} = [r_{ij}]$ is an $S \times S$ matrix.

In what follows we denote by $\boldsymbol{v}(m) \equiv \boldsymbol{v}^{(1)}(m)$, resp. by $\boldsymbol{v}^{(2)}(m)$, the $S \times 1$ vector of total expected rewards, resp. of the sum of one-stage second moments of rewards, earned in the m-next transitions (its i-th element, denoted $v_i(m) \equiv v_i^{(1)}(m)$, resp. $v_i^{(2)}(m)$, is the total expected reward, resp. sum of expected one-stage second moments, earned provided the chain starts in state $i \in \mathcal{S}$). Obviously

$$\boldsymbol{v}^{(1)}(m) = \boldsymbol{v}(m) = \sum_{n=0}^{m-1} \boldsymbol{P}^n \boldsymbol{r}, \qquad \boldsymbol{v}^{(2)}(m) = \sum_{n=0}^{m-1} \boldsymbol{P}^n \boldsymbol{s}. \tag{3}$$

If we assume that the reward obtained at time n is discounted by β^n, then from (**??**) we get:

$$\boldsymbol{v}(\beta, m) = \sum_{n=0}^{m-1} [\beta \boldsymbol{P}]^n \boldsymbol{r}, \qquad \boldsymbol{v}^{(2)}(\beta, m) = \sum_{n=0}^{m-1} [\beta \boldsymbol{P}]^n \boldsymbol{s}. \tag{4}$$

Recall that $\beta \in (0,1)$ is the discount factor and for the corresponding interest rate ρ we have $\rho = \frac{1-\beta}{\beta} \Longleftrightarrow \beta = \frac{1}{1+\rho}$.

Letting $m \to \infty$ we get from (**??**), (**??**) the following formulas for the average (or mean) reward $\boldsymbol{g} = \boldsymbol{g}^{(1)}$ and for the average second moment of one-stage return $\boldsymbol{g}^{(2)}$

$$\boldsymbol{g}^{(1)} = \boldsymbol{g} = \lim_{n \to \infty} \frac{1}{n} \sum_{k=0}^{n-1} \boldsymbol{P}^k \boldsymbol{r} = \boldsymbol{P}^* \boldsymbol{r}, \qquad \boldsymbol{g}^{(2)} = \lim_{n \to \infty} \frac{1}{n} \sum_{k=0}^{n-1} \boldsymbol{P}^k \boldsymbol{s} = \boldsymbol{P}^* \boldsymbol{s}. \tag{5}$$

Moreover, if $\boldsymbol{P}$ has one class of recurrent states in virtue of (**??**) the rows of $\boldsymbol{P}$ are identical and $\boldsymbol{g}$, resp. $\boldsymbol{g}^{(2)}$, are constant vectors (i.e., elements of $\boldsymbol{g}$, resp. $\boldsymbol{g}^{(2)}$, are equal to g, resp. $g^{(2)}$).

Similarly, letting $m \to \infty$ we get from (**??**) the following formulas

$$\lim_{m \to \infty} \boldsymbol{v}(\beta, m) = \boldsymbol{v}(\beta) = \boldsymbol{r} + \beta \boldsymbol{P} \boldsymbol{v}(\beta) = (\boldsymbol{I} - \beta \boldsymbol{P})^{-1} \boldsymbol{r} \tag{6}$$

$$\lim_{m \to \infty} \boldsymbol{v}^{(2)}(\beta, m) = \boldsymbol{v}^{(2)}(\beta) = \boldsymbol{s} + \beta\, \boldsymbol{P} \boldsymbol{v}^{(2)}(\beta) = (\boldsymbol{I} - \beta \boldsymbol{P})^{-1} \boldsymbol{s}. \tag{7}$$

The following facts are well known to workers in dynamic programming.

There exist $S \times 1$ vectors $\boldsymbol{w}$, resp. $\boldsymbol{w}^{(2)}$, that are (unique) solutions to

$$\boldsymbol{g} = \boldsymbol{P} \boldsymbol{g}, \qquad \boldsymbol{w} + \boldsymbol{g} = \boldsymbol{r} + \boldsymbol{P}\, \boldsymbol{w}, \qquad \boldsymbol{P}^* \boldsymbol{w} = \boldsymbol{0} \tag{8}$$

resp. to

$$\boldsymbol{g}^{(2)} = \boldsymbol{P} \boldsymbol{g}^{(2)}, \qquad \boldsymbol{w}^{(2)} + \boldsymbol{g}^{(2)} = \boldsymbol{s} + \boldsymbol{P}\, \boldsymbol{w}^{(2)}, \qquad \boldsymbol{P}^* \boldsymbol{w}^{(2)} = \boldsymbol{0}. \tag{9}$$

Observe that if $\boldsymbol{P}$ has one class of recurrent states, the first equation in (**??**), (**??**) is trivially fulfilled and the last equation is identical with $\boldsymbol{p}^* \boldsymbol{w} = 0$ or $\boldsymbol{p}^* \boldsymbol{w}^{(2)} = 0$.

It can be verified that $\boldsymbol{g} = \boldsymbol{P}^* \boldsymbol{r}$, $\boldsymbol{w} = (\boldsymbol{Z} - \boldsymbol{P}^*) \boldsymbol{r} = \boldsymbol{Z} \boldsymbol{r} - \boldsymbol{g}$, is the (unique) solution to (**??**); obviously, the same holds also for (**??**).

Iterating (**??**), (**??**) from (**??**) we can verify that

$$\boldsymbol{v}(m) = m \boldsymbol{g} + \boldsymbol{w} - \boldsymbol{P}^m \boldsymbol{w}, \qquad \boldsymbol{v}^{(2)}(m) = m \boldsymbol{g}^{(2)} + \boldsymbol{w}^{(2)} - \boldsymbol{P}^m \boldsymbol{w}^{(2)}. \tag{10}$$

Futhermore, it can be shown (cf. Miller and Veinott [**?**] or Sladký [**?**]) that for β sufficiently close to unity

$$(\boldsymbol{I} - \beta \boldsymbol{P})^{-1} = \left(\frac{1}{1-\beta}\right) \boldsymbol{P}^* + \frac{1}{\beta} \sum_{k=0}^{\infty} \left(\frac{1-\beta}{\beta}\right)^k \boldsymbol{Z}^{k+1} (\boldsymbol{I} - \boldsymbol{P}^*) \tag{11}$$

and hence, since $\boldsymbol{v}(\beta) = (\boldsymbol{I} - \beta \boldsymbol{P})^{-1}\boldsymbol{r}$, $\boldsymbol{P}^*\boldsymbol{r} = \boldsymbol{g}$, $\boldsymbol{Z}(\boldsymbol{I} - \boldsymbol{P}^*) = \boldsymbol{w}$

$$\boldsymbol{v}(\beta) = \frac{1}{1-\beta}\boldsymbol{g} + \frac{1}{\beta}\boldsymbol{w} + \frac{1}{\beta}\sum_{k=1}^{\infty}\left(\frac{1-\beta}{\beta}\right)^k \boldsymbol{Z}^k\boldsymbol{w} \tag{12}$$

In particular,

$$\boldsymbol{v}(\beta) = \frac{1}{1-\beta}\boldsymbol{g} + \frac{1}{\beta}\boldsymbol{w} + \frac{1}{\beta}\left(\frac{1-\beta}{\beta}\right)\tilde{\boldsymbol{w}} + o(1-\beta)^2\boldsymbol{e} \tag{13}$$

with $\tilde{\boldsymbol{w}} = \boldsymbol{Z}\boldsymbol{w}$.

3 Reward variance in Markov chains

Obviously, the random reward obtained from the m-th to the n-th transition is given by $\xi^{(m,n)} = \sum_{k=m}^{n} \xi_{X_k,X_{k+1}}$. The expected value of $\xi^{(m,n)}$, resp. of $(\xi^{(m,n)})^2$, provided that at the m-th transition the chain is in state i is given by $v_i^{(m,n)} = \mathsf{E}_{X_m=i}\sum_{k=m}^{n}\xi_{X_k,X_{k+1}}$, resp. by $u_i^{(m,n)} = \mathsf{E}_{X_m=i}\left[\sum_{k=m}^{n}\xi_{X_k,X_{k+1}}\right]^2$, and the corresponding variance $[\sigma_i^{(m,n)}]^2 = u_i^{(m,n)} - [v_i^{(m,n)}]^2$. In particular, if the chain starts in state $i \in \mathcal{S}$ for the expectations of $\xi^{(0,n)}$ and $[\xi^{(0,n)}]^2$ we get the following formulas

$$v_i^{(0,n)} = r_i + \sum_{j=1}^{S} p_{ij}v_j^{(1,n)}, \qquad u_i^{(0,n)} = s_i + 2\sum_{j=1}^{S} p_{ij}r_{ij}v_j^{(1,n)} + \sum_{j=1}^{S} p_{ij}u_j^{(1,n)}. \tag{14}$$

Employing (**??**) the variance of the random reward earned in the n following transitions, provided the chains starts in state i is given by (recall that $\mathsf{E}[\xi^2] = \mathsf{Var}[\xi] + \{\mathsf{E}[\xi]\}^2$)

$$[\sigma_i^{(0,n)}]^2 = s_i + 2\sum_{j=1}^{S} p_{ij}r_{ij}v_j^{(1,n)} + \sum_{j=1}^{S} p_{ij}\{[\sigma_j^{(1,n)}]^2 + [v_j^{(1,n)}]^2\} - [v_i^{(0,n)}]^2 \tag{15}$$

that can be also written as (recall that $s_i = \sum_{j=1}^{S} p_{ij}\{[r_{ij}]^2 + [\sigma_{ij}]^2\}$)

$$[\sigma_i^{(0,n)}]^2 = \sum_{j=1}^{S} p_{ij}\{[r_{ij} + v_j^{(1,n)}]^2 + [\sigma_{ij}]^2\} - [v_i^{(0,n)}]^2 + \sum_{j=1}^{S} p_{ij}[\sigma_j^{(1,n)}]^2 \tag{16}$$

Obviously, $\boldsymbol{v}(\beta, m) = \boldsymbol{v}(\beta) - [\beta\,\boldsymbol{P}]^m\boldsymbol{v}(\beta)$ for any m, and (**??**) takes on the following form:

$$[\sigma_i^{(0,m)}(\beta)]^2 = \sum_{j=1}^{S} p_{ij}\,[\beta\,\sigma_j^{(1,m)}(\beta)]^2 + \sum_{j=1}^{S} p_{ij}\{[r_{ij} + \beta v_j^{(1,m)}(\beta)]^2 + [\sigma_{ij}]^2\} - [v_i^{(0,m)}(\beta)]^2 \tag{17}$$

i.e., in matrix notation, $[\boldsymbol{\sigma}^{(0,m)}(\beta)]^2 = \tilde{\boldsymbol{s}}^{(0,m)}(\beta) + \boldsymbol{P}[\beta\,\boldsymbol{\sigma}^{(1,m)}(\beta)]^2$ where the i-th element of $\tilde{\boldsymbol{s}}^{(0,m)}(\beta)$ is equal to $\tilde{s}_i^{(0,m)}(\beta) = \sum_{j=1}^{S} p_{ij}\{[r_{ij} + \beta\, v_j^{(1,m)}(\beta)]^2 + [\sigma_{ij}]^2\} - [v_i^{(0,m)}(\beta)]^2$.

As $m \to \infty$ elements of $[\boldsymbol{\sigma}^{(0,m)}(\beta)]^2$ (and of $\boldsymbol{v}^{(0,m)}(\beta)$, $\tilde{\boldsymbol{s}}^{(0,m)}(\beta)$ respectively) converge geometrically in m to $[\boldsymbol{\sigma}(\beta)]^2$ (and to $\boldsymbol{v}(\beta)$, $\tilde{\boldsymbol{s}}(\beta)$ respectively). Then (cf. (**??**)):

$$\begin{aligned}[\sigma_i(\beta)]^2 - \sum_{j=1}^{S} p_{ij}\,[\beta\,\sigma_j(\beta)]^2 &= \sum_{j=1}^{S} p_{ij}\,\{[r_{ij} + \beta\, v_j(\beta)]^2 + [\sigma_{ij}]^2\} - [v_i(\beta)]^2 \\ &= s_i + 2\beta\sum_{j=1}^{S} p_{ij}r_{ij}v_j(\beta) + \beta^2\sum_{j=1}^{S} p_{ij}[v_j(\beta)]^2 - [v_i(\beta)]^2 \end{aligned} \tag{18}$$

i.e., in matrix notations ($[\boldsymbol{A}]_{\rm dg}$ results from $\boldsymbol{A}$ by setting off-diagonal entries to 0)

$$[\boldsymbol{\sigma}(\beta)]^2 = [\boldsymbol{I} - \beta^2\boldsymbol{P}]^{-1}\,\tilde{\boldsymbol{s}}(\beta) \tag{19}$$

where $\tilde{\boldsymbol{s}}(\beta) = \boldsymbol{s} + 2\beta[\boldsymbol{P}[\boldsymbol{v}(\beta)]_{\rm dg}\boldsymbol{R}^{\rm T}]_{\rm dg}\boldsymbol{e} - \{\boldsymbol{I} - \beta^2\boldsymbol{P}\}[\boldsymbol{v}(\beta)]_{\rm sq}$.
In virtue of (**??**) for β sufficiently close to unity

$$[\boldsymbol{I} - \beta^2\boldsymbol{P}]^{-1} = \frac{1}{1-\beta^2}\boldsymbol{P}^* + \frac{1}{\beta^2}\sum_{k=0}^{\infty}\left(\frac{1-\beta^2}{\beta^2}\right)^k \boldsymbol{Z}^{k+1}(\boldsymbol{I} - \boldsymbol{P}^*)$$

and hence (provided that the convergence is guaranteed) also

$$[\boldsymbol{\sigma}(\beta)]^2 = \frac{1}{1-\beta^2}\boldsymbol{P}^*\tilde{\boldsymbol{s}}(\beta) + \frac{1}{\beta^2}\sum_{k=0}^{\infty}\left(\frac{1-\beta^2}{\beta^2}\right)^k \boldsymbol{Z}^{k+1}(\boldsymbol{I} - \boldsymbol{P}^*)\tilde{\boldsymbol{s}}(\beta) \tag{20}$$

4 Small interest rate

In what follows we shall focus our attention on the following relation

$$\lim_{\beta\to 1_-}(1-\beta^2)[\boldsymbol{\sigma}(\beta)]^2 = \lim_{\beta\to 1_-}[\boldsymbol{P}^*\tilde{\boldsymbol{s}}(\beta) + \sum_{k=0}^{\infty}\left(\frac{1-\beta^2}{\beta^2}\right)^{k+1} \boldsymbol{Z}^{k+1}(\boldsymbol{I} - \boldsymbol{P}^*)\tilde{\boldsymbol{s}}(\beta)] \tag{21}$$

Recalling the well known Tauberian theorems the above limit is equal to the average variance of the rewards in infinite horizon (provided it exists).

Examining the above relation (**??**) we can easily see that it is crucial to simplify the following expression

$$\begin{aligned} &\lim\nolimits_{\beta\to 1_-}\quad \left(\frac{1-\beta^2}{\beta^2}\right)^{k+1}\tilde{s}_i(\beta) = \\ = \;&\lim\nolimits_{\beta\to 1_-}\quad \left(\frac{1-\beta^2}{\beta^2}\right)^{k+1}\left(\sum_{j=1}^{S} p_{ij}\left\{[r_{ij}+\beta v_j(\beta)]^2 + [\sigma_{ij}]^2\right\} - [v_i(\beta)]^2\right) \end{aligned} \tag{22}$$

for each $i = 1, \ldots, S$ and $k = 0, 1, 2$.

To simplify the analysis we make the following assumption.

Assumption GA. The transition probability matrix $\boldsymbol{P}$ contains one class of recurrent state (hence $\boldsymbol{g}$ is a constant vector).

To this end we insert (**??**) into $\tilde{s}_i(\beta)$ of (**??**). In particular, first of all we simplify the following expression:

$$\sum_{j=1}^{S} p_{ij}\left\{r_{ij} + \frac{\beta}{1-\beta}g + w_j + \frac{1-\beta}{\beta}\tilde{w}_j(\Pi) + o(1-\beta)^2\right\}^2 +$$
$$\sum_{j=1}^{S} p_{ij}[\sigma_{ij}]^2 - \left\{\frac{1}{1-\beta}g + \frac{1}{\beta}w_i + \frac{1-\beta}{\beta^2}\tilde{w}_i + o(1-\beta)^2\right\}^2.$$

That is

$$\sum_{j=1}^{S} p_{ij}\left\{[r_{ij}]^2 + \frac{\beta^2}{(1-\beta)^2}[g]^2 + [w_j]^2 + \frac{(1-\beta)^2}{\beta^2}[\tilde{w}_j]^2 + 2\frac{\beta}{1-\beta}r_{ij}g + 2r_{ij}w_j + \right.$$

$$\left. +2\frac{1-\beta}{\beta}r_{ij}\tilde{w}_j + 2\frac{\beta}{1-\beta}gw_j + 2g\tilde{w}_j + 2\frac{1-\beta}{\beta}w_j\tilde{w}_j\right\} + \sum_{j=1}^{S} p_{ij}[\sigma_{ij}]^2 - \left\{\frac{1}{(1-\beta)^2}[g]^2\right.$$

$$\left. +\frac{1}{\beta^2}[w_i]^2 + \frac{(1-\beta)^2}{\beta^4}[\tilde{w}_i]^2 + 2\frac{1}{\beta(1-\beta)}gw_i + 2\frac{1}{\beta^2}g\tilde{w}_i + 2\frac{1-\beta}{\beta^3}w_i\tilde{w}_i\right\} + o(1-\beta)^2.$$

Since $\tilde{\boldsymbol{w}} = \boldsymbol{Z}\boldsymbol{w}$ also $\boldsymbol{P}\tilde{\boldsymbol{w}} = (\boldsymbol{Z} - \boldsymbol{I} + \boldsymbol{P}^*)\boldsymbol{w} = \boldsymbol{w} + \tilde{\boldsymbol{w}}$ (cf. (**??**), (**??**)), hence for i-th element we have $\sum_{j=1}^{S} p_{ij}\tilde{w}_j = \tilde{w}_i - w_i$. Recalling that $r_i = \sum_{j=1}^{S} p_{ij}r_{ij}$ and using (**??**) we conclude that $\tilde{s}_i(\beta)$ is equal to

$$\sum_{j=1}^{S} p_{ij}\left\{[r_{ij}]^2 + \frac{\beta^2}{(1-\beta)^2}[g]^2 + [w_j]^2 + \frac{(1-\beta)^2}{\beta^2}[\tilde{w}_j]^2 + 2\frac{\beta}{1-\beta}r_i g + 2r_{ij}w_j + 2\frac{1-\beta}{\beta}r_{ij}\tilde{w}_j\right.$$

$$\left. +2\frac{\beta}{1-\beta}[g]^2 + 2\frac{\beta}{1-\beta}gw_i - 2\frac{\beta}{1-\beta}r_i g + 2g\tilde{w}_i - 2gw_i + 2\frac{1-\beta}{\beta}w_j\tilde{w}_j\right\} + \sum_{j=1}^{S} p_{ij}[\sigma_{ij}]^2$$

$$-\frac{1}{(1-\beta)^2}[g]^2 - \frac{1}{\beta^2}[w_i]^2 - \frac{(1-\beta)^2}{\beta^4}[\tilde{w}_i]^2 - 2\frac{1}{\beta(1-\beta)}gw_i - 2\frac{1}{\beta^2}g\tilde{w}_i - 2\frac{1-\beta}{\beta^3}w_i\tilde{w}_i.$$

Thus

$$\tilde{s}_i(\beta) = \sum_{j=1}^{S} p_{ij}\left\{[r_{ij}]^2 + [w_j]^2 + \frac{(1-\beta)^2}{\beta^2}[\tilde{w}_j]^2 + 2r_{ij}w_j + 2\frac{1-\beta}{\beta}r_{ij}\tilde{w}_j + 2\frac{1-\beta}{\beta}w_j\tilde{w}_j\right\}$$

$$+\sum_{j=1}^{S} p_{ij}[\sigma_{ij}]^2 - [g]^2 - \frac{1}{\beta^2}[w_i]^2 - \frac{(1-\beta)^2}{\beta^4}[\tilde{w}_i]^2 - 2\frac{1+2\beta}{\beta}gw_i - 2\frac{1-\beta^2}{\beta^2}g\tilde{w}_i - 2\frac{1-\beta}{\beta^3}w_i\tilde{w}_i.$$

Then for every $i \in S$ is

$$\lim_{\beta\to 1_-} \tilde{s}_i(\beta) = \sum_{j=1}^{S} p_{ij}\left\{[r_{ij}]^2 + [w_j]^2 + 2r_{ij}w_j + [\sigma_{ij}]^2\right\} - [g]^2 - [w_i]^2 - 6gw_i$$

and

$$\lim_{\beta\to 1_-}\left(\frac{1-\beta^2}{\beta^2}\right)^{k+1}\tilde{s}_i(\beta) = 0, \quad k \ge 0.$$

Thus

$$\lim_{\beta\to 1_-}(1-\beta^2)[\boldsymbol{\sigma}(\beta]^2 = \lim_{\beta\to 1_-}\boldsymbol{P}^*\tilde{\boldsymbol{s}}(\beta) = \tag{23}$$

$$\boldsymbol{P}^*[\boldsymbol{s} + \boldsymbol{P}[\boldsymbol{w}]_{\rm sq} + 2[\boldsymbol{P}[\boldsymbol{w}]_{\rm diag}\boldsymbol{R}^T]_{\rm diag}\boldsymbol{e} - [g]^2\boldsymbol{e} - [\boldsymbol{w}]_{\rm sq} - 6g\boldsymbol{w}]$$
$$= [g^{(2)} - [g^{(1)}]^2]\boldsymbol{e} + 2\boldsymbol{P}^*[\boldsymbol{P}[\boldsymbol{w}]_{\rm diag}\boldsymbol{R}^T]_{\rm diag}\boldsymbol{e}$$

using (**??**) and properties of the matrix $\boldsymbol{P}$.

It corresponds to the formula derived in [**?**].

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Mgr. Milan Sitař, Ing. Karel Sladký, CSc.,
Institute of Information Theory and Automation
Academy of Sciences of the Czech Republic
Pod vodárenskou věží 4, CZ 182 08 Praha 8, Czech Republic
e-mails: sitar@utia.cas.cz, sladky@utia.cas.cz

ASSESSMENT OF COOPERATIVES EFFICIENCY USING STOCHASTIC PARAMETRIC APPROACH

ZLATA SOJKOVÁ, ŠTEFAN COVACI
SLOVAK AGRICULTURAL UNIVERSITY IN NITRA, SLOVAKIA

1. Introduction

Significant changes, which take place in transition economies, are accompanied with strong requirements of efficient performance of entrepreneurial subjects. Efficient performance in accordance with production economics can be simply expressed as maximisation of outputs from a given set of inputs or minimisation of inputs given the outputs. The standard measure of this kind of efficiency is called *technical efficiency*. In professional journals there are published results of several analysises of efficiency on commodity, firm, sector, regional, and macroeconomic level. Mostly nonparametric methodology known as Data Envelopment Analysis (DEA) is used. In Slovakia Fandel (2000a, 2000b) has applied it in agriculture on firm and sector level. DEA itself cannot implicate influence of qualitative factors such as social or demographic variables. Usually it is necessary to apply Tobit regression analysis. As an alternative to DEA parametric approach known as stochastic frontier production function is used. It has not been applied in Slovakia yet.

2. Methods and Material

From the methodological point of view in the paper stochastic parametric approach has been applied to estimate production frontier from which output orientated technical efficiency measures have been derived. Aigner, Lowell and Smith (1977) and Meeusen and Van den Broeck (1977) have simultaneously proposed model of stochastic production frontier. They assumed random shocks outside the control of producers, which can affect output and thus measure of efficiency. They divided shocks into two components.

The analysis assumes cross sectional data of the quantities of N inputs used to produce a single output, which are available for each n producer. The stochastic production frontier can be written as follows:

$$y_i = f(x,\beta).\ exp\ (v_i)\ TE_i \qquad (1.1)$$

where

y_i is the output of producer i, $i=1,2,....n$

x_i is a vector of N inputs used by producer i

$f(x,\beta\, exp(v_i))$ is the stochastic production frontier

β is a vector of parameters to be estimated

TE_i is the output-oriented technical efficiency of producer i

where $f(x,\beta)\ .\ exp\ (v_i)$ is the stochastic production frontier. The stochastic production frontier consists of two parts: a deterministic part $f(x,\beta)$ and common to all producers and a producer specific part $exp(v_i)$, which capture*s* the effect of random shocks on each producer. If the production frontier is specified as being stochastic, equation *(1.1)* becomes

$$TE_i = \frac{y_i}{f(x,\beta\,)\,.\,exp(v_i)} \qquad (1.2)$$

From equation given about we can rewrite TE_i as the ratio of observed output to maximum feasible output. $TE_i < 1$ gives a measure the proportion (shortfall) of observed output from maximum feasible output. The value $TE_i = 1$ means the achievement of technical efficiency, when the observed and maximum feasible values are identical. Impact of environmental specific dissimilarities between individual producers and excluding the random shocks from technical efficiency is the principal reason to prefer the stochastic approaches for assessment the technical efficiency in the last years. In the paper maximum likelihood method for estimation of stochastic production frontier and software FRONTIER 4.1 (Coelli, 1994) has been used. If we assume that $f(x,\beta)$ takes the log-linear Cobb-Douglas form, then stochastic production frontier model given in equation *(1.1)* can be written as

$$ln\ y_i = \beta_0 + \Sigma\beta_k\, ln\ x_{ki} + v_i - u_i \quad , \qquad where \quad u_i \geq 0\ ,$$

where v_i is white noise

u_i nonnegative component of the technical efficiency

The technical efficiency is computed as $TE = exp\ (-u_i),$

where $u_i = [\beta_0 + \Sigma\beta_k\, ln\ x_{ki} + v_i] - ln\ y_i$

Comparative analysis has been performed on the cross sectional data for 61 cooperatives in Slovakia in the year 1998 operated on the different soils quality As an aggregate output added value (AV) in thousand SK is used. In the model three inputs are included: labor (L), capital (C) in the thousand SK and farmland (V) in ha. Different farmland quality is incorporated into model by dummy variable for two groups of cooperative farms operating in worse and in better quality farmland.

3. RESULTS AND DISCUSSION

Because of the character of the C-D function all data have been log transformed. Due to all farms were operating on various soil quality, the quality was expressed by zero-one (D) variable which was defined according to land price group (SCP). Farms were divided into two groups: those operating on worse land (SCP 1-15, *D=0*) and farms operating on better land (SCP 15-20, *D=1*). In the first group we have 35 farm and the in the second group we have 26 farms.

By the parametric stochastic approach the stochastic model is used, which enables implementation of quality of soils through dummy variable (D). The interactive terms which enable to estimate the variously impact of the inputs in the better or worse soils quality conditions (it means different elasticity of inputs) are in the model included. Log - linear model C-D of the production function has the following form:

$$ln\ AV_{odh} = \beta_0 + \beta_1 ln\ L + \beta_2 ln\ C + \beta_3 ln\ V + \beta_4 D + \beta_5 lnL\ .D + \beta_6\ lnC\ .\ D + \beta_7 lnV.\ D + (v_i - u_i)$$

Maximum likelihood method estimations are shown in table 1. From the model we can derive two models, one for better and one for worse soil quality:

worse soils quality (D=0) $lnAV = 4,343+1,162lnL+0,404lnC-0,583lnV$

better soils quality (D=1) $lnAV = 3,663+0,456lnL+0,204lnC+0,309lnV$

Table 1. Parameters of Cobb-Douglas production function and its verification

Parameters		Coefficients.	Standard error	t-ratio
Intercept	beta 0	4,343	1,413	3,07
lnL	beta 1	1,162	0,267	4,35
lnC	beta 2	0,404	0,157	2,58
lnV	beta 3	-0,583	0,177	-3,30
Du	beta 4	-0,680	2,797	-0,24
lnL*D	beta 5	-0,706	0,359	-1,97
lnC*D	beta 6	-0,200	0,442	-0,45
lnV*D	beta 7	0,892	0,397	2,25

The parameters by the individual variables represent the coefficient of elasticity Interesting is the comparative analysis of the coefficients for the better and worse quality of soil. While in the worse soil condition increasing inputs of the labour by 1% leads to the consequence of increasing the added value by 1.16%, in the better of the soils condition that mans increasing only by 0.46% Increasing of capital by 1% increases value added in worse conditions by 0,40%, but in better conditions only by 0,20%. One percent increase of the farm size in worse conditions decreases value added by 0,58%, but in better condition by 0,31%.

Analysis of technical efficiency

From the estimated stochastic production frontier technical efficiency measures are derived. Table 2 presents basic statistics of the technical efficiency according to various soil quality.

Table 2. Comparison of basic statistic characteristics of technical efficiency

Characteristics	minimum	maximum	average	stand. deviation
worse condition	0,092	0,944	0,650	0,209
better condition	0,554	0,907	0,727	0,107

As it is evident from the table 2, farms operating on worse soil achieve average technical efficiency of 0,65 and farms operating on better soil achieve average technical efficiency of 0,73. It can be interpreted that farms operating in worse conditions achieve only 65% of the maximal output (value added) and farms in better conditions achieve from the given inputs 75% of output. Analysis showed that in the group of worse conditions farms we can find least efficient farm (TE=0,092), but also the farm with the highest technical efficiency score (TE=0,944). An efficiency measure of the farms operating on better soil varies within the interval from 0,554 to 0,907.

Interval distribution of efficiency measures is shown in picture 1. At the picture 2 efficiency measures of all firms are showed. Also from the figure 2 it is evident the higher variations of TE of the farms operating on worse condition and lower average technical efficiency

Figure 1 Distribution of technical efficiencies of cooperatives operating by worse land quality and better land quality

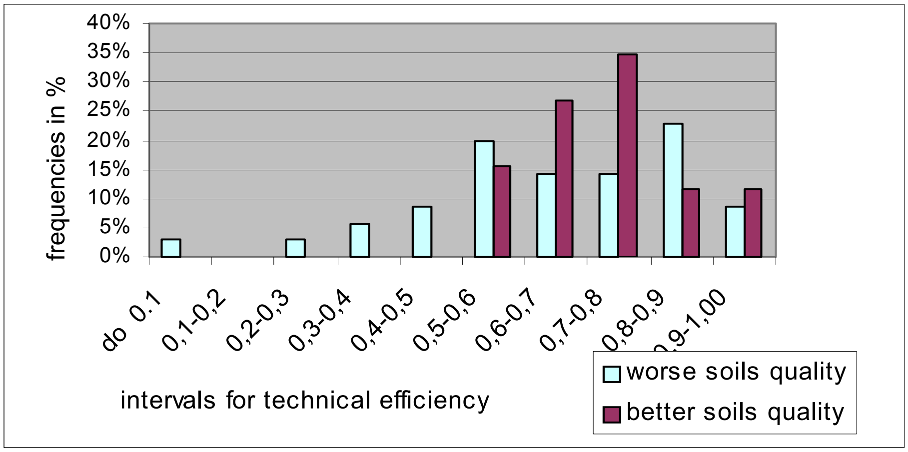
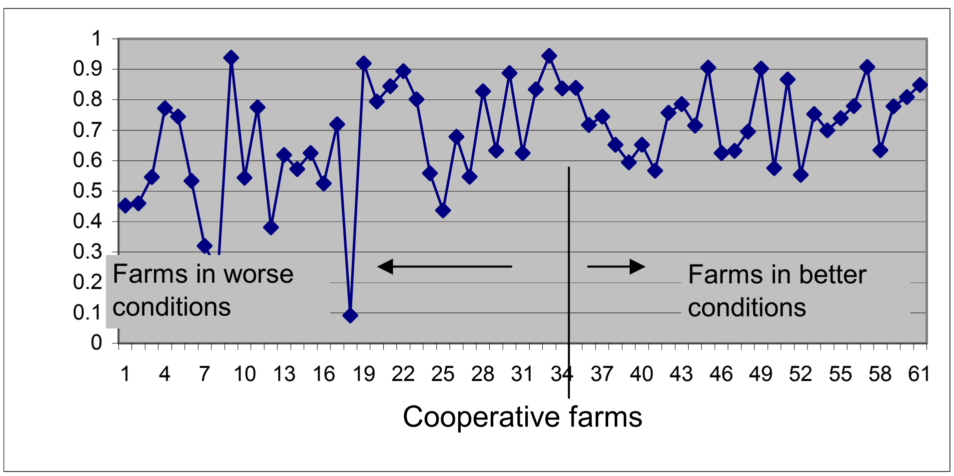


Figure 2 Technical efficiencies for all cooperative farms sorted by quality of farmland



We have learned that efficiency variability of farms operating in worse conditions is significantly bigger than of farms operating in better conditions (significance level = 0,0004). One-sided test assuming equal means showed significance of 0,034.

The paper present alternative approach to measure output-orientated measures of technical efficiency via estimation of the stochastic frontier production function expressed by a Cobb-Douglas production frontier. None of the methodologies is accurate enough. The advantage of the presented approach is nonlinearly of the modeled relation between inputs and outputs and ability to simultaneously implement not only quantitative, but also qualitative variables, so called environmental, variables which are out of control of the manager and variables related to the farm management. From the analysis it is possible to conclude that not only relevant inputs determine efficiency measure but also quality of management. The weak point of the presented analysis is law number of farms, which was determined by the access to the data. Expanded data set can increase information power of the results. It may be interested from the methodological point of view to compare result of the non-parametric approach with the parametric one. This topic however exceeds the focus of this article.

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doc.Ing. Zlata Sojková, CSc., Štefan Covaci

Department of Statistics and Operational Research, Faculty of Economics and Management, Slovak Agricultural University, Tr. A. Hlinku 2, 949 01 Nitra, Slovakia, e-mail: zlata.sojkova@fem.uniag.sk, stefan.covaci@fem.uniag.sk

APPLICATION OF THE FUZZY DELPHI IN CONSTRUCTION OF DESCRIPTOR´S BORDERS

BEÁTA STEHLÍKOVÁ, MILAN-MIHAI HAJDUK
SLOVAK AGRICULTURAL UNIVERSITY NITRA, SLOVAKIA

Abstract

In this paper the possibilities of the fuzzy Delphi method in the work with plant genetic resources are considered. There are describing possibilities for application of fuzzy set theory in creation of borders of descriptors on a base of standpoint of experts, obtained results are natural and by this better reflex of reality as a classical build up borders of descriptors. There are two differences in our technology for construction of descriptor´s borders. In the fuzzy Delphi method the experts are asked for forecasting and the data are presented in a form of triangular numbers. In our procedure the experts are asked for descriptor´s borders and the data given by the experts are presented in a form of trapezoidal numbers.

At present with increasing interest for conservation of existing plant genetic resources, the need for improving of information already existing collection of plants is increasing. One of the ways is gradual description of collections with applications of different types of descriptor lists. By usage of descriptor list is effort to describe in details specific biological material. On description are used different descriptors. During evaluation very often arose situation when very similar objects are arranged to the different levels of descriptor, where the reason for it is unsuitable determination of borders for level of descriptor. The standard procedure is described by Zajcev (1984). Following procedure with the help of fuzzy sets is general. It enables criterion of descriptors of base of experts´ standpoints. Stehlíková, Gažo (1998) construct border of descriptors using expected value of Simpson´s distribution.

1. Material and Methods

Fuzzy Dephi method is a generalization of the classical method for forcasting known as Delphi method. The essence of Delphi method can be described as follows. Experts may be asked to forecast the general state of the economy, science, busines, etc.. The data which have subjective character are analyzed by finding their average. The results are told to the experts. The experts review the results and provide new estimates. This process could be repeated again and again until the outcome forecast converges to a reasonable solution.

The fuzzy Delphi method was introduced by Kaufman ad Gupta (1998). In the fuzzy Dephi method the experts are asked to provide the posible realization of a certain event (the earlist date, the most plausible date, the latest date). The data given by the experts are presented in a form of triangular numbers.

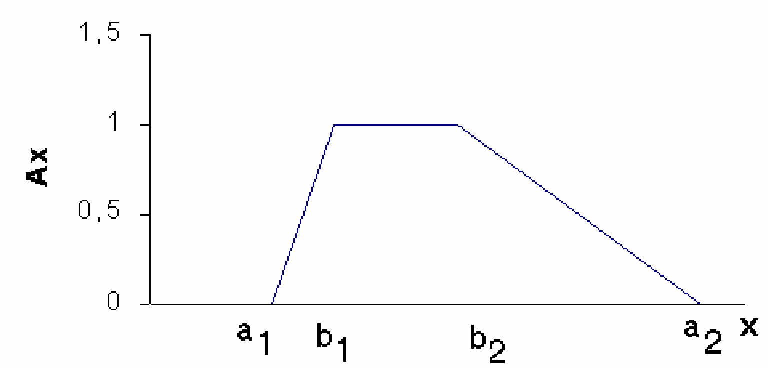
There are two diferences in our new technology for construction of descriptor´s borders. The experts are asked for descriptor´s borders and the data given by the experts are presented in a form of trapezoidal numbers.

A trapezoidal fuzzy number A is defined on real numbers by

$$A = \begin{cases} \dfrac{x - a_1}{b_1 - a_1} & \text{for } a_1 \le x \le b_1, \\ 1 & \text{for } b_1 \le x \le b_2, \\ \dfrac{x - a_2}{b_2 - a_2} & \text{for } b_2 \le x \le a_2, \\ 0 & \text{otherwise} \end{cases}$$

Triangular fuzzy number is a particular case of trapezoidal fuzzy number for $b_1 = b_2$.

Figure 1 Trapezoidal fuzzy number $A = (a_1, b_1, b_2, a_2)$



Asked experts determine four points: point a_1 which certainly to the category still do not belong, point a_2 which already do not belong and such poits b_1 and b_2 that values between them to be investigated category certainly belong. Let A_i i = 1, 2, ..., n represent the standpoint of experts. If $A_i = (a_1^{(i)}, b_1^{(i)}, b_2^{(i)}, a_2^{(i)})$ i = 1, 2, ..., n are trapezoidal numbers , then trapezoidal average is

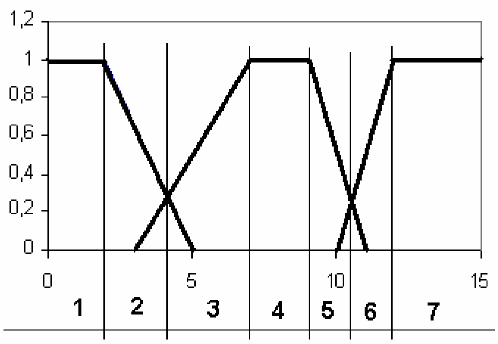
$$A_{ave}^{1} = \left(\frac{\sum a_1^{(i)}}{n}, \frac{\sum b_1^{(i)}}{n}, \frac{\sum b_2^{(i)}}{n}, \frac{\sum a_2^{(i)}}{n}\right).$$

Then for each expert the deviation between A_{ave}^{1} and A_i is computed. The deviation $A_{ave}^{1} - A_i$ is send back to the expert i for reexamination. Each expert i (i = 1, 2, ..., n) presents a new are trapezoidal numbers. The process could be repeated again and again until two trapezoidal averages become reasonably close. The criterion of „reasonably close“ is that the difference between two defuzzifying trapezoidal average is less as $\varepsilon > 0$.

The defuzzification c of the trapezoidal fuzzy number $A = (a_1, b_1, b_2, a_2)$ as center of area can be performed using formula $c = \frac{a_1 + b_1 + b_2 + a_2}{4}$.

Determination of the borders of characters for scale of descriptor is evident from figure 2.

Figure 2 Determination of the borders



We can show procedure on a simulated example (Stehlíková, Gažo, 1998).

2. Resuls

Our task is to determine borders of descriptor of the medium high content of the mater X. Let ε be 0,20. Data of five experts are given in table 1.

Table 1 Data of experts

Number of expert	a_1	b_1	b_2	a_2
1	15	25	50	60
2	13	21	48	51
3	12	15	40	46
4	20	25	35	40
5	20	25	45	50

The trapezoidal average is A^1_{ave} = (16; 22,2; 43,6; 49,4). The deviations between A^1_{ave} and A_i are presented in table 2.

Table 2 Deviation $A^1_{ave} - A_i$

	Deviation for			
Number of expert	a_1	b_1	b_2	a_2
1	-1	2,8	6,4	10,6
2	-3	-1,2	4,4	1,6
3	-4	-7,2	-3,6	-3,4
4	4	2,8	-8,6	-9,4
5	4	2,8	1,4	0,6
defuzzification c_1	32,8			

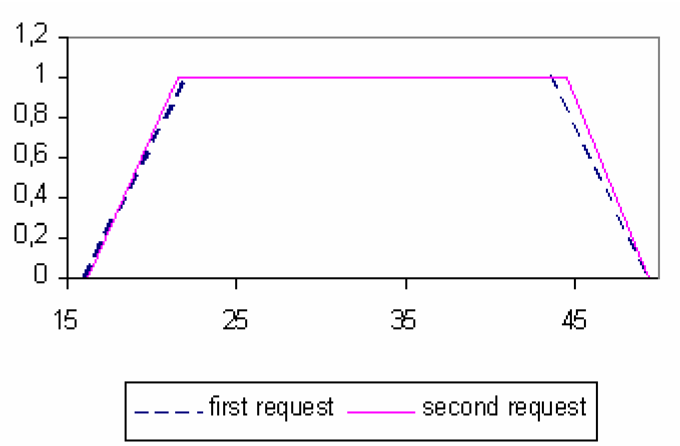
Table 3 Trapezoidal numbers presented by experts (second request)

Number of expert	a_1	b_1	b_2	a_2
1	15	22	45	55
2	15	21	45	50
3	13	17	45	50
4	18	23	43	42
5	20	25	45	50
defuzzification c_2	32,95			

The trapezoidal average is A^2_{ave} = (16,2; 21,6; 44,6; 49,4). We see, that $|c_1 - c_2| = 0,15 < 0,20 = \varepsilon$. The borders of descriptor of the medium high content of the mater X are

(16,2; 21,6; 44,6; 49,4).

Figure 3 Average trapezoidal numbers



Important part of biodiversity as a total is created by genetic resources of useful plant species. They represent important share in provision of food, health and other increasing demands of manking. Plant genetic resources became target of economical and strategic interests. The most important activities is description and obtaining information about individual samples. Described procedure is ideal for construction of descriptors.

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doc. RNDr. Beáta Stehlíková, CSc.
Department of Statistics and Operational Research, Faculty of Economics and Management, Slovak Agricultural University, Tr. A. Hlinku 2, 949 01 Nitra, Slovakia, e-mail: Beata.Stehlikova@uniag.sk
Milan-Mihai Hajduk
Department of Statistics and Operational Research, Faculty of Economics and Management, Slovak Agricultural University, Tr. A. Hlinku 2, 949 01 Nitra, Slovakia, e-mail: milan.hajduk@fem.uniag.sk,

LOGISTIC SERVICE PROVIDER (LSP) TASK CHANGES IN E-COMMERCE

AXEL STOLZE
MARTIN-LUTHER-UNIVERSITY HALLE-WITTENBERG, GERMANY

1. Relevant data to the Problem

In the past few years and still today, the words and expressions such as:

- Globalization
- Shortening of product lifecycles
- Concentration on core competencies
- Implementation of the customer into production processes
- Atomization of quantity in delivery services
- Invention time-reduction in the Information and communication technology
- Creation of global network

or discussions about E-categories like

- E-Business
- E-Commerce
- E-Logistics

have been influencing the development driven decision-process of enterprises and business service offerings.

In spite of some ancient economic appearances, especially at the end of the past year, we can suppose that these mentioned expressions, will stipulate and mark considerably the economic evolution in the years to come. This hypothesis is endorsed in numerous studies.
The existing companies on the market are forced to face those challenges if they want to survive. They especially must do so (to face these fast changing development in business driven areas), if the companies want to strengthen or to evolve their current position. 1)
The companies from the logistics area are marked by these innovations in a particular way, because according to the industry, affected logistics have become an unusually highly considered success factor.

Already a few years ago it was recognized that the logistics area of a company plays a key role in the potential of improvement, which was till a very recent time badly underestimated in its volume/capacity.

The logistics can considerably contribute to the realization of requirements, which rise from the already named highly actual and relevant face of affairs.

This realization of requirements by companies from the logistics area leads to enormous change in assignment/task –spectrum, offering structure and variety of the logistics providers.

To make these assignments clear, it is important and meaningful to define some terms/concepts having the possibility to restrict their contents.

2. Contents restriction

While the logistics has used and till now still uses the efficient outlay of the physical business processes of the Information and Communication systems and develops them, does the electronically supported trade in form of E-Business becomes more and more known. At the same time is the logistics a necessary condition for an economically effective and gain-driven internet-based trade. 2)

E-Business

We understand by E-Business an electronically realized trade. E-Business encloses hereby the exchange of Informations between businesses (B2B), respectively between businesses and customers (B2C), see picture 1, and uses them to manage and realize market transactions trough private and public networks. 3)

PICTURE 1: Demonstration of the relevant E-Business applications 4)

A component of E-Business is the category of E-Commerce.

E-Commerce

Trough this intersection with E-Business has the potential customer the possibility to access trough internet the production system of the Company/ Business (B2C) and can receive Information about the Business and its products, and/or order and pay for the products and services. 5)

E-Commerce has not only changed the art of selling products, but has also considerably influenced the art of delivery of these products. 6)

As it will be shown later, have the LSP in the concept of E-Commerce, especially, but not exclusively, to face to a big variety of new challenges/tasks, which can contribute under their proper perception to the reinforcement of the providers' market position.

The demonstrated categories

- E-Business and
- E-Commerce

need a good elaborated/ well designed, mature and e-market oriented E-Logistics.

E-Logistics

"E-Logistics includes the strategic planning and development of all necessary logistics-systems and processes for the electronic trade as well as theirs administrative and operative structure for physical realization." 7) The fundamental assumption hereby is the competence at work with networks, which represent the base for these processes. 8)

From the short-held descriptions is clear, that the content and therewith connected physical layout of the logistics processes has an enormous importance.

3. New tasks of the LSP

Following the studies and research made in the USA at the beginning of the new millenary, causes a 100 US $ turnover up to 25 US $ logistics costs on the way to the end-customer. 9)

Whereas the use of electronic media is today a usual and normal thing in the business/trade, is the physical realization of the signed businesses very often abnormally difficult.

Beside the problems in connection with the perception of new activities of the LSP in the purchase and distribution-process, remains a very big need in the realization of new logistic-oriented ideas in the production processes of potential ordering customers. This fact is bemoaned since a long time, because for the providers of integrated logistics systems it offers best chances concerning the strengthening and enlarging of the market position and with it a stabilization of the company as well. The logistics providers are of course still active in the same core-competence business fields. To this core-businesses belong of course the transportation, transshipping and storage. With the globalization of business processes and with the fast evolution of the IT sector have risen in the market new competencies which cause that the LSP looks and acts like a system provider.

A few words should be said at least to some of these new competences.

Stockmanagement

LSP are responsible for providing and management of the warehouse for the producer.

Commissioning (commission business)

The physical realization of orders, which are ordered trough E-Business, represents in many ways a problem. Especially the area of commission businesses can represent the main reason for delayed delivery. Companies, mainly from the conveying area, practice therefore more and more outsourcing of the whole commission work. 10)

In place mounting

LSP, who are charged with the delivery of a good, are prevailingly changing to a modus, where they can offer and produce add-value services whereby delivered results/products are set-up in place and are connected to all delivery/settlement services.

C-part management

he modern C-part management refers to the resources of a LSP. 11)

Return-management

A domain, which should be included in the offer of modern LSP, is the return-management.

Structuring and using of continuos Information and Communication systems (ICS)

The demonstrated process of tasks described in this report/contribution which is connected with the development of conventional forwarding agent to a complex LSP, needs in connection with all here demonstrated activities an intensive information and communication networking of all production process partners. 12)

Tracking & Tracing

This provided service from a LSP enables the customer at any time to check the status and place of his order. 13)

Data analysis

Logistics tasks are anchored in the whole supply chain. This means a big area, which includes numerous information flows and flows of goods, from the procurement up to the consumer market. Extraordinary quantities of data have to be handled, analyzed and evaluated. With the use of sophisticated software, find the high-developed LSP's here another new task area. 14)

E-Fulfillment

A very complex E-Fulfillment is as a management concept of the logistics companies the more and more important. 15)

As a part of this concept, does the LSP the wholly or partially out-sourced task realization in charge of a company. 16)

Logistics management of the whole supply chain

With the diverse market and economical changes and especially with the growing concentration of active companies from the production process on core competencies, evolved during the last years the LSP's in a surprising manner. As stated before, this evolution was carried out, trough the realization of classical logistics services up to the offering of extensive system servicing.

These system providers (3PL) are very often strongly included in the logistics concepts of their ordering customer. 17)

The customer asks more often the most accomplished complex logistics providers for overtaking/offering the whole supply chain management instead of providing just some partial solution for the production process. 18) 19)

In this trend emerge "Fourth Party Logistics Providers" (4PL). 20)

Virtual logistics-marketplace administration

Another task, which could be handled by the complex LSP, is the preparation and managing of virtual logistics-marketplaces. 21) 22)

4. Conclusion

Against this background and chosen examples should be clear, that the logistics companies have very good development/success chances on markets which are becoming more and more global if they start offering extensive services and are ready to integrate themselves into the complex structure of the production and service providers area.

The signification of the logistics as a strategic factor of the company/lead-management is generally recognized. The only problem is, today as well as before, the realization of numerous, theoretical opportunities.

A wide international research in connection with the evolution of the logistics area/branch in the coming five to then years has given the following results: 23)

- Logistics costs are for the companies a very important cost-factor, which calls for the attention of the top management. Investments into efficient logistics are profitable.
- The logistics tasks are still now as before handled separately one from each other. A resort exceeding sight/overview approach is missing.
- This causes unnecessary costs. Extensive economical reserves can be detached by solving these discrepancies.
- The art and level of logistics task assignment to qualified LSP vary considerably between branches and companies.
- With the out-sourcing of logistics services gain the extensive strategic co-operations on importance.

From the research is also apparent, that in the long-run only those logistics companies will have a chance to survive, which:

- are considering themselves and act not only as a provider of services, but as a part of a high complex logistics network,
- dispose of a high responsibility in the management of complex flows of goods,
- by all means are about to apply the most modern information and communication technologies.

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PD Dr. Axel Stolze

Martin-Luther-Universität Halle-Wittenberg, Große Steinstr. 73, D-06108 Halle, GERMANY, e-mail: stolze@wiwi.uni-halle.de

MATHEMATICAL PROGRAMMING MODEL OF THE CRITICAL CHAIN METHOD

TOMÁŠ ŠUBRT, PAVLÍNA LANGROVÁ
CUA, SLOVAKIA

Abstract

Currently there is increasingly indicated that most of classical project management methods is not suitable for working with ill structured and not enough quantified inputs. Using classical methods (CPM, MPM, PERT, GERT) precise results from the inaccurate numbers are obtained. Applying such numbers on a real problem points to the non-performance of project duration through the critical path overrun. New methods that are able to work with not enough quantified inputs start to gain ground. Beside soft approach based on fuzzy modeling one rather new method called Critical Chain will be mentioned in this paper. Critical Chain method has mainly empirical character, it has a relatively sufficient software support, but it is still waiting for detail analysis and for precise mathematical description. The objective of the following paper is to present one possibility of how to model and solve a Critical Chain problem using discrete programming tools.

1. Introduction

Classical approaches to project management and their applications are commonly used, but there still exist a lot of factors, which can negatively influence task duration, and commensurate with these durations it influences project duration too. Using Theory of Constrains and their direct application to project management known as Critical Chain, some negative factors could be eliminated and so the project schedule can be finished in time without any overrun. The aim of the Critical Chain method is to estimate task durations and to advert all factors, which can influence (extent) these durations. Among them human behavior and recourse availability could be mentioned. Both of these factors play an important role and as we can see they play a big role in duration estimations.

The Critical Chain method doesn’t follow the duration of a task as a whole, but it divides this duration into two different parts. The first part is a time needed for task completion and the second one has a meaning of a reserve, protecting the task against delay arising from turn up for the book. The time estimate is given like a sum of these two parts. While the first part is changeless, the second part is very variable and it is dependent on person making the estimate – on his/her knowledge, experiences and propensity to risk. These estimations are not single numbers but rather statistical entities, reflecting the probability of task completion in a certain amount of time. An aggressive estimate, reflecting only the amount of work (the first part) might have a 50% level of confidence, while a longer realistic estimate, against which the resource is comfortable committing to, might be closer to an 85-95% range of confidence. The difference between 50% and 95% estimates is a safety and because of it the task estimate includes a plenty of safety. Even thought this safety is often the longer part of estimate, it won't be used, because in most cases it will get wasted. The main reason of it is that work on task is usually started later than planned and on the other hand when the work is finished earlier, saved time is not used, because the resources are often allocated on other places of projects.

The Critical Chain methodology requires the schedule to be built within only the time for completion without any safety. The main idea of this approach is to protect the project and not individual tasks, because the most important think is finishing the project in time. The whole approach can be described in following steps: 1) Creating the project network 2) Resource leveling, 3) Identifying the Critical Chain ("Critical Chain is a resource leveled critical path reflecting the task and resource dependences that determinate project duration") 4) Creating and inserting buffers. For purposes of our further approach let's precise a little bit more the fourth step dealing with buffers. In the phase of buffers creating there are used individual safeties for each task. The safety associated with the critical chain tasks can be shifted to the end of the chain, protecting the project deadline from variation in the critical chain tasks. This concentrated aggregation of safety is called "project buffer" and it protects project finish date from Critical Chain variation. The safety of individual tasks allows for everything to be bad with the same probability as everything to be well. It means that only in 50% of cases the task is finished earlier and in other 50% later. So the project buffer can be half duration smaller than the sum of its parts (tasks). Using project buffer tasks on critical chain are protected, but we have to keep in mind, that there are noncritical tasks in project schedule too. According to the traditional approach they have to start as soon as possible and using their slacks all other tasks are protected. The demerit of the slack is that it can get waste. In the Critical Chain method this problem is eliminated in the same way using another type of buffer called "feeding buffer". The feeding buffer is placed whenever any feeding chain (noncritical chain) joins the critical chain and its function is to protect all tasks on the feeding chain. In the worst case it can be expended all feeding buffers, but the due date (project finish date) is still protected from project buffer.

"Resource buffers" usually in the form of an advance warning, are placed whenever a resource has a job on the critical chain, and the previous critical chain activity is done by a different resource.

A very moot point deals with the size of safety for individual tasks. The statement that the project buffer is a half of sum of safety of Critical Chain and the feeding buffer is approximately one third of feeding chain seams to be very simplified. Their size depends on many factors. For example it is very important if the task is unique or if it is common and was used many time before. The unique task will probably have longer safety time that a common one. We have to keep in mind, that there are also huge differences in safety between short and long tasks because. These factors can influence each other and so it is very difficult to estimate this safety for each task.

The Critical Chain schedule is primary based on inserting buffers and on Critical Chain identification. Compared with traditional critical path methods the Critical Chain schedule is often longer, but there exists a high probability of project completion on time.

Mathematical Programming Model of an Activity on Node Critical Path Problem as a Fundament for CC Model Definition

Even thought the majority of project management mathematical methods are based on activity on arc (AOA) approach we shall apply an activity on node (AON) network model for the formal project interpretation because all software realization of CC method are derived from it. In AON graphs all project tasks are represented by nodes and their relationships are represented by arcs. When finding critical path, each project task can either be critical or noncritical. The arc joining two tasks (nodes) is a part of a critical path if and only if both tasks at its ends are critical too. Let's define two kinds of variables – x_i and x_{ij}, where $x_i = 1$ when task *i* is critical, $x_i = 0$ otherwise and $x_{ij} = 1$ when the arc *ij* lies on a critical path, $x_{ij} = 0$ otherwise. The first and the last task must lie on a critical path and so $x_1 = 1$, $x_n = 1$. Each task has a parameter – t_i in the meaning of its duration and each arc has only one parameter – t_{ij} in the meaning of a lag (or lead time when negative), i.e. the time interval between the end of previous task and the beginning of the next one. The set of feasible solutions *X*, $(x_i \in X, x_{ij} \in X)$ is defined using following constraints:

$$-x_i + \sum_{j \in R_i} x_{ij} = 0; \quad i = 1, 2, ..., (n-1)$$

$$x_j - \sum_{i \in P_j} x_{ij} = 0; \quad j = 2, 3, ..., n$$

$$x_1 = 1; \ \ x_n = 1$$

$$x_i \in \{0;1\}$$

$$x_{ij} \in \{0;1\} \tag{1}$$

where

i…index of predecessor or of a start node

j…index of successor or of an end node

x_i, x_j… variables representing nodes (tasks)

x_{ij}… variables representing arcs (task dependencies)

P_j…set of j-th task predecessors

The objective function differs could be described by the following way:

$$\sum_{i=1}^{n} (t_i x_i + \sum_{j \in R_i} t_{ij} x_{ij}) \rightarrow \max \tag{2}$$

where

t_i …i-th task duration

t_{ij}…time interval between the end of i-th task and

With respect to a special type of constraint coefficient matrix **A** = $(a_{ij})_{m.n}$ where $a_{ij} \in \{-1;0;1\}$ and to a type of RHS vector **b** = $(b_i)_m$ where $b_i \in \{0;1\}$ (matrix is completely unimodal) the bivalent solution is guaranteed when solving this mathematical programming problem using standard simplex algorithm. Solving this problem we obtain a set of critical tasks and consequently the whole critical path. The objective function value determines a length of critical path. For analyzing noncritical tasks parameters we can use sensitivity analysis of cost coefficients. Applying standard cost coefficient sensitivity analysis algorithms for linear problems we obtain intervals of stability for both types of costs coefficients, i.e. $t_i \in \left\langle \underline{t_i}; \overline{t_i} \right\rangle$ and $t_{ij} \in \left\langle \underline{t_{ij}}; \overline{t_{ij}} \right\rangle$. Using intervals of stability for task duration – t_i – we can define a total slacks for noncritical tasks. A noncritical task is represented either by a non-basic variable or by a basic variable with zero value. A task becomes critical when its duration (its cost coefficient value) exceeds a maximum limit (upper bound of stability interval). The total slack can be expressed as

$$s_i^1 = \overline{t_i} - t_i \tag{3}$$

where s_i^1 is a total slack of *i-th* task and $\overline{t_i}$ is an upper bound of an interval of stability for t_i.

CC Mathematical Programming Model

For the Critical Chain mathematical model let's assume, that the project is represented by a classical AON network graph (most of all it means it is continuous and it has one starting node and one ending node) and is really resource leveled. Furthermore let's assume each project ends by a dummy node connected with last project task by a dummy arc (this "last arc" will be used for project buffer inserting later in this text) and so the number of nodes in CC model (n) is one unit higher than the number of project task is. We have to keep in mind that each task is protected by a reserve (safety). The amount of such reserve time is implicated in objective function cost coefficients. According to the assumptions mentioned above, a task duration t_i can be divided into two parts: q_i – task reserve, it means task protection from delay and (t_i - q_i) – the pure (or clarified) duration. Furthermore let's assume that all dependency lags (t_{ij}) are equal to 0. (In the next phase of the model these lags (objective function cost coefficients of arcs) will be used for CC buffers. Accepting this a mathematical programming model for the first phase (critical chain definition and project buffer establishment) can be defined as follows

$$\sum_{i=1}^{n}((t_i - q_i)x_i + \sum_{j \in R_i} t_{ij}x_{ij}) \rightarrow \max \qquad (x_i \in X, x_{ij} \in X) \tag{4}$$

where
q_i ... task duration reserve

Solving this model we obtain a critical path – its structure and its length. In this phase of model solving the critical path determines a critical chain as well.

Project Buffer

According to the conclusions of CC method, the amount of time needed for project finish date protection in the form of project buffer is determined by the formula

$$PB = \sum_{i=1}^{n} q_i x_i \tag{5}$$

Because $x_i = 1$ only on a critical path, this formula guarantees summarization of CC reserves only. In the next step of the model this buffer will be interpreted as a cost coefficient of "last arc" $x_{n-1,n}$ it means $t_{n-1,n} = \sum_{i=1}^{n} q_i x_i$.

Feeding Buffers

The aim of feeding buffer is to protect critical chain from potential delays on non-critical chain activities (supporting paths) and it has to be placed whenever a non-critical activity joins the critical chain. According to the classical CC approach a feeding buffer protects only the longest non-critical sequence (chain). Let's modify this assumption and suppose, that feeding buffer protects all non-critical activities lying between two critical ones, it means all zero variables placed in the network between two nonzero ones (two nodes). Such set of task will be called "feeding subgraph". We suppose that realistic value for each feeding buffer will be a value summarizing all reserves on a feeding subgraph plus the largest total slack on this subgraph. The reason for such preposition is that respecting any smaller slack could precociously consume buffer.

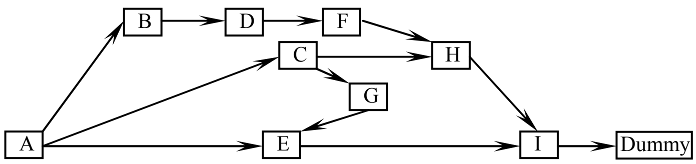
For description of feeding buffer let's define a feeding subgraph P^r, its starting nodes (represented by variables $x_k^{r(s)}$), and its unique end node (represented by variable $x_k^{r(f)}$). Each variable can be a starting (end) point of a feeding subgraph satisfying:

$$(x_k = x_k^{r(s)}) \Leftrightarrow \forall (x_i, x_{ij}) : (i \in B \wedge ij \in R)$$
$$(x_k = x_k^{r(f)}) \Leftrightarrow \exists (x_j, x_{ij}) : (j \in B \wedge ij \in R) \qquad (5)$$

where
B... set of basic nonzero variables indexes
R... set of zero variables

Using these variables and terms an *r-th* feeding subgraph can be defined as a set of all paths, starting in any node $V^{r(s)}$ and ending in a unique node $V^{r(f)}$.

Example: *Let' s have a following graph and let's suppose, that critical chain is formed by tasks A, E, I*



On this graph two feeding subgraph are defined: P_1 as {B-D-F-H; C-H}, and P_2 as {C-G}. Cost coefficients of arcs G-E and H-I will be reserved for placement of feeding buffers and cost coefficient of arc I-Dummy for placement of project buffer.

In our approach let's model feeding buffers using cost coefficient of last arc in each subgraph P^r, it means as a cost coefficient of arc $x_{k,j}^{r(f)}$, where $j \in R$ and according to previous definition it could be described as

$$FB_{P^r} = t_{k,j}^{r(f)} = f(q_i)$$
$$f(q_i) = \sum_{i \in R_r} (q_i) + \max_{i \in R^r} (s_i) \qquad (6)$$

where
R^r... set of indexes of r-th feeding chain variables
f(q_i)... function for FB calculation

For $f(q_i)$ calculation we assume summarization of all feeding chain tasks' safeties (reserves) but as an acceptable calculation we can take a sum of all reserves on the longest feeding path (classical Goldratt's approach) or applying the maximum size of task reserve on the whole feeding subgraph etc.

Defining all these buffers we obtain a mathematical programming model with the same definition of feasible solution set, but with another objective function cost coefficients. Solving this model we obtain either the same structure of basic variables vector or we obtain another basic solution. In such case (majority of cases) a classical critical path includes inserted buffers too. If the length of CP is far away from former CC length, we have to think about the size of task reserves. After solution analysis we should begin with working on a project, track its progress, consume buffers and potentially change the CC structure too.

2. Conclusion

The objective of our paper was to show one possibility of how to solve Critical Chain method models using mathematical programming tools. The main effect of this approach is in AON graph interpretation of this model. Because of real activities are on nodes and buffers are modeled using arcs, no changes in graph structure (and no changes in mathematical programming model definition) need to be done. All buffers inserting are made only using changes in cost coefficients of arcs. Using standard Excel linear programming tools an optimal solution of CC model can be obtained.

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Dr. Ing. Tomáš Šubrt, Ing. Pavlína Langrová
Dept. of Operations Research and System Analysis, Faculty of Economics and Management, Czech University of Agriculture in Prague , subrt@pef.czu.cz, langrova@pef.czu.cz

COMPARING OF DECISION SYSTEMS

MATĚJ TURČAN
VŠB-TU OSTRAVA, CZECH REPUBLIK

Abstract

In practice we usually need to compare different models of decision-making systems, based on an expert knowledge base in form of IF-THEN rules. In this paper two approaches to comparing of such decision systems are shown. They are applied on General Fuzzy Decision System (GFDS), i.e. universal decision-making system that can describe all those systems.

1. Introduction

Let be (**U**,**C**) a decision space, where **U** is a set of variants u and **C** is a set of criteria c_i. The goal of every model of decision making process is to create a decision (or "utility") function h: $\mathbf{U} \to [0,1]$ that enables choosing of one or more variants from **U**. A variant of general theory of decision systems is introduced in the paper of J. Močkoř [5]. A global utility function h for a set **K** of goals g is determined by an appropriate aggregation of (local) utility functions h_g for $g \in \mathbf{K}$, where h_g is constructed by a general fuzzy decision system (GFDS) and a general type of utility function. In this theory the GFDS is defined as a standard mathematical structure:

$$R_g = (G_g, \boldsymbol{W}_{g,\mathbf{C}}, \boldsymbol{V}_{g,\mathbf{C}}, s, f),$$

which is based on a formal notion of implication and some formal expert knowledge base of m rules and n criteria c_i

$$\boldsymbol{V}_{g,\mathbf{C}} = \{ \vec{V}_j = (V_{jc_1}, V_{jc_2}, \dots, V_{jc_n}, V_{j(n+1)}) \mid c_i \in \mathbf{C}, 1 \le j \le m\}.$$

The set

$$\boldsymbol{W}_{g,\mathbf{C}} = \{(w_{c_1}(u), w_{c_2}(u), \dots, w_{c_n}(u)) \mid, u \in \mathbf{U}\}$$

describes how well variants u satisfy criteria c_i and the fuzzy set G_g represents required level of satisfaction of the goal g. It is considered that the more fuzzy sets $w_{c_i}(u)$ and G_g are similar to a crisp set {1} in the interval [0,1], the higher level of satisfaction is expressed. In order to evaluate variants into this system we need some fuzzy inclusion s and some aggregation function f (see [3],[4],[5]). A local utility function h for R_g, could be than defined for any variant $u \in \mathbf{U}$ as follows:

$$h_{R,g}(u) = \sup_{j=1}^{m} t\left[f(s(w_{c_1}(u),V_{jc_1}),...,s(w_{c_n}(u),V_{jc_n})),s(V_{n+1},G_g)\right],$$

where t is t-norm that most often represents operations "$\wedge$" or "$\cdot$" and aggregation function f is *min*. The higher the value of $h_{R,g}(u)$ is, the better is u with regard to R_g.

As GFDS is a mathematical structure, it is possible to investigate properties of decision systems by using of mathematical methods. More information about GFDS can be found in [1], [2], [3], [5], [6]. One of the results that were proved is that to each function h: $\mathbf{U} \rightarrow [0,1]$ there exists a GFDS R_g such that its utility function $h_{R,g}$ for the goal g is equal to the function h, i.e. $h(u) = h_{R,g}(u)$.

2. Comparing of GFDS

In practice we often have to decide which of decision systems is better, which can better differentiate one variant from another. I would like to show two approaches how to solve this problem. Both use some pre-ordering on the set of all GFDS over decision space ($\mathbf{U}$,$\mathbf{C}$).

Definition 1

Let $\mathbf{K}$ *be a set of goals, then the general fuzzy decision system for the set* $\mathbf{K}$ *is a set* $R_{\mathbf{K}} = \{R_g \mid g \in \mathbf{K}\}$, *where* R_g *are general fuzzy decision systems for goals g from the set* $\mathbf{K}$, *with the same s and f. Let the symbol* $\mathbf{R}(\mathbf{U},\mathbf{C})$ *denotes the set of all general fuzzy decision systems* $R_{\mathbf{K}}$ *over decision space* ($\mathbf{U}$,$\mathbf{C}$).

2a Pre-ordering on R(U,C) based on relation "to be coarser"

Definition 2

Let $R_1,R_2 \in \mathbf{R}(\mathbf{U},\mathbf{C})$ *be two general fuzzy decision systems with the same goals, i.e.* $\boldsymbol{K} = \boldsymbol{K}_1 = \boldsymbol{K}_2$ *and* $G_{1,g} = G_{2,g}$ *for all* $g \in \mathbf{K}$. *Then we say that* R_1 *"is coarser than"* R_2 (*in symbols* $R_1 \leq R_2$), *if for any goal* $g \in \mathbf{K}$ *and variants* $u \in \mathbf{U}$ *one of the following conditions holds:*

$$h_{R_2,g}(u) - h_{R_2,g}(v) \geq h_{R_1,g}(u) - h_{R_1,g}(v) \geq 0\,,$$

$$h_{R_2,g}(v) - h_{R_2,g}(u) \geq h_{R_1,g}(v) - h_{R_1,g}(u) \geq 0\,.$$

It is clear that a quality of GFDS depends in some sense on its ability to separate elements from $\mathbf{U}$. Hence the GFDS, that can better separate these elements, will be called finer. It is clear that ($\mathbf{R}(\mathbf{U},\mathbf{C})$, $\leq$) is a pre-ordered set. It means that there could exist two systems $R_{1,g}$ and

$R_{2,g}$ such that $R_{1,g} \le R_{2,g}$ and in the same time $R_{2,g} \le R_{1,g}$, but they are not the same $R_{1,g} \ne R_{2,g}$. There is no problem to find the coarsest GFDS. It is the one that have

$$h_{R,g}(u) = \quad f(s(\{1\},\{1\}),...,s(\{1\},\{1\})).s(G_g, G_g) = 1, \text{ for all } u \in \mathbf{U}.$$

It could be proved that for any GFDS from **R**(**U**,**C**) there exists the coarsest one (in the sense that it is the minimal element in (**R**(**U**,**C**), ≤)). (See [3] and [5]).

And what about the maximal element, the finest GFDS? The following proposition shows some properties of these systems.

Proposition 1

Let $R \in$ **R**(**U**,**C**).

1. *Let* $0,1 \in h_{R,g}(\mathbf{U})$ *for some goal* $g \in \mathbf{K}$. *Then R is a maximal element in* (**R**(**U**,**C**), ≤)).
2. *Let R be a maximal element in* (**R**(**U**,**C**), ≤) *and let there exists such* $g \in \mathbf{K}$ *that for any* $\vec{V}_j \in \boldsymbol{V}_{g,\mathbf{C}}$, $c \in \mathbf{C}$ *and* $u \in \mathbf{U}$ *we have* $s(w_c(u), V_{j,c}) < 1$. *Then* $0,1 \in h_{R,g}(\mathbf{U})$.

Proof see [3] and [5].

2b Pre-odering based on R(U,C) based on ordering of U

In the second part of this section we introduce another pre-ordering on the set **R**(**U**,**C**), of fuzzy decision systems over decision space (**U**,**C**). Let's start with definition of ordering on the set of variants **U**.

Definition 3

Let $R \in$ **R**(**U**,**C**) *be a general fuzzy decision system,* $g \in \mathbf{K}$ *be the goal from the set of goals of* GFDS *R,* $h_{R_g}(u)$ *be the local utility function and* $u,v \in \mathbf{U}$ *be two variants. Then we say that u "is worse than" v, symbolically* $u \le_{R_g} v$, *if* $h_{R_g}(u) \le h_{R_g}(v)$. *We denote an ordered set of variants by symbol* $(\mathbf{U}, \le_{R_g})$.

We use this ordering to define the equivalence on the set **U**.

Definition 4

Let $R \in$ **R**(**U**,**C**), $g \in \mathbf{K}$, $h_{R_g}(u)$ *be the local utility function and* $u,v \in \mathbf{U}$. *Then an equivalence relation* $\mathrm{E}_{R_g} \subseteq \mathbf{U} \times \mathbf{U}$ *is defined as follows:*

$$\forall\, u,v \in \mathbf{U},\ (u,v) \in \mathrm{E}_{R_g},\ \textit{symbolically}\ u\, \mathrm{E}_{R_g} v \Leftrightarrow h_{R_g}(u) = h_{R_g}(v).$$

Now we can define a partition of the set **U**, for any goal g∈**K**. We denote a factor set as **U**/E_{R_g} and its elements (classes) as $u\,\mathrm{E}_{R_g}$. An ordering on the set **U**/E_{R_g} could be defined as follows:

$$u\,\mathrm{E}_{R_g} \leq_{R_g} v\,\mathrm{E}_{R_g} \Leftrightarrow u \leq_{R_g} v.$$

Let's define the pre-ordering on set **R**(**U**,**C**) now.

Definition 5

Let g∈**K** *be a goal,* $R_{1,g}, R_{2,g} \in$ **R**(**U**,**C**) *be two fuzzy decision systems and* $G_{1,g} = G_{2,g}$. *Then we say that* $R_{2,g}$ *"is better than"* $R_{1,g}$, *symbolically* $R_{1,g} \leq_g R_{2,g}$, *if the following conditions hold*:

(i) $\mathrm{E}_{R_{2,g}} \subseteq \mathrm{E}_{R_{1,g}}$

(ii) $\forall\, u,v \in$ **U**: $u\,\mathrm{E}_{R_{1,g}} \leq_{R_{1,g}} v\,\mathrm{E}_{R_{1,g}} \Rightarrow u\,\mathrm{E}_{R_{2,g}} \leq_{R_{2,g}} v\,\mathrm{E}_{R_{2,g}}$,

where $\leq_{R_{1,g}}$ *or* $\leq_{R_{2,g}}$ *is the pre-ordering on the set* **U**$/\mathrm{E}_{R_{1,g}}$ *or* **U**$/\mathrm{E}_{R_{2,g}}$ *respectively.*

Remark 1

If $R_{1,g} \leq_g R_{2,g}$ and $R_{2,g} \leq_g R_{1,g}$, then we say that $R_{1,g}$ and $R_{2,g}$ are equivalent in the sense, that they both decided equivalently. It is denoted by $R_{1,g} \approx_g R_{2,g}$.

It is evident that the relation $\leq_g$ is different from the above-mentioned pre-ordering. Decision system, that is greater (better) in this ordering, is able to separate more variants than the smaller (worse) one. Pre-ordering can't guarantee this property i.e. implication $R_{1,g} \leq R_{2,g} \Rightarrow R_{2,g} \leq_g R_{1,g}$ (where $\leq$ resp. $\leq_g$ is pre-ordering, resp. ordering relation) doesn't hold. It's evident that reverse implication doesn't hold, but the following lemma holds.

Lemma 1

Let $R_{1,g}, R_{2,g} \in$ **R**(**U**,**C**) *be two fuzzy decision systems and g*∈**K** *be a goal. If* $R_{1,g} \leq R_{2,g}$ *and* $R_{2,g} \leq R_{1,g}$ *then* $R_{1,g} \approx_g R_{2,g}$.

Reverse lemma doesn't hold. Let's describe some properties of ordering relation $\leq_g$.

Lemma 2

Let u,v ∈ **U** *and* $R_{1,g} \leq_g R_{2,g}$, *where g*∈**K** *be a goal, then*

$$u\,\mathrm{E}_{R_{1,g}} <_{R_{1,g}} v\,\mathrm{E}_{R_{1,g}} \Rightarrow u\,\mathrm{E}_{R_{2,g}} <_{R_{2,g}} v\,\mathrm{E}_{R_{2,g}}.$$

Lemma 3

Let $R_{1,g}, R_{2,g} \in \mathbf{R}(\mathbf{U},\mathbf{C})$ *and* $R_{1,g} \le_g R_{2,g}$ *for* $g \in \mathbf{K}$*, then holds*

(*i*) $\forall u,v,w \in \mathbf{U}$:

$u\mathrm{E}_{R_{1,g}} \le_{R_{1,g}} v\mathrm{E}_{R_{1,g}} \le_{R_{1,g}} wE_{R_{1,g}} \Rightarrow u\mathrm{E}_{R_{2,g}} \le_{R_{2,g}} v\mathrm{E}_{R_{2,g}} \le_{R_{2,g}} wE_{R_{2,g}}$,

(*ii*) $E_{R_{2,g}} \subset E_{R_{1,g}} \Leftrightarrow \exists u,v \in \mathbf{U}$: $u\mathrm{E}_{R_{1,g}} =_{R_{1,g}} v\mathrm{E}_{R_{1,g}} \Rightarrow u\mathrm{E}_{R_{2,g}} <_{R_{2,g}} v\mathrm{E}_{R_{2,g}}$,

(iii) $\forall o,p,q \in \mathbf{U}$: $oE_{R_{1,g}} \le_{R_{1,g}} pE_{R_{1,g}} =_{R_{1,g}} qE_{R_{1,g}} \Rightarrow$

$oE_{R_{2,g}} \le_{R_{2,g}} pE_{R_{2,g}} \le_{R_{2,g}} qE_{R_{2,g}}$ *or* $oE_{R_{2,g}} \le_{R_{2,g}} qE_{R_{2,g}} \le_{R_{2,g}} pE_{R_{2,g}}$.

Remark 2

According to (*ii*) we can define sharp ordering $<_g$ on **R**(**U**,**C**) for the goal *g*:

$$(R_{1,g} <_g R_{2,g}) \Leftrightarrow (R_{1,g} \le_g R_{2,g} \wedge E_{R_{2,g}} \subset E_{R_{1,g}}).$$

If we can separate at least one more couple of variants in decision system $R_{2,g}$ than in $R_{1,g}$, then $R_{2,g}$ is sharply greater (better) than $R_{1,g}$.

Similarly to the first approach we can discuss the problem of minimal and maximal elements of the set **R**(**U**,**C**) of all GFDS.

Lemma 4

For each set of GFDS there exists minimal element (the worst GFDS).

Lemma 5

For each set **R**(**U**,**C**) *of GFDS there exists maximal element (the best GFDS).*

Up till now we have defined ordering on **R**(**U**,**C**) for just one goal.We can extend this definition for the set of goals **K**.

Definition 5

Let $R_1, R_2 \in \mathbf{R}(\mathbf{U},\mathbf{C})$ *be two fuzzy decision systems and* $G_{1,g} = G_{2,g}$ *for each goal* $g \in \mathbf{K}$. *Then we say that* R_2 *"is better than"* R_1, *symbolically* $R_1 \le R_2$, *if* $R_{1,g} \le_g R_{2,g}$ *hold for each goal* $g \in \mathbf{K}$.

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Matěj Turčan

Department. of Mathematical Methods in Economics, Faculty of Economics, VŠB-TU Ostrava, Sokolská třída 33, 701 21 Ostrava, Czech Republic, e-mail: matej.turcan@vsb.cz

HETEROGENEOUS AGENT MODEL WITH LEARNING

MILOSLAV VOŠVRDA, LUKÁŠ VÁCHA*
INSTITUTE OF INFORMATION THEORY AND AUTOMATION
ACADEMY OF SCIENCES OF THE CZECH REPUBLIC, CZECH REPUBLIC

Abstract

The Efficient Markets Hypothesis provides a theoretical basis for trading rules. Technical trading rules provide a signal of when to buy or sell asset based on such price patterns to the user. Technical traders tend to put little faith in strict efficient markets. Fundamentalists rely on their model employing fundamental information basis to forecasting of the next price period. The traders determine whether current conditions call for the acquisition of fundamental information in a forward looking manners, rather than relying on past performance. This approach relies on heterogeneity in the agent information and subsequent decisions either as fundamentalists or as chartists. Changing of the chartist's profitability and fundamentalist's positions is the basis of cycles behaviour. It was shown that a level of profitability particular agent patterns is very sensitive on the structure of memory weights and the memory lengths. It is shown that different values of these memory coefficients can significantly change the preferences of trader strategies.

In this paper is shown an influence of the learning agents process on a level of agent pattern profitability.

Keywords: efficient markets hypothesis, technical trading rules, heterogeneous agent model with memory and learning, asset price behaviour

JEL classifications: **C061; G014; D084**

1. Introduction

Assumptions about rational behaviour of agents, homogeneous models, and efficient market hypothesis were paradigms of economic and finance theory for the last years. After empirical data analysis on financial markets and economic and finance progress these paradigms are gotten over. There are phenomena observed in real data collected from financial markets that cannot be explained by the recent economic and finance theories. One paradigm of recent economic and finance theory asserts that sources of risk and economic fluctuations are

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exogenous. Therefore the economic system would converge to a steady-state path, which is determined by fundamentals and there are no opportunities for speculative profits in the absence of external shocks prices. It means that the other factors play important role in construction of real market forces as heterogeneous expectations. Since agents no have sufficient knowledge of a structure of the economy to form correct theoretical expectations, it is impossible for any formal theory to postulate unique value expectations that would be held by all agents (Gaunersdorfer [6]). Prices are partly determined by fundamentals and partly by the observed fluctuations endogenously caused by non-linear market forces. This implies that technical trading rules need not be systematically bad and may help in predicting future price changes. Developments in the theory of non-linear dynamic systems have contributed to new approaches in economics and finance (Brock [3]). Introducing non-linearity in the models may improve research of a mechanism generating the observed movements in the real financial data. Financial markets are considered as systems of the interacting agents processing new information immediately. A heterogeneity in expectations can lead to market instability and complicated dynamics.

Our approach assumes that agents are intelligent having no full knowledge about the underlying model in the sense of the rational expectation theory and no having the computational equipment can interpret the same information by different way. Therefore prices are driven by endogenously market forces. The approach Adaptive Belief Approach by Brock and Hommes [1] is employed in this paper. Agents adapt their predictions by choosing among a finite number of predictors. Each predictor has a performance measure. Based on this performance agents make a rational choice between the predictors. Brock and Hommes showed that the adaptive rational equilibrium dynamics incorporates a general mechanism, which may generate local instability of the equilibrium steady state and complicated global equilibrium dynamics.

We focus on a version of the model with two types of trades, i.e., fundamentalists, and technical traders. Technical traders tend to put little faith in strict efficient markets. Fundamentalists rely on their model employing fundamental information basis to forecasting of the next price period. The traders determine whether current conditions call for the acquisition of fundamental information in a forward looking manners, rather than relying on post performance. This approach relies on heterogeneity in the agent information and

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subsequent decisions either as fundamentalists or as chartists. Changing of the chartist's profitability and fundamentalist's positions is a basis of the cycles behaviour. A more detailed analysis is introduced in the Brock and Hommes model. The model with memory was analysed in [13]. This model presents in a form of evolutionary dynamics, which is called **Adaptive Belief System**, in a simple present discounted value (PDV) pricing model. Such model without memory (and only one period) was presented by Brock and Hommes [2][1]. Let us consider an asset-pricing model with one risky asset and one risk-free asset. Let p_t be the share price (ex dividend) of the risky asset at time t, and let $\{y_t\}$ be an i.i.d. the stochastic dividend process of the risky asset. The risk free asset is perfectly elastically supplied at gross return $R > 1$. The dynamics of wealth can be written as

$$W_{t+1} = R \cdot W_t + \left(p_{t+1} + y_{t+1} - R \cdot p_t\right) \cdot z_t, \tag{1.1}$$

where z_t denotes the number of shares of the asset purchased at time t. E_t and V_t are the conditional expectation and conditional variance operators, based on the publicly available information set consisting of past prices and dividends, i.e., on the information set $\mathcal{F}_t = \{p_t, p_{t-1}, \ldots; y_t, y_{t-1}, \ldots\}$. Let $E_{h,t}$, $V_{h,t}$ denote **beliefs** of investor of type h about the conditional expectation and conditional variance. The conditional variance of wealth is

$$V_{h,t}\left(W_{t+1}\right) = z_t^2 \cdot V_{h,t}\left(p_{t+1} + y_{t+1} - R \cdot p_t\right). \tag{1.2}$$

We assume that beliefs about the conditional variance of excess returns are constant for all investor types h

$$V_{h,t}\left(p_{t+1} + y_{t+1} - R \cdot p_t\right) \equiv \sigma_h{}^2 = \sigma^2. \tag{1.3}$$

Assume each investor type is a **myopic mean variance maximizer.** So for type h, the demand for shares z_{ht} is solved as follows

$$\max_z \left\{E_{h,t} W_{t+1} - \frac{a}{2} V_{h,t}\left(W_{t+1}\right)\right\}, \tag{1.4}$$

i.e.,

$$E_{h,t}\left(p_{t+1} + y_{t+1} - Rp_t\right) - a\sigma^2 z_{s,t} = 0, \tag{1.5}$$

[1] The model was inspired by the model of Lucas [7]

$$z_{h,t} = \frac{E_{h,t}\left(p_{t+1} + y_{t+1} - R \cdot p_t\right)}{a \cdot \sigma^2}. \qquad (1.6)$$

The risk aversion a is here assumed to be the same for all traders.

This paper is organized as follows. In Section 2 this model is studied with memory in the performance measure. This case is sensitive on the structure of memory weights and the memory lengths (see [13]).

In Section 3, this case shows high profitability of learning strategy at the described model. Results of numerical analysis are introduced.

2. The Dynamics of Fractions

Let us concentrate on the adoption of beliefs, i.e., on dynamics of the fractions $n_{h,t}$ of different trader types. Next, let us change slightly the timing of updating beliefs, i.e.,

$$R \cdot x_t = \sum_h n_{h,t-1} \cdot f_h\left(x_{t-1}, \ldots, x_{t-L}\right) \equiv \sum_h n_{h,t-1} \cdot f_{h,t}, \qquad (2.1)$$

where $n_{h,t-1}$ denotes the fraction of trader type h at the beginning of period t, before than the equilibrium price x_t has been observed. Now the **realized excess return** over period t to the period $t+1$ is computed,

$$R_{t+1} = p_{t+1} + y_{t+1} - R \cdot p_t. \qquad (2.2)$$

$$R_{t+1} = x_{t+1} + p^*_{t+1} + y_{t+1} - R \cdot x_t - R \cdot p^*_t. \qquad (2.3)$$

$$R_{t+1} = x_{t+1} - R \cdot x_t + p^*_{t+1} + y_{t+1} - E_t\left(p^*_{t+1} + y_{t+1}\right) + E_t\left(p^*_{t+1} + y_{t+1}\right) - R \cdot p^*_t. \qquad (2.4)$$

From the equation (2.4) we get

$$E_t\left(p^*_{t+1} + y_{t+1}\right) - R \cdot p^*_t = 0, \text{ and } \delta_{t+1} = p^*_{t+1} + y_{t+1} - E_t\left(p^*_{t+1} + y_{t+1}\right),$$

which is a martingale difference sequence with respect to $\mathcal{F}_t$ i.e., $E_t\left(\delta_{t+1} | \mathcal{F}_t\right) = 0$ for all t. So the expression (2.4) can be written as follows

$$R_{t+1} = x_{t+1} - R \cdot x_t + \delta_{t+1}. \qquad (2.5)$$

The decomposition of the equation (2.5) as separating the 'explanation' of realized excess returns R_{t+1} into the contribution $x_{t+1} - R \cdot x_t$ of the theory is investigated here and the conventional Efficient Markets Theory term δ_{t+1} is shown.

Let the fitness measure (or the performance measure) $\pi\left(R_{t+1}, \rho_{h,t}\right)$ be defined by

$$\pi_{h,t} = \pi\left(R_{t+1}, \rho_{h,t}\right) = R_{t+1} \cdot z\left(\rho_{h,t}\right) = \left(x_{t+1} - R \cdot x_t + \delta_{t+1}\right) \cdot z\left(\rho_{h,t}\right), \qquad (2.6)$$

so the fitness is given by the realized profits for trader h. In the following paragraphs, numerical simulations with a stochastic dividend process $y_t = \bar{y} + \varepsilon_t$, where ε_t is i.i.d.[2], with a uniform distribution on an interval $\langle -\varepsilon, +\varepsilon \rangle$ will be used.

Now write type *h* beliefs $\rho_{h,t} = E_{h,t}\left(R_{t+1}\right) = f_{h,t} - R \cdot x_t$ in the deviations form. Let the updated fractions $n_{h,t}$ be given by the discrete choice probability

$$n_{h,t} = \exp\left(\beta \cdot \pi_{h,t-1}\right) / Z_t, \text{ where } Z_t = \sum_h \exp\left(\beta \cdot \pi_{h,t-1}\right). \tag{2.7}$$

The parameter β is the **intensity of choice** measuring how fast agents switch between different prediction strategies. The parameter β is a measure of trader's rationality. The variable Z_t is just a normalization so that fractions $n_{h,t}$ sum up to 1. If the intensity of choice is infinite $(\beta = +\infty)$, the entire mass of traders uses the strategy that has the highest fitness. If the intensity of choice is zero, the mass of traders distributes itself evenly across the set of available strategies.

The timing of predictor selection is important. The fractions $n_{h,t}$ depend upon fitness π and return *R* at the time $t - 1$ in order to ensure that depends only upon observable deviations x_t at time *t*. The timing ensures that past realized profits are observable quantities that can be used in predictor selection.

3. Learning process and memory in the performance measure

For the case with memory and learning process in the performance measure the fitness is not given by the most recent past (last period), but by summation of more values of fitness measure in the past with different weights for these values. The weights sum up to one.

$$n_{h,t} = \exp\left(\beta \cdot \sum_{p=1}^{m} \eta_{h,t} \cdot \pi_{h,t-p}\right) / Z_t, \; Z_t = \sum_h \exp\left(\beta \cdot \sum_{p=1}^{m} \eta_{h,t} \cdot \pi_{h,t-p}\right). \tag{3.1}$$

where *m* denotes the memory length, η is the vector of memory weights. All beliefs - learning process (formation of expectations) or learning schemes with different lag lengths will be of the following form

$$f_{h,t} = g_h \cdot \frac{1}{L_h} \sum_{p=1}^{L_h} x_{t-p} + b_h \tag{3.2}$$

where g_h denotes the trend, b_h the bias of trader type *h*, and L is number of lags. If $b_h = 0$, the agent *h* is called a **pure trend chaser** if $g > 0$ (strong trend chaser if $g > R$) and a **contrarian** if $g < 0$ (strong contrarian if $g < -R$). If $g_h = 0$, type *h* trader is said to be

[2] In this case we have $\delta_{t+1} = \varepsilon_{t+1}$

purely biased. He is upward (downward) biased if $b_h > 0$ $(b_h < 0)$.

In the special case $g_h = b_h = 0$, type *h* trader is called **fundamentalist** i.e., the trader is believing that prices return to their fundamental value. Fundamentalists do have all past prices and dividends in their information set, but they do not know the fractions $n_{h,t}$ of the other belief types.

Now we derive the fitness measure for the simple belief type $V_{h,t}(W_{t+1}) = z_t^2 \cdot V_{h,t}(p_{t+1} + y_{t+1} - R \cdot p_t)$. Rewriting the equation $z_{h,t} = \dfrac{E_{h,t}(p_{t+1} + y_{t+1} - R \cdot p_t)}{a \cdot \sigma^2}$ in deviations form yields the demand for shares by type *h* by assumption that all beliefs are of the form

$$E_{h,t}(p_{t+1} + y_{t+1}) = E_t(p^*_{t+1} + y_{t+1}) + f_h(x_{t-1}, \dots, x_{t-L}).$$

$$z_{h,t-1} = \frac{E_{h,t-1}(p_t + y_t - R \cdot x_{t-1})}{a \cdot \sigma^2} = \frac{f_{h,t-1} - R \cdot x_{t-1}}{a \cdot \sigma^2}. \tag{3.3}$$

Now the fitness function (2.6) can be rewritten, hence the realized profit is

$$\pi_{h,t-1} = R_t \cdot z_{h,t-1} = (x_t - R \cdot x_{t-1} + \delta_t) \cdot \left(g_h \cdot \frac{1}{L_h} \sum_{p=1}^{L_h} x_{t-1-p} + b_h - R \cdot x_{t-1} \right) / (a \cdot \sigma^2). \tag{3.4}$$

An example of the price equation with memory in performance measure and learning process with arithmetical mean scheme is the following formulation

$$x_{t+1} = \frac{1}{R} \sum_{h=1}^{4} \left(\frac{\exp\left(\beta \cdot \sum_{p=1}^{m} \eta_{h,p} \cdot \pi_{h,t-p} \right)}{\sum_{h=1}^{4} \exp\left(\beta \cdot \sum_{p=1}^{m} \eta_{h,p} \cdot \pi_{h,t-p} \right)} \left(g_h \cdot \frac{1}{L_h} \sum_{p=1}^{L_h} x_{t+1-p} + b_h \right) \right). \tag{3.5}$$

4. Numerical Analysis of the model under learning process with the memory in the performance measure

This section demonstrates numerically an importance of the memory in the performance measure and the presence of learning process for the behavior of this model. We show that there are significant differences in profitability of trader's strategies as memory length is changed and learning process is implemented to the beliefs of traders.

Here, a numerical analysis is focused only on the model with four types of trader's strategies, each with different beliefs. We examine a case, where investor's types are fundamentalists that interact with other technical trader's types such as trend chasers with bias and pure trend chasers.

For the case in this section we add noise to a dividend process. A noise has a uniform distribution on the interval $\langle -0.005, +0.005 \rangle$. The equation (4.1) is used for a memory less system, i.e., decision-making procedure is formulated using one period only, and the equation (4.2) for the system with a memory where *m* denotes the memory length. The sum of memory weights $\eta_{j,p}$'s must add up to one. The equation (4.3) describes the system with memory in the performance measure with adoption of the learning process with the length of lag L_h.

Memory less system generates the following price formulation

$$x_{t+1} = \frac{1}{R}\sum_{h=1}^{4}\left(\frac{\exp\left(\beta \cdot \pi_{h,t-1}\right)}{\sum_{h=1}^{4}\exp\left(\beta \cdot \pi_{h,t-1}\right)}\left(g_h x_t + b_h\right)\right). \tag{4.1}$$

System with memory, where *m* denotes memory length and η memory weights, generates the following price formulation

$$x_{t+1} = \frac{1}{R}\sum_{h=1}^{4}\left(\frac{\exp\left(\beta \cdot \sum_{p=1}^{m}\eta_{h,p} \cdot \pi_{h,t-p}\right)}{\sum_{h=1}^{4}\exp\left(\beta \cdot \sum_{p=1}^{m}\eta_{h,p} \cdot \pi_{h,t-p}\right)}\left(g_h x_t + b_h\right)\right). \tag{4.2}$$

System with memory *m* and with the linear learning process with lag L_h generates the following price formula

$$x_{t+1} = \frac{1}{R}\sum_{h=1}^{4}\left(\frac{\exp\left(\beta \cdot \sum_{p=1}^{m}\eta_{h,p} \cdot \pi_{h,t-p}\right)}{\sum_{h=1}^{4}\exp\left(\beta \cdot \sum_{p=1}^{m}\eta_{h,p} \cdot \pi_{h,t-p}\right)}\left(g_h \frac{1}{L_h}\sum_{p=1}^{L_h} x_{t+1-p} + b_h\right)\right). \tag{4.3}$$

where the performance measure is described by equation (4.4).

$$\pi_{h,t-1} = R_t \cdot z_{h,t-1} = \left(x_t - R \cdot x_{t-1} + \delta_t\right) \cdot \left(g_h \cdot \frac{1}{L_h}\sum_{p=1}^{L_h} x_{t-1-p} + b_h - R \cdot x_{t-1}\right) / \left(a \cdot \sigma^2\right). \tag{4.4}$$

Case: Fundamentalists, Trend Chasers with bias, Pure Trend Chasers

Table 1: *Parameters of the system*

Type	Parameters	
N1	$g_1 = 0$ $b_1 = 0$	Fundamentalists
N2	$g_2 = 1.1$ $b_2 = 0.2$	Trend with upward bias
N3	$g_3 = 0.9$ $b_3 = -0.2$	Trend with downward bias
N4	$g_4 = 1$ $b_4 = 0$	Pure Trend Chasers

Figure 1 shows the numerical analysis of the system with parameters in table 1 is without memory in the performance measure, i.e. agents make decisions according to the last period of the performance measure. For values of beta larger than 90 there arise chaotic price fluctuations and the trading strategy of trend chasers N2 (see table 1) becomes dominant on the market, see figure 1. We do not want to explore dynamic features in the sense of chaotic behaviour but mainly the presence of traders on the market.

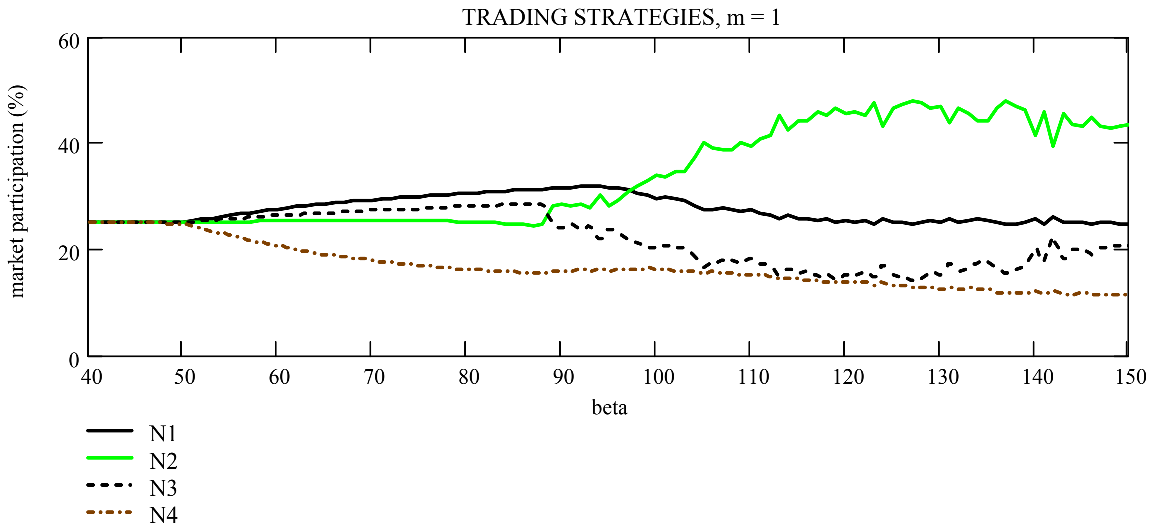

Figure 1: *Participation of trading strategies on the market with the constant memory length m = 1 for N1, N2, N3, N4, and different values of the parameter β.*

An effect of memory in the performance measure (for all the strategies) is displayed in figure 2. With longer memory in the performance measure (m=20), the system is more stable, the price is less volatile, and its amplitude is smaller. Fundamentalists become the dominant strategy as β is rising. There is a significant change with comparison to the figure 1 where fundamentalists are not the most profitable strategy on the market.

Figure 3 displays simulations with equal memory length in the performance measure for all trading strategies (m=20), but with learning process implemented into the beliefs of strategy N4 (4.5).

$$f_{4,t} = g_4 \cdot \frac{1}{L_4} \sum_{p=1}^{L_4} x_{t-p} = 1 \cdot \frac{1}{30} \sum_{p=1}^{30} x_{t-p} \ . \tag{4.5}$$

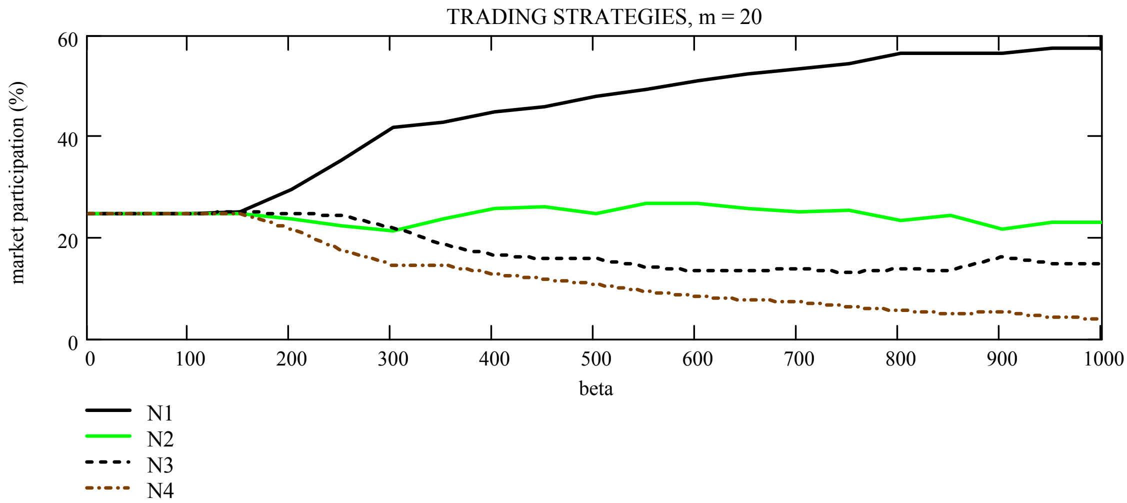

Figure 2: *Participation of trading strategies on the market with the constant memory length m = 20 for N1, N2, N3, N4, without any learning process (L = 1) and with different values of the parameter β.*

The analysis shows, for different values of the intensity choice parameter β, remarkable result – the adoption of the learning process (4.5) for the strategy N4 increases its profitability on the market.

Similar but more apparent changes in the traders' profitability occur when we add learning process to trading strategies N2 and N3 (L_2 = 30, L_3 = 30), see figure 4. In this case the strategies N2 and N3 are taking the lead as the parameter β is rising. On the other hand Fundamentalists N1 are loosing its profits that they earned with adding the memory to the performance measure.

We can see here strong advantage of the strategies that use the learning process in comparison to the strategies that do not.

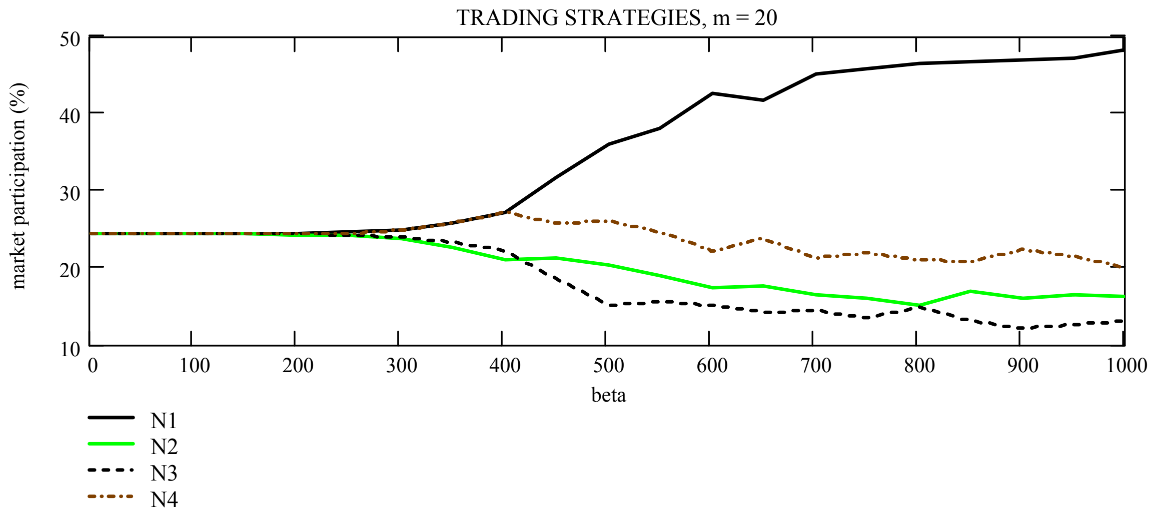

Figure 3: *Participation of trading strategies on the market with the constant memory length $m = 20$ for N1, N2, N3, N4, learning process for N4 ($L_4 = 30$) and different values of the parameter β.*

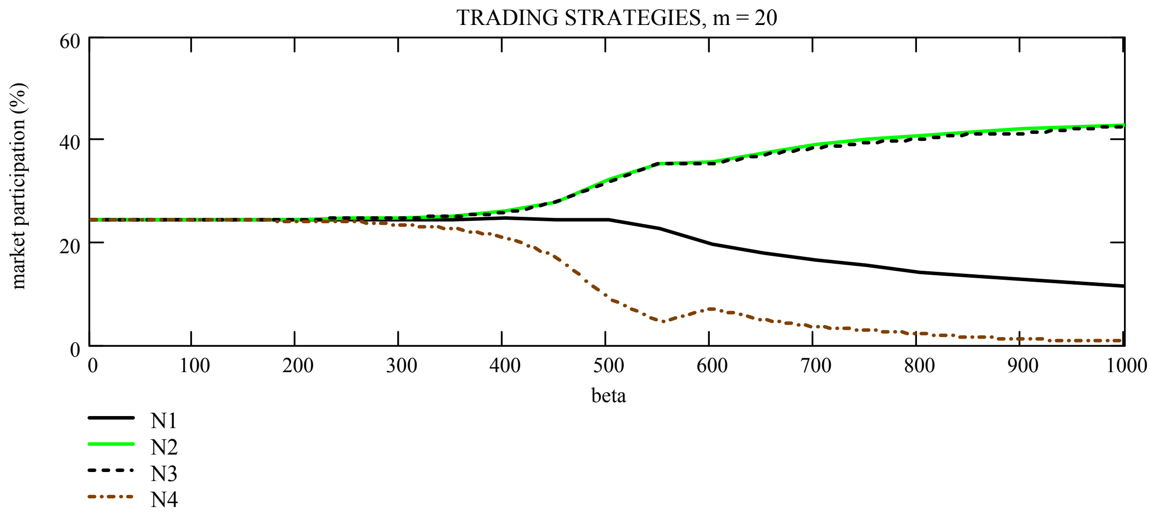

Figure 4: *Participation of trading strategies on the market with the constant memory length $m = 20$ for N1, N2, N3, N4 learning process for N2, N3 (L_2, $L_3 = 30$) and different values of the parameter β.*

5. Conclusion

In this paper we showed that this system with memory in the performance measure is more stable than the memory-less system (for more details see [13]). Memory adding helps fundamentalists to increase profits, i.e., to increase their participation on the market. It was demonstrated here that the implementation of the learning process into the traders beliefs changes the proportions on the market significantly The strategies that use the learning process showed here are increase their profitability in the market.

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Miloslav Vošvrda, Lukáš Vácha
Institute of Information Theory and AutomationAcademy of Sciences of the Czech Republic,

Prague, Czech Republic, e-mail: vosvrda@utia.cas.cz

ON SOME MULTIPLE OBJECTIVE LOCATION PROBLEMS.

KAREL ZIMMERMANN

CHARLES UNIVERSITY PRAGUE, CZECH REPUBLIK

1. Introduction.

We shall consider location problems with m customers $X^{(i)}$, $i \in S \equiv \{1, \ldots, m\}$ and a service centres S_j , $j \in N \equiv \{1, \ldots, n\}$; there are given n roads $A_j B_j$, $j \in N$ connecting places A_j , B_j the distances $d_j \equiv \rho (A_j , B_j)$ are given; on each of the roads exactly one sevice centres must be placed, itd position is determined by $x_j \equiv \rho (A_j , S_j)$, so that $x_j \in [0, d_j]$ and $\rho (S_j , B_j) = d_j - x_j$. We consider bicriterial optimization problems, inwhich a balance between two max-min-type objective functions is found. The problems are appropriate in situations, in which a big difference between the two objective functions should be avoided. One simpler problem is investigated in ldetail, for some other problems are only briefly discussed. The problems are modifications of problems considered in [1], [2], [3], [4] and generalize the problem considered in [2], [4], [5].

2. Problem Formulation.

We assume that if $x_j = \rho (A_j , S_j)$, the there exist two routes along which the customer $X^{(i)}$ can be reached: the first one via A_j having the length $x_j + a_{ij}$, where $a_{ij} \equiv \rho (X^{(i)} , A_j)$ is given, and the second one via B_j having the length $d_j - x_j + c_{ij}$, where $c_{ij} \equiv \rho (X^{(i)} , B_j)$ is given. We assume further that the shorter from the two routes can be chosen, so that the travelling distance between S_j and $X^{(i)}$ is

$$r_{ij} (x_j) \equiv \min (a_{ij} + x_j , b_{ij} - x_j),$$

where we set $b_{ij} \equiv c_{ij} + d_j$.

We shall consider the following objective functions, which characterize from different point of view the quality of the service provided by the a-tuple of service cetres S_j , if $\rho (A_j , S_j) = x_j$:

$$f (x) \equiv \max_{i \in S} f_i (x) \qquad (2.1)$$

where $f (x) \equiv \max_{j \in N} r_{ij} (x_j)$;

$$g (x) = \max_{i \in S} g_i (x) \qquad (2.2)$$

where $g_i (x) \equiv \min_{j \in N} r_{ij} (x_j)$;

$$h(x) = \min_{i \in S} h_i(x) \qquad (2.3)$$

where $h_i(x) \equiv \min_{j \in N} r_{ij}(x_j)$.

The first objective function represents the „pessimistic“ point of view, in which we assume that each customer will be served by the most distant service cetre, the second represents the „optimistic“ estimation assuming that each customer may be srved by the service centre, which is the nearest to him and the third function characterizes the smallest distance between a service centre and a customer.

We shall consider in the sequel the following optimization problem:

Minimize α

subject to

$$\left.\begin{aligned} f(x) &\le \alpha \\ h_i(x) &\ge \beta \text{ for all } i \in S \\ 0 \le x &\le d \end{aligned}\right\} \quad (P1)$$

This problem may occur if the service centres are somehow „obnoxious: and should not be therefore placed too close to the customers or if it is desirable that the difference between the farthest and the nearest centre should be within certain limits.

3.Properties and Solution of (P1).

Since the max-operators in (P1) are interchangable, i.e.

$$f(x) = \max_{j \in N} \max_{i \in S} r_{ij}(x_j),$$

and since if $f_i.(x) \le \alpha$ and $h_i(x) \ge \beta$ implies that $f_i(x) - h_i(x) \le \alpha - \beta$, we can proceed as follows. We shall find the smallest value of α-β, under the constraints $f(x) \le \alpha$, $h(x) \ge \beta$, $0 \le x \le d$; let us denote the corresponding values of α,β by α^{opt}, β^{opt} and set $\lambda^{opt} \equiv \alpha^{opt} - \beta^{opt}$. .If the given $\alpha - \beta < \lambda^{opt}$, then (P1) has no feasible solution. If $\alpha - \beta \ge \lambda^{opt}$, then we shall find the optimal value of α over the set of x satisfying the constraits of (P1) for α, β. The interchangebility of the max-operators in f enables us to decompose the problem in a subproblems $(P1)_j$ of the following form for each $j \in N$:

Minimize $\lambda \equiv \alpha - \beta$

subject to

$$\left.\begin{aligned} &\max_{i \in S} r_{ij}(x_j) \le \alpha , \\ &\min_{i \in S} r_{ij}(x_j) \ge \beta , \\ &0 \le x_j \le d_j \end{aligned}\right\} \quad (P1)_j$$

If $\lambda(j) = \alpha(j) - \beta(j)$ is the optimal solution of $(P1)_j$, then $\lambda^{opt} = \max \{ \lambda(j) \mid j \in N \}$.

It can be easily seen that

$$\min_{i \in S} r_{ij}(x_j) \ge \beta \iff l_j(x_j) = \min(a_{rj} + x_j , b_{sj} - x_j) \ge \beta, \qquad (3.1)$$

where $a_{rj} = \min \{ a_{ij} \mid i \in S \}$ and $b_{sj} = \min \{ b_{ij} \mid i \in S \}$.

Therefore if $\beta > \beta' \equiv (a_{rj} + b_{sj}) / 2$, then $(P1)_j$ has no feasible solution. Otherwise the set of x_j satisfying (3.1) is an interval $[\beta - a_{rj} , b_{sj} - \beta]$. Function $l_j(x_j)$ will be referred to as the lower envelope function. If we set $v = a_{rj}$ and $w = b_{sj}$, then the lower envelope function can be described as follows:

$$l_j(x_j) = \begin{cases} v + x_j & \text{if } 0 \le x_j \le y_j \\ w - x_j & \text{of } y_j < x_j \le d_j , \end{cases}$$

where $y_j = (w - v) / 2$, i.e.y_j is the point of the maximum of $l_j(x_j)$ on $\{ 0 , d_j]$, so that $l_j(y_j) = \beta'$.

We shall investigate now the local minima of the upper envelope function

$$u_j(x_j) \equiv \max_{i \in S} r_{ij}(x_j) \qquad (3.2)$$

The following procedure determines all points of local minima of $u_j(x_j)$ on $[0, d_j]$

PROCEDURE I

1. $F := S$, $p := 1$;
2. $a_{fj} := \max \{ a_{ij} \mid i \in F \}$;
3. $t := \min \{ (b_{fj} - a_{fj}) / 2 , d_j \}$
4. $G := \{ i \in S \mid t < (b_{ij} - a_{ij}) / 2 < d_j \}$
5. If $G = \emptyset$, then $x_j^{(p)} := t$, stop;

6. $a_{sj} := \max \{ a_{ij} \mid i \in G \}$;

7. $t' := \min \{ (b_{fj} - a_{sj}) / 2 , d_j \}$, $x_j^{(p)} := \max \{ 0 , t' \}$;

8. $F := G$, $p := p + 1$, go to 2

As a result of this procedure we obtain all points $x_j^{(p)}$ local minima of the upper envelope function $u_j(x_j)$ and it holds : $0 \le x_j^{(1)} < x_j^{(2)} < x_j^{(k)} \le y_j < x_j^{(k+1)} < \ldots < x_j^{(r)} \le d_j$, where $r \le m + 2$.

It follows immediately from the properties of the upper and lower envelope functions that the difference function $q_j(x_j) \equiv u_j (x_j) - l_j (x_j)$ is a quasiconvex piecewise linear function and it attains its minimum in the point y_j so that $\lambda(j) = q_j(y_j)$.

Let t be such index from S that $u_j (x_j^{(p)}) = r_{tj} (x_j)$ and $y_j^{(t)} = (b_{tj} - a_{tj}) / 2$, i.e. $y_j^{(t)}$ is the point at which function r_{tj} attains its maximum on $[0, d_j]$. Then it holds:

If $x_j^{(p)} \le y_j < y_j^{(t)} < x_j^{(p+1)}$, then $q_j (x_j) = \lambda(j)$ for all $x_j \in [x_j^{(p)} , y_j]$ (3.3)

If $x_j^{(p)} < y_j^{(t)} < y_j < x_j^{(p+1)}$, then $q_j (x_j)= \lambda(j)$ for all $x_j \in [y_j , x_j^{(p+1)}]$ (3.4)

It is therefore $\alpha(j) = x_j^{(p)}$ if (3.3) occurs and $\alpha(j) = x_j^{(p+1)}$, in case of (3.4) .

If $\lambda > \lambda(j)$, then the corresponding optimal value of α, $\alpha_j (\lambda)$ and the corresponding $x_j (\lambda)$ can be found as the minimal value of the upper envelope function over the set of of x_j's, in which the value of the difference function $q_j(x_j)$ is smaller or equal to λ. This set is a subinterval of $[0, d]$, so that it can be done by checking all local minima of the upper envelope in this interval and its values in the corresponding end points.

If we found in this way $\alpha_j (\lambda)$ for all $j \in N$, the optimal value of the objective function in (P1) is $\alpha^{opt} = \max \{ \alpha_j(\lambda) \mid j \in N \}$. It is attained in the point $x(\lambda) = (x_1(\lambda) , \ldots, x_n(\lambda))$.

4. Remarks on Some Other Problems.

Another problem, which can be solved in a similar way is the following:

$$\left.\begin{array}{l} \text{Minimize } \lambda \equiv \alpha - \beta \\ \text{subject to} \\ f(x) \le \alpha ,\ h(x) \ge \beta \\ 0 \le x \le d \end{array}\right\} \quad (P2)$$

Let us remark that if $\alpha = \max \{\alpha(j) \mid j \in N \}$, then the optimal value of λ in (P2) is λ^{opt}.

In situations, in which the „optimistic“ estimation of the performance is used, it may happen that the nearest service centre to $X^{(i)}$ is engaged and $X^{(i)}$ must be served from some other service centre. In such a situation it may be required that $f (x) \le \alpha$, where α is a given constant so that we have an estimation of the worst service quality, which may happen in such case. The corresponding optimization problem is then the following.

$$\left.\begin{array}{l} \text{Minimize } g(x) \\ \text{subject to} \\ f(x) \leq \alpha \\ 0 \leq x \leq d \end{array}\right\} \quad (P3)$$

This problem is in general NP-hard (see [3], but it can be effectively solved by making use of the procedure suggested in [6] if for each fixed $j \in N$ for any two indices $r, s \in S$, either $(a_{rj}, b_{rj}) \leq (a_{sj}, b_{sj})$ or $(a_{sj}, b_{sj}) \leq (a_{rj}, b_{rj})$ holds. By a similar procedure the following „symmetric“ problem can be solved:

$$\left.\begin{array}{l} \text{Minimize } f(x) \\ \text{subject to} \\ g(x) \leq \lambda \\ 0 \leq x \leq d \end{array}\right\} \quad (P4)$$

Another approach to solving multiple objective location problems based on the approach from [7], which uses the method suggested in [8] will be briefly discussed in the concluding part of the contribution.

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Karel Zimmermann

Charles University Prague, Faculty of Mathematics and Physics, Malostranské náměstí 25, CZ-118 00 Praha 1, e-mail: zimm@kam.ms.mff.cuni.cz