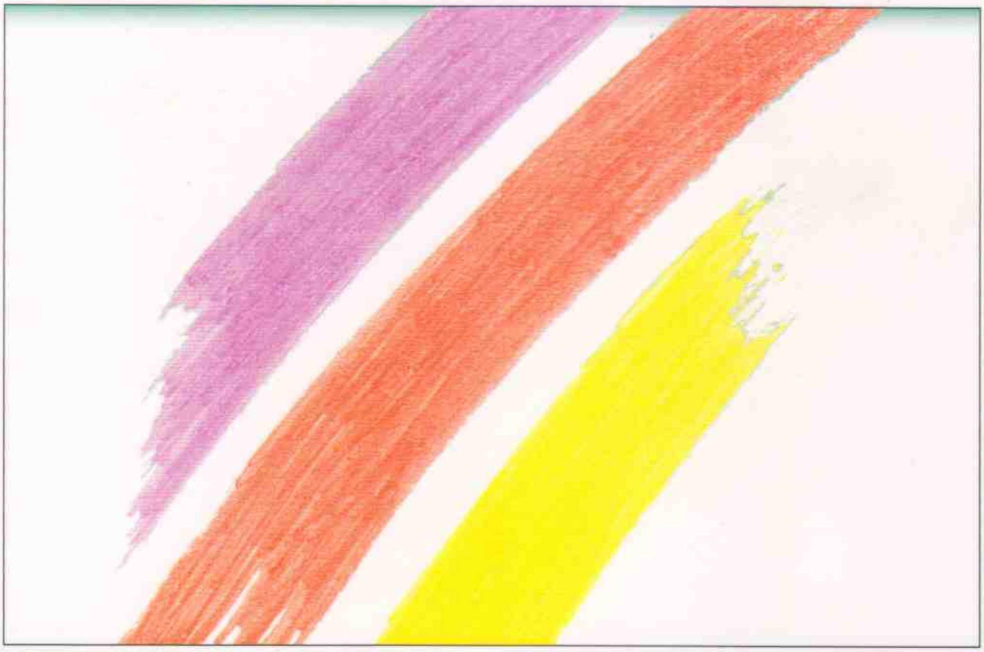
Quantitative Methods in Economics
(Multiple Criteria Decision Making XIII)



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REPORTS

STORAGE LOCATION PROBLEM

IVAN BREZINA - JURAJ PEKÁR - ZUZANA ČIČKOVÁ

Abstract: The Storage Location Problem frequently appears in a business practice. Evidently, using of relevant methods could lead to reduction of logistic costs and in that way to reduction of total cost of economic subject. In this paper, some approaches of quantitative methods for solving storage location problem, based on optimization techniques and heuristics procedures, are presented as a simply way for support of manager's decision making. Presented approach was tested on real data of the business company in Slovakia. The results of this article show that this theoretical approach is fully applicable in practice and makes the convenient support for decision making in many production companies, warehouses, logistic firms etc.

INTRODUCTION

A decision about new storage building is often based only on expert's guessing. Problem of new storage building is known as Plant Location Problem in literature (e.g. [1], [2], [4]), and here, approaches of integer or dynamic programming as well as many heuristics can be used successfully.

To make a decision about storage placement definitely, we consider the fact that the final location depends on minimum rutted distance (assuming the known vehicles capacities, customer demands, and distances between customers and potential storage locality). That can be realized by employing special methods for determining of goods delivery among all supposed localities.

Sequence of the decision:

1. Localization on the Euclid plane,
2. Localization on network nodes ,
3. Optimization of distribution channels

Using method of localization on the Euclid plane it is possible to choose the point in principle with shortest distances to others nodes of consumption. Applying methods for centre of gravity determination leads to the point, which position preserves the best localization of central point for new storage building.

Because finding of that point doesn't guarantee the really shortest distance between potential locality under the given customers demand, determination of the mass quantity units to kilometer could be useful. Evidently, this technique could be realized on basis of different distribution requirements. Results for shortest distance could not be identical to results for quickest or preferred roads (highways, motorways etc.) This approach leads to detecting of more alternative localities.

So this approach provides a determination of several potential localities and gives the basis for optimization of distribution channels. It is obvious that for that decision partial quantitative approaches (exact methods or heuristics) can be used. Simulation of distribution from individual potential locality can lead to emplacement on the basis of shortest (quickest or preferred) distances.

1. LOCALIZATION ON THE EUCLID PLANE

The object localization depends substantially on its distance to the others nodes of consumption. Since the days of Euclid, the shortest distance between two points has been a straight line.

The new airport building taking into consideration existing airports is typical example given by literature (e.g. [3]). It is a work of necessity to build new airport in principle shortest distances to others airports. Leave infrastructure out of consideration (building the new roads is no a significant part of the total costs), it is possible to built new airport in any point of space.

We consider the distance between two points, the searching point (new locality) $\mathbf{x^0}$ and $\mathbf{x^i}$ (others points, e.g. customers) for $i = 1, 2, ..., n$. So the distance is known as the Euclidean metric (L2 metric) defined by equation:

$$d(\mathbf{x^i}, \mathbf{x^0}) = \sqrt{(x_1^i - x_1^0)^2 + (x_2^i - x_2^0)^2} \tag{1}$$

where $[x^0{}_1, x^0{}_2]$ and $[x^i{}_1, x^i{}_2]$ for $i = 1, 2, ..., n$ are related Cartesian coordinate.

The goal is to find the point that is in totally minimal distance to all other points. In that way, the problem can be formulated as the tasks of extreme finding:

$$\min f(x^0{}_1, x^0{}_2) = \sum_{i=1}^{n} \sqrt{(x_1^i - x_1^0)^2 + (x_2^i - x_2^0)^2} \tag{2}$$

Considering different importance of points (customers) the weight is associated with every point, which represents the importance of each point (e.g. frequency of required visits of the point, customers demands etc.) Following this idea we need to find weighted minimum of distances that can be formalized as:

$$\min f(x^0{}_1, x^0{}_2) = \sum_{i=1}^{n} w_i \sqrt{(x_1^i - x_1^0)^2 + (x_2^i - x_2^0)^2} \tag{3}$$

This begs the question of weights determination. One way is to consider proportion of each point (customer) to capacity assumed with the new storage.

Solution based on Euclidean distances could lay the foundation for storage location based on existing infrastructure. Then, for chosen locality we can use graph theory algorithms for finding of centre of gravity that can be formulated by equations:

$$x_1^0 = \frac{\sum_{i=1}^{n} w_i \, x_1^i}{\sum_{i=1}^{n} w_i} \qquad x_2^0 = \frac{\sum_{i=1}^{n} w_i \, x_2^i}{\sum_{i=1}^{n} w_i} \qquad (4)$$

Equations (4) determine the coordinate of new point as weighted average of the other points coordinates.

2. LOCALIZATION ON NETWORK NODES

Taking economic view, most important case of storage location seems to be location on network nodes. Basic principle lies in selection of candidate locality, so that only a in a given nodes is possible to built a new storage because of existing infrastructure. Practical examples are new storage, branch or factory location. The goal lies in minimizing of distribution costs. By this approach, a must is that node (customers) demands (quantity of material, raw material, products etc.), in e.g. tones, square meters etc. are known. The outcome is vector size (n x 1). Multiplying this vector by minimal distance matrix results to vector in tonkilometers that gives distributed capacity in case of chosen related node to new storage locality. Of that account best result belong to minimal component of final vector.

3. OPTIMIZATION OF DISTRIBUTION CHANNELS

Logical prerequisite in practice is that the new storage needs to be chosen from concrete locality set because of existing infrastructure. Approximate location can be realized on the basis of previous discussion in centre of gravity of all the customer nodes.

Assuming complete net for transportation in which n customers must be served from a unique depot (centre, central depot etc.). Each customer asks for a given quantity of goods and a vehicle of capacity K is available to deliver goods. Since the vehicle capacity is limited, the vehicle has to periodically return to the depot for reloading.

From a graph theoretical point of view this problem may be stated as follows: Let G be a complete graph with node set $u_0, u_1, u_2, ..., u_n$ (u_0 – central depot, $u_1, u_2, ..., u_n$ – other customers to be served) size n+1 and arc set size a $n.(n+1)/2$. Each node is associated with a fixed quantity q_i of goods to be delivered (a quantity $q_o = 0$ is associated to the depot u_0). To each arc (u_i, u_j), i.j = 0,1,…n, is associated a value c_{ij} that represents the travel distance between u_i and u_j , shown in minimum distance matrix $\mathbf{C} = \{c_{ij}\}$ size $(n+1)$x$(n+1)$. The goal is to find a set of tours of minimum total travel distance. Each tour starts from and terminates at the depot u_0, each node u_i (i = 1,2,..., n) must be visited exactly once, and the quantity of goods to be delivered on a route should never exceed the vehicle capacity K. This problem described above is known as Vehicle Routing Problem in literature.

Further condition is $q_i \leq K$ (in case of violating this condition the drive with quantity qi = K is add to the tasks). Thus, each customer demand could be realized by exactly once vehicle visit. The goal is to find the shortest way of vehicle so that all customer demands will be realized and on no road the vehicle capacity exceeds the limit. This problem could be solved by various algorithms. One group of approaches to this problem consists of analytical (exact) methods but for more difficult computation they are not suitable for solving practical, usually very complex and extensive issues (see [2]). The second major group consists of heuristics that may not lead to optimal solution, but guarantee a relatively simple and economical way to find effective solution. Many heuristics approaches have been developed to solve vehicle routing problem There are many ways to classify heuristics for solving such problems, for example by its implementation principle (e.g. [1], [3]) to methods that work on stepwise adding nodes to existing route with current inspection of validity (Clark –Wright algorithm, Multi-route Improvement Heuristics etc., and to methods that work on decomposition of original problem to subproblems and they try to find individual solutions for each subproblem (Sweep algorithm etc.).

4. GENERALIZATION OF TECHNIQUES FOR NEW STORAGE LOCALIZATION

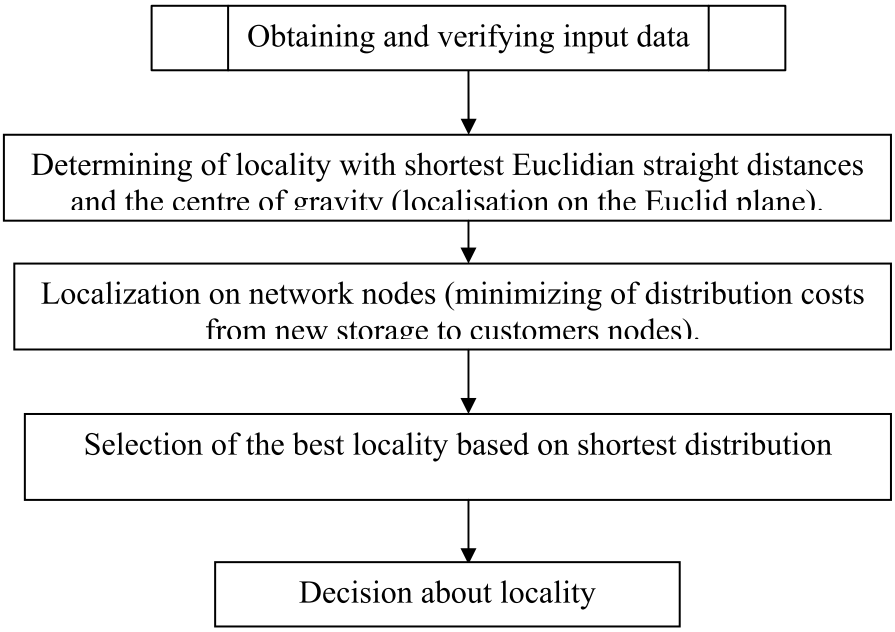
Techniques mentioned above could be generalized into following algorithm:

1. Decision about new storage building, obtaining the input data and verifying the source of information (count of customer nodes with its quantity to be delivered, common knowledge about shortest, quickest or preferred roads and distances etc,), input data analysis.
2. Are the coordinate of constituent elements with its quantity to be delivered at disposal? (required for determining of weights in tasks (3)). If yes, determining of locality with shortest Euclidian straight distances (tasks (2) a (3)) and the centre of gravity between customers nodes (equation (4)) (Localization on the Euclid plane**)**. If no, using of other methods for new storage localization (expert's guessing etc.).
3. Are quantity to be delivered of each node at disposal? If no, go to step 5. Otherwise, localization on network nodes (minimizing of distribution costs from new storage to customers nodes). Multiplying vector of customer demands in tonkilometers by minimal distance matrix results and choosing the minimal component.
4. Specification of several potential localities on the basis of steps 1. to 3. and selection of the best locality based on shortest distribution (the vehicles capacities supplementation) Using of optimization methods (dynamic or

integer programming) possibly uses of heuristics, (Clark –Wright algorithm, Multi-route Improvement Heuristics, Sweep algorithm etc.).

5. Decision about locality of new storage building.

Scheme of generalized procedure of new storage localization



CONCLUSION

Decision about new storage locality is conditioned by many factors. Basics of methods consist of finding the locality associated with lowest costs, considering as most important criteria minimization of distribution costs. Cost minimizes was prerequisite for our analysis.

At the beginning a must is in initial point determination, as a base for following steps, and here the methods for determination of locality associated with minimal straight distances and of centre of gravity between individual customers depot can be used successfully.

Minimal total distribution costs from central depot to customers could be used for obtaining the sequence of customer nodes. By this part, a real distance among all discussed nodes, but no vehicle capacity limit was taken into consideration.

In case the customers demand and vehicle capacity are known, it is possible to determine full distribution tabling connected with relevant costs. Methods mentioned in step 4. can be employed to solve this problem. That results in best locality finding.

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QUANTITATIVE METHODS OF COST EFFICIENCY EVALUATION

MARTIN DLOUHÝ

Abstract: Traditional econometric analysis, frontier analysis, and survivor analysis are three methods of cost-efficiency evaluation. The strength of econometric analysis is a highly developed methodology of hypothesis testing; the weakness is that it is necessary to deal with many problems related to estimation technique. The strength of stochastic frontier analysis is that it incorporates random shocks in efficiency evaluation; on the other hand, strong assumptions about the distribution of efficiency have to be made. The advantages of survivor analysis are its simplicity and the possibility to include the factors that are otherwise hard to measure; the disadvantages are the application only in the long-term studies, and the provision of no specific information on the character of cost function in the studied industry.

1. INTRODUCTION

A cost function describes the relationship between output and cost; it shows the minimal cost of producing the given level of output. The total cost function can be written as $TC = c(y)$, where y is the level of output. The cost of production can be also related to inputs and prices of these inputs. In this case, the total cost function is expressed as $TC = p_1x_1 + p_2x_2 + ... + p_kx_k$, where x_j (j=1, 2,…, k) are the quantities of inputs (factors of production) and p_j (j=1, 2,…, k) are the prices of these inputs. In empirical studies, researchers also relate cost with external factors, such as the location of hospital, the type of ownership, or the chain affiliation. Estimation of the cost function offers: (1) to determine marginal cost; (2) to explore the existence of scale economies; (3) to explore the existence of economies of scope; (4) to evaluate the cost efficiency of production unit, which is the topic of this paper. Traditional econometric analysis, frontier analysis, and survivor analysis are three methods of cost-efficiency evaluation that I will describe in the paper.

2. TRADITIONAL ECONOMETRIC ANALYSIS AND FRONTIER ANALYSIS

A basic tool of econometric analysis is the regression analysis that studies the dependence of one variable, called the dependent variable, on one or more other variables, called the independent or explanatory variables. The cost function is correctly specified if all important variables are included and if the right functional form is chosen. The cost function is usually specified as a multiple regression equation, with the dependent variable being either the total cost or the average cost. In many studies, the recurrent cost is only used because of problems with the capital cost, which is measured in historical prices. Depending on the objective of the study and the data availability, the empirical econometric studies include a great variety of explanatory variables. It is possible to divide these explanatory variables into three main categories: (1) input-related explanatory variables; (2) output-related explanatory variables; (3) external factors, for example, the type of ownership, the market competitiveness, location (urban or rural), etc.

If larger firms are able to achieve scale economies, the average cost curve is *L*-shaped. An alternative assumption is that, at some point, the average cost begins to grow due to the inefficient management control over a large organization. Such average cost curve is *U*-shaped and implicitly assumes that there exists an optimum size of a firm.

The interpretation of a regression model is simple if one assumes that the residual represents only inefficiency. If there is no recognition of statistical error, the level of efficiency is calculated as the distance between the observation and the regression line. If the residual is positive, the unit is relatively efficient, and vice versa, if the residual is negative, the production unit is relatively inefficient. The shortcoming of such approach is that it concentrates on the estimation of average behavior, not on the best performance. The classical regression analysis was, therefore, extended to the *stochastic frontier analysis* (see Kumbhakar, Lovell, 2000).

Let us define that E_i is the total expenditure of production unit i, y_i is the vector of outputs produced by unit i, x_i is the vector of inputs, w_i is the vector of input prices faced by unit i, and β is the vector of unknown technology parameters to be estimated. Let us suppose that outputs and inputs are nonnegative and prices are positive. The total expenditure of ith unit is $E_i = w_i^T x_i$. The cost frontier $c(y_i, w_i;\beta)$ is common for all production units. The cost efficiency of ith unit, CE_i, may be expressed as the ratio of minimum feasible cost to expenditure:

$$CE_i = \frac{c(y_i, w_i; \beta)}{E_i}.$$

For a cost efficient unit, the observed expenditure E_i equals to the minimum feasible cost $c(y_i,w_i;\beta)$, and therefore the cost efficiency $CE_i = 1$. If the observed expenditure E_i is higher than the minimum feasible cost, the production unit is not cost efficient and $CE_i < 1$. This formulation is *a deterministic cost frontier*, which ignores random shocks and attributes the higher expenditure of the unit to cost inefficiency. Notice that cost efficiency of the unit can be estimated without observing the input vector x_i.

A *stochastic cost frontier* is formulated as $[c(y_i,w_i;\beta)\ exp\{v_i\}]$, where $c(y_i,w_i;\beta)$ is the deterministic part and $exp\{v_i\}$ is the unit-specific stochastic part of the frontier. The input-oriented cost efficiency is then given by the ratio

$$CE_i = \frac{c(y_i, w_i; \beta) \exp\{v_i\}}{E_i}.$$

There are two differences between the production frontier and cost frontier models. First, the composed error term ε_i in the stochastic cost frontier model is defined as $v_i + u_i$, where v_i is the two-sided random-noise component, and u_i is the nonnegative cost inefficiency component. In the production frontier model, the composed error ε_i is defined as $v_i - u_i$. In both the stochastic production and cost frontier models, the composed error ε_i is asymmetric and positively skewed because $u_i \geq 0$. Second, the cost frontier must be linearly homogeneous in input prices: $c(y_i,\lambda w_i;\beta) = \lambda c(y_i,w_i;\beta)$ for $\lambda > 0$. One solution is the restriction that the sum of the technology parameters β_j equals one, or another solution is that the cost frontier model is reformulated. Let us assume that the stochastic cost frontier takes the Cobb-Douglas functional form. The reformulated stochastic cost frontier is then written as

$$ln\left(\frac{E_i}{w_{ki}}\right) = ln\,\beta_0 + \beta_y\, ln\, y_i + \sum_{j=1}^{k-1} \beta_j \left(\frac{w_{ji}}{w_{ki}}\right) + v_i + u_i\ .$$

where w_{ji} is the price of jth input faced by ith unit, and k is the number of inputs. The first part of equation measures the relation between the expenditure E_i and output y_i, and the second part of equation measures the relation between the expenditures and input prices faced by unit i. The measure of cost efficiency for the ith production unit is calculated as $CE_i = exp\{-u_i\}$. The estimates of cost efficiency can be obtained by the mean or the mode point estimators $E(u_i|\varepsilon_i)$ and $M(u_i|\varepsilon_i)$. They are given by

$$E(u_i|\varepsilon_i) = -\sigma_* \left[\frac{\varphi(\varepsilon_i\lambda/\sigma)}{1-\Phi(\varepsilon_i\lambda/\sigma)} - \left(\frac{\varepsilon_i\lambda}{\sigma}\right)\right],$$

and by

$$M(u_i|\varepsilon_i) = \begin{cases} -\varepsilon_i\left(\dfrac{\sigma_u^2}{\sigma^2}\right) & \text{if } \varepsilon_i \leq 0, \\ 0 & \text{otherwise,} \end{cases}$$

where ε_i is the composed error term, σ_v^2 and σ_u^2 are distribution parameters of v_i and u_i, $\Phi(\cdot)$ is the cumulative density function of standard normal distribution, $\varphi(\cdot)$ is the density function of standard normal distribution, $\sigma = (\sigma_v^2+\sigma_u^2)^{1/2}$,

$\lambda = \sigma_u/\sigma_v$, and $\sigma_*=(\sigma_v^2\sigma_u^2)/\sigma^2$. When the point estimates of u_i are obtained, the cost efficiency is estimated by $exp\{-u_i\}$.

3. SURVIVOR ANALYSIS

A survivor analysis was developed by George I. Stigler in 1958. The idea of the method is straightforward: those categories that relatively grow to the rest of the industry are assumed to have some advantage over the other ones. In the long-run, the distribution of firms should tend toward an optimum, which is, by the analysis, identified as the category (-ies) with the fastest growth. Categories may be defined by the size of firm, by the type of ownership, by location, and so forth.

An advantage of classical, univariate survivor analysis is that the method includes both the factor to be investigated and all other factors. The analysis thus includes factors that are hard to measure in econometric studies of cost function. On the other hand, a limitation of the survivor analysis is that it is not able to isolate the effects of those factors. This limitation can be moderated by taking an explicit account of such factors in an expanded, multivariate survivor analysis. The linear version of the multivariate survivor analysis takes the form
$s_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots + \beta_k x_{ki} + u_i$,
where s_i is the change in market share of group i, and $x_{1i}, x_{2i}, \ldots, x_{ki}$ are the explanatory variables (factors). An alternative is a binary growth model, but in this type of model, the information is lost in converting a continuous variable into a binary one. The univariate survivor analysis has one advantage over other methods: it is its simplicity. But both the univariate and multivariate survivor analyses, like the other methods of cost analysis, are not able to overcome the fact that the governments, not the market forces play a significant role in some industries. Changes in the structure of the market may rather demonstrate governmental interventions than a market competition.

Koutsoyiannis (1979) argues that the survivor analysis suffers from serious limitations. The survivor analysis assumes that: (a) the firms pursue the same objectives; (b) the firms operate in similar environments so that they do not have locational (or other) advantages; (c) prices of factors and technology are not changing; (d) the firms operate in a competitive market structure, that is, there are no barriers to entry or collusive agreements, since under such conditions inefficient (high-cost) firms would probably survive for long periods of time. Another shortcoming of the survival analysis is that it is not able to explain cases where the size distribution of firms remains constant over time. If the share of the various plant sizes does not change over time, this does not imply that all scales of plant are equally efficient.

4. CONCLUSION

We described and evaluated three methods of cost-function analysis: traditional econometric analysis, frontier analysis, and survivor analysis. The strength of classical econometric analysis is a highly developed methodology of hypothesis testing: both the selection of variables and the selection of functional form can be tested. The weakness of econometric analysis is that it is necessary to deal with many problems related to estimation technique, such as multicollinearity, autocorrelation, heteroscedasticity, etc. The strength of stochastic frontier analysis is that it incorporates random shocks in efficiency evaluation. On the other hand, strong assumptions about the distribution of efficiency have to be made. The advantages of survivor analysis are the simplicity and the possibility to include the factors that are otherwise hard to measure. The disadvantages of survivor analysis are: the method can be applied only in the long-term studies (it is not applicable to cross-sectional data); it does not provide specific information on the character of cost function in the studied industry. Hence results of survivor analysis are not controlled for possible confounding factors. Different approaches have strengths and weaknesses and the choice of the appropriate method depends, of course, on the objective of the study.

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MODELLING OF REASONABLE PROFIT IN REGULATED INDUSTRIES

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Abstract: The existence of pure monopoly in network industries increases the role of regulation mechanisms in connection with objectification and increase in their social effectiveness. The objective of regulation mechanisms is to find an appropriate proportion between price and product supply of network industry under assumption of the existence competitive market. With regard to analysis of equilibrium in network industries models it is important to point out that except for competition policy protection the state fulfils another specific task – regulation of network industries. The state influences proportional relations between price and supply of network industry production.

The aim of the paper is to examine the equilibrium conditions in the market of network industries. This issue bears an increased attention under conditions of economic influence of network industries in the national economies since the second half of the twenties of the last century. This tendency is especially visible particularly in the context of economic globalization and increased significance of the supranational entities. The conditions for equilibrium of network industries and methods of their regulations will be examined in the paper. The stress will be laid on the regulation on the base of returns – Rate of Return Regulation (ROR). Attention will be paid to the ways of calculation reasonable profit in regulated industries.

Keywords: Network industries, regulated prices, reasonable profit in regulated industries, rate of return regulation, Averch-Johnson model, Regulatory Office for Network Industries, regulated prices, reasonable profit in regulated industries.

1. INTRODUCTION

Creation of new regulatory framework was the important part of restructuring market with electricity, gas and other goods of network industries. In August 2001 the Regulatory Office for Network Industries (hereinafter referred as to "RONI") was established which task was an issue of licenses and regulation of prices and quality standards for goods of network industries. RONI started issuing licenses and creating quality standards in year 2001, but it started executing the price regulation from 1.1.2003. The primary aim of RONI was to prepare new regime of price regulation for goods of network industries.

RONI already defined regime of new price regulation for electricity distribution and worked out the system of new regulation rules for price making of goods of network industries. Models for determination of maximum prices and tariffs of goods of network industries were created over the years 2001-2002 and on their basis RONI proceeded in creation of regulatory and legal framework within the area of price regulation, which determined the method of calculation of maximum prices and tariffs for an item of goods or of a services, which delivery and provision is considered as a performing of activities subject to regulation.

A traditional methodological tool for price regulation, which is applied by price regulators at determining the price of products of network industries is the regulation on the basis of the rate of return, through which the production prices in the most developed economies are regulated, i.e. of electricity, gas and other companies.

The aim is to ensure the regulated subject to determine the price of production or services for his customer so that he can cover from its revenue all its *reasonable and providently arisen costs* as well as the regulated recoverability of its *provident* investment.

2. RATE OF RETURN REGULATION

Let us now derive the allowable rate of cost return for investments *RoR* of the regulated firm analytically. Let us suppose that the firm produces a homogenous product in production volume q, which it realizes at a relevant market for the price p. Let us further suppose that the firm uses two production factors, namely labour force with consumption level L by the labour price w and the capital with consumption level K by the capital price r.

The profit of the firm is generally defined as the difference between yields and costs in form

$$\pi(q) = t(q) - n(q)$$

where

$t(q) = p \times q$ – function of proceeds of the firm, $t: R \rightarrow R$

$n(q) = n_v(q) + n_F$ – function of the total costs of the firm, $n: R \rightarrow R$

$n_v(q)$ – function of variable costs of the firm, $n_v : R \rightarrow R$

n_F - fixed costs of the firm, $n_F \in R$

If we substitute the general cost function by a cost function on the basis of consumption of production factors, we get a profit function in the following form

$$\pi(q) = p \times q - w \times L - r \times K$$

If we further express the production volume q on the basis of the production function in the form

$$q = f(K, L)$$

and the production price p on the basis of the price-demand function in the form

$$p = p(q)$$

so we can express the profit function in the form

$$\pi(q) = p(q) \times q - w \times L - r \times K$$

and after further modification in the form

$$\pi(q) = p(f(K, L)) \times f(K, L) - w \times L - r \times K$$

A non-regulated firm can set its endogenous decision parameters in any way. So it chooses an optimum output volume q^*, an acceptable optimum price p^* and corresponding consumptions of the production factors labour L and capital K in order to reach the maximum profit. It calculates the optimum output and optimum price by solving the following task of mathematical programming

$$\pi(q) = p(f(K, L)) \times f(K, L) - w \times L - r \times K \to \max$$
$$K, L \in R_{\geq 0}$$

So in this case the non-regulated firm has no formal obstacles for setting the parameters guaranteeing its maximum profit. On the other hand the regulated form must respect the boundaries defined by the regulator. The model of price regulation on the basis of the rate of return consists in the fact that through the exogenously defined control variable RoR the allowable level of the quotient of the proceeds of the firm $p \times q$ reduced by its non-capital expenditures $L \times w$ and of the volume of consumed capital K is regulated.

The firm can optimise, respectively freely determine the consumption levels of labour L, capital K by the market prices of production factors w, r on one side and on the other side the level of its production q and the production price p. But the firm must respect the rate of return defined by the regulator, i.e. the validity of the relation

$$RoR \geq \frac{p \times q - w \times L}{K} \tag{1}$$

Let us now explore more in detail the relation between the rate of return of the capital expenditure and the profit of the regulated subject. The profit can be analytically expressed as the difference between the proceeds and the costs of the firm in the form

$$\pi(q) = p \times q - w \times L - r \times K \tag{2}$$

Let us deduct from both sides of the relation (1) the price of the capital *r*. We get the relation

$$RoR - r \geq \frac{p \times q - w \times L}{K} - r$$

After another modification we get

$$RoR - r \geq \frac{p \times q - w \times L}{K} - \frac{r \times K}{K}$$

$$RoR - r \geq \frac{p \times q - w \times L - r \times K}{K} \tag{3}$$

From the comparison of the relations (2) and (3) we get the relation

$$RoR - r \geq \frac{\pi(q)}{K}$$

$$(RoR - r) \times K \geq \pi(q) \tag{4}$$

We can see from the relation (4), that the regulated subject can set its system parameters only so, that its reached profit does not exceed the value of the capital evaluated by the difference between the rate of return defined by the regulator *RoR* and the price of capital *r*.

The regulated firm can set its controlled, respectively endogenous decision parameters only in a way in which it respects the condition determined by the regulator. It determines the regulated volume of output q_R, the acceptable regulated price p_R and the corresponding consumptions of production factors labour *L* and capital *K* so that it reaches the maximum profit and at the same time it respects the condition of the regulator (4) about not-exceeding the reasonable profit level. The regulated output and the regulated price are calculated by the solution of the following task of mathematical programming

$$\pi(q) = p(f(K,L)) \times f(K,L) - w \times L - r \times K \to \max \tag{5}$$

subject to

$$p(f(K,L)) \times f(K,L) - w \times L - r \times K - (RoR - r) \times K \leq 0 \tag{6}$$

$$K, L \in R_{\geq 0} \tag{7}$$

The solution of this optimisation task is the optimum level of consumption of the production factors labour L^* and capital K^*, on the basis of which subsequently the regulated optimum level of output q_R^* is quantified using the production function based on the relation

$$q_R^* = f(K^*, L^*)$$

and the regulated optimum price p_R^* with the use of the price-demand and production function based on the relation

$$p_R^* = p(q_R^*) = p(f(K^*, L^*)$$

whereby the rate of return of the capital of the firm defined by the parameter *RoR,* i.e. the exogenous control parameter determined by the regulator is respected.

In the situation when the firm would not be regulated and it would have an exclusive position at a relevant market, it would choose such an optimum volume of consumption of the variable input *labour* L^* and *capital* K^**,* that would ensure a maximum profit $\pi(q) = p(f(K^*, L^*)) \times f(K^*, L^*)$. On the basis of the optimum consumption of variable inputs it would determine its optimum supply $q^* = f(K^*, L^*)$ and the optimum price of production $p^* = p(q^*) = p(f(K^*, L^*)$.

In case the firm is regulated, it can choose only such a combination of production factors, so that the corresponding volume of supply and price of production generates the so-called *reasonable profit*, i.e. this relation is valid

$$(RoR - r) \times K \geq p(f(K, L)) \times f(K, L) - w \times L - r \times K$$
$$(RoR - r) \times K \geq \pi(q)$$

So in the end result the regulated form can produce in such a way that its *reasonable profit* does not exceed *RoR – r* multiple of the level of the variable input capital. This condition is called the *boundary of reasonable profit of a regulated firm* in professional literature.

However, this form of price regulation contains one serious risk, that it often motivates the firm to use a greater volume of the variable input capital than in a non-regulated firm.

3. AVERCH – JOHNSON MODEL OF REGULATION EFFECTS

A non-regulated monopolistic firm can set its optimal profit upon a solution of the following task on uncontrained extremum:

$$\pi(q) = t(q) - w \times L - r \times K \rightarrow \max$$
$$q, K, L \in R_{\geq 0} \qquad (8)$$

A solution of optimisation task is an optimal monopoly supply $q^*$and optimal consumption levels of production factors L^*, K^*. In case of exercise of regulation based on yields corresponding the rate of cost return for investment RoR a monopoly must respect the condition in the form

$$RoR \geq \frac{t(q) - w \times L}{K} \Leftrightarrow t(q) - w \times L \leq RoR \times K \qquad (9)$$

or after modification

$$t(q) - w \times L - r \times K \leq RoR \times K - r \times K$$

$$\underbrace{t(q) - w \times L - r \times K}_{\pi(q)} \leq RoR \times K - r \times K$$

$$\pi(q) \leq (RoR - r) \times K$$

It is evident that if a regulator determines for a regulated subject his rate of return RoR being higher than the price of capitak r, and is valid at the same time

$$RoR > r \Rightarrow RoR - r > 0$$

so then the firm has a guaranteed positive profit $\pi(q) > 0$ for each positive unit of invested capital K.

This conclusion really indicates a possibility, how the firm can increase its allowable "reasonable" profit by a non-reasonable and useless (inefficacious) accumulation of capital investments. Let us now explore this assumption in more detail. A regulated firm calculates its optimal parameters of behaviour by solving the following optimisation task of mathematical programming

$$\pi(q) = t(q) - w \times L - r \times K \rightarrow \max_{q,L,K}$$

subject to (10)

$$w \times L + RoR \times K - t(q) \geq 0$$

$$f(K, L) - q = 0$$

and also under the condition

$$RoR > r$$

Now we formulate the Langrange fuction for a task of mathematical programming on constrained extremum (10) in the following way

$$\mathsf{L}(q,K,L,\lambda,\mu)=\left[t(q)-w\times L-r\times K\right]+\lambda\left[w\times L+RoR\times K-t(q)\right]+\mu\left[f(L,K)-q\right] \quad (11)$$

Let us now explore in detail the validity of the Kuhn-Tucker optimality conditions for the Lagrange functions (11). We formulate the optimality conditions in the way

$$\frac{\partial \mathsf{L}}{\partial q}=0 \quad (a) \qquad \frac{\partial \mathsf{L}}{\partial L}=0 \quad (b) \qquad \frac{\partial \mathsf{L}}{\partial K}=0 \quad (c)$$

$$\lambda\times\frac{\partial \mathsf{L}}{\partial \lambda}=0 \quad (d) \qquad \mu\times\frac{\partial \mathsf{L}}{\partial \mu}=0 \quad (e)$$

$$\lambda,\mu\geq 0 \quad (f)$$

After a further modification of relations in Kuhn – Tucker conditions for optimal values of decision variables like volume of output q^*, consumption of production factors L^*, K^* and optimal values of Langrange multipliers λ^*, μ^*, we get the following

$$\left(1-\lambda^*\right)\frac{dt^*(q)}{dq}=\mu^* \quad (a1)$$

$$\mu^*\frac{\partial f^*(L,K)}{\partial L}=\left(1-\lambda^*\right)w \quad (b1)$$

$$\mu^*\frac{\partial f^*(L,K)}{\partial K}=r-\lambda^*\times RoR \quad (c1)$$

$$\lambda^*\left(w\times L^*+RoR\times K^*-t(q^*)\right)=0 \quad (d1)$$

$$\mu^*\left(f(L^*,K^*)-q^*\right)=0 \quad (e1)$$

$$\lambda^*\geq 0,\mu^*\geq 0 \quad (f1)$$

Let us now analyse the features of optimal solution of task (11), possibly the features of vector (q^*, K^*, L^*, λ^*, μ^*), that suits the optimality conditions (a1), ..., (f1). In case, the firm wants to reach the level of profit on a boundary defined by a regulator, he must set its optimal solution so that the condition on the upper level of allowable rate of return for investments $w \times L + RoR \times K - t(q) \geq 0$ was fulfilled as equality. However, it is definitely fulfilled only under the condition, if optimal value of Langrange multiplier λ^* is positive. Then, it is valid based on the relation (d1)

$$\lambda^* > 0 \Rightarrow w \times L^* + RoR \times K^* - t(q^*) = 0 \qquad (12)$$

The conditions (12) illustrates Averch – Johnson efect. The primary issue is, how a regulated firm responds to toughening a regulation regime by a regulator, i.e. to decreasing the allowable rate of rerurn for investments. Most probably, it results in the volume of investment into capital K realised by the firm. A change of level of investments into capital dK^* caused by a change of allowable rate of rerurn $dRoR$ can be formulated upon a derivative of relation (12) by the variable RoR as follows:

$$\frac{dK^*}{dRoR} = \frac{K^*}{\frac{dt(q^*)}{dq}\frac{\partial f(K^*,L^*)}{\partial K}} < 0 \qquad (13)$$

what implies that, if the regulator cuts the allowable rate of return for invested capital, then the regulated firm will apparently tend to redundant and inefficious increase of capital investments.

4. CONCLUSION

Based on the analysis of the behaviour of the firm in the conditions of regulation on the basis of the return of the used capital we have shown that in this regulation scheme the firm has the tendency to react to the tightening of the regulation conditions by the increase of the volume of used capital. However, the increase of the volume of the used capital is far from the aim of the system of regulation. The objective of regulation is rather to influence the values of other indicators important for the firm and for the economy, such as the volume of production, the level of product sale, respectively the cost level.

There are of course also other forms of price regulation, which influence the reasonable profit of the firm directly on the basis of the volume of its production, the level of product sale of the regulated firm, respectively on the basis of the amount of its total costs. The aim is to support the effective development of the regulated subject by help of regulation mechanisms.

Based on the results of analysis of Averch-Johnson model we can formulate the following important conclusions relating to behaviour of the firm being regulated upon the principle of regulation of rate of return for used capital.:

1. The firm being regulated upon the principle of regulation of rate of return for used capital is, in its natural effort to increase its “reasonable” profit, strongly motivated for redundant and inefficious growth of capital investments.

2. By reduction of rate of return for capital expenditures and under the condition it is still valid $RoR > r$, the firm responds by increasing capital expenditures in order to retain the volume of its profit (natural effort).
3. We can say, that reduction of rate of return for capital expenditures means a toughening the regulation conditions for the regulated firm.

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THE SIMULATED ANNEALING FOR LAGRANGEAN MULTIPLIER ADJUSTMENT IN THE CAPACITATED FACILITY LOCATION PROBLEM

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Abstract: The design of the distribution system can be formulated as the facility location problem, in which often must be taken into account limits on terminal capability. They constitute severe obstacles in exact solving processes. We focused on study of approximate methods based on Lagrangean relaxation of the capacity constraints, which has several advantageous properties. We used that the relaxed problem, known as the uncapacitated location problem, is possible to solve exactly even for real sized instances. The second useful property of the Lagrangean relaxation is that the objective function value of the optimal solution of the relaxed problem provides lower bound of the optimal solution of the original problem. We present combination of subgradient method and the simulated annealing algorithm for adjustment multipliers.

1. INTRODUCTION

Cost optimal design of the most of distribution and servicing systems consists in decisions on number and locations of facilities, from which customer's demands are satisfied. When distribution system is designed, limits on terminal capability often must be taken into account. To avoid the complications with this limits and to obtain a good solution of the capacitated facility location problem in sensible time.

We employ approximate methods based on Lagrangean relaxation of the capacity constraints, which has several advantageous properties. One of these properties is that the relaxed problem known as the uncapacitated location problem can be solved exactly by an algorithm based on Erlenkotter's approach [2], [3]. Real sized instances of the uncapacitated problem were broadly tested [4], [7] and it was proved that it is possible to obtain their optimal solution in a sensible time. The next useful property of the Lagrangean relaxation, which can be exploited with the previous one, is that the objective function value of optimal solution of the relaxed problem provides lower bound of the optimal solution of the original problem. In this way, the obtained approximate solution can be compared with the estimated value of the optimal one.

We present two methods for obtaining suitable values of Lagrangean multipliers. The classical one is based on a subgradient method applied on capacity constraints after their special adjustment. The second method is designed as an adaptive method with random experiments for determination of candidates for move from a current solution to the next one. Both approaches make use of the strengthening constraint, which considerably improves quality of the obtained lower bound. Both methods were tested and compared in this paper.

2. MATHEMATICAL MODEL OF THE SOLVED PROBLEM

Let J denote a finite set of customers and if a quantity of customer's demand can be expressed by a real number, then the demand of customer $j \in J$ be denoted by b_j. Let I denote a finite set of possible facility locations, the decision on facility location at place $i \in I$ is modelled by zero-one variable $y_i \in \{0,1\}$, which takes value 1 if a facility should be located at i and value 0 otherwise. Variable $z_{ij} \in \{0,1\}$ models a decision on assigning or not assigning customer j to facility location i. The matematical model of the cost minimal capacitated facility location problem can be formulated as follows:

$$\text{Minimize} \quad \sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{j \in J} c_{ij} z_{ij} \tag{1}$$

$$\text{Subject to} \quad \sum_{i \in I} z_{ij} = 1 \qquad \text{for } j \in J \tag{2}$$

$$y_i - z_{ij} \geq 0 \qquad \text{for } i \in I, j \in J \tag{3}$$

$$\sum_{j \in J} b_j z_{ij} \leq a_i \qquad \text{for } i \in I \tag{4}$$

A fixed charge for location of a facility for one year at possible location $i \in I$ is denoted by f_i. The cost of *j-th* customer yearly demand satisfaction via facility located at place i is denoted by coefficient c_{ij}. This coefficient includes all transportation costs for goods transport from the primary source to the facility location, from this location to the customer and manipulating cost in the facility.

In this integer programming model, constraints *(2)* ensure that each customer demand must be satisfied from exactly one facility location and constraints *(3)* force out the placement of a facility at location i whenever a customer is assigned to this facility location. Constraints *(4)* ensure that the total demand satisfied via facility location i doesn't exceed given capacity a_i. Having omitted or relaxed constraints *(4),* the problem *(1)-(3)* is model the uncapacitated facility location problem. It is effectively solved making use of implementation of the branch and bound method with Erlenkotter's lower bounding [2].

The performed reformulation of constraints *(5)* has proved to be a very efficient form for subsequent Lagrangean relaxation.

$$\sum_{j\in J} b_j\, z_{ij} \le a_i\, y_i \qquad \textit{for } i \in I \qquad (5)$$

3. LAGRANGEAN RELAXATION

The measure of the capacity constraint dissatisfaction is enumerated as the difference between the capacity and the demand, which is modeled by the expression on the left-hand-side of the constraint. This overload of a facility is weighed by a nonnegative weight called Lagrangean multiplier. Having denoted $u_i \ge 0$ the Lagrangean multiplier of the capacity constraint $i \in I$ [1]. The reformulation of the objective function is to be showed:

$$\sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J} c_{ij} z_{ij} + \sum_{i\in I} u_i \Big(\sum_{j\in J} b_j z_{ij} - a_i y_i\Big) =$$
$$\sum_{i\in I} (f_i - a_i u_i) y_i + \sum_{i\in I}\sum_{j\in J} (c_{ij} + b_j u_i) z_{ij} \qquad (6)$$

The strengthening constraint is derived from the capacities and is laid upon the number of located facilities. It is obvious that at least such a number of facilities must be located to cover all customer demands. If a denotes $\max \{a_i : i \in I\}$ and B denotes sum of all customer demands, then minimal number r of the necessary located facilities is $r = B / a$ and the strengthening constraint can be constructed in this way:

$$\sum_{i\in I} y_i \ge r \qquad (7)$$

We get the form of the uncapacitated location problem:

$$\textit{Minimize} \sum_{i\in I} (f_i - a_i u_i - v)\, y_i + \sum_{i\in I}\sum_{j\in J} (c_{ij} + b_j u_i)\, z_{ij} + r\, v \qquad (8)$$

$$\textit{Subject to} \sum_{i\in I} z_{ij} = 1 \qquad \textit{for } j \in J \qquad (9)$$

$$y_i - z_{ij} \ge 0 \qquad \textit{for } i \in I, j \in J \qquad (10)$$

The constraint *(7)* can be relaxed as well as the other capacity constraints using Lagrangean multiplier $v \ge 0$ with reformulation the objective function *(6)*.

The associated algorithms of the both approaches was designed with the goal to maximize a lower bound of the objective function value of an optimal solution of the original capacitated facility location problem:

3.1 SUBGRADIENT METHOD

This iterative process improves the lower bound by particular steps, in which it adjusts the values of the Lagrangean multipliers u_i [1], [6]. The overloads of facilities with dissatisfied capacity constraints are penalized by higher values of the associated Lagrangean multipliers and hence forced to minimize size of dissatisfaction. This process may lead to a complete capacity constraint satisfaction or, which is a more frequent case, to such solution, which is only slightly infeasible.

The subgradient method maximizes function $F(\mathbf{u})$, the value of which is equal to the minimal value of expression *(8)* subject to *(9), (10)* and integer nonnegative values of variables y_i and z_{ij} for the given values of $\mathbf{u} = <u_1, u_2, ... >$. Value $F(\mathbf{u})$ for the given $\mathbf{u}$ is obtained using the above mentioned dichotomy algorithm applied on the branch and bound method developed for the uncapacitated location problem. The algorithm starts from an initial point of the Lagrangean multipliers, usually it is zero.

A move from current point $\mathbf{u}^k$ to next point $\mathbf{u}^{k+1}$ is made along a direction, in which the function increases in the neighborhood of point $\mathbf{u}^k$. The steepest increase of $F(\mathbf{u})$ in the neighborhood of point $\mathbf{u}^k$ can be found along the gradient of this function at point $\mathbf{u}^k$. The coordinates of the gradient can be enumerated as values of partial derivatives of *(8)* or *(6)* by the individual multipliers u_i with consequent substitution of values u_i^k. In this way, the *i*-th component takes the value of the associated located facility overload.

To follow the direction of the gradient, the move should be performed in accordance with equality $\mathbf{u}^{k+1} = \max\{\mathbf{0}, \mathbf{u}^k + \alpha.\mathbf{grad}/\|\mathbf{grad}\|\}$, where

$grad_i = \sum_{j \in J} b_j z_{ij} - a_i y_i \text{ , } for\ i \in I.$

That is why the move is not performed exactly along the gradient, but along the direction of subgradient. The length of the step is given by parameter α, the value of which is chosen from interval $<\alpha_{min}, \alpha_{max}>$. After each step, values of $F(\mathbf{u}^k)$ and $F(\mathbf{u}^{k+1})$ are compared, and if $F(\mathbf{u}^k) \geq F(\mathbf{u}^{k+1})$ holds, then the return to $\mathbf{u}^k$ is done and the move is repeated with a lower value of α. The process terminates if parameter α reaches value of α_{min} or if the resulting improvement of the last step is less than the given value ε.

3.2 SIMULATED ANNEALING ALGORITHM

The metaheuristic method improves the lower bound by particular steps in which it adjusts the values of the Lagrangean multipliers u_i [5]. An algorithm based on this principle processes solutions of neighbourhood of a current solution in a given order. This method can search through a larger area of feasible solutions and increase the probability that better solution will be met. Any output variable of the experiment takes only one of two possible values, when probability of the value, which evokes the move, is indirectly proportional to the difference between the objective function values of the examined and current solutions. The probability is proportional to the parameter *T*, which is called the temperature of process. An usual form of the expression, which is used to enumerate the probability P_{move} of the move from the current solution $\boldsymbol{u}^k$ to the examined solution $\boldsymbol{u}^{k+1}$ at temperature T is $P_{move} = e^{-\frac{F(u^k)-F(u^{k+1})}{T}}$, where $F(\boldsymbol{u}^{k+1})$ and $F(\boldsymbol{u}^k)$ denote the objective function values of the examined and current solutions respectively. The probability is defined when $F(\boldsymbol{u}^k) \geq F(\boldsymbol{u}^{k+1})$. It starts from an initial point, which can be set to zeros.

At the beginning of searching process, the temperature *T* is set at a high value and the probability of move to a bad solution is high. The temperature adjustment can be done in accordance to the statement: $T = T/(1+\beta)$, where β is a parameter of the cooling process. The temperature is reduced and the probability of a move to a worse solution drops. The stopping criterion is the searching process after the given number of moves, sequence of which has been accompanied by no change of the best-found solution.

4. NUMERICAL EXPERIMENTS

The associated algorithms were implemented using Delphi 7 programming environment. The numerical experiments were performed using Pentium 4, 3.0 GHz, 1 GB with data originating at the Slovak road network (2907 dwelling places as customers, 71 centres of the possible facility locations). The number of the test problems is 35 with different values of the initial fixed charge and the cost coefficients.

The some results are reported in the table where the first attribute of the solution quality (LB) is resulting lower bound *(8)* of the objective function value of an optimal solution of the original problem. Two parameters of the quality of the solution were chosen to evaluate this possible capacity constraint violation: (Max Over.) is ratio of the maximum value of positive differences between the workloads of facilities and their capacities and the associated capacity and likewise (Sum Over.) for the sum of positive differences. (Time) is the computational time in seconds.

We were tested three approaches - the subgradient method, Simulated Annealing Algorithm and their the combination in which metaheuristic method starts from the initial point as the resulting of the subgradient method.

Table: The some results from 35 the test problems (use the bold font for better solution)

Problem	Method	LB	Max Over.	Sum Over.	Time[s]
The solutions of the Simulated Annealing and the combination correspond					
LAS*071322911	subgradient	23195365	0,89	1,31	4
	simulated annealing	396855578	0,54	0,54	138
	combination	396855578	0,54	0,54	52
LAS*071442911	subgradient	35466124	0,97	1,36	6
	simulated annealing	405981780	0,48	0,48	54
	combination	405981780	0,48	0,48	53
The Simulated Annealing is better solution					
LAS*071722911	subgradient	18062008	0,61	1,96	5
	simulated annealing	**390055578**	**0,54**	**0,54**	**45**
	combination	21621064	7,17	11,13	51
LAS*071462911	subgradient	50328934	0,97	1,01	4
	simulated annealing	**417268930**	**0,44**	**0,44**	**62**
	combination	75495587	10,22	13,79	538
The combination is better solution					
LAS*071542911	subgradient	29172384	0,97	1,38	2
	simulated annealing	93982507	14,79	14,79	54
	combination	**399581780**	**0,48**	**0,48**	**57**
LAS*071552911	subgradient	33011641	0,96	2,01	5
	simulated annealing	329157787	14,63	14,63	144
	combination	**366204381**	**11,59**	**11,59**	**47**
LAS*071662911	subgradient	32497717	0,96	1,00	2
	simulated annealing	42943702	5,67	9,52	45
	combination	**364659561**	**3,1**	**4,89**	**190**

5. CONCLUSION

As concerns lower bound of the original problem, the above- mentioned approaches to the capacitated facility location problem promise a good solution, especially if constraints *(5), (7)* are employed. The Simulated Annealing process wins in 25% of the cases for 35 the test problems and the combination two approachs wins in 45%. In 30% of the problems is no unambiguous winner. This effect evokes an idea for the future research, where composed heuristic should be designed and studied.

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MONETARY MODELS OF THE SKK/EUR EXCHANGE RATE

MICHAELA CHOCHOLATÁ

Abstract: This paper deals with presentation and estimation of the three versions of monetary models of SKK/EUR exchange rate in order to forecast its future development.

INTRODUCTION

The monetary approach to the exchange rate determination tries to explain how exchange rates respond to changes in monetary policy. Monetary models of exchange rate determination assume a strong connection between the nominal exchange rate and a set of monetary fundamentals. The monetary model implies that price level of a country is determined by its supply and demand for money and that the price level in different countries should be the same when expressed in the same currency (purchasing power parity). According to this approach the prices are assumed to be flexible in the short - run and it is also assumed that the purchasing power parity holds continuously – through the purchasing power parity are monetary policy changes transmitted to exchange rates.

This paper will concentrate on the three versions of monetary models: the flexible – price monetary model (first presented by Frenkel [5] and Bilson [1]), the sticky – price monetary model (suggested by Dornbusch [3]) and the real interest rate differential monetary model (pioneered by Frankel [4])[1]. The monetary approach in its various forms has been widely tested in order to evaluate the model's ability to explain and predict the path of exchange rates (see e.g. [2], [6], [7], [8]).

The aim of this paper is to estimate the presented versions of monetary models for the SKK/EUR exchange rate for the period 1. quarter 1998 – 4. quarter 2005 and after positive economic and statistical verification, calculation of the exchange rate forecasts for the individual quarters of the 2006.

THE FLEXIBLE – PRICE MONETARY MODEL

The flexible – price monetary model is the simplest one of the presented models. In this model we assume that the purchasing power parity (PPP) holds all times, i.e. contiunuously, which means that the exchange rate - s_t (expressed

[1] By presentation of the above mentioned monetary models we will suppose the existence of a domestic and foreign country (variables concerning the foreign country will be denoted by an asterisk (*)). All variables with exception of interest rates will be expressed in logarithms.

in units of domestic currency per one unit of foreign currency) will immediately adjust to the relative prices of domestic (p_t) and foreign (p_t^*) goods (hence the term flexible - price monetary model):

$$s_t = p_t - p_t^* \qquad (1)$$

Furthermore the model assumes a stable money demand function in domestic and foreign country. The money market equilibrium conditions in domestic and foreign country are supposed to depend on the logarithm of real income (y_t and y_t^*, respectively), the logarithm of price level (p_t and p_t^*, respectively) and nominal interest rate (i_t and i_t^*, respectively). Monetary equilibria in the domestic and foreign country are given by equations (2) and (3):

$$m_t = p_t + \beta_2 y_t - \beta_3 i_t \qquad (2)$$

$$m_t^* = p_t^* + \beta_2 y_t^* - \beta_3 i_t^* \qquad (3)$$

where m_t and m_t^* are the domestic and foreign money supply respectively. It is assumed that both the income and interest rate elasticities of the demand for money (β_2 and β_3, respectively) are the same in both countries. After solving equations (2) and (3) for p_t and p_t^* and substituting it into the equation (1) we arrive at the flexible – price monetary model of the exchange rate:

$$s_t = \left(m_t - m_t^*\right) - \beta_2\left(y_t - y_t^*\right) + \beta_3\left(i_t - i_t^*\right) + \varepsilon_t \qquad (4)$$

where ε_t is a disturbance term.

According to the uncovered interest rate parity it holds that the interest rate on a domestic bond equals the interest rate on a foreign bond, adjusted for the expected rate of appreciation or depreciation of the foreign currency ($s_{t+1}^e - s_t$):

$$s_{t+1}^e - s_t \approx i_t - i_t^* \qquad (5)$$

If PPP holds continuously, it must be true that the expected change in the exchange rate will be equal to the difference in expected inflation rates between the two analysed countries:

$$s_{t+1}^e - s_t \approx \pi_{t+1}^e - \pi_{t+1}^{*e} \qquad (6)$$

Taking into account the formulas (5) and (6) we will furthermore assume that

$$i_t - i_t^* = \pi_{t+1}^e - \pi_{t+1}^{*e} \qquad (7)$$

Equation (4) can be also rewritten as:

$$s_t = \left(m_t - m_t^*\right) - \beta_2\left(y_t - y_t^*\right) + \beta_3\left(\pi_{t+1}^e - \pi_{t+1}^{*e}\right) + \varepsilon_t \qquad (8)$$

which is another form of the flexible – price monetary model.

The coefficient of the relative money supply is positive and equals to one based on the neutrality of money. An X percent increase in the domestic money supply will cause the increase of prices by the same percentage. On condition that PPP holds continuously, this would mean a depreciation of the domestic currency (i.e. increase in s_t) by the same amount, in order to restore equilibrium.

In the flexible – price monetary model real forces influence exchange rates but only indirectly to the extent that they first affect the demand for domestic money. A rise in the domestic real income creates an excess demand for the domestic currency. Assuming the domestic money supply is fixed, an increase in the real demand for domestic money could only be satisfied if the domestic price level (p_t) falls, thereby restoring monetary equilibrium. Then taking into account the validity of the PPP, a decline in the domestic price level results in an appreciation of the domestic currency.

Furthermore, a relative rise in domestic interest rates, which reflects a rise in domestic inflationary expectations (see equation (7)), will give rise to a depreciation of the domestic currency's value (i.e. an increase in s_t).

THE STICKY – PRICE MONETARY MODEL

In the sticky – price monetary model the prices are assumed to be sticky in the short – run. This model is also based on the PPP validity, but only in the long - run, which means that long – run equilibrium exchange rate ($\bar{s}_t$) is determined by relative price levels ($\bar{p}_t$ and $\bar{p}_t^*$) in a domestic and foreign country[2]:

$$\bar{s}_t = \bar{p}_t - \bar{p}_t^* \qquad (9)$$

From equation (10) received through the combination of equations (4) and (9) we can see that the interaction between the supply of and demand for the respective monies will now determine the exchange rate only in the long run, i.e.

$$\bar{s}_t = \left(\bar{m}_t - \bar{m}_t^*\right) - \beta_2\left(\bar{y}_t - \bar{y}_t^*\right) + \beta_3\left(\bar{i}_t - \bar{i}_t^*\right) \qquad (10)$$

In the sticky – price monetary model it is furthermore assumed that the expected rate of depreciation of the exchange rate is a positive function of the gap between the long – run equilibrium rate and the current exchange rate

$$s_{t+1}^e - s_t = \alpha\left(\bar{s}_t - s_t\right) \qquad (11)$$

where α is the speed of adjustment to equilibrium. This equation states that the current exchange rate will, over time, gradually gravitate towards its long – run equilibrium level ($\bar{s}_t$) at the rate of α.

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[2] A „bar" over a variable denotes long-run equilibrium.

To solve for the current level of the exchange rate, let's assume the validity of the uncovered interest rate parity (5). After combining the equations (5) and (11) we receive

$$i_t - i_t^* = s_{t+1}^e - s_t = \alpha(\bar{s}_t - s_t) \quad (12)$$

Solving for s_t results in

$$s_t = \bar{s}_t - \frac{1}{\alpha}\left(i_t - i_t^*\right) \quad (13)$$

Combining equations (10) and (13), adding of the disturbance term ε_t and solving for s_t leads to the Dornbusch sticky – price monetary model

$$s_t = \left(\overline{m}_t - \overline{m}_t^*\right) - \beta_2\left(\overline{y}_t - \overline{y}_t^*\right) + \left(\beta_3 - \frac{1}{\alpha}\right)\left(\bar{i}_t - \bar{i}_t^*\right) + \varepsilon_t \quad (14)$$

The equation (14) is after substitution $\beta_4 = \beta_3 - \frac{1}{\alpha}$ actually identical to the reduced equation of the flexible – price monetary model, thus the sticky – price monetary model reduces to a flexible - price monetary model in the long – run (let's assume current values for the explanatory variables are long – run equilibrium levels):

$$s_t = \left(m_t - m_t^*\right) - \beta_2\left(y_t - y_t^*\right) - \beta_4\left(i_t - i_t^*\right) + \varepsilon_t \quad (15)$$

The models (4) and (15) are almost the same. The only difference between them is that whereas the coefficient β_3 in model (4) has a positive sign, the coefficient β_4 in model (15) is expected to be negative. This means that in case of model (15) an increase in the domestic interest rate leads to a capital inflow which increases the demand for the domestic currency and, in turn, leads to the appreciation of the domestic currency (i.e. a decrease in s_t).

THE REAL INTEREST RATE DIFFERENTIAL MONETARY MODEL

The real interest rate differential monetary model is a combination of the two above mentioned models. It draws upon the sticky – price model by assuming that PPP holds only in the long – run. The only difference between these models concerns the factors influencing exchange rate expectations. The expected change in exchange rate in the real interest rate differential model is assumed to be influenced both by the gap between the spot and the long – run equilibrium exchange rate (as in the sticky – price monetary model), and by the long – term expected inflation differential (as in the flexible – price monetary model).

$$s_{t+1}^e - s_t = \alpha(\bar{s}_t - s_t) + \left(\pi_{t+1}^e - \pi_{t+1}^{*e}\right) \quad (16)$$

Combining equations (10) and (16) and solving for s_t , after substitutions $\beta_4 = 1/\alpha$, $r_t = i_t - \pi^e_{t+1}$ and $r^*_t = i^*_t - \pi^{*e}_{t+1}$ we arrive at the more general monetary model (ε_t denotes the disturbance term)

$$s_t = \left(m_t - m^*_t\right) - \beta_2\left(y_t - y^*_t\right) + \beta_3\left(\pi^e_{t+1} - \pi^{*e}_{t+1}\right) - \beta_4\left(r_t - r^*_t\right) + \varepsilon_t \tag{17}$$

„The real interest rate differential monetary model is a more general model of exchange rate determination in that it allows for (a) the once-and-for-all direct impact of money growth on exchange rates embodied in all monetary models, (b) the indirect effect of expectations of higher or lower inflation on exchange rates embodied in the flexible – price model, and (c) the liqudity-induced effect on real interest rates, capital movements and, therefore, on the exchange rate embodied in the sticky – price model." [7]

MONETARY MODELS OF THE SKK/EUR EXCHANGE RATE – APPLICATION

In this part we estimated three versions of the following general monetary model

$$s_t = \beta_0 + \beta_1\left(m_t - m^*_t\right) + \beta_2\left(y_t - y^*_t\right) + \beta_3\left(\pi^e_{t+1} - \pi^{*e}_{t+1}\right) + \beta_4\left(i_t - i^*_t\right) + \varepsilon_t \tag{18}$$

The analysis was done on quarterly data for the period 1. quarter 1998 – 4. quarter 2005 which is 32 observations. Symbol s_t denotes the logarithm of the SKK/EUR exchange rate; m_t and m^*_t denote the logarithms of monetary base M2 in the SR (bil. SKK) and in the EMU[3] (bil. EUR), respectively; y_t and y^*_t are the logarithms of real income expressed as logarithms of real GDP in constant prices of 1995 in the SR (bil. SKK) and in the EMU (bil. EUR)[4], respectively; π^e_{t+1} and π^{*e}_{t+1} are expected inflation rates in next period measured by consumer price index in the SR and harmonized consumer price index in the EMU (the same quarter of the previous year = 1), respectively; i_t and i^*_t are finally nominal interest rates – we used the 3-month BRIBOR and 3-month EURIBOR.

[3] EMU – European Monetary Union

[4] Since for the EMU only the seasonally adjusted dates for monetary base M2 and real GDP were available, we use also for Slovakia seasonally adjusted (moving averages) dates for the monetary base M2 and real GDP.

Estimations of the three versions of monetary models[5] (with t-statistics in parenthesis) together with the coefficient of determination R^2, adjusted coefficient of determination R^2adj, F-statistic and statistics testing the character of residuals (uncorrelatedness[6] - Ljung-Box Q-statistic, homoscedasticity - LM statistic and normality - Jarque-Bera statistic) are in the Table 1.

When we compare the calculated Ljung-Box Q-statistic, LM statistic $T.R^2$ (T=number of observations) and Jarque-Bera statistic with critical values ($\chi^2_{0,05}(12) = 21{,}026$, $\chi^2_{0,05}(1) = 3{,}8415$, $\chi^2_{0,05}(2) = 5{,}9915$) it is clear that residuals in all three models had (after application of the Box-Jenkins methodology) the character of the white noise, i.e. were on the significance level 0,05 up to the lag 12 mutually uncorrelated, homoscedastic (without ARCH effect) and normally distributed.

In all three models we can reject the hypothesis that $\beta_1 = 1$ and in case of the model II we can even see that β_1 was not statistically significant different from 0. The statistical significance was also a problem of parameter β_3 in model II and also the signs of parameters β_3 and β_4 in this model were not correct. Considerably better results were received in cases of models I and III.

All parameters of model I were statistically significant on any significance level[7]. The negative sign of β_2 correctly indicates that a relative rise in domestic economic activity results in a rise in the domestic currency's value. The positive sign of β_4 is also in a flexible-price monetary model correct, because a relative rise in domestic interest rates results in a depreciation of the domestic currency's value[8].

The estimation of model III was based on validity of formula (7). The signs of all estimated parameters are correct and all parameters were statistically significant on significance level 0,05.

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[5] The whole analysis was done in econometric software EViews 5.1.

[6] In order to ensure the uncorrelatedness of residuals the Box-Jenkins methodology ARIMA (autoregressive integrated moving average) was used.

[7] With exception of β_1 which was significant on the significance level 0,0812.

[8] In case of sticky-price monetary model should be the sign negative.

Table 1

I.	$\hat{s}_t = 3{,}179 + 0{,}229(m_t - m_t^*) - 0{,}462(y_t - y_t^*) + 0{,}0084(i_t - i_t^*)$ (9,646) (1,817) (−7,732) (5,032) $\hat{\varepsilon}_t = 0{,}616\hat{\varepsilon}_{t-1} + \nu_t - 0{,}908\nu_{t-2}$ (5,716) (−18,530) $R^2 = 0{,}934$; R^2adj = 0,921; F = 71,000; Q(12) = 10,262; T.R^2 = 0,071; J-B = 1,773
II.	$\hat{s}_t = 3{,}074 + 0{,}196(m_t - m_t^*) - 0{,}483(y_t - y_t^*) - 0{,}0943(\pi_{t+1}^e - \pi_{t+1}^{*e}) +$ $+ 0{,}0085(i_t - i_t^*)$ (8,407) (1,450) (−7,174) (−0,734) (4,873) $\hat{\varepsilon}_t = 0{,}609\hat{\varepsilon}_{t-1} + \nu_t - 0{,}913\nu_{t-2}$ (5,511) (−19,252) $R^2 = 0{,}936$; R^2adj = 0,920; F = 58,166; Q(12) = 7,512; T.R^2 = 0,090; J-B = 1,336
III.	$\hat{s}_t = 2{,}996 + 0{,}272(m_t - m_t^*) - 0{,}595(y_t - y_t^*) + 0{,}399(\pi_{t+1}^e - \pi_{t+1}^{*e})$ (12,064) (2,636) (−9,722) (2,203) $\hat{\varepsilon}_t = \nu_t + 0{,}283\nu_{t-1} - 0{,}528\nu_{t-3} - 0{,}744\nu_{t-4}$ (2,074) (−3,793) (−5,798) $R^2 = 0{,}907$; R^2adj = 0,884; F = 40,411; Q(12) = 6,591; T.R^2 = 0,206; J-B = 0,354

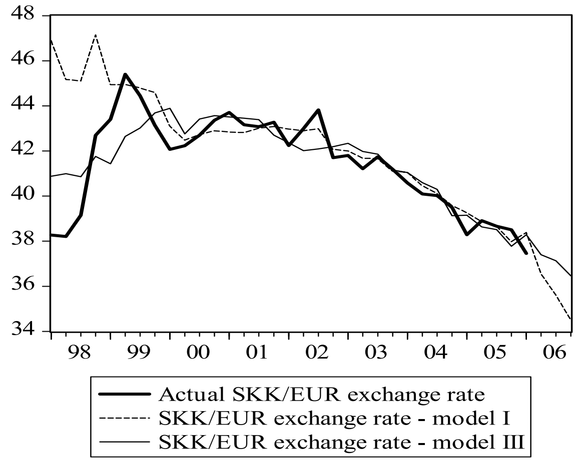
According to the above mentioned results it is possible to say that models I and III are quite good and therefore were used to calculate forecasts for the individual quarters of 2006. The explanatory variables were forecasted through the trend models in combination with ARIMA models and then used to forecast the SKK/EUR exchange rate. The forecasted values of the SKK/EUR exchange rates based on models I and III are in Table 2:

Table 2

	2006:1	**2006:2**	**2006:3**	**2006:4**
Model I	38,387	36,557	35,608	34,506
Model III	38,283	37,404	37,131	36,469

Actual values of the SKK/EUR exchange rate and fitted values of this exchange rate calculated by models I and III together with the forecasted values are depicted in Graph 1:

Graph 1



CONCLUSION

The SKK/EUR exchange rate forecasts were calculated through the two versions of monetary models with satisfying economic and statistical characteristics. Both models predict the appreciation of the SKK during the year 2006 which is a generally expected tendency. The already known values of the SKK/EUR exchange rate for the 1. and 2. quarter 2006 were 37,456 and 37,667, so the SKK actually slowly depreciated against the EUR which is probably attributable to reaction on results of parlamentary elections in June. According to the actual development and also according to the ability of the concrete model to copy the past development we can consider the results received by model III to be more presumptive.

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MODELING OF COORDINATION PROBLEMS IN NETWORKS

PETR FIALA

Abstract: The paper is devoted to modeling of network coordination. There are some approaches to model and analyze network effects. In the model are used some important features of game theory approaches. The combination of network structure modeling and modeling of coordination processes can be a powerful instrument of performance analysis of networks.

Keywords: network economy, coordination, games, multiple decision-makers, performance

1. NETWORK ECONOMY

The network economy is a term for today's global relationship among economic agents characterized by massive connectivity (Economides,1996, Shapiro, Varian, 1999). The central act of the new era is to connect everything to everything in deep web networks at many levels of mutually interdependent relations, where resources and activities are shared, markets are enlarged and costs of risk are reduced. Network industries play a crucial role in modern life.

Traditionally, economics has considered the interactions between agents as taking place within anonymous institutions, typically the markets. However, in the last thirty years the theory of industrial organization has drawn attention to the importance of strategic interaction among agents,

Networks are presented as environments in which strategic interaction takes place. The networks' topologies and the characteristics of links influence the strategies of all agents and economic results consequently. Networks play a crucial role in the formation of economic structures, in the circulation of information and in the emergence of competition and cooperation among agents.

Coordination is the process of managing interdependencies between activities. An agent is a subject that has some knowledge about the situation and can perform actions. The problem of coordination occurs when an agent has a choice in its actions, and the choice, the order and the time at which actions are carried out affects performance. A multiagent system is a collection of agents that coexist in an environment, interact with each other, and try to optimize a performance measure. Research in multiagent systems aims at providing principles for the construction of complex systems containing multiple independent agents and focuses on behavior issues in such systems. A key aspect in such systems is the problem of coordination: the process that ensures that the individual decisions of the agents result in jointly optimal decisions for the group. In principle game theoretic techniques can be applied to solve the coordination problem. Whenever agents should choose an identical action to receive better performance, we speak about a coordination game. As a typical example think about competing technology standards. When agents are capable of agreeing on the technology to apply, the performance is high. However, in the lack of a common technology standard the performance is lower.

2. MODELING INSTRUMENTS

The modeling instruments for analyses of network coordination are:

- game theory models,
- oligopoly models,
- negotiation models.

Game theory provides a collection of tools for predicting outcomes for a group of interacting units. It is the basis for development of network coordination models, cooperative and non-cooperative models as well. We will use normal form games and also extensive form games for analyzing cooperative and non-cooperative behavior of units in the network. In normal form games all units choose their actions simultaneously, whereas in extensive form games units may choose their actions in different time periods. There are some special types of games to illustrate network economy problems as coordination games. A coordination game is a situation with interdependent decisions, a coincidence of interests, and at least two proper coordination equilibria. Nash equilibrium concept is used for solution of non-cooperative models. A basic definition is this: If there is a set of strategies for a game with the property that no player can benefit by changing his strategy while the other players keep their strategies unchanged, then that set of strategies and the corresponding payoffs constitute Nash equilibrium.

The Cournot, Stackelberg and Bertrand models are representations of oligopolistic behavior. Cartel models are representations of cooperative behavior. Negotiations take place in cooperative solutions of competition problems. There are approaches based on game theory and other approaches including ones based on multicriteria evaluations.

The theoretical framework serves as a common basis for developing special models for analyzing specific features in network coordination.

3. BASIC MODEL OF COORDINATION GAME

Coordination games attract many theoretically and experimentally oriented economists. The basic model is a simple symmetric normal form 2×2 games which are characterized by having two equilibria in pure strategies. In

coordination games equilibria in pure strategies are characterized by the requirement that players choose the same strategy. Both symmetric equilibria may be candidates for strategy selection.
The game is characterized by the payoff table

	X	*Y*
X	b,b	c,d
Y	d,c	a,a

with $a > b > c > d > 0$ and $(b - d) > (a - c)$, i.e. (Y, Y) is Pareto dominant equilibrium and (X,X) is the risk dominant equilibrium.
Let $N = \{1, 2, \ldots, n\}$ be the set of agents. Agents play an n-player game. Agents are embedded in a graph G that represents the pattern of partnerships. The link between i and j is denoted by g_{ij} such that $g_{ij} = 1$ when i and j are partners and $g_{ij} = 0$ otherwise.
The game is played in large populations with players who are pairwise matched in network structures. Players are allowed to choose their neighbors by themselves and to select simultaneously their neighbors in the population and the strategy in the basic game.
Decision making is supposed to be deterministic. Opening a new link to a member of the population is supposed to generate constant connection costs per link for the agent who initiates the link. Let k denote the linking costs. The relative size of linking costs compared with the payoffs of the game has a big impact on the resulting equilibrium network in the population. The resulting equilibrium networks are characterized by non directed graphs. Depending on the particular value of linking costs k we obtain different graphs for coordination games.
a) If $k > a$ then the unique equilibrium network G is the empty network and the action choice of each player in the coordination game is not determined.
b) If $k < d$ holds then the unique equilibrium network G is the complete graph and all players choose the equilibrium strategy X or all players choose the equilibrium strategy Y.
c) If $d < k < c$ holds then either the equilibrium is the one obtained in part b) or there is an equilibrium such that the resulting network is characterized by a graph G with two sets of vertices N_1 (agents play the strategy X) and N_2 (agents play the strategy Y) such that all vertices in N_1 are connected with all vertices in N_2 but not with each other in N_1 while all vertices in N_2 are also connected with each other.
d) If the relation $c < k < b$ holds then either the equilibrium is the one obtained in part b) or an equilibrium induces a disconnected graph G with two components where each component is a complete graph and players in one

component (N_1) choose action X, players in the other component (N_2) choose action Y.
e) If $b < k < a$ holds then an equilibrium network G is the complete graph with all players choosing Y. An alternative equilibrium is the empty graph with all players choosing X.

Fig. 1. Typical equilibrium networks

The approaches for analyzing the coordination problem and network formation process can be divided to two groups: cooperative and non-cooperative approaches.

4. COOPERATIVE APPROACHES

An important first paper in the literature is by Myerson (1977). Myerson started from cooperative game theory. The model is a game ($N; v$), where N is a set of players and v is a characteristic function denoting the worth $v(S)$ of each coalition S. He defines a cooperation structure as a non-directed graph G among the agents. A graph represents a list of which individuals are linked to each other, with the interpretation that if individual i is linked to individual j in the graph, then they can communicate and function together.
Thus, a network G partitions the set of individuals into different groups who are connected to each other. The value of a coalition S under the network G is the sum of the values of the sub-coalitions of S across the partition of S induced by G.
The cooperative approach, introduced by Myerson (1977), can be considered as a natural development of the theory that studies the formation of coalitions: this theory analyzes the external structure of coalitions by focusing on the allocation of players within the various groups only; the purely cooperative approach goes further and considers the set of communication components within each coalition.
Myerson's approach was followed by Aumann and Myerson (1988) who introduced two-stage game: in the first stage links are formed, and in the second stage the players receive payoffs which depend on the value of the network, according to an exogenous rule. They use the extensive form to represent the game, assume that players don't bear any cost while forming links.

Jackson and Wolinsky (1996) introduce the concept of pairwise stability in order to analyze the network formation process. The game is given by the finite set of players *N*, function $v : G \rightarrow R$ gives the value of a network *G*, while $\pi(G, v)$ is the allocation rule describing the way in which the value of the network is allocated among players. A network *G* is pairwise stable if, given any two linked agents, none of them benefit from deleting the link, and if, given any two unlinked agents, it cannot be that both of them find it convenient to form the link.

Let *G* be a network; $(i, j) \in G$ *g* means that players *i* and *j* are linked to each other, $(i, j) \notin G,$ means the opposite, $g + ij$ is the network obtained when the link between *i* and *j* is set and $g - ij$ means just the opposite.

A network *G* is pairwise stable if both of the following conditions hold

(*i*) $\forall (i, j) \in G, \pi_i(G, v) \geq \pi_i(G - (i, j), v), \pi_j(G, v) \geq \pi_j(G - (i, j), v)$

(*ii*) $\forall (i, j) \notin G,$ if $\pi_i(G + (i, j), v) \geq \pi_i(G, v),$ then $\pi_j(G + (i, j), v) < \pi_j(G, v)$

The concept of pairwise stability is very helpful to both evaluate a given network topology and determine the set of equilibria, but it says nothing about their desirability. Pairwise stable networks are not always efficient.

5. NON-COOPERATIVE APPROACHES

Bala and Goyal (2000) studied the strategic effects of local interaction for a coordination game with agents exogenously located on network nodes. There are some assumtions of the approach. Players can add or delete links unilaterally; thus there is non need for an agreement between the players in order to a link to be set. Consider two different patterns of information transmission: in the one way flow models, the information goes from j to i, while in the two-way flow models, the information goes from and to both agents.There are some basic results:

Nash Network is based on Nash equilibrium concept for the network G:

- In the one-way flow models, a Nash Network is either empty or minimally connected.
- In the two-way flow models, a Nash Network is either empty or minimally pairwise-connected.

Bala and Goyal show that, in general, the number of Nash networks increases very rapidly with *n*.That leads to focus on strict Nash networks,that are networks supported by a strict Nash equilibrium.

The set of Strict Nash Networks is significantly more restrictive:

- In the one-way flow models, a Strict Nash Network is either empty or a wheel.
- In the two-way flow models, a Strict Nash Network is either empty or a center-sponsored star.

Fig. 2. Typical strict Nash networks

6. EXTENSIONS OF BASIC CONCEPTS

The assumptions of basic concepts can be extended to get more realistic models of coordination problems. The approaches for analyzing the coordination problems can be modified also.

We mention only some of these possible extensions:

- indirect links,
- imperfect reliability,
- costs and benefits,
- system dynamics,
- combined approaches

The importance of the above results is weakened by the strong assumption that information obtained through indirect links has the same value as that obtained through direct links. There are some possibilities how to model dependency of value of agent *j*'s information to *i* on distance between *i* and *j*. The concept of information decay introduces the decay parameter to modify the value of information by distance between agents.

In order to represent the assumption of imperfect reliability and to add realism there is possible to construct a stochastic network model. By introducing a probability parameter p into stochastic network model each link works with probability p, while fails with probability $1-p$.

Standard models can be modified by introducing cost and benefit structures according the real situation. The situation can differ in assumptions who will share link formation costs and related benefits and what are rules of sharing.

Problems of coordination on networks are dynamic problems very often. For example network externalities as positive or negative are dynamic effects. The situation can be described by standard instruments like differential equations. The network structure of the problems introduces the possibility of using extensive form games and repeated games for describing the dynamic processes.

Combined approaches for analyzing coordination problems are much promised because they exploit advantages from both theories - cooperative and non-cooperative ones.

7. CONCLUSION

Network economy is today reality. Network coordination is the important subject of an intensive economic research. The paper presents a basic theoretical framework for analyses of network coordination and some problems and approaches for coordination activities. The framework makes possible to develop simple models for analyzing specific features in network coordination. Extensions of the basic framework are possible and very useful to be more realistic. Than it can be construct simple models for analyzing coordination problems. The simple models have very important managerial implications indeed. The combination of such models can give more complex views on the problem of network coordination.

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AHP MODEL FOR MEASURING EFFICIENCY OF PRODUCTION UNITS – A CASE STUDY[9]

JOSEF JABLONSKÝ

1. INTRODUCTION

The importance of pension funds is very high in current economic conditions in the Czech Republic. It is given by several factors. Not very positive demographic perspectives of the Czech population belong among them to the one of the most significant ones. That is why the responsibility of each individual is increasing and the state supports the individuals investing into pension funds in this situation. Since 1998 there were established together 12 pension funds in the Czech Republic. All of them offer standard services of long-term saving of money bringing usually the above-average rate of return comparing to other low risk investment instruments. The investing into the pension funds is characterized by the following:

- it is a saving instrument with a very long time horizon based on monthly saving of very low amounts leading to a significant target amount when the pension age is reached,
- without investing into the pension funds the life standard of the most individuals can be significantly lower after they reach the pension age,
- the higher rate of return together with fixed monthly state support makes the investing into the pension funds one of the most economic and at the same time the safest investment opportunities.

The aim of the paper is to describe one of the approaches for evaluation and analysis of the current situation in pension market and estimate the efficiency of pension funds. The standard modeling tool for evaluation of efficiency is data envelopment analysis (DEA). The standard DEA models split the evaluated units into the efficient and inefficient ones and they are not able to differentiate among the efficient ones. The classification of efficient units can be done by so called super-efficiency DEA models. Our aim is to develop an AHP model for evaluation of pension funds (generally production units) indicated as efficient ones by means of a DEA model used in the first stage of the analysis and compare the results given by super-efficiency DEA models with conclusions given by AHP model.

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2. DATA ENVELOPMENT ANALYSIS MODELS

Data envelopment analysis (DEA) is a tool for measuring the relative efficiency and comparison of decision making units (DMU). The DMUs are usually described by several inputs that are spent for production of several outputs. Let us consider the set E of *n* decision making units E = {DMU_1, DMU_2, …, DMU_n}. Each of the units produces *r* outputs and for their production *m* inputs are spent. Let us denote $\mathbf{x}^j = \{x_{ij}, i=1,2,\dots,m\}$ the vector of inputs and $\mathbf{y}^j = \{y_{ij}, i=1,2,\dots,r\}$ the vector of outputs of the DMU_j. Then **X** is the (*m*, *n*) matrix of inputs and **Y** the (*r*, *n*) matrix of outputs. The basic principle of the DEA in evaluation of efficiency of the DMU_q, $q\in\{1,2.,,,.n\}$ consists in looking for a virtual unit with inputs and outputs defined as the weighted sum of inputs and outputs of the other units in the decision set - $\mathbf{X}\lambda$ a $\mathbf{Y}\lambda$, where $\lambda=(\lambda_1, \lambda_2,\dots, \lambda_n)$, $\lambda>\mathbf{0}$ is the vector of weights of the DMUs. The virtual unit should be better (or at least not worse) than the analyzed unit DMU_q. The problem of looking for a virtual unit can be formulated as a standard linear programming problem:

$$\begin{aligned}
\text{minimize} \quad & z = \theta - \varepsilon(\mathbf{e}^T\mathbf{s}^+ + \mathbf{e}^T\mathbf{s}^-),\\
\text{subject to} \quad & \mathbf{Y}\lambda - \mathbf{s}^+ = \mathbf{y}^q, \qquad (1)\\
& \mathbf{X}\lambda + \mathbf{s}^- = \theta\mathbf{x}^q,\\
& \lambda, \mathbf{s}^+, \mathbf{s}^- \geq \mathbf{0},
\end{aligned}$$

where $e^T = (1,1,\dots,1)$ and ε is a infinitesimal constant (usually 10^{-8}). The variables s^+, s^- are just slack variables expressing the difference between virtual inputs/outputs and appropriate inputs/outputs of the DMU_q. Obviously, the virtual inputs/outputs can be computed using the optimal values of variables of the model (1) as follows:

$$\mathbf{x}^{q'} = \mathbf{x}^q\theta^* - \mathbf{s}^-,$$
$$\mathbf{y}^{q'} = \mathbf{y}^q + \mathbf{s}^+.$$

The DMU_q is to be considered as efficient if the virtual unit is identical with evaluated unit (does not exist a virtual unit with better inputs and outputs). In this case $\mathbf{Y}\lambda = \mathbf{y}^q$, $\mathbf{X}\lambda = \mathbf{x}^q$ and the minimum value of the objective function z = 1. Otherwise the DMU_q is not efficient and minimum value of $\theta<1$ can be interpreted as the need of proportional reduction of inputs in order to reach the efficient frontier. The presented model is input oriented model because its objective is to find a reduction rate of inputs in order to reach the efficiency. The output oriented models maximize the expansion rate of outputs in order to reach the efficient frontier. The mathematical formulation of output oriented model can be as follows:

$$\begin{aligned}
\text{maximize} \quad & z = \phi + \varepsilon(\mathbf{e}^T\mathbf{s}^+ + \mathbf{e}^T\mathbf{s}^-),\\
\text{subject to} \quad & \mathbf{Y}\lambda - \mathbf{s}^+ = \phi\mathbf{y}^q, \qquad (2)\\
& \mathbf{X}\lambda + \mathbf{s}^- = \mathbf{x}^q,\\
& \lambda, \mathbf{s}^+, \mathbf{s}^- \geq \mathbf{0}.
\end{aligned}$$

In model (2) the evaluated unit DMU_q is efficient if the optimal objective value z = 1, ie. $\phi = 1$ and all the slack variables equal to zero. The optimal objective function for inefficient units is greater than 1.

Models (1) and (2) suppose constant returns to scale – it is considered that a percentual change of inputs leads to the same percentual change of outputs. The modification of the above models taking into account variable return to scale is derived from them by adding the convexity constraint $\mathbf{e}^T\lambda=1$. The efficiency score in standard DEA models is limited to unit (100%) but the number of efficient units identified by DEA models and reaching the maximum efficiency score 100% can be relatively high and especially in problems with a small number of decision units the efficient set can contain almost all the units. In such cases it is very important to have a tool for a diversification and classification of efficient units. That is why several DEA models for classification of efficient units were formulated. In these models the efficiency score of inefficient units remains lower than 100% but the efficiency score for efficient units can be higher than 100%. The DEA models that relax the condition for unit efficiency are called super-efficiency models.

3. INTERVAL AHP MODELS

The Analytic Hierarchy Process (AHP) is a powerful tool for analysis of complex decision problems. The AHP organizes the decision problem as a hierarchical structure containing always several levels. The topmost level defines a main goal of the decision problem and the last (lowest) level describes usually the decision alternatives or scenarios. The levels between the first and the last level can contain secondary goals, criteria and subcriteria of the decision problem. Let us consider a simple three-level hierarchy that can represent a standard decision problem with finite set of alternatives - evaluation of n-alternatives $X_1, X_2, \ldots, X_n$, by k-criteria $Y_1, Y_2, \ldots, Y_k$. The decision maker expresses his preferences or compares importance of the elements on the given level with respect to the element of the preceding level. The information resulting from decision maker's judgements in the given level of the hierarchy is synthesised onto the local priorities. They can express, e.g. relative importance of criteria (weight coefficients - v_j, i=1,2,...,k) or preference indices of the units with respect to the given criterion (w_{ij}, i=1,2...,n, j=1,2,...,k). In the standard AHP model the decision maker judgements are organised into paired comparison matrices at each level of the hierarchy. The judgements are point estimates of the preference between two elements of the level. Let us denote the paired comparison matrix $\mathbf{A} = \{a_{ij} | \, a_{ji} = 1/a_{ij}, a_{ij}>0, i,j=1,2,...,k\}$, where k is the number of elements of the particular level. The local priorities are derived by solving the following eigenvector problem

$$\mathbf{A.v} = \lambda_{max}\mathbf{v},$$
$$\sum_{i=1}^{k} v_i = 1, \tag{3}$$

where λ_{max} is the largest eigenvalue of **A** and **v** is the normalised right eigenvector belonging to λ_{max}. In standard AHP models decision makers always specify point estimates that express their preference relations between two elements in the given hierarchical level. It can often be very difficult to fulfil this condition for decision makers. They feel much better and closer to have the possibility to express their preferences as interval estimates. For instance, instead of giving that the i-th element is four times as preferable as the j-th element, he can assert that the i-th element is at least two but no more than five times as preferable as the j-th element.

AHP models with interval decision maker's judgements are usually called interval AHP (IAHP) model. They are characterised by interval pairwise comparison matrices:

$$A = \begin{bmatrix} 1 & <p_{12}, q_{12}> & \cdots & <p_{1k}, q_{1k}> \\ <p_{21}, q_{21}> & 1 & \cdots & <p_{2k}, q_{2k}> \\ \vdots & \vdots & \cdots & \vdots \\ <p_{k1}, q_{k1}> & <p_{k2}, q_{k2}> & \cdots & 1 \end{bmatrix} \tag{4}$$

where p_{ij} is lower bound and q_{ij} upper bound for preference relation (a_{ij}) between the i-th and j-th element. Due to the reciprocal nature of the pairwise comparison matrices the relation $p_{ij}.q_{ji} = 1$ holds for all i,j=1,2,...,k.

The judgements in IAHP models can be considered as random variables defined over the given interval. In this way the IAHP changes from the deterministic model to the model with some stochastic features. That is why it cannot be analysed in the traditional way – by solving the eigenvector problem (3). The random variables for description of interval judgements can be selected from the available probabilistic distributions. We will use the uniform distribution defined over the interval $<p_{ij}, q_{ij}>$ and in our numerical experiments below. The preferences of elements derived form from matrix A are random variables as well. Their characteristics can be computed by several approaches – one of them is Monte Carlo simulation that is very simple and offers lower and upper bounds for preferences and other useful characteristics.

4. EVALUATION OF PENSION FUNDS - A CASE STUDY

DEA models are based on maximization of individual efficiency of the evaluated unit under the constraints that the efficiency of all the other units is lower than 1 (100%). With regard to the number of units of the evaluated set on one hand and the number of inputs and outputs on the other hand, the number of efficient units can be relatively high. That is why several super-efficiency models were proposed in order to make it possible to classify the efficient units. In the super-efficiency models the units indicated originally as efficient become the super-efficiency measure greater than one and this measure can be used for their discrimination and classification. Our aim was to compare the results given by super-efficiency DEA models with multiple criteria decision making methodology that can be represented very well by an AHP model. We used an IAHP model because it makes it possible to incorporate to the analysis an appropriate level of uncertainty that is typical for economic decision making problems.

The discussion concerning the both approaches will be demonstrated on a small numerical example with real economic background. It is the problem of evaluation of efficiency of available pension funds in the Czech Republic. We worked with the data set for 12 pension funds, each of them was characterized by the following seven criteria:

- INP 1 - the number of customers [thousands],
- INP 2 - total assets [mil. CZK],
- INP 3 - equity capital [mil. CZK],
- INP 4 - total costs [mil. CZK],
- OUT 1 - appreciation of the customer deposits for the last year (2003) [%],
- OUT 2 - average appreciation of the customer deposits for the last three years (2001 – 2003) [%],
- OUT 3 - net profit [mil. CZK].

For DEA analysis, first four criteria were taken as inputs and the remaining ones as outputs of the model. The criterion matrix is given in Table 1.

	#of cst.	**assets**	**equity**	**tot.costs**	**appr. 1**	**appr. 3**	**profit**
Allianz	106	4095	77.0	49.5	3.00	3.69	1.29
Credit Suisse	611	22592	549.0	454.1	3.36	3.67	5.22
CSOB Progres	18	452	56.0	15.1	4.30	4.15	1.13
CSOB Stabilita	304	8508	298.6	203.3	2.30	2.83	10.87
Generali	23	789	74.0	15.5	3.00	3.90	0.45
ING PF	346	9767	289.1	221.7	4.00	4.27	0.26
CP PF I	225	6348	290.7	184.7	3.34	3.65	6.83
CP PF II	518	12441	522.5	297.3	3.10	3.37	6.90
CS PF	401	10954	223.5	238.8	2.64	3.31	1.10
KB PF	285	11776	441.6	166.0	3.40	4.14	6.40
PF Ostrava	19	935	71.0	18.2	2.44	2.68	0.04
PF Zemsky	14	468	87.9	23.2	4.01	4.24	2.03

Table 1: Pension funds – criterion matrix.

The funds listed in the previous table are of different nature. Four of them are very small (CSOB Progres, Generali, PF Ostrava and PF Zemsky) and the remaining ones are significantly bigger. That is why we decided to analyze them separately. In Table 2, there are results of DEA analysis of both groups of funds (large and small). We used the basic envelopment DEA model with variable returns to scale with output orientation and the super-efficiency model under the same assumptions. First column of Table 2 contains efficiency scores of the evaluated units – the presented score is a reciprocal value of the optimal score given by the model because in output oriented models the score of inefficient units is greater than one. Our transformation can be better explained – higher score corresponds to more efficient unit. The same holds for super-efficiency scores presented in the last column of Table 2. Of course the super-efficiency scores are available for units indicated as efficient by the standard model only. The word "infeasible" for super-efficiency score of Allianz fund means that the corresponding VRS super-efficiency model has no feasible solution. This situation can occur in VRS super-efficiency models very often and in this case it disables the possibility to classify the efficient units.

DEA/VRS model	**efficiency score**	**super-eff score**
Large funds		
Allianz	100.00	infeasible
Credit Suisse	96.12	
CSOB Stabilita	100.00	159.24
ING PF	100.00	119.87
CP PF I	100.00	122.40
CP PF II	95.64	
CS PF	82.14	
KB PF	100.00	112.40
Small funds		
CSOB Progres	100.00	infeasible
Generali	93.72	
PF Ostrava	64.06	
PF Zemsky	100.00	infeasible

Table 2: Efficiency measures given by DEA models.

The IAHP model for evaluation of efficiency is very simple - it is presented on Figure 2. In the model q_1 is the total weight of inputs and q_2 is the total weight of outputs, $q_1 + q_2 = 1$. qi_j, j=1,2,...,m, are the weights of single inputs and qo_j, j=1,2,...r, are the weights of single outputs. Preference indices u_{ij}, i=1,2,...,n, j=1,2,...,m+r, express the preference of the i-th alternative (pension fund) with respect to the j-th input/output. The global preference indices of alternatives p_i, i=1,2,...,n, are synthesized from previous hierarchical levels as follows:

$$p_i = \sum_{j=1}^{m+r} u_{ij},\ i = 1,2,...,n.$$

In our analysis the weights of the inputs and outputs were not derived by the AHP model but they were either to set up directly as constants or optimized as variables of proposed simulation model. In the model the pairwise comparisons of alternatives (pension funds) with respect to all inputs and outputs were given as random variables with uniform distribution defined on the interval <p, q>. The comparisons reflect given criterion values but by this way it is possible to use different returns to scale for different inputs and outputs. It is one of the advantages of this approach. Our numerical experiments were realized on the set of five big pension funds identified as efficient by DEA model with variable returns to scale.

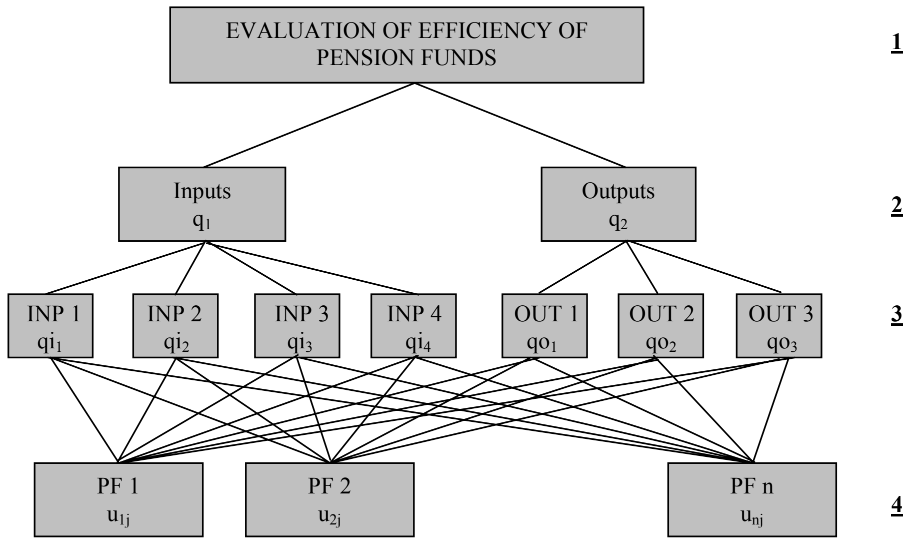

Figure 2: AHP model for evaluation of efficiency.

The DEA models maximize the individual efficiency of evaluated units by looking for optimum weights of inputs and outputs. That is why we did not work with weights derived by the IAHP model but we tried to optimize the weights in order to reach the best efficiency score of the evaluated alternative. We used the following requirements to the set of the weights:

1. The sum of weights equals to 1, all the weights have to be greater than 0.05 (AHP I).
2. The sum of weights of inputs equals to 0.5, the same holds for the weights of outputs. All the weights have to be greater than 0.05 (AHP II).
3. All the weights are fixed to value 1/(m+r), ie. 1/7 in our example (AHP III).

Pension funds	**DEA/VRS super-eff.**	**AHP I**	**AHP II**	**AHP III**
Allianz	infeasible	0.4574	0.3262	0.3334
CSOB Stabilita	159.24	0.3608	0.2583	0.1449
ING PF	119.87	0.3652	0.2764	0.1894
CP PF	122.40	0.2123	0.2056	0.1699
KB PF	112.40	0.2397	0.2138	0.1625

Table 3: Comparison of super-efficiency measures.

The optimization run was realized by means of Crystal Ball which is an MS Excel add-in application for Monte Carlo experiments. Crystal Ball contains a special tool for optimization under stochastic conditions called OptQuest. This tool can find optimum values of variables (weights of inputs and outputs in our case) in stochastic environment that can be modeled within MS Excel. The optimization criterion is the efficiency score p_i, i=1,2,...n, of the evaluated alternative that is to be maximized. Because the efficiency score under our stochastic conditions is a random variable we tried to maximize its mean value. We always used a five minutes optimization run for all the alternatives with 100 trials per one simulation. The results are presented in Table 3. The first column of this table contains super-efficiency scores computed by the Andersen and Petersen DEA model with variable returns to scale, the remaining three columns contain maximized efficiency score of pension funds given by the presented IAHP model with different weight constraints (weight sets I, II and III).

5. CONCLUSIONS

The aim of the paper was to verify how the AHP models can be used for efficiency evaluation of production units and compare the results given by proposed interval AHP model with efficiency scores computed by DEA models. In contrary to super-efficiency DEA models the advantage of the IAHP approach consists in several points:

- the IAHP model can use different scales for different inputs and outputs according to the decision maker preferences,

- in super-efficiency DEA models with VRS not all the units receive their super-efficiency score (the problems need not be always feasible),
- the IAHP model can deal with categorical inputs and outputs without any transformation,
- the IAHP model offers a possibility of sensitivity analysis of results with respect to the inputs and/or outputs.

The main disadvantage of the AHP (IAHP) models comparing to the DEA models consists in preparing of data for the analysis (pairwise comparison matrices) and in time consuming length of the optimization analysis. Nevertheless, by using the AHP model the decision maker can receive new information useful for the global analysis of the efficiency of the evaluated set of units.

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TIME ACCESSIBILITY AND PUBLIC SERVICE SYSTEMS

JAROSLAV JANÁČEK

1. INTRODUCTION

The needs and requirements of human society or particular social groups form various demands, which are usually spread over a geographical area. An effective satisfaction of the demands is possible only if the corresponding service provider concentrates its sources at several places of the served area and provides the service in or from these places only. Many of public service systems should ensure equity of their “customers” in access to the service, which is provided by the service system. In some cases this equity may be formalized by a time limit, to which an occurring customer demand is to be satisfied. If the satisfaction is accompanied by some traveling in a transportation network from or to a service facility location, then it is obvious that the limit can be exceeded with some probability. We suggest several criteria how to evaluate the time accessibility in the service system and offer an optimization approach to the system design. We restrict ourselves on the problem, in which a medical emergency service system is designed. This system design belongs to the family of location problems, in which it must be decided, where ambulance vehicles should be placed. Our suggestions are supported by preliminary numerical experiments performed on the road network of Slovak Republic. The studied problem arose from the necessity to locate approximate 250 depots of the ambulance vehicles in towns of the Slovak Republic so that each dwelling place of this region is accessible in fifteen minutes.

2. THE QUALITY CRITERIA OF SUGESSED EMERGENCY SYSTEM

The original requirement for the system design is that each inhabited place must be reachable within the time T^{max} from at least one service centre placed in some location from the set I of feasible centre locations. The design of this public system is equivalent to determination of a feasible solution of the maximum distance problem [10] under the assumption that a number of possible service centres is given. Nevertheless, the average speeds on the links of road network are not constant, but they depend on weather, traffic volume and other dynamically changing conditions. Considering this condition variability, no system design ensures full satisfaction of the original requirement and that is for a measure of the design quality must be applied.

We make use of the proposals formulated in [7] and choose the following two basic criteria for the next study:

$$\sum_{\substack{j \in J \\ t_{i(\boldsymbol{v},j),j}(\boldsymbol{v}) > T^{max}}} b_j \qquad (1)$$

$$\sum_{\substack{j \in J \\ t_{i(\boldsymbol{v},j),j}(\boldsymbol{v}) > T^{max}}} b_j (t_{i(\boldsymbol{v},j),j}(\boldsymbol{v}) - T^{max}) . \qquad (2)$$

In the above-formulated criteria, the following denotations are used:

J – the set of dwelling places (customers),

b_j – the number of inhabitants at the dwelling place j,

$\boldsymbol{v}$ – the vector of average speeds, which corresponds to the particular road link classes,

$i(\boldsymbol{v}, j)$ – the located centre, which is the time-nearest one to j considering the link speeds given by $\boldsymbol{v}$,

$t_{ij}(\boldsymbol{v})$ – the time of traversing from the place i to j considering the speed vector $\boldsymbol{v}$.

The first criterion expresses the size of the part of population, which is out of the time limit T^{max} considering the time nearest service centre.

The second criterion is the sum of positive differences between the shortest access time of and the time limit T^{max}.

In addition to the basic criteria, the other criteria can be applied to express other characteristics of the emergency system design. In [7], a composed criterion was proposed to evaluate stability of the design considering the variety of speeds and the objective function of maximum number of double-covered dwelling places is presented in [3].

In the frame of this paper, we try to estimate the suggested system resistance to a double accident occurrence by the above-mentioned basic criteria computed for the half or third of original access time T^{max}.

3. THE AMBULANCE LOCATION PROBLEM AND A SOLVING TECHNIQUE

The considered optimization problem can be formulated to decide on location of p emergency centres at some places from the set I of possible ambulance locations so that the value of chosen criterion is minimal. If we express price c_{ij} of the j-th dwelling place assignment to possible location i accordingly to [7], then we obtain $c_{ij}^1(\boldsymbol{v}) = b_j$, if $t_{ij}(\boldsymbol{v}) > T^{max}$ and $c_{ij}^1(\boldsymbol{v}) = 0$ otherwise; or $c_{ij}^2(\boldsymbol{v}) = b_j(t_{ij}(\boldsymbol{v}) - T^{max})$, if $t_{ij}(\boldsymbol{v}) > T^{max}$ and $c_{ij}^2(\boldsymbol{v}) = 0$ otherwise for the criterion *(1)* and *(2)*respectively.

Afrer these preliminaries, we can formulate the associated model in the following form:

Minimize $\sum_{i \in I} \sum_{j \in J} c_{ij} z_{ij}$ *(3)*

Subject to $\sum_{i \in I} z_{ij} = 1$ *for $j \in J$* *(4)*

$z_{ij} \leq y_i$ *for $i \in I$ and $j \in J$* *(5)*

$\sum_{i \in I} y_i \leq p$ *(6)*

$z_{ij} \in \{0,1\}$ *for $i \in I$ and $j \in J$* *(7)*

$y_i \in \{0,1\}$ *for $i \in I$.* *(8)*

In this model, the decision on ambulance location at place $i \in I$ is modelled by the zero-one variable $y_i \in \{0,1\}$, which takes the value 1 if an ambulance should be located at *i* and it takes the value 0 otherwise. Furthermore, the auxiliary variables $z_{ij} \in \{0,1\}$ for each $i \in I$ and $j \in J$ are introduced to assign the dwelling place *j* to the possible location *i* ($z_{ij}=1$).

The associated problem can be solved by the approach reported in [6] or [7], where the Lagrangean multiplier is introduced for constraint *(6)*, and the constrain is relaxed. Then the problem takes form of the uncapacitated location problem. To solve this problem, the procedure *BBDual* [8] was designed and implemented based on principle presented in [4]. The procedure was embedded into the dichotomy algorithm, which was used to find propper value of the Lagrangean multiplier.

4. NUMERICAL EXPERIMENTS

We follow two goals in this experimental part of work. First, we attempted to analyse the current medical emergency system of Slovak Republic from the point of the double or triple accident occurrence (see Section 2). Second, we tried to answer the question "How much is it possible to improve the emergency system by an implementation of optimization technique?" To prepare the associate experiments, we employed the electronic road map of Slovak Republic, in which the set of towns and villages forms a part of node set. The numbers of inhabitants of the dwelling places were given together with other attributes of the nodes. As concerns the links of road network we were given by the length and class of each link. The road network contains 2906 dwelling places. The analysed current medical emergency system consists of 259 places, but 46 of them duplicate or triplicate locations at some bigger cities and they have no influence on the studied characteristics.

That is why the 213 points (locations) were taken into consideration only as concerns the original system. Two speed scenarios of the vehicle speeds were used to complete the test problem instances. The used scenarios are denoted by the symbols "Op" and "So" and, in accordance to [7], the speeds associated with the particular link classes are plotted in Table 1, where the speeds of a particular scenario are given in kilometres per hour.

Table 1

Speed scenarios

Scenario	Speed0	Speed1	Speed2	Speed3	Speed4
Op	105	95	75	75	60
So	115	105	85	85	60

"Speed0" is assumed for highways, "Speed1", 2 and 3 for the roads of the first, second and third class and "Speed4" is considered along the local roads. The scenarios "Op" and "So" correspond to good and optimal traffic conditions.

To perform the analysis of current emergence system, we have calculated values of the formulae *(1)* and *(2)* (Criterion 1 and 2 respectively) for the both scenarios and for T^{max} = 15, 8 and 4 minutes. The resulting values are shown in Table 2.

Table 2

The access time and criterion evaluation

Scenario	T^{max} [min]	Criterion1	Criterion2
Op	15	8158	39764
Op	8	453625	1089856
Op	4	1595457	5577762
So	15	5010	23056
So	8	259544	568970
So	4	1439207	4232068

Criterion1 is number of inhabitants, which are out of the accessibility limit of 15 minutes and Criterion2 is given in minutes and its value represents sum of the surplus accessibility times of individual inhabitants, which are not accessible in the given limit. The results in Table 2 show that there are some places, which are not accessible in the original access time T^{max}= 15 even under good and very good traffic condition in the road network. It evokes the question: "What is the minimum access time, for which each place of the Slovak Republic is accessible from the current ambulance locations considering one of the used speed scenarios?" We have performed another series of the experiments for gradually increasing T^{max} for the both scenarios. The associated results are given in Table 3.

Table 3
The real access time determination

Scenario	T^{max} [min]	Criterion1	Criterion2
Op	20	2745	14112
Op	27	820	1558
Op	30	0	0
So	20	1385	6729
So	24	820	1189
So	26	0	0

Next, we have focused on the evaluation of optimization technique contribution to the emergence system improvement. To established proper instances of the optimization problems, we made use of the town population analysis published in [7] and overtook the set *I* of 449 possible locations. Then, we solved the problem *(3) – (8)* for the instances described in Table 2 for p=213.

Table 4
The real access time determination using optimization technique

Scenario	T^{max} [min]	Optimum1	No. Loc.	Time [s]	Optimum2	No. Loc.	Time [s]
Op	15	6192	154	0	25204	154	1
Op	8	259887	213	297	656284	213	5792
Op	4	1201021	213	8	4381717	213	321
So	15	3806	131	0	12638	131	1
So	8	124008	213	8005	303400	213	11558
So	4	1047757	213	13	3197912	213	295

The results are plotted in Table 4, where the column denotations have the following meaning. “Optimum1” and “Optimum2” denote optimal values of the objective functions created accordingly to *(1)* and *(2)* respectively for the corresponding speed scenario. Corresponding values are given in the same units as “Criterion1” and “Criterion2” in the Table 2. Symbol “No. Loc.” denotes number of emergency vehicle locations, which were suggested in the frame of the optimal solution. The column “Time [s]” denotes the computational time of solving algorithm in seconds, which was necessary for obtaining the optimal solution. The computations were performed on PC Pentium 4, 2.8 GHz, 512 MB.

The last series of experiments was devoted to searching for the minimum value of T^{max}, for which each place of Slovak Republic is covered taking into account the above-defined set of possible locations *I*. Similarly to the approach used in the analysis of current system, the access time T^{max} was gradually increased and the problem *(3) – (8)* was solved by the optimization technique until the criterion value is equal to zero. The associated results are shown in Table 5.

Table 5

Results of optimization in accordance to both criteria

Scenario	T^{max} [min]	Optimum1	No. Loc.	Time [s]	Optimum2	No. Loc.	Time [s]
Op	20	1574	101	1	6112	101	1
Op	27	0	64	0	0	64	0
Op	30	0	55	1	0	55	0
So	20	565	79	0	2260	79	1
So	24	0	62	0	0	62	0
So	26	0	55	0	0	55	0

In this table, there are reported the results for each scenario, which correspond with T^{max}=20 minutes and then the results for such T^{max} follow, for which the zero values of “Optimum” and “Criterion” respectively were obtained first time.

5. CONCLUSIONS

The presented results of numerical experiments show that the current ambulance locations don’t ensure the declared access time of 15 minutes even under the good and very good traffic conditions (see Table 2). It has been found that this assess time should have to be two times larger to cover each dwelling place in the Slovak Republic (see Table 3).

In addition to these results, the usefulness of optimization technique has been demonstrated. It can be found that the access time for fully coverage can be reduced by more than seven percent for the both scenarios (see Table 5). As concerns the possibility of double coverage studied by reducing of the access time, the optimization technique enables to reduce the number of double-uncovered inhabitants by more than forty percent (compare Table 2 and Table 4).

Other improvement of the emergency system design could be achieved by employing of the decision support tool, which comprehends the both optimization algorithm and GIS tool displaying the results in a graphical form. Such decision support system could enable to form a sufficient set of possible locations by man-machine approach and provide an excellent set of ambulance locations, which is resistant to double-accident occurrence.

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IDENTIFICATION OF NATURAL DISTRIBUTION CENTERS IN REGION

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Abstract: This paper deals with the selection of natural centers from the possible location set. An improved modification of the Erlenkotters algorithm (*BBDual*) was used for this purpose.

1. INTRODUCTION

When the structure of distribution is designed, the strategic decisions are made on building of centers at the places of their possible locations. Together with these strategic decisions, the tactical decisions are performed, which assign each customer to some centre. The associated problems are mostly NP-hard and the particular parts of objective function describing the total costs are in mutual contradiction. E.g. it the number of located centers is increased, then the cost of center building and keeping grow up, but, on the other side, the cost of goods distribution decrease. Vice versa, when the number of located centers decreases, then the distribution cost increase.

In our contribution, we deal with the natural center determination in real networks and in the networks with deformed structure.

2. NATURAL CENTERS

Solving the location problems, it can be found that some places in the given network are often used for the centre location even if various distribution systems are designed. It could be caused by the fact that some places are more admissible for the centre location than the other at least for the most of distribution systems on the considered network. These places can be called the "natural" centers.

Several different approaches can be used, when the set of natural centers is sought for. Some approaches enable only to determine the number of centre (e.g. the continuous approximation method) but other ones are able to provide the centre location or, in addition, the assignment of customers to the centre locations. The limits of particular methods follow from the principles, on which the particular methods are based.

Some of the methods are allowed to place a centre only at some place from a finite set of possible locations. In the contrast to them, some other methods can place the centre wherever in the given area. One of the important sub-problems solved in the connection with distribution system structure design is the uncapacitated facility location problem.

There are exact algorithms, which are able to solve large problems of the distribution system design in a short time. One possibility of uncapacitated location problem solving is the algorithm named *BBDual*, what is a modification of Erlenkotter's algorithm [1]. This algorithm is based on the branch and bound method.

We add to the basic problem the condition, which enables to serve a customer from such centre, distance of which is less than a given value. This problem is called the uncapacitated location problem with maximal service distance. After a conversion we obtain a set of problem instances with deformed networks [3] and we can solve it with the algorithm mentioned above.

3. EXPERIMENTS

In our research project we solved problems with real and deformed networks. To form the problem instances, we used the road network of Slovak Republic. This network consists of 2907 nodes. From these nodes we selected the sets, which differ in the cardinalities of possible location sets. We preferred the localities with greater number of inhabitants. We solved these problems for different values of the fixed charges and for the additional constraints. The demands of customers in all the problems were proportional to the number of inhabitants in localities.

In this paper we present only results of some experiments. We selected 10 variants of the networks with 200 candidates for possible locations of the centers. The particular variants differed in the structure of set of possible location of the centers. We used 7 different values of the fixed charges for each network. Each of these 70 networks we extended by the deformation of transportation network. Each customer could be served only from such center, where the distance between the customer and the center should be less than the given constant (D_{max}). We used 11 different values of D_{max} and so we obtained 770 different problems. The value of fixed charge and the constant D_{max}, had an influence on the number of optimal locations selected from the nominated set of candidates. We computed the average of results from each 10 problem variants with the same parameters.

In table 1 we show the numbers of optimal locations (*P)* for the different values of D_{max} [km] and for the fixed charges in order 10^7.

Tab. 1

$\mathbf{D_{max}}$	47	75	104	132	161	189	218	246	275	303	332
P	28	11	5	4	3	2	2	2	2	1	1

The result for the highest value D_{max} is proportional to result of the problem with nondeformed network. The smaller values of fixed charges brought bigger number of optimal locations of centers in real and deformed network as well.

Bigger number of centers denoted the best access to the customers. The upper bound of D_{max} became lower due to this fact. The difference between upper and lower bounds of D_{max} became lower and the differences among the results of deformed and real network were fade away.
The numbers of optimal locations (*P)* solved on deformed network for different values of fixed charges we show in the table 2.

Tab. 2

Fixed charges	10^7	40 000	15 000	8 000	4 000	2 000	0
P	1	33	67	100	133	167	200

In the cases, when the optimal set of located centers contains only one element, the centre was located at the town Martin or Ružomberok. We produced the list of optimal centre locations using the results of all 110 problems referred in the table 1. This list is plotted in the table 3.

Tab. 3

Name of the center	**Number of inputs**	**Number of outputs**	**Success in %**
Veľké Zálužie	110	42	38.18
Prešov	110	33	30.00
Martin	110	26	23.64
Šoporňa	66	13	19.70
Vranov nad Topľou	110	21	19.09
Veľký Krtíš	110	19	17.27
Košice – Staré Mesto	110	16	14.55
Bardejov	110	15	13.64
Humenné	110	14	12.73
Bystré	55	7	12.73
Rožňava	110	13	11.82
Kežmarok	110	13	11.82
Pata	44	5	11.36
Hanušovce nad Topľou	66	7	10.61
Žilina	110	11	10.00
Ružomberok	110	11	10.00
Rimavská Sobota	110	10	9.09

"The number of inputs" denotes the number of town or village occurrence in the lists of possible locations in the set of problems.
"The number of outputs" gives the number of town or village occurrence in the optimal sets of locations, in which a centre was placed.

In the problems with the lower fixed charges, the towns with the largest population have formed the optimal sets of used locations. We refer only on those, which occurred in the optimal solution, of each solved problem. The list of towns follows: Banská Bystrica, Bardejov, Bratislava, Brezno, Čadca, Dunajská Streda, Košice, Lučenec, Martin, Michalovce, Námestovo, Nitra, Piešťany, Poprad, Považská Bystrica, Prešov, Prievidza, Rimavská Sobota, Ružomberok, Senica, Spišská Nová Ves, Stará Ľubovňa, Trenčín, Trnava, Žilina.

4. CONCLUSIONS

Having obtained the results of experiments, we depicted them in graphic way together with the underlying network. As concerns the geographical view, the distribution of located centers was approximately uniform and corresponded to the distribution of associated population.

The different choice of optimal location sets in the problems with the fixed charges in order to 10^7 (see the table 3) is obviously caused by the constraint of customer accessibility, which accompanies the deformed distribution networks. Even in these cases, the optimal locations take places near to the big cities.

Considering the presented results, it is obvious that the number of used locations in the solutions of corresponding problems is almost the same. If the fixed charges were equal in order to 4.10^4, then the number of located centers was approximately equal to 1/6 of the number of possible locations and the number of located centers increases by the increment of 1/6 for each next value of fixed charges. For the zero value of fixed charges, the optimal set of used locations consists of all possible locations.

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UNEMPLOYMENT PROBLEM VIA MULTISTAGE STOCHASTIC PROGRAMMING

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Abstract: Multistage stochastic programming problems correspond to many practical situations that can be reasonably considered with respect to some finite "discrete" time interval and simultaneously there exists a possibility to decompose them with respect to the individual time points. We employ this type of the problems to analyze an economic and social problem of unemployment, including the problem of restructuralization.

Keywords: Multistage stochastic programming problems, unemployment problem, restructuralization, random element, probability constraints.

1. INTRODUCTION

An unemployment problem belongs to one of the most serious problems of many world economics. This fact has been recognized already in a period between the world wars. A relationship between a number of unemployment and some other economic characteristics has been investigated in an economic and social literature. Moreover, specialists in many countries have begun to influence a development of unemployment number to be in "acceptable" bounds. However, a comparison of these efforts has been very complicated since there didn't exist a common definition of an unemployment person. This definition has been accepted (by most countries) erst in the last years.

It has been mentioned above that most societies try (by suitable decisions) to influence the development of unemployment. There is no doubt, that a development of unemployment depends on a random factor and, moreover, the decisions (to correct the future number of unemployment) are taken in the case when the real state is known. In particular, a sequence of decisions x^k and a knowledge of the number of unemployment u^k (corresponding to time points $k = 0,1,...,$) follow the following scheme:

$$u^0 \longrightarrow x^0(u^0) \longrightarrow u^1 \longrightarrow x^1(x^0, u^1) \longrightarrow u^2 \longrightarrow ...... \longrightarrow x^{M-1}(\overline{x}^{M-2}, \overline{u}^{M-1}) \longrightarrow$$
$$\longrightarrow u^M \longrightarrow x^M(\overline{x}^{M-1}, \overline{u}^M) \longrightarrow u^{M+1} \longrightarrow ..., \tag{1}$$

where $\overline{x}^k = [x^1, ..., x^k], \overline{u}^k = [u^1, ..., u^k], k = 1, ..., M, ...$ The symbol $x := x(\cdot, \cdot)$ denotes a dependence, it means that a decision x^k can depends on the former decisions $\overline{x}^{k-1}$ and the unemployment numbers $\overline{u}^k$. However, assuming that the development of unemployment u^k can be described by a prescription of sequence in which a random element ξ^k appears, we can see that the relation (1) is equivalent to the following one:

$$u^0 \longrightarrow x^0(u^0) \longrightarrow \xi^0 \longrightarrow x^1(x^0, \xi^0) \longrightarrow \xi^1 \longrightarrow ...... \longrightarrow x^{M-1}(\overline{x}^{M-2}, \overline{\xi}^{M-2}) \longrightarrow$$
$$\longrightarrow \xi^{M-1} \longrightarrow x^M(\overline{x}^{M-1}, \overline{\xi}^{M-1}) \longrightarrow \xi^M \longrightarrow ... \tag{2}$$

The random element ξ^k corresponds there to the development of unemployment u^k in the time point $k+1$, $\overline{x}^k = [x^1, ..., x^k], \overline{\xi}^k = [\xi^1, ..., \xi^k], k = 1, ..., M, ...$ Evidently this change is without loss of generality; moreover, we assume that the decision x^0 depends only on u^0.

It follows from the scheme (2) that a decision x^k can depends on the decisions and the random element realizations to the time point $k-1$, however, it can not depend on the decisions taken in the time points $k+1, ...$ and the random element realizations corresponding to the time points $k, k+1, ..., M ....$ This assumption is realistic; from the mathematical point of view it says that the decision has to be nonanticipative. Furthermore, it can be assumed that the above mentioned state of the process (at every time point k) can be evaluated by some function, say $c^k(\overline{x}^k, \overline{\xi}^k)$ and that the decisions have to fulfil constraints that can generally depend on the probability measure. Furthermore, surely it is “reasonable" to investigate the process only to a finite time horizon; say to a discrete time interval $(0, M)$.

Analyzing the above mentioned economic and social process (from the mathematical point of view) we can recognize that a multistage stochastic programming model corresponds very well to it. Namely, multistage ($M+1-$ stage) stochastic programming problems correspond just to practical situations in which a random element exists and, moreover, it is reasonable to treat them with respect to some finite discrete time interval. In particular, this type of the problems corresponds to practical situations that can be considered with respect to some time interval and, simultaneously, they can be decomposed with respect to the individual time points. More precisely, a decision x^k at the time point k can depend on the decisions x^j and the random elements realizations ξ^j to the time point $k-1$, however it can not depend on the decision corresponding to the time points $k+1,...M$ and the random element realization corresponding to the time points $k, k+1,...,M$. However, the decision x^k can depend on the random elements $\xi^k,...,\xi^M$ through the corresponding probability measure.

2. MATHEMATICAL DEFINITION OF THE MULTISTAGE PROBLEM

To introduce the multistage ($M+1$–stage, $M \geq 1$) stochastic programming problem, let for $k=0,...,M$, $\xi^k := \xi^k(\omega)$ be an s–dimensional random vector defined on the probability space (Ω, S, P) ; $F^{\xi^k}(z^k)$, $F^{\bar{\xi}^k}(\bar{z}^k)$, $F^{\xi^k|\bar{\xi}^{k-1}}(z^k \mid \bar{z}^{k-1})$ ($F^{\xi^0|\xi^{-1}}(\cdot|\cdot) := F^{\xi^0}(\cdot)$) be the distribution functions of $\xi^k, \bar{\xi}^k$ and the conditional distribution function (ξ^k conditioned by $\bar{\xi}^{k-1}$); $x^k \in R^n, z^k \in R^s$, $\bar{x}^k = [x^0, x^1,...,x^k]$, $\bar{z}^k = [z^0, z^1,...,z^k]$, $\bar{\xi}^k = [\xi^0, \xi^1,...,\xi^k]$. Let, furthermore, $K^{k+1}(\bar{x}^k, \bar{z}^k)$, $k=0,1,...,M-1$ denote a multifunction mapping from $R^{n(k+1)} \times R^{s(k+1)}$ into the space of nonempty, closed subsets of R^n; $g_0^M(\bar{x}^M, \bar{z}^M)$ be a real–valued function defined on $R^{n(M+1)} \times R^{s(M+1)}$; $K^0 \subset R^n$ be a nonempty set. ($R^n, n \geq 1$ denotes an n–dimensional Euclidean space, the symbol $P_{F^{(\cdot)}}$ is reserved for the probability measure corresponding to the distribution function $F^{(\cdot)}$.)

A general multistage ($M+1$–stage, $M \geq 1$) stochastic programming problem can be introduced recursively (see e.g. [3]) as the problem

Find

$$\inf\{\mathrm{E}_{F^{\xi^0}} g_F^0(x^0, \xi^0) \mid x^0 \in K^0\}, \qquad (3)$$

where the function $g_F^0(x^0, z^0)$ is defined recursively

$$g_F^k(\bar{x}^k, \bar{z}^k) = \inf\{\mathrm{E}_{F^{\xi^{k+1}|\bar{\xi}^k = \bar{z}^k}} g_F^{k+1}(\bar{x}^{k+1}, \bar{\xi}^{k+1}) \mid x^{k+1} \in K^{k+1}(\bar{x}^k, \bar{z}^k)\},$$

$$k = 0,...,M-1,$$

$$g_F^M(\bar{x}^M, \bar{z}^M) = g_0^M(\bar{x}^M, \bar{z}^M). \qquad (4)$$

Observe that (in (3), (4)) the decision corresponding to the time point $k \in (0, M\rangle$ can depend on the decisions x^j and the random elements realization ξ^j only for $j < k$. (The symbol $\mathrm{E}_{\bar{F}}$ denotes the operator of the mathematical expectation corresponding to the distribution function $\bar{F}$.)

Evidently, the problem (3), (4) depends on the system of (mostly conditional) probability measures or equivalently on the system of distribution functions

$$F = \{F^{\xi^0}(z^0), F^{\xi^{k+1}|\bar{\xi}^k}(z^{k+1} \mid \bar{z}^k), k = 0,1,\ldots,M-1\} \qquad (5)$$

through the operators of mathematical expectation. Multifunctions $K^{k+1}(\bar{x}^k, \bar{z}^k),\ \ k = 0,1,\ldots,M-1$ (corresponding to the constraints sets) are in "classical" multistage problems defined by a linear or nonlinear system of algebraic inequalities. However, in applications very often

$$K^{k+1}(\bar{x}^k, \bar{z}^k) = K_F^{k+1}(\bar{x}^k, \bar{z}^k), k = 0,1,\ldots M-1$$

can also depend on the system (5). The constraints sets then usually correspond to the probability constraints (see e.g. [9]). In this contribution we admit also the type of this constraints.

3. MODEL CONSTRUCTION

Evidently, to construct the model (or equivalently to construct a "suitable" multistage stochastic programming problem), it is necessary first to determine a time horizon M and, furthermore, to construct:

- a type of the sequence corresponding to a development of the number u^k of unemployment persons,
- a function $g_0(\bar{x}^M, \bar{z}^M)$ corresponding to an evaluation of the process in the time interval $(0, M)$. We assume that the functions $g_F^k(\bar{x}^k, \bar{z}^k)$ in (5) can be written in the form:

$$g_F^k(\bar{x}^k, \bar{z}^k) = \min_{x^{k+1}} \mathrm{E}_{F^{\xi^{k+1}|\bar{\xi}^k = \bar{z}^k}} \{c^{k+1}(\bar{x}^{k+1}, \bar{\xi}^{k+1}) + g_F^{k+1}(\bar{x}^{k+1}, \bar{\xi}^{k+1})\},$$

- constraints sets $K^{k+1}(\bar{x}^k, \bar{z}^k),$ that can generally depend on the system (5):

$$K^{k+1}(\bar{x}^k, \bar{z}^k) = K^{k+1}_{F^{\xi^{k+1}|\bar{\xi}^k = \bar{z}^k}}(\bar{x}^k, \bar{\xi}^k).$$

A construction of all these characteristics has to be done with respect to many factors. A model complication has to depend on the possibility to treat with it; computational techniques and data existence. In particular, in the year 1998 (according to a short data sequence; an unemployment process started about 1990 in the Czech Republic) the third steps time sequence has been employed in [1] to "estimate" the number of unemployment. Consequently, the development of the number of the unemployment u^k in the time point t has been estimated by a sequence:

$$u^t = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \bar{\alpha}_t, \qquad (6)$$

where the parameter $\bar{\alpha}_t$ corresponds to a random element, a_1, a_2, a_3 are deterministic parameters.

Since there has existed a longer sequence of data in 2004, a SARIMA model

$$(1-b_1B^1)(1-b_2B^6)(1-b_3B^{12})(1-B)u^t = \varepsilon^t, \qquad (7)$$

has been employed in [4]; ε^t corresponds to a white noise. b_1, b_2, b_3 are deterministic coefficients (for more details see e.g. [2]).
However, others approaches have been also considered in the literature. A Markov model has been suggested e.g. in [8].
The construction of $g_0(\bar{x}^M, \bar{z}^M), K^{k+1}_{F^{\xi^{k+1}|\bar{\xi}^k=\bar{z}^k}}(\bar{x}^k, \bar{\xi}^k)$ evidently depends on given political and economic possibilities. The model suggested in [6], [7] has been constructed according to a transformation period in which a restructuralization problem has been essential. Consequently, in this time a question has arisen: how long it is necessary (or suitable) to support unproductive factories. The model has been constructed with respect to this question and under the assumption that there exists a requalification process organized and supported by government. The aim of this model has been to minimize the sum of expenses that the society spend for the problem of unemployment, including a social impact. Moreover, it was assumed that the value of the support to one unemployment person can be included in the objective function as one component of the decision vector. A worse value of the support has been corrected by a penalty functions. Moreover, it was there supposed that the same support has been done to every unemployment person. This assumption could approximately correspond to the situation in the first years of the transformation.
To introduce the model, we denote for $k = 0,1,\ldots,M$, u^k the number of the unemployed persons, v^k the support given by the society to one unemployed person; d^k the number of employees working in the unproductive factory, y_2^k the support to one person in the unproductive factory, δ^k the number of persons that must leave their position in the unproductive factory, y_1^k the support that must obtain one person leaving the unproductive factory.
Furthermore, we denote by N^k the labor size, $k_1^k 100\%$ the acceptable percentage of unemployment, $k_2^k 100\%$ the acceptable percentage of persons that can stay in the unproductive factory; $C(k)$ the total amount that the society can spend to settle up the problem, ζ^k a random element connected with a form of development of number of unemployment, r^k the number of requalificated persons, a^k the cost per one person requalification, η^k a random element connected with a form of development of number of people working in unproductive factory. In this approach it was assumed that

$$d^{k+1} = d^k - \delta^k + \eta^k.$$

Evidently, η^k includes persons that leave voluntarily unproductive factory and persons that come to help with the closing the factory. The objective function has been defined by the relation

$$g_F^M(\overline{x}^M, \overline{z}^M) \quad = \quad \sum_{k=0}^{M} \{u^k v^k + r^k a^k + [d^k - \delta^k] y_2^k + y_1^k \delta^k + q_2^k u^k (v^k - \underline{b}^k)^- + q_3^k u^k (z^k - \overline{b}^k)^+\} : \tag{8}$$

where for the time point $k \in \{0, \ldots, M\}$

- $u^k v^k$ corresponds to the support to the unemployment persons,
- $r^k a^k$ corresponds to the cost of requalification,
- $[d^k - \delta^k] y_2^k + y_1^k \delta^k$ corresponds to the cost connected with the unproductive factory,
- $q_2^k u^k (v^k - \underline{b}^k)^-$, $q_3^k u^k (v^k - \overline{b}^k)^+$ correspond to penalties corresponding to the difficulties caused by a "bad" chosen support value.

q_2^k, q_3^k, $\underline{b}^k, \overline{b}^k, k = 0, 1, \ldots, M$ are real–valued known constants; they have to be determined on the bases of the given situation.

Evidently, the decision vector x^k and the random element ξ^k fulfil the relations

$$x^k = (v^k, \delta^k, r^k), \quad \xi^k = (\zeta^k, \eta^k).$$

The constraints set K_F^{k+1} has been considered as a composition of "deterministic" ("usual") constraints given by algebraic inequalities:

$$\begin{aligned} & u^k v^k + [d^k - \delta^k] y_1^k + \delta^k y_2^k + r^k a^k \le C(k), \\ & r^k - \delta^k \le u^k, \quad \underline{x}^{*k} \le v^k \le \overline{x}^{*k}, \quad \delta^k \le d^k, \\ & \qquad r^k \le \overline{r}^k, \quad x^k, \delta^k, r^k \ge 0 \end{aligned} \tag{9}$$

and by individual probability constraints

$$\begin{aligned} & P_{F^{\zeta^{k+1}|\overline{\zeta}^k}} \{\hat{u}^{k+1} \le k_1^{k+1} N^{k+1}\} \ge \alpha_1, \quad P_{F^{\eta^{k+1}|\overline{\eta}^k}} \{d^{k+1} \le k_2^{k+1} d^0\} \ge \alpha_2, \\ & \qquad \hat{u}^{k+1} = u^{k+1} + \hat{k}_\delta \delta^k - \hat{k}_r r^k, \end{aligned} \tag{10}$$

where $\alpha_1, \alpha_2 \in \langle 0, 1 \rangle, \underline{x}^{*k}, \overline{x}^{*k}, \overline{r}^k \ge 0$ are given constants; the symbol $\hat{u}^{k+1}$ denotes the number of unemployment after requalification and reduction in the unproductive factory, the coefficients $\hat{k}_\delta, \hat{k}_r$ are assumed to be deterministic and not depending on k. $F^{\zeta^{k+1}|\overline{\zeta}^k}$, $F^{\eta^{k+1}|\overline{\eta}^k}$ denotes the corresponding conditional distribution functions. Evidently, the influence of requalification can appear in the longer time interval, however, we neglected this fact.

It follows from the elementary properties of the mathematical statistics that employing definition of quantils, the inequalities (10) can be rewritten also in the form of two algebraic inequalities. According to this fact, the constraints set is completely given by a system of linear inequalities, of course with some coefficients depending on the “underlying" probability measure. The constraints set K^0 is assumed to be “deterministic", known and given e.g by the system of linear inequalities.

Employing the results of the paper [5] we can find conditions under which the constructed multistage problem fulfils (from the numerical point of view) “pleasant" properties. The models constructed in [1] and [4] are more realistic. It was supposed (in these models) that the values v^k are determined by the government. Moreover, others instruments have been employed in these works. We can mention “public" employment, youth programme for new schools absolvent, subsidized places, however also considered disabled measures. A numerical results has been achieved there, for more details see [4].

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APPROXIMATIVE METHODS FOR „MANY-TO-MANY“ DISTRIBUTION SYSTEM DESIGN

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Abstract: The many-to-many distribution system is a special case of transportation system, where flows of goods from primary sources to customers are concentrated in terminals to create a bigger flow between terminals. This model belongs to discrete quadratic programmes and the optimal solution can't be found because of time purposes. To solve this problem the model must be reformulated to the form of linear programming problem to be able to use solving algorithms for integer programming problems. One of the method is an approximate linearization of the model.
Keywords: Many-to-many distribution system, linearization, transportation system

1. INTRODUCTION

A transportation system, which has approximately the same number of primary sources as number of customers, seems to be a marginal case of a distribution system. This case includes such instances as National Postal Network or cargo railway system [5], which provides transport of carriages between railway stations. In these cases, demands of customers form a matrix of yearly flows from sources to places of destination. We denote this matrix as $\boldsymbol{B} = \{b_{sj}\}$, for $s \in S$ and $j \in J$, where S is a set of sources and J denotes set of customers. The fact that unit cost of transportation is smaller when bigger bulks of items are transported, approves concentration of flows between different pairs of source and customer to stronger flows at least on a part of their way. This flow concentration needs terminals, in which transshipment of transported items is performed and bigger bulks are formed or, on the other side, where bulks are split into smaller groups designated to different customers.

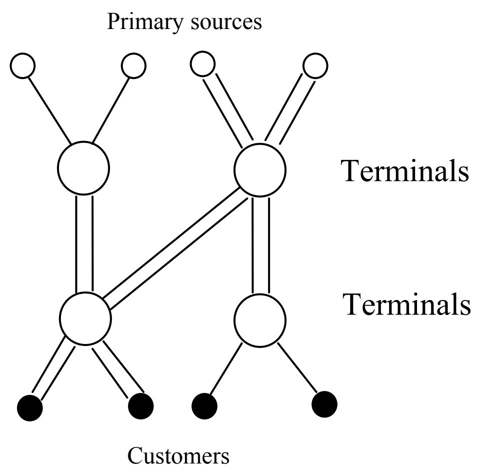

Figure 1

On the contraries to the classical distribution systems, in which big bulks leave primary source, another situation emerges in the many-to-many distribution systems. Primary sources send relative small bulks of items and it is useful to concentrate them to bigger bulks in the terminals located near the sources and then to send these bigger bulks to remote terminals and to split them there (see Figure 1).

We restrict ourselves here to the distribution system, in which a customer/source is assigned to only one terminal and an exchange of the consignments between the customer/source and other primary sources or customers is done via this assigned terminal, as it is shown in Fig. 1. Furthermore, we consider the general case, in which any source is also a customer, what is the case of post offices, railway stations and so on. We do not make any difference between a primary source and a customer hereafter and we introduce the set $J' = \{1, ..., n\}$, of customer-sources, for which matrix $\boldsymbol{B}$ gives by coefficients b_{sj} the yearly volume of the consignments, which are sent from the object s to the object j and it gives by coefficients b_{js} the total yearly volume sent from the object j to the object s. In the next section we try to model a symmetrical many-to-many distribution system with unique assignment of customers to terminals.

2. MODEL OF MANY-TO-MANY DISTRIBUTION SYSTEM DESIGN PROBLEM

Let us consider a case with a linear cost estimation function with unit cost e_0 for one item transport along unit distance on the way from a primary source to a terminal or from a terminal to a customer. Next, let us consider unit cost e_1 for one item transport along unit distance on the way between terminals. Furthermore we denote possible terminal locations by symbol $i=1, ..., m$, where each place i is associated with yearly fixed charges f_i for building and performance of the terminal i and with unit cost g_i for transhipment of one unit in the terminal. In accordance to the previous definition, we denote by $s \in J'$ object which sends consignments with the yearly total amount b_{sj} from s to $j=1, ..., n$. Symbol d_{pq} denotes the distance between objects $p \in J'$ and $q \in J'$. Our goal is to assign each sending or receipting object to exactly one terminal so that the total yearly cost of the designed system be minimal. If we denote by $y_i \in \{0, 1\}$ for $i= 1, ..., m$ the bivalent variable, which corresponds to the decision if a terminal will $(y_i =1)$ or will not $(y_i =0)$ be built at place i and if we introduce the variable $z_{ij} \in \{0, 1\}$ for $i =1, ..., m$ a $j =1, ..., n$, which says if object j will $(z_{ij} =1)$ or will not $(z_{ij} =0)$ be assigned to place i, then we can establish following mathematical programming model of the problem.

$$Minimise \sum_{i=1}^{m} f_i y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} (e_0 d_{ij} + g_i)(\sum_{s=1}^{n} b_{js} + \sum_{s=1}^{n} b_{sj}) z_{ij} +$$

$$+ \sum_{i=1}^{m} \sum_{k=1}^{m} e_1 d_{ik} \sum_{j=1}^{n} \sum_{s=1}^{n} b_{sj} z_{ij} z_{ks} \ . \qquad (1)$$

Subject to $\sum_{i=1}^{m} z_{ij} = 1$ *for j=1, ..., n,* *(2)*

$z_{ij} \leq y_i$ *for i=1, ..., m, j=1, ..., n,* *(3)*

$y_i \in \{0, 1\}$ *for i=1, ..., m,* *(4)*

$z_{ij} \in \{0, 1\}$ *for i=1, ..., m, j=1, ..., n.* *(5)*

The model belongs to discrete quadratic programmes due the third term of *(1)*.

3. APPROXIMATE LINEARIZATION OF THE MODEL

To be able to use a general solver for integer programming problems, the model of the problem must be reformulated to the form of linear programming problem

Let us concentrate on the particular ways of the quadratic term linearization, which consists in replacing the sum in brackets by some term t_{is}, which doesn't depend on index *k* and which could be good approximation of the sum.

$$\sum_{i \in I} \sum_{k \in I} e_1 d_{ik} \sum_{j \in J'} \sum_{s \in J'} b_{sj} z_{ij} z_{ks} = e_1 \sum_{i \in I} \sum_{j \in J'} \sum_{s \in J'} b_{sj} (\sum_{k \in I} d_{ik} z_{ks}) z_{ij} \qquad (6)$$

This replacing is done subject to constraint $\sum_{k \in I} z_{ks} = 1$.

It is obvious that exactly one of the distances d_{ik} for $k \in I$ will be result of the sum and the estimated value ranges over the interval $<min\{ d_{ik} : k \in I \}, max\{ d_{ik} : k \in I \}>$.

The simplest way, how to estimate the d_{is} is an average of all distances from *i* to all terminals. Then

$$t_{is} = (\sum_{k \in I} d_{ik}) / |I| \qquad (7)$$

This estimation doesn't take regards to position of the place *s* to place *k.* It gives the same chance to the most efficient assignment as to fully disadvantageous one.

Further way of the estimation makes use of so-called set *K(i, s)* of relevant locations to which object (customer-source) *s* is allowed to be assigned. It should be noted here that prime cost of one unit transport from terminal *i* via terminal *k* to object *s* is $e_1 d_{ik} + e_0 d_{ks}$. This transportation chain is economically sensible only if this prime cost is less than cost of the direct transport from *i* to *s*, what is $e_0 d_{is}$. This idea enables to establish the set of relevant locations $K(i, s) = \{k \in I: e_1 d_{ik} + e_0 d_{ks} < e_0 d_{is}\}$ and to take into consideration only *k's* from this set. We can define $t_{is} = \beta_{is} d_{is}$, where

$$\beta_{is} = (\sum_{k \in K(i,s)} d_{ik} / d_{is}) / |K(i,s)| \qquad (8)$$

, what is average of the relevant ratios.
Instead of β_{is}, less precise coefficients β_i or β may be used. These coefficients can be determined as an average or weighted average of the relevant ratios over set of all objects – customer-source $s \in J'$ and for given i or over the set of all pairs $(i, s) \in I \times J'$ respectively.

$$\beta_i = \frac{\sum_{s \in J'} ((\sum_{k \in K(i,s)} d_{ik} / d_{is}) / |K(i,s)|)}{|J'|} \qquad (9)$$

$$\beta = \frac{\sum_{i \in I} \frac{\sum_{s \in J'} ((\sum_{k \in K(i,s)} d_{ik} / d_{is}) / |K(i,s)|)}{|J'|}}{|I|} \qquad (10)$$

Obtaining estimation t_{is} the term *(12)* can be rewritten as follows: $e_1 \sum_{i \in I} \sum_{j \in J'} \sum_{s \in J'} b_{sj} (\sum_{k \in I} d_{ik} z_{ks}) z_{ij} = e_1 \sum_{i \in I} \sum_{j \in J'} (\sum_{s \in J'} b_{sj} t_{is}) z_{ij}$, what is a linear expression. Then problem *(1), (2)-(5)* takes form of a linear uncapacitated location problem, which is easy to solve using BBDual procedure[1].

4. NUMERICAL EXPERIMENTS

The set of test problems was created from real network and from real many-to-many problem, which was formulated in the frame of the project [5] in which railway infrastructure and yearly carriage flows were analyzed. This original problem, in which 53 possible locations and almost five hundred railway stations-customers were considered, was cut into sequence of smaller subproblems, in which 5, 9 or 15 possible locations were considered and where the number of considered railway stations ranged from ten to eighty by step of ten. The original fixed charges were adjusted for a particular subproblem proportionally to its total flow size to obtain non-trivial results. This way, three instances came into being for each problem size. The first one has the original fixed charges and the next two instances with modified fixed charges, what led to different solutions.
A goal of this investigation was to compare, how precise are these linearization methods in comparision wiht exact solutions and which of this method gives more precise upper bound of solution in this problem. For compariton were used optimal solutions of this subproblem from [3], [5]. Experiments were made in Delphi 7 enviroment.

In *Tables 1-3* are solutions for three various fixed charges with dimension of 9 possible locations, in *Tables 4-5* for three various fixed charges with dimension of 15 possible locations. In all tables column *Size* is the size of solved subproblem, in column *Optimal* are values of optimal solutions of appropriate subproblem, in other columns are values of solutions of appropriate subproblem using linearization methods and comparision of these solutions with optimal value.

Table 1: Fixed charges 1, dimension of 9 locations

Size	Optimal	(7)		(8)		(9)		(10)	
	obj.value	obj.value	%	obj.value	%	obj.value	%	obj.value	%
9x10	91425,03	91425	**100**	91425	**100**	91425	**100**	91425	**100**
9x20	206237	206381	**100,07**	206237	**100**	206237	**100**	206237	**100**
9x30	276266,8	276411	**100,05**	276267	**100**	276267	**100**	276267	**100**
9x40	328374,3	328584	**100,06**	328374	**100**	328374	**100**	328374	**100**
9x50	361425,6	361728	**100,08**	361426	**100**	361426	**100**	361426	**100**
9x60	430776,2	433781	**100,7**	433697	**100,68**	433781	**100,7**	433768	**100,69**
9x70	455919,7	465018	**102**	464952	**101,98**	465018	**102**	465023	**102**
9x80	464807	473794	**101,93**	473723	**101,92**	473794	**101,93**	473794	**101,93**

Table 2: Fixed charges 2, dimension of 9 locations

Size	Optimal	(7)		(8)		(9)		(10)	
	obj.value	obj.value	%	obj.value	%	obj.value	%	obj.value	%
9x10	13197,94	13198	**100**	13198	**100**	13198	**100**	13198	**100**
9x20	57380,34	57380	**100**	57380	**100**	57380	**100**	57380	**100**
9x30	97141,85	98147	**101,03**	98147	**101,03**	98147	**101,03**	98616	**101,52**
9x40	119528,1	120608	**100,9**	120608	**100,9**	120575	**100,88**	121026	**101,25**
9x50	136475	137506	**100,76**	137502	**100,75**	137483	**100,74**	138098	**101,19**
9x60	166187,1	167269	**100,65**	167265	**100,65**	167242	**100,63**	167798	**100,97**
9x70	179908,9	181007	**100,61**	181003	**100,61**	180985	**100,6**	181528	**100,9**
9x80	186775,7	187819	**100,56**	187814	**100,56**	187796	**100,55**	188394	**100,87**

Table 3: Fixed charges 3, dimension of 9 locations

Size	Optimal	(7)		(8)		(9)		(10)	
	obj.value	obj.value	%	obj.value	%	obj.value	%	obj.value	%
9x10	37313,01	37313	**100**	37313	**100**	37313	**100**	38143	**102,22**
9x20	121758,9	124433	**102,2**	124433	**102,2**	124396	**102,17**	124932	**102,61**
9x30	166299,3	173567	**104,37**	173567	**104,37**	173567	**104,37**	178060	**107,07**
9x40	196387,9	198526	**101,09**	198526	**101,09**	198492	**101,07**	198923	**101,29**
9x50	216112	216832	**100,33**	216827	**100,33**	216808	**100,32**	217404	**100,6**
9x60	246621,7	248214	**100,65**	248209	**100,64**	248186	**100,63**	247991	**100,56**
9x70	262003,3	263610	**100,61**	263606	**100,61**	263588	**100,6**	263420	**100,54**
9x80	268912,2	270424	**100,56**	270419	**100,56**	270401	**100,55**	270381	**100,55**

Table 4: Fixed charges 1, dimension of 15 locations

Size	Optimal obj.value	(7) obj.value	(7) %	(8) obj.value	(8) %	(9) obj.value	(9) %	(10) obj.value	(10) %
15x20	168266,2	184482	**109,64**	184482	**109,64**	184482	**109,64**	184482	**109,64**
15x30	224795,8	252203	**112,19**	252203	**112,19**	252203	**112,19**	252547	**112,35**
15x40	257311,6	290158	**112,77**	287878	**111,88**	287878	**111,88**	291971	**113,47**
15x50	277600,6	312318	**112,51**	312318	**112,51**	312318	**112,51**	312532	**112,58**

Table 5: Fixed charges 2, dimension of 15 locations

Size	Optimal obj.value	(7) obj.value	(7) %	(8) obj.value	(8) %	(9) obj.value	(9) %	(10) obj.value	(10) %
15x20	34001,03	47512	**139,74**	47512	**139,74**	47512	**139,74**	47542	**139,83**
15x30	60234,19	80724	**134,02**	80724	**134,02**	80724	**134,02**	81002	**134,48**
15x40	77792,78	100228	**128,84**	100215	**128,82**	100215	**128,82**	100502	**129,19**
15x50	88533,93	113533	**128,24**	113535	**128,24**	113535	**128,24**	113842	**128,59**

5. CONCLUSION

In smaller problems these methods of linearization give values near optimal value. When the problem is bigger, precision of solution depends on type of network and fixed costs. In comparision between methods better solutions give us method using *(8)* or *(9)*, but differences between all these four methods are small. In the future research in this field the attention could be paid to ways of investigation of sensibility of parameters β, using this investigation in heuristic methods and improve this solutions by using other parameters to estimate distance d_{is}.

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THE FIXED CHARGE TRANSPORTATION PROBLEM

PETER LOŠONSKÝ

Abstract: The fixed charge transportation problem (FCTP), which is known to be NP-hard, is an extension of the classical transportation problem (TP) in which a fixed cost is incurred, independent of the amount transported, along with a variable cost that is proportional to the amount shipped. FCTP can be solved by heuristic which use linearization of objective function, so then it can be solved as a TP. Several algorithms for solving TP will be evaluated and mutually compared according to their efficiency for solving FCTP problems.

Keywords: Fixed-charge, Transportation problem, Heuristic algorithm

1. INTRODUCTION

The fixed charge transportation problem (FCTP) is a mathematical programming problem in which a fixed cost is incurred if another related variable assumes a nonzero value. The problem has a wide variety of classic applications that have been documented in the scheduling and facility location literature.

FCTP can be solved by heuristic which use linearization of objective function. The computational merit of this approach is that it approximates a solution for FCTP by solving successive LP problems with recursively updated objective functions. These LP problems can be solved efficiently since the set of constraints is not changed. Accordingly, one can surely reduce the tedious computation to search the tree in branch and bound type methods by removing all the binary variables from the classical mixed integer formulation. This can be done by finding a linear factor that effectively reflects the current marginal variable cost and fixed cost at the same time depending on the current level of each variable.

Several algorithms (MODI, Hungarian method) for solving TP will be evaluated and mutually compared according to their efficiency for solving FCTP problems.

2. THE FIXED CHARGE TRANSPORTATION PROBLEM

The fixed charge transportation problem can be stated as a distribution problem in which there are M suppliers (warehouses or factories) and N customers (destinations or demand points). Each of the M suppliers can ship to any of the N customers at a variable cost per unit c_{ij} (unit cost for shipping from supplier i to customer j) plus a fixed cost f_{ij} , assumed for opening this route -

y_{ij}. Each supplier has a_i units of supply, and each customer has a demand of b_j units. The objective is to determine which routes are to be opened and the size of the shipment on those routes, so that the total cost of meeting demand, given the supply constraints, is minimized.

$$\min \ Z^* = \sum_{i=1}^{M} \sum_{j=1}^{N} (c_{ij}.x_{ij} + f_{ij}.y_{ij}) \tag{1}$$

st

$$\sum_{j=1}^{N} x_{ij} = a_i \qquad i = 1 .. M \tag{2}$$

$$\sum_{i=1}^{M} x_{ij} = b_j \qquad j = 1 .. N \tag{3}$$

$$x_{ij} \le U.y_{ij} \qquad \forall \ i, j \tag{4}$$

$$x_{ij} \ge 0 \qquad \forall \ i, j \tag{5}$$

$$y_{ij} \in \{0,1\} \qquad \forall \ i, j \tag{6}$$

Constraints (2) and (3) are classical flow conservation constraints, while constraints (4) are the forcing constraints that guarantee that y_{ij} takes value one whenever x_{ij} is positive.

Despite its similarity to a standard TP problem, FCTP is significantly harder to solve because of the discontinuity in the objective function Z introduced by the fixed costs.

3. INITIAL FEASIBLE SOLUTION

Balinski [1] observed that there exists an optimal solution to the relaxed version of FCTP (formed by relaxing the integer restriction on y_{ij}), with the property that

$$y_{ij} = \frac{x_{ij}}{m_{ij}} \tag{7}$$

where

$$m_{ij} = \min(a_i, b_j) \tag{8}$$

So, the relaxed transportation problem (RTP) of an FCTP would be simply a standard TP with unit transportation costs as

$$C_{ij} = c_{ij} + \frac{f_{ij}}{m_{ij}} \tag{9}$$

and RTP is possible to write as follows:

$$\min\ Z = \sum_{i=1}^{M}\sum_{j=1}^{N} C_{ij}.x_{ij} \tag{10}$$

st

(2), (3), (5)

The optimal solution $\{X_{ij}^{'}\}$ to the RTP problem can be easily modified into a feasible solution of $\{X_{ij}^{'}, y_{ij}^{'}\}$of FCTP as follows:

$$y_{ij}^{'} = 0 \quad \text{ak} \quad X_{ij}^{'} = 0 \tag{11}$$

$$y_{ij}^{'} = 1 \quad \text{ak} \quad X_{ij}^{'} > 0 \tag{12}$$

Balinski shows that the optimal value $\{X_{ij}^{'}\}$ of RTP provides a lower bound on the optimal value Z* of FCTP and modified feasible solution $\{X_{ij}^{'}, y_{ij}^{'}\}$ provides an upper bound:

$$\sum\sum C_{ij}.X_{ij}^{'} \le Z^{*} \le \sum\sum (c_{ij}.X_{ij}^{'} + f_{ij}.y_{ij}^{'}) \tag{13}$$

4. DYNAMIC SLOPE SCALING PROCEDURE

The initial motivation of this approach is to find a linear factor that effectively reflects the variable cost c_{ij} and marginal fixed cost f_{ij} simultaneously. This can be done by effectively updating the linear factor [4]. The efficiency of this approach is associated with an initial solution produced by a certain type. Two types of candidates for initial solutions are considered as follows:

Type I:

$$C_{ij}^{0} = c_{ij} \qquad \forall i, j \tag{14}$$

The idea in the Type I scheme is to solve the original FCTP by finding an initial solution without regarding fixed costs.

Type II:

$$C_{ij}^{0} = c_{ij} + \frac{f_{ij}}{\min(a_i, b_j)} \qquad \forall i, j \tag{15}$$

The idea in the Type II scheme is closer described in previous chapter.

Starting with the initial solution produced by either Type I or Type II and the initial coefficients, the main iterative procedure is executed solving the RTP with the updated dynamic slope scaling factor $\overline{C}_{ij}^{k}$, $k = 1, 2, ...$ At each iteration k is used following updating scheme:

$$c_{ij}^{k+1} = \begin{cases} c_{ij} + \dfrac{f_{ij}}{x_{ij}^k} & \text{if } x_{ij}^k > 0 \\ \\ c_{ij}^r & \text{if } x_{ij}^k = 0 \end{cases} \tag{16}$$

where r is the index of the most recent value of slope scaling factor when $x_{ij}^{r-1} > 0$. In [4] there is mentioned another updating scheme. The stopping criterion for the procedure is that any consecutive solutions of RTP are exactly the same:

$$x_{ij}^{k-1} = x_{ij}^k \qquad \forall i, j \tag{17}$$

5. METHODS FOR SOLVING TRANSPORTATION PROBLEM

5.1 MODI

MODI provides a new means of finding the unused route with the largest negative improvement index. Once the largest index is identified, we are required to trace only one closed path. This path helps determine the maximum number of units that can be shipped via the best unused route. Detailed explanation of MODI method can be found in [4].

The structure of the transportation constraint coefficients allows us to use one of several special starting procedures for transportation problems – northwest corner starting procedure (NWM), least cost starting procedure (LCM), Vogel's aproximation method starting procedure (VAM), Frequency method (FQM) starting procedure. Detailed explanation of mentioned starting procedures can be found in [2].

5.2 HUNGARIAN METHOD

The idea behind the Hungarian algorithm is to reduce the matrix so that only nonnegative costs exist and at least one 0 remains in each row and column; then we attempt to make a complete assignment using only the 0 costs. If this is unsuccessful, we apply a systematic matrix reduction procedure that creates a new matrix of costs, in which we reduce to 0 an entry that previously had a positive cost. Then we make another attempt to find a complete 0 cost assignment. We repeat the process until we find an assignment of only 0 entries. Detailed explanation of Hungarian method can be found in [4].

6. COMPUTATIONAL EXPERIMENTS

Availability of the FCTP testbed, access to the Sun et al. TS implementation [6], and the fact that the Sun et al. tabu search method was established as the most effective approximation approach for the FCTP has motivated me to carry out comparative testing using the same FCTP testbed. Glover et al. in [3] were using the same FCTP testbed and their heuristic brought better results, so I decided to compare my solutions with their results.

This testbed includes eight problem types, A through H, each in seven problem sizes. For a given problem size, problem types differ from each other by the range of fixed costs, which increases upon progressing from problem type A through problem type H. Each problem type includes 15 randomly generated problems. The variable costs range over the discrete values from 3 to 8. The seven problem types present different levels of difficulty for alternative solution approaches. In my computational experiments I was using 10 test problems of problem type A, where the lower limit for fixed costs was 50 and upper limit 200. In the following two tables there are results of mentioned heuristic using two types of candidates for initial solutions. There is also execution time of whole heuristic with using MODI method with different starting procedures or Hungarian method. DSSP results were compared to the best found solutions mentioned in [3].

Table 1 – DSSP with Type I initial solution

Problem ID	**B.F.S.**	**MODI**								**Hung. method**	
		NWM		**LCM**		**FQM**		**VAM**		B.F.S	
		B.F.S.		B.F.S		B.F.S.		B.F.S.			
		Time [s]	GAP [%]	Time [s]	GAP [%]	Time [s]	GAP [%]	Time [s]	GAP [%]	Time [s]	GAP [%]
N3000	168057	**172305**		172181		173604		173579		173263	
		175,7	**2,53**	45,3	2,45	66,9	3,3	58,6	3,29	5,6	3,09
N3001	166678	171626		**171547**		173152		171858		171857	
		226,7	2,97	55,7	**2,92**	66,5	3,88	43,4	3,11	7,1	3,11
N3002	167919	**171449**		172599		171602		172173		172785	
		143,9	**2,1**	40,2	2,79	47,3	2,19	39,4	2,53	4,2	2,89
N3003	168434	173898		174185		**173515**		173659		173859	
		118,9	3,24	34,5	3,41	34,3	**3,02**	42,9	3,1	6,7	3,22
N3004	167275	174172		172878		173809		172457		**172111**	
		170,5	4,12	28,5	3,35	44,6	3,91	42,4	3,1	5,6	**2,89**
N3005	167639	172529		**171706**		172728		172292		172525	
		120,5	2,92	32,5	**2,42**	57,7	3,03	36,6	2,78	5,4	2,91
N3006	165862	170095		170708		**170057**		170656		170559	
		137,7	2,55	36,6	2,92	46,3	**2,53**	27,1	2,89	4,3	2,83
N3007	167364	172019		172526		**171478**		172020		172025	
		108,5	2,78	46,9	3,08	45,7	**2,46**	48,5	2,78	3,9	2,78
N3008	165576	169619		169849		169516		170318		**169510**	
		182,6	2,44	29,8	2,58	45,2	2,38	29,4	2,86	4,7	**2,37**
N3009	167193	171499		171158		172132		172102		**170916**	
		152,5	2,58	30,9	2,37	52,7	2,95	26,1	2,93	4,6	**2,23**
Avg.											
		153,8	2,83	38,1	2,83	50,7	2,97	39,5	2,93	**5,2**	**2,83**

Table 2 – DSSP with Type II initial solution

Problem ID	**B.F.S.**	**MODI**								**Hung. method**	
		NWM		**LCM**		**FQM**		**VAM**		B.F.S	
		B.F.S.		B.F.S		B.F.S.		B.F.S.			
		Time [s]	GAP [%]	Time [s]	GAP [%]	Time [s]	GAP [%]	Time [s]	GAP [%]	Time [s]	GAP [%]
N3000	168057	171889		**171064**		171168		172243		171718	
		183,4	2,28	50,3	**1,79**	51,3	1,85	45,1	2,49	4,2	2,18
N3001	166678	169410		171099		169299		171021		**169083**	
		226,5	1,63	48,2	2,65	67,5	1,57	54,2	2,6	5,4	**1,44**
N3002	167919	170900		171625		170888		170996		**170276**	
		130,2	1,78	36,1	2,21	49,2	1,77	40,7	1,83	6,2	**1,4**
N3003	168434	173307		173378		173268		173184		**172810**	
		143,3	2,89	27,9	2,94	44,5	2,87	40,7	2,82	5,8	**2,6**
N3004	167275	170690		170690		170710		170473		**170199**	
		170,1	2,04	30,4	2,04	45,9	2,05	38,7	1,91	5,6	**1,75**
N3005	167639	171163		171193		171595		171275		**171110**	
		131,9	2,1	31,2	2,12	50,9	2,36	39,4	2,17	5,8	**2,07**
N3006	165862	169654		168826		169791		169635		**168181**	
		113,3	2,29	40,2	1,79	41,6	2,37	32,3	2,27	4,6	**1,39**
N3007	167364	171757		171851		171841		171766		**171238**	
		133,3	2,62	27,1	2,68	40,6	2,68	32,4	2,63	5,2	**2,3**
N3008	165576	168393		168610		168393		168479		**168371**	
		177,6	1,7	26,4	1,83	54,2	1,7	33,5	1,75	4,1	**1,69**
N3009	167193	170966		170904		171037		171037		**170352**	
		151,5	2,26	34,6	2,22	42,6	2,3	58,7	2,3	5	**1,89**
Avg.											
		156,1	2,16	35,3	2,23	48,8	2,15	41,6	2,28	**5,2**	**1,87**

As you can see, in both tables the fastest algorithm for solving relaxed transportation problem there is Hungarian method, which seems to be eight times faster than MODI with best starting procedure. DSSP with Type II initial solution provides almost 1% improvement comparing to DSSP with Type I initial solution, so in the next computational experiments I would recommend using only that Type II initial solution.

7. CONCLUSION

The computational results show that the proposed solution approach (DSSP) is highly efficient and reliable. Using of Hungarian method can speed up whole heuristic in comparison to MODI method. There are some differencies in results even if the same values can be expected for different solving methods. This is due to a fact that the interval of variable costs $c_{ij} \in <3,8>$ is small and thus transportation problem can have several optimal solutions (the same value of objective function, different basis variables). Ratio between the fixed cost and variable cost in used test problems was from interval (6, 70). In the future it might be interesting to determine sensitivity of DSSP to higher ratio between those costs.

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THE LONG-RUN DETERMINANTS OF MONEY DEMAND IN SLOVAKIA

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Abstract: The paper verifies the long-run determinants of the demand for money in Slovakia. Approvingly to theoretical assumptions, this paper confirms money demand positively responding to an increase in real income and short-run interest rate and negatively to a rising in long-run interest rate, real effective exchange rate and the rate of inflation. This work is the first part of the complex study in demand for money, in which we can make an answer to question about a relevancy of the quantitative theory of money in Slovak money market.

1. INTRODUCTION

The demand for money is obviously explained according to main functions of money. Portfolio theories emphasize storing of value function and are relevant only for M2 or M3 money. Transactions theories on the other hand emphasize medium of exchange function and are applicable also for M1 money. Our analysis investigates primary determinants of money demand and these theories could be tested as simple hypotheses in our model.

Our research is supported by a lot of studies that show off the importance of a stable long-run money demand function. Because it could be shown that the deviations from long-run money helps to forecast future changes in output growth and inflation.

The theoretic basis for formation of money demand function is bottom on base created by Laidler (1993) and Hoffman and Rasche (2001). In their works money demand is specified as a function of real income, a long-run interest rate on substitutable non-money financial assets, a short-run rate of interest on money itself, and the inflation rate. Ericsson (1998) suggests this log-linear specification:

$$\left(m^d - p\right)_t = \beta_0 + \beta_1 y_t + \beta_2 RL_t + \beta_3 \pi_t + \beta_4 RS_t + u_t, \qquad (1)$$

where m^d is nominal money demanded, p is the price level, y is the real GDP, RL is the long-run rate of return on assets outside of money, RS is the short-run rate of interest on money itself and π is the rate of inflation. All variables in lowercase are in logs and the remaining variables are in levels.

A parameter β_1 denotes income elasticity of the demand for money and parameters β_2, β_3 a β_4 denote semi-elasticises of the interest rates and the inflation rate with respect to money demand. The expected signs and magnitudes of the coefficients are as follows:

- The income elasticity of the money demand provides testing hypotheses:
  if it equals 1, the quantity theory applies,
  if it equals 0,5, the Baumol-Tobin approach is disposable.
- The semi-elasticity of the long-run interest rate is expected negative, because the rise of interest rate other than money penalizes their holding.
- The semi-elasticity of the short-run interest rate should be positive because of the positive correlation short-run rate with money demand.
- The annualised rate of inflation (four lag differentia of logarithm of a price index) is considered as a proxy to measure the return on holdings of goods, and its coefficient should be negative, if goods are an alternative to money.

The empirical research of many authors denotes the fact, that coefficients β_2 and β_4 are equal with the opposite signs. That's why the demand for money can be written as:

$$\left(m^d - p\right)_t = \beta_0 + \beta_1 y_t + \beta_2 \left(RL - RS\right)_t + \beta_3 \pi_t + u_t \,. \qquad (2)$$

A parameter β_2 should take the negative sign and is interpreted as measure of opportunity costs of money holding.

The inclusion of the rate of inflation in these functions of money demand is a little controversy. Valadkhani (2002) in his study summarises opinions about the inclusion or exclusion of this variable. As a support of inclusion he stated inflation is a good proxy for the opportunity cost of holding money rather than real assets and that the inclusion or exclusion is a dynamic specification issue, which should be subject to an empirical testing.

Previous specifications don't include any impact of foreign economies. So, in the small economy as Slovakia is, we are expected some measures like an exchange rate or a foreign interest rate. Thus, due to an illogical impact of Slovak economy to foreign interest rates (in our VAR model), the long-run demand for money is specified only as:

$$\left(m^d - p\right)_t = \beta_0 + \beta_1 y_t + \beta_2 RL + \beta_3 RS_t + \beta_4 \pi_t + \beta_5 \xi_t + u_t \,, \qquad (3)$$

where every symbol represents the same thing as before and ξ is the real effective exchange rate. Its parameter β_5 denotes exchange rate elasticity of the demand for money and may be positive as well as negative.

The time period used for an analysis is from the first quarter 1996 till the fourth quarter 2005. The data are from the internet database SLOVSTAT published by The Statistical Office of the Slovak Republic and from the web pages and the internal resources of The National Bank of Slovakia.

In our research we analyze M2 money in actual exchange rates in milliards Sk stated at the end of the quarter. We use also values of seasonally adjusted gross domestic product measured in constant prices in milliards Sk, the consumer price index as average of monthly data, the interest rates measured as fractions and definitely the real effective exchange rates.

The real effective exchange rate is calculated by NBS for two groups of countries. The first group is created by the Czech Republic (0,58), Germany (0,197), Italy (0,043), Austria (0,083), France (0,023), Netherlands (0,022), Switzerland (0,014), Great Britain (0,017) and USA (0,021). There are nine countries – the main foreign trade partners of Slovakia. In the brackets there are weights of the individual trade share of trade turnover in the year 1993. The second group is created by the same countries without the Czech Republic. The values are available CPI deflated and we use average of monthly data. The first group is marked with number nine and the second with number eight.

2. METHODOLOGY

In our analysis of the money demand the Johansen (1991) cointegration technique is used to test the presence of a long-run equilibrium among the mentioned variables. The specification of the definite form is opened, we investigate the various possibilities.

Before testing for cointegration, the order of integration of the individual time series must be determined. Tests for unit roots are performed on all of the data using the Augmented Dickey-Fuller (ADF) test. The Schwarz Information Criterion (SC) has been used to determine the optimal lag length in the ADF regression. These lags augment the ADF regression to ensure that the error term is without serial correlation. The null hypothesis of ADF is that the variable under investigation has a unit root, against the alternative that it does not. The more negative values of the reported test statistic lead to rejection of the null hypothesis. The critical values of the test statistics are discussed in Dickey and Fuller (1979).

For the Johansen method, there are two test statistics for the number of cointegrating vectors: the lambda trace and maximum eigenvalue statistics. The cointegration rank is obtained from the lambda max statistic.

The deterministic trend specification is important, because of its impact on the relevant statistics used in cointegration analysis. Suppose the vector error correction model:

$$\Delta \mathbf{y}_t = \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 t + \mathbf{B}\mathbf{y}_{t-1} + \sum_{j=1}^{p-1} \mathbf{B}_j \Delta \mathbf{y}_{t-j} + \mathbf{Q}\mathbf{x}_t + \boldsymbol{\varepsilon}_t , \qquad (4)$$

where $\mathbf{y}$ is a vector of the variables integrated of order one ($\mathbf{y} \sim I(1)$), $\mathbf{x}$ is a vector of the stationary variables and $\mathbf{B}$ can be written as $\mathbf{B} = \mathbf{D}\mathbf{C}^{\mathrm{T}}$.$\mathbf{B}$ is the product of the cointegrating matrix $\mathbf{C}$ and the matrix of adjustment $\mathbf{D}$. The matrix $\mathbf{Q}$ represents the parameters of the stationary variables.

If there is in (4) that $\mathbf{y_t} \sim I(1)$, then by presence of the cointegration have to be $\mathbf{C^T y_t} \sim I(0)$, and if in that $\Delta\mathbf{y_t}$, $\Delta\mathbf{y_{t-1}}$, ..., $\Delta\mathbf{y_{t-j}} \sim I(0)$ and $\mathbf{Q^T x_t} \sim I(0)$, then all the variables are stationary and there exists five known possible deterministic trend specifications.

3. EMPIRICAL RESULTS

Table 1: ADF unit root test

Variable	trend + constant	constant	no constant
	t-stat. (*lag*)	*t*-stat. (*lag*)	*t*-stat. (*lag*)
m2-p	-3,096 (4)	-2,393 (4)	0,736 (4)
Δ(*m2-p*)	-2,372 (3)	-2,655 (3)	-2,588 (3)
y	0,200 (0)	1,502 (0)	10,80 (0)
Δ*y*	-5,437 (0)	-	-
RL	-2,414 (1)	-0,804 (1)	-1,133 (1)
Δ*RL*	-3,302 (0)	-3,310 (0)	-
RS	-1,448 (1)	0,336 (1)	-1,403 (1)
Δ*RS*	-4,340 (0)	-	-
R	-2,569 (1)	-1,150 (1)	-1,132 (1)
Δ*R*	-3,406 (0)	-3,460 (0)	-
π	-3,255 (2)	-1,940 (0)	-0,985 (0)
Δ*π*	-5,568 (0)	-	-
ξ9	-2,578 (1)	-0,357 (0)	1,329 (0)
Δ*ξ9*	-4,987 (0)	-	-
ξ8	-1,989 (0)	0,270 (0)	2,916 (0)
Δ*ξ8*	-5,455 (0)	-	-

The highlighted value verifies the stationarity of the variable. The critical value of 5% significance level and 40 observations is for the tested equation included constant and trend -3,527, for the equation included only constant -2,937 and for the equation without constant -1,949. *R* represents the difference between *RL* and *RS*.

The tests are performed sequentially. Every odd row of the table 1 reports tests of stationarity of the levels of the time series. The reported statistics indicate that the null hypothesis cannot be rejected for any variable. Every even row of the table shows tests of stationarity on first differences of the variables. The null hypothesis is rejected for all the time series, which means, all the variables are integrated of the same order one.

The sample data are quarterly but indeed relative short (40 observations), that’s why we use just four lags as an upper frontier of VAR lag length. Table 2 shows two the most important selection criteria for specification of the optimal lag length of the VAR model. We prefer Schwarz information criterion, so we are decided for one lag length of our VAR model.

Table 2: The optimal lag length of the model – selection criteria

Lag	SC	HQ
0	-28.20619	-28.36793
1	-38.32901*	-39.46115
2	-36.87906	-38.98161
3	-35.78074	-38.85371
4	-36.72682	-40.77020*

* indicates lag order selected by the criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

An assumption, that the analyzed time series don't encompass any trend, is illogical and because of unit root, we can expect all the trends are stochastic. That's why we prefer the most used deterministic trend specification. It supposes linear trend in the data, and an intercept but no trend in the cointegrating equation.

As we have mentioned earlier the cointegration rank is obtained from the lambda max statistic. The table 3 shows EViews listing of this test.

Table 3: Johansen test for cointegration – listing from EViews

Sample (adjusted): 1996Q1 2005Q4
Included observations: 40 after adjustments
Trend assumption: Linear deterministic trend
Series: m2-p y RL RS π ξ8
Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.669723	44.31299	40.07757	0.0157
At most 1	0.494592	27.29557	33.87687	0.2479
At most 2	0.377523	18.96191	27.58434	0.4175
At most 3	0.331676	16.11926	21.13162	0.2180
At most 4	0.226337	10.26477	14.26460	0.1951
At most 5	0.075500	3.140087	3.841466	0.0764

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

As we can see at the 5% significance level there is identified exactly one cointegrating equation.

Then we are looking for the product of the cointegrating matrix **C** and the matrix of adjustment **D** in the equation (4), thus matrix **B**. We examine as the first model A – the function of money demand with the inflation rate and the real effective exchange rate recalculated for 8 countries.

Table 4: The cointegrating vector and the adjustment coefficients – model A

Cointegrating vector	Coefficients of matrix **C**	Standard deviation	*t*-statistic	VEC equation	Coefficients of matrix **D**	Standard deviation	*t*-statistic
m2(-1)-p(-1)	1			Δ(m2-p)	-0,678633	0,17343	-3,91304
y(-1)	-0,490851	0,17413	-2,81884	Δ(y)	-0,096452	0,03585	-2,69078
RL(-1)	3,273860	0,31671	10,3370	Δ(RL)	-0,002459	0,03308	-0,07434
RS(-1)	-9,707586	1,49837	-6,47878	Δ(RS)	0,015458	0,00835	1,85084
π(-1)	0,667345	0,23532	2,83591	Δ(π)	0,173901	0,11343	1,53305
ξ8(-1)	0,108214	0,15660	0,69101	Δ(ξ8)	-0,026735	0,16767	-0,15946
Const	0,191268						

The highlighted value shows the t-statistic of the real effective exchange rate parameter in cointegrating vector. We have to refuse this model. The second model B is the function of money demand with the inflation rate and the real effective exchange rate recalculated for 9 countries.

Table 5: The cointegrating vector and the adjustment coefficients – model B

Cointegrating vector	Coefficients of matrix **C**	Standard deviation	*t*-statistic	VEC equation	Coefficients of matrix **D**	Standard deviation	*t*-statistic
m2(-1)-p(-1)	1			Δ(m2-p)	-0,216648	0,09582	-2,26098
y(-1)	0,057284	0,16285	0,35175	Δ(y)	-0,065366	0,01565	-4,17783
RL(-1)	4,907818	0,65417	7,50237	Δ(RL)	-0,016650	0,01643	-1,01325
RS(-1)	-13,87957	3,22625	-4,30208	Δ(RS)	0,005807	0,00429	1,35348
π(-1)	1,951382	0,49642	3,93094	Δ(π)	0,091373	0,05651	1,61700
ξ9(-1)	-0,467455	0,22624	-2,06617	Δ(ξ9)	-0,006735	0,09501	-0,07088
Const	-0,024580						

The highlighted value emphasizes the t-statistic of the income parameter in cointegrating vector. We also have to refuse this model. The third model C represents the function of money demand with the inflation rate however without the real effective exchange rate.

Table 6: The cointegrating vector and the adjustment coefficients – model C

Cointegrating vector	Coefficients of matrix **C**	Standard deviation	*t*-statistic	VEC equation	Coefficients of matrix **D**	Standard deviation	*t*-statistic
m2(-1)-p(-1)	1			Δ(m2-p)	-0,845552	0,18256	-4,63161
y(-1)	-0,463423	0,06686	-6,93158	Δ(y)	-0,078088	0,04151	-1,88140
RL(-1)	2,937408	0,29143	10,0794	Δ(RL)	-0,025831	0,03647	-0,70833
RS(-1)	-10,50365	1,15793	-9,07106	Δ(RS)	0,015742	0,00913	1,72364
π(-1)	0,976247	0,15769	6,19106	Δ(π)	0,147356	0,12437	1,18479
Const	0,609954						

In our analysis there is potential even more alternative. As an example we can present the model D – the money demand without the inflation rate however with the real effective exchange rate recalculated for 9 countries.

Table 7: The cointegrating vector and the adjustment coefficients – model D

Cointegrating vector	Coefficients of matrix **C**	Standard deviation	*t*-statistic	VEC equation	Coefficients of matrix **D**	Standard deviation	*t*-statistic
m2(-1)-p(-1)	1			Δ(m2-p)	-0,833444	0,19698	-4,23107
y(-1)	-0,526058	0,07846	-6,70510	Δ(y)	-0,097312	0,04325	-2,24990
RL(-1)	2,245012	0,29830	7,52594	Δ(RL)	0,048027	0,03939	1,21914
RS(-1)	-5,189577	1,14534	-4,53104	Δ(RS)	0,016615	0,01014	1,63815
ξ9(-1)	0,357280	0,07726	4,62434	Δ(ξ9)	-0,230028	0,22575	-1,01894
Const	-0,761626						

Before the interpretation of the results it's necessary to remind, the parameters of cointegrating vector are written with the opposite signs as they are in the long-run equation. The tables also involve coefficients of the adjustment matrix, which represent the speed of the short-run reactions to the deviations from the long-run equilibrium. Seeing that the subject of our analysis is the money demand, there is important, the error correction term in both models (C and D) at the variable $\Delta(m2\text{-}p)$ is statistical significant. It confirms the initial assumption about the needfulness of the application of the cointegration approach.

4. CONCLUSION

From the results obtained from the Table 6 and 7, the long-run demand for the real money is positive related to the income and short-run interest rate and negatively related to the long-run interest rate and the inflation rate or the real effective exchange rate. Because of contradictoriness of the inflation rate and the real effective exchange rate in one model, we estimated two different models: one with the inflation and without exchange rate and one without the inflation and with exchange rate. The final decision between them will be realised after testing for weak exogeneity by imposing zero restrictions on the coefficients of the adjustment matrix with respect to the variable $(m2\text{-}p)$.

As can be seen in Table 8 from the separate zero restriction on the corresponding parameter the null hypothesis is rejected only at term $d_{\mathrm{m2\text{-}p}}$. With respect to the joint zero restrictions coefficients, d_{RL} and d_{RS} are significant in the model with inflation. These results indicate that while the interest rates are weakly exogenous with respect to real money balances, the income and the inflation are not. It is contra the economic theory, we refuse this model. In the model without the inflation all the coefficients are insignificant and any variable isn't weakly exogenous with respect to real money.

Table 8: Testing for restrictions in model with and without inflation rate

Model with the inflation rate		Model without the inflation rate	
Null hypothesis	**Test statistics**	**Null hypothesis**	**Test statistics**
$d_{m2-p} = 0$	$\chi^2(1) = 12{,}90\ (0{,}000)$	$d_{m2-p} = 0$	$\chi^2(1) = 6{,}211\ (0{,}013)$
$d_y = 0$	$\chi^2(1) = 3{,}011\ (0{,}083)$	$d_y = 0$	$\chi^2(1) = 2{,}247\ (0{,}134)$
$d_{RL} = 0$	$\chi^2(1) = 0{,}522\ (0{,}470)$	$d_{RL} = 0$	$\chi^2(1) = 1{,}212\ (0{,}271)$
$d_{RS} = 0$	$\chi^2(1) = 3{,}079\ (0{,}079)$	$d_{RS} = 0$	$\chi^2(1) = 2{,}568\ (0{,}109)$
$d_\pi = 0$	$\chi^2(1) = 1{,}409\ (0{,}235)$	$d_{\xi 9} = 0$	$\chi^2(1) = 0{,}949\ (0{,}330)$
$d_y = d_{RL} = d_{RS} = d_\pi = 0$	$\chi^2(4) = 9{,}211\ (0{,}056)$	$d_y = d_{RL} = d_{RS} = d_{\xi 9} = 0$	$\chi^2(4) = 4{,}872\ (0{,}301)$
$d_{RL} = d_{RS} = d_\pi = 0$	$\chi^2(3) = 6{,}731\ (0{,}081)$	$d_{RL} = d_{RS} = d_{\xi 9} = 0$	$\chi^2(3) = 3{,}229\ (0{,}358)$
$d_y = d_{RL} = d_{RS} = 0$	$\chi^2(3) = 9{,}112\ (0{,}028)$	$d_y = d_{RL} = d_{RS} = 0$	$\chi^2(3) = 4{,}756\ (0{,}191)$
$d_y = d_{RS} = d_\pi = 0$	$\chi^2(3) = 5{,}249\ (0{,}154)$	$d_y = d_{RS} = d_{\xi 9} = 0$	$\chi^2(3) = 3{,}582\ (0{,}310)$
$d_y = d_{RL} = d_\pi = 0$	$\chi^2(3) = 5{,}163\ (0{,}160)$	$d_y = d_{RL} = d_{\xi 9} = 0$	$\chi^2(3) = 4{,}640\ (0{,}200)$
$d_y = d_{RL} = 0$	$\chi^2(2) = 4{,}578\ (0{,}101)$	$d_y = d_{RL} = 0$	$\chi^2(2) = 4{,}429\ (0{,}109)$
$d_y = d_{RS} = 0$	$\chi^2(2) = 4{,}986\ (0{,}083)$	$d_y = d_{RS} = 0$	$\chi^2(2) = 3{,}472\ (0{,}176)$
$d_y = d_\pi = 0$	$\chi^2(2) = 3{,}626\ (0{,}163)$	$d_y = d_{\xi 9} = 0$	$\chi^2(2) = 2{,}609\ (0{,}271)$
$d_{RL} = d_{RS} = 0$	$\chi^2(2) = 6{,}262\ (0{,}044)$	$d_{RL} = d_{RS} = 0$	$\chi^2(2) = 2{,}901\ (0{,}234)$
$d_{RL} = d_\pi = 0$	$\chi^2(2) = 2{,}020\ (0{,}364)$	$d_{RL} = d_{\xi 9} = 0$	$\chi^2(2) = 1{,}984\ (0{,}371)$
$d_{RS} = d_\pi = 0$	$\chi^2(2) = 3{,}713\ (0{,}156)$	$d_{RS} = d_{\xi 9} = 0$	$\chi^2(2) = 2{,}851\ (0{,}240)$
$c_{m2-p} = 0{,}5$	$\chi^2(1) = 0{,}208\ (0{,}648)$	$c_{m2-p} = 0{,}5$	$\chi^2(1) = 0{,}027\ (0{,}869)$
$c_{m2-p} = 1$	$\chi^2(1) = 18{,}519\ (0{,}000)$	$c_{m2-p} = 1$	$\chi^2(1) = 7{,}924\ (0{,}005)$
$-c_{RL} = c_{RS}$	$\chi^2(1) = 11{,}638\ (0{,}001)$	$-c_{RL} = c_{RS}$	$\chi^2(1) = 1{,}568\ (0{,}211)$

At the same table there are tests of the values 0,5 or 1 for the long-run income elasticity, which mean the confirmation of some theory. As we can see we reject the hypothesis of the quantity theory, however it is not possible to reject the hypothesis of Baumol-Tobin approach. Test of the equality with the opposite signs of the interest rate parameters also can not reject this hypothesis. The conclusions of this model with the interest rate spread are the same to the model of the separate interest rate, and this model provides worse level of the quality of the determination.

The estimated cointegrating equation can be written as:

$$\left(m2-p\right)_t = 0{,}762 + 0{,}526 y_t - 2{,}245 RL_t + 5{,}190 RS_t - 0{,}357 \xi 9_t . \qquad (5)$$

One percent increase in real income stimulates the real demand for M2 by 0,526 percent. The intercept coefficient in equation (5) captures things like technological change in financial instruments and increases in wealth. Given that the estimated coefficients of –2,245 and 5,190 are the semi-elasticities for RL and RS, it cannot be rejected the same magnitude. The impact of an increase in the expected inflation rate on real money balances is as expected. The cointegrating vector also shows that a depreciation of Slovak Crown can encourages agents to diversify their portfolios in the economy by acquiring foreign financial assets.

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APPLICATION OF DYNAMIC MODELS AND AN SV MACHINE TO INFLATION MODELLING

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Abstract:. Based on work [1] we investigate the quantifying of statistical structural model parameters of inflation in Slovak economics. Dynamic and SVM´s (Supper Vector Machine) modelling approaches are used for automated specification of a functional form of the model in data mining systems. Based on dynamic modelling, we provide the fit of the inflation models over the period 1993-2003 in the Slovak Republic, and use them as a tool to compare their forecasting abilities with those obtained using SVM´s method. Some methodological contributions are made to dynamic and SVM´s modelling approaches in economics and to their use in data mining systems. The study discusses, analytically and numerically demonstrates the quality and interpretability of the obtained results. The SVM´s methodology is extended to predict the time series models.

Keywords: Support vector machines, data mining, learning machines, time series analysis and forecasting, dynamic modelling.

1. INTRODUCTION

The paper is about learning from a database. In contemporary statistical data mining systems, potential inputs are mainly chosen based on traditional statistical analysis. These include descriptive statistics, data transformations and testing. Input selection relies mostly on correlation and partial autocorrelation, cross-correlation analysis, cluster analysis, classification techniques, statistical tests and other statistical tools [3], [4], [5]. Although all these tools are in reality linear, they are deemed to provide a useful tool for the determination of the input lag structure and the selection of inputs.

In economic and finance, where our understanding of real phenomena is very poor and incomplete, it seems to be more realistic and more useful, instead of making unrealistic mathematical assumptions about functional dependency, to take the data as they are and try to represent the relationships among them in such a way that as much information as possible would be preserved. Very frequently, in such cases more sofisticated approaches are considered. Knowledge discovery in databases is a non trivial process for identifying valid, novel, potentially useful and ultimately understandable patterns in data. The usual steps in this process are data selection, data preprocessing, data transformation, data mining, evaluation and interpretation of found knowledge.

Our purpose is to present quantitative procedures for use in data mining systems that routinely predict values of variables important in decision processes and to evaluate their fit to the data and forecasting abilities.

This contribution discusses determining an appropriate causal model for inflation modelling and forecasting in the Slovak Republic. We use these models as a tool to compare their fit to the data and forecasting abilities with those obtained using SVM´s method. The paper is organised as follows. In the next section of this article, we briefly describe the analysed data. Section 3 discusses building a structural model by modelling strategy described as being a "specific to general" methodology and provides a fit of the SV regression model, discusses the circumstances under which SV regression outputs are conditioned and corresponding interpretation of SV regression results is also considered. Section 6 extends the SVM´s methodology for economic time series forecasting. A section of conclusions will close the paper.

2. ANALYSED DATA

The character of econometric models is mainly determined by theoretical economic approaches, which are more or less target used in constructing them. The period from 1993 to 1996 of the Slovak economy, in the field of econometric modelling, was a period of enormous growth of data quality and quantity. In this period at the Institute of Economics of the Slovak Academy of Science was constructed a database with approximately 160 time series of relevant macroeconomic indicators. This database contains variables relating to the following block of the economy.

Examples of some variables (for details see [7]):

- Monetary block:
  - demand deposits of enterprises,
  - insurance companies,
  - households - disposable incomes, final consumption, unconsumed" incomes/volumes, interest rates of time and savings deposits, interest rates of time and savings deposits, volume of time and savings deposits,
  - liquid liabilities,
  - volume of credit enterprises and households,
  - net domestic asset,
  - aggregate credit rate.
- State Budget block:
  - state budget deficit/revenues/expenditures,
  - tax (income tax of physical/legal persons), consumption taxes, customs duties,
  - expenditures (current, capital, other).
- Population block:

- wages incomes of population (real, social, nominal).
- Block Prices and Labour productivity:
  - labour supply economically active population,
  - labour demand (employment number, increment of fixed capital),
  - unemployment (economically active population, number of unemployment rate),
  - consumer price index.
- GDP block. Its main components are:
  - private/public consumption,
  - public gross capital formation,
  - net export.
- Foreign Trade block. Its main components are:
  - exchange rates and prices of partner imports,
  - domestic production,
  - prices of production (imports and exports).

This text focuses primarily on quarterly data series. Occasionally other frequencies may be used. A part of quarter year time series of the period 1993 – 2004 was gathered together and included in a data matrix form into our information system. The data required by the forecasting system are tested for reliability and analysed to detect obvious or likely mistakes. Observations can be excluded or corrected through an appropriate management person, who may then decide whether or not to include the observation in the forecasting process. The some approach is used in analysing historical data in order to select the model form and develop the initial values for estimating the model parameters or improving forecasting performance.

3. AUTOMATED MODELLING STRATEGY

The strategy for selecting an appropriate model is based on so called a "specific to general" methodology [2]. This strategy is well known under the common name Dynamic Modelling in Economics (DME). The DME methodology leads to two stage modelling procedure. In the first stage the researcher use simple economic theory or can incorporate some prior knowledge which might be used to formulate and estimate a model and, if found to be unsatisfactory, in the second phase is generalised until it is acceptable.

Next, we will demonstrate these phases for modelling economic time series, say inflation which may be explained by the behaviour of another variables. According to the inflation theory [8], the variable inflation is explained by the unemployment rates and wages. In this section we will present the DME approach in the modelling and investigating of the relationship between the dependent variable of inflation measured by *CPI* (Consumption Price Index) and two independent variables the unemployment rate (*U*), and aggregate wages (W) in the Slovak Republic. In the next section the SV regression (SVM´s method) is applied. Finally, the results are compared between a dynamic model based on statistical modelling and the SV regression model.

To study the modelling problem of the inflation quantitatively, the quarterly data from 1993Q1 to 2003Q4 was collected concerning the consumption price index *CPI*, aggregate wages *W* and unemployment *U*. These variables are measured in logarithm, among others for the reason that the original data exhibit considerable inequalities of the variance over time, and the log transformation stabilises this behaviour. Fig. 1a illustrates the time plot of the *CPI* time series. This time series shows a slight decreasing trend without apparent periodic structure. Using simple economic inflation theory the model formulation may be

$$CPI_t = \beta_0 + \beta_1 W_t + \beta_2 U_t + u_t \qquad (1)$$

where u_t is a white noise disturbance term, $\beta_0, \beta_1, \beta_2$ are the model parameters (regression coefficients). Using time series data the model is estimated as

$$\hat{CPI}_t = 11.3302 - 1.355 W_t + 1.168 U_t$$

$$R^2 = 0.374, \quad DW = 0.511 \qquad (2)$$

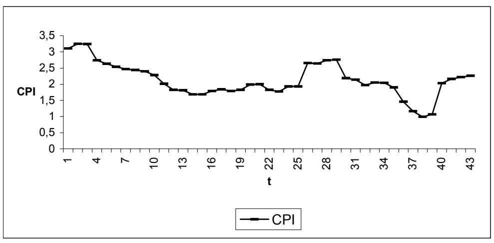
Model (2) is not satisfactory. The estimated coefficients do not have the correct signs, there is evidence of first order positive autocorrelation. The model does not well fit the data inside the estimation period. R^2 is often refered loosely as the amount of variability in the data explained or accounted for by the regression model (only 37 percent of the variance in CPI_t is explained by the model).

There are various methods and criteria for automated selecting the lag structure of dynamic models from a database. As we have mentioned above, autoregression, partial autoregression and cross-correlation functions can provide powerful tools to determine the relevant structure of a dynamic model. So after the data are transformed, the first thing to do is to perform a suitable differencing of the input and output series, and to analyse autocorrelation, partial autocorrelation and cross-correlation function of the series to produce an appropriate model of the input-output series (transfer function models). In this procedure the orders of AR processes are usually determined within the procedure itself using an information criterion, e.g. Bayesian or Akaike information criterion. Experimenting with these methods [1], the following reasonable model formulation was found

$$\hat{CPI}_t = 0.5941 - 0.0295 W_{t-1} - 0.00359 U_{t-1} + 0.84524 CPI_{t-1},$$

$$(0.229) \qquad (0.3387) \qquad (0.1035) \qquad R^2 = 0.7762 \qquad (3)$$

where the standard deviations of the model parameters are presented in parentheses.



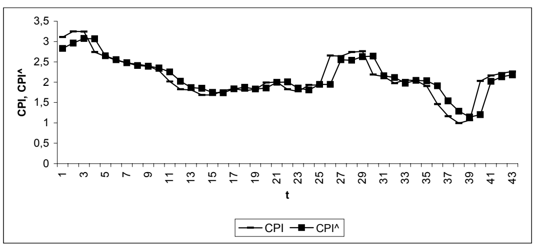


Fig. 1a Natural logarithm of quarterly inflation from January 1993 to December 2003

Fig. 2b Natural logarithm of actual and fitted inflation values (model (4))

Finally another attempt was made supposing a more sophisticated dependence of current inflation on the previous observation performed with the help of SV regression. As it is well known, we can not have a model in which the coefficients are statistically insignificant. We made an arbitrary decision which deleted "insignificant" explanatory variables. Then the equation (3) becomes the first-order autoregressive process, i.e.

$$C\hat{P}I_t = \beta_0 + \beta_1\, CPI_{t-1} = 0.292 + 0.856\, CPI_{t-1}. \qquad (4)$$

$(0.158) \quad (0.072)\, R^2 = 0.776$

A graph of the historical and the fitted values for inflation is presented in Fig. 1a. The model follows the pattern of the actual very closely.Statistical modelling approach based on dynamic models have found extensive practical application. These models naturally arise in areas where either a correlative or causal structure exists between variables that are temporally or spatially related. These models are also useful in many types of process and quality-control problems and everywhere, where the value of the dependent variable at time t is related to the adjustment to the controllable process variables at previous time periods t *-1*.

4. CAUSAL MODELS, EXPERIMENTING WITH NON-LINEAR SV REGRESSION

In this section we will discuss the problem of selecting the appropriate functional form of the SV regression model [6]. We demonstrate here the use of the SV regression framework for estimating the model given by Eq. (4). If CPI_t exhibits a curvilinear trend, one important approach for generating an appropriate functional non-linear form of the model is to use the SV regression in which the CPI_t is regressed either against CPI_{t-1} or the time by the form

$$C\hat{P}I_t = \sum_{i=1}^{n} w_i \varphi_i(\mathbf{x}_t) + b \qquad (5)$$

where $\mathbf{x}_t = (CPI_{t-1}, CPI_{t-2}, ...)$ is the vector of time sequence of the regressor variable (regressor variable CPI_{t-1} - causal model) or $\mathbf{x}_t = (1,2,...,43)$ is the vector of time sequence (time series model), b is bias, $\varphi(.)$ is a non-linear function (kernel) which maps the input space into a high dimensional feature space, w_i are the that are subject of learning. Our next step is the evaluation of the goodness of last three regression equations to the data insite the estimation period expressed by the coefficient of determination (R^2), and the forecast summary statistics the Root Mean Square Error (RMSE) for each of the models out of the estimation period.

One crucial design choice in constructing an SV machine is to decide on a kernel. The choosing of good kernels often requires lateral thinking: many measures of similarity between inputs have been developed in different contexts, and understanding which of them can provide good kernels depends on the insight into the application´s domains. Tab.1shows SVM´s learning of the historical period illustrating the actual and the fitted values by using various kernels and presents the results for finding the proper model by using the quantity R^2. As shown in Tab. 1, the model that generate the "best" $R^2 = 0.9999$ is the time series model with the RBF kernel and quadratic loss functions. In the cases of causal models, the best R^2 is 0.9711 with the exponential RBF kernel and ε -insensitive loss function (standard deviation $\sigma = 0.52$). The choice of σ was made in response to the data. In our case, the CPI_t, CPI_{t-1} time series have $\sigma = 0.52$. The radial basis function defines a spherical receptive field in $\Re$ and the variance σ^2 localises it.

Tab. 1 SV regression results of three different choice of the kernels and the results of the dynamic model on the training set (1993Q1 to 2003Q4). In two last columns the fit to the data and forecasting performance respectively are analysed. See text for details.

Fig.	MODEL	KERNEL	σ	DEGREE-d	LOSS FUNCTION	R^2	*RMSE*
5	causal	Exp. RBF	1		ε - insensitive	0.9711	0.0915
5	causal	RBF	1		ε - insensitive	0.8525	0.0179
5	causal	RBF	0.52		ε - insensitive	0.9011	0.0995
5	causal	Polynomial		2	ε - insensitive	0.7806	0.0382
5	causal	Polynomial		3	ε - insensitive	0.7860	0.0359
5	time series	RBF	0.52		quadratic	0.9999	1.1132
4	dynamic					0.7760	0.0187

5. CONCLUSION

In Support Vector Machines (SVM´s), a non-linear model is estimated based on solving a Quadratic Programming (QP) problem. The use of an SV machine is a powerful tool to the solution many economic problems. It can provide extremely accurate approximating functions for time series models, the solution to the problem is global and unique.

In this paper, we have examined the SVM´s approach to study linear and non-linear models on the time series of inflation in the Slovak Republic. The benchmarking of that model was performed between traditional statistical techniques and SVM´s method in regression tasks. The SVM´s approach was illustrated on the conventional regression function. As it visually is clear from Tab. 1, this problem was readily solved by a SV regression with excellent fit of the SV regression models to the data. Tab. 1 present also forecast statistics (RMSEs) for the ex post time periods. From the Tab. 1 is shown that too many model parameters results in overfitting, i.e. a curve fitted with too many parameters follows all the small fluctuations, but is poor for generalisation . Our experience shows that SV regression models deserve to be integrated in the range of methodologies used by data mining techniques, particularly for control applications or short-term forecasting where they can advantageously replace traditional techniques.

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ASSESSING THE PREDICTIVE ACCURACY OF BAYESIAN METHODS

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Abstract: A recurring problem concerning marketing operatives at all levels is a forecast of the demand for new products. This article discusses a technique, Bayesian method, by which the forecast of such products can be calculated. This technique is used to examine the forecast accuracy of lightning current arristers during the year 2004.

Key words: Bayesian estimation, prior and posterior distribution, ex ante forecast, double exponential smoothing.

1. INTRODUCTION

Econometric and statistical forecasting techniques are widely used in managing production systems. They have also found frequent application in a variety of other problem areas, including financial planning, investment analysis, distribution planning, marketing. For example, consider the two common economic theories of demand and production. Each one is based on elementary economic principles and, if enough historical data are available, provides a basis for a theoretical multivariate regression model. Usually the demand function for a good relates the quantity of a good sold to several explanation variables. The theoretical foundation for demand analysis is based on the modern theory consumer behaviour. In determining the demand the important roles play the price of the good and the consumer's real income. Other variables that should be included are the price of other goods (either substitutes or complements) in consumption, the size of the market, the expected future price of the good and tastes and preferences.

Nevertheless, it is likely that the price of the good, the income, the size of market and the price of substitutes play the most important role in determining demand. It is important to remember that the model is being developed for forecasting purposes. Therefore, it is extremely important to keep the model relatively simple in terms of the number of the explanatory variables. Remember that, for each period in an ex ante forecast of the dependent variable, it is also necessary to provide a forecast for each of the independent variables.

Most dependent variables may be related to time or may be explained by simple mathematical functions of time. Regression models are statistical techniques for modelling an investigating the relationship between two or more variables. As we mentioned above, regression analysis requires that enough historical data of dependent and independent variables are available. In many decision problems there are little or non useful historical information available at the time of decision making. In such situations Bayesian methods are often useful to determine the early forecast.

The next section contains a brief description of forecasting models based on Bayesian approach that are linear in the unknown parameters. The data, fitted model and prediction results of the Bayesian procedure are given in section 3. Assessing the predictive accuracy of models using Bayesian procedures and exponential smoothing methods is discussed in section 4. The final section contains a summary of our results.

2. MODELS

Retail and whole sales are important components of the business company. One of the most basic problems facing the marketing or product manager is how to predict sales of new products in situations where no historical data are available. Marketing decisions must be made in the context of insufficient information about processes that are dynamic, stochastic and downright difficult.

This article discusses a technique, Bayesian method in forecasting, which should prove helpful in modelling of such situations. The methodology of Bayesian method, i.e. Bayesian parameter estimation and prediction for general time series model is described in [5]. We will this methodology apply and further extend for general (causal) model.

The functional form of the causal regression model has the form

$$y_t = b_0 + b_1 x_{1t} + b_2 x_{2t} + \dots, b_k x_{kt} + u_t \, , \qquad (1)$$

where $\{x_i\}_{i=1}^{k}$ represent a series of independent variables for i = 1, 2, ..., k and t = 1, 2, ..., n, $\{b_i\}_{i=1}^{k}$ are partial regression coefficients (the model parameters), $\{y_t\}_{t=1}^{n}$ is dependent variable, $\{u_t\}_{t=1}^{n}$ is random error term.

It is convenient to express the model (1) in matrix notation of the following form

$$\mathbf{y} = \mathbf{Xb} + \mathbf{u} \, , \qquad (2)$$

where $\mathbf{y} = \{y_t\}_{t=1}^{n}$ is a column vector of the n dependent values, $\mathbf{b} = (b_0, b_1, \dots, b_k)$ is a column vector of the k + 1 unknown parameters, $\mathbf{u} = \{u_t\}_{t=1}^{n}$ is an column vector of the errors, and $\mathbf{X}$ is a matrix of the independent values of the form

$$\mathbf{X} = \begin{pmatrix} 1, \ x_{11}, \ x_{21}, \dots, x_{k1} \\ 1, \ x_{21}, \ x_{22}, \dots, x_{k2} \\ \dots \\ 1, \ x_{1n}, \ x_{2n}, \dots, x_{kn} \end{pmatrix}$$

As a practical example, consider a firm which mass-produces new systems of lightning current arresters. The marketing manager wishes to forecast monthly sales of these products of new unknown foreign markets. The functional form of this process relates the size of sales to two explanatory variables: $\{x_{1t}\}_{t=1}^{n}$ - the advertising expenditures, $\{x_{2t}\}_{t=1}^{n}$ - service and cost of repairs. No similar item has been sold in the past. Thus, we have the model

$$y_t = b_0 + b_1 x_{1t} + b_2 x_{2t} + u_t\,, \quad t = 1, 2, ..., n \qquad (3)$$

The manager expects that the sales during first three months do not fall under the specified level. Using this information the last-squares model parameter estimates $\hat{\mathbf{b}} = (\hat{b}_0, \hat{b}_1, \hat{b}_2)$ can be calculated.

The model (3) is the classical linear regression equation. To generate forecasts of future observations using Bayes methodology we will assume that variance of the random component σ_u^2 is known. Then the vector $\hat{\mathbf{b}} = (\hat{b}_0, \hat{b}_1, \hat{b}_2)$ follows the multivariate normal prior distribution with mean $E(\mathbf{b})$ = $\overline{\mathbf{b}}'$ = $(\overline{b}_0', \overline{b}_1', \overline{b}_2')$, the variance *var* $(\{b_i\}_{i=1}^{k})$ = $\sigma_{b_i'}^2$ and the covariance $cov(b_i', b_j')$, i.e.

$$N(\overline{\mathbf{b}}', \mathbf{V}') \equiv (2\pi)^{-(\frac{1}{2})} \left|\mathbf{V}'^{-1}\right|^{1/2} \exp\{-\tfrac{1}{2}[\mathbf{b} - \overline{\mathbf{b}}']^T \mathbf{V}'^{-1} [\mathbf{b} - \overline{\mathbf{b}}']\}\,, \qquad (4)$$

where $\mathbf{V}'$ is the variance-covariance matrix of the prior distribution. This probability distribution measures our subjective information above $\mathbf{b}$.

After one period, or generally after T periods, we have observed $y_1, y_2, ..., y_T$. The problem is to modify the estimate $\overline{\mathbf{b}}'$ and the measure of uncertainty expressed by *var* $(\{b_i\}_{i=1}^{k})$ in light of this information. This can be done using the normal posterior of the parameters with mean $\overline{\mathbf{b}}''$ and variance-covariance matrix $\mathbf{V}''$ as [5]

$$\overline{\mathbf{b}}'' = \mathbf{G}''^{-1}(\mathbf{G}'\overline{\mathbf{b}}' + \mathbf{X}^T \mathbf{X}\hat{\mathbf{b}})\,, \qquad (5)$$

where $\mathbf{G}' = \sigma_u^2 \mathbf{V}'$, $\mathbf{G}'' = \mathbf{G}' + \mathbf{X}^T \mathbf{X}$. From the expression (5) is seen that the parameters of the posterior can be determined from simple algebraic combination of the prior parameters and the results of last-squares analysis of the time series data.

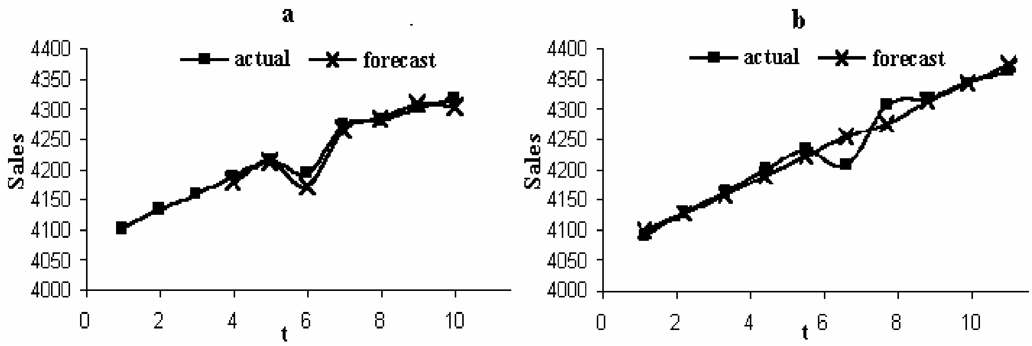
3. THE DATA AND RESULTS

The data were taken from 10 consecutive months of lightning current arresters purchasing history (y_t). The data for each independent variable (x_{1t}, x_{2t}) are collected by firm (see [3]). Table 1 presents the monthly data for all variables (y_t - lightning current arristers purchasing, x_{1t} - the advertising expenditures, x_{2t} - service and cost of repairs).

Tab. 1: Dependent, Independent and forecast values when Bayesian method is used for a causal linear model (see text for details)

T	1	2	3	4	5	6	7	8	9	10
y_t	4101	4132	4158	4188	4214	4193	4272	4281	4301	4317
x_{1t}	121	121	128	120	130	1125	138	140	142	140
x_{2t}	97	99	108	113	110	105	114	116	120	121
$\hat{y}_t$				4179	4212	4171	4265	4284	4308	4302.

Fig. 1: Sales – actual and forecast values a) Bayesian method b) single



exponential smoothing

The calculation of forecast values for a given example can be summarised as follows (detailed procedures can be found in [5]

1. *Assumptions of the model - prior distribution.*

The manager supposes that the initial sales rate will be between 3300 – 4300 per month. But with the advertising program planed, the manager expects the sales rate to increase steadily and estimate the amount of increase from month to month at between 80 – 120. Analogously, with the service program planed, the amount of steadily increase of the sales rate from month to month is estimated at between 100 – 140. The assumption about disturbance term u_t is, $u_t \equiv N(0,\sigma_u^2)$ with σ_u^2 = 2500. The prior variances of the parameters are assumed to be one-sixth of the ranges, i.e. $\sigma_{b_0'}^2$ = (4300 – 3300)^2/6 = 27777.778. Analogously $\sigma_{b_1'}^2$ = (140 – 100)^2/6 = 44.44, $\sigma_{b_2'}^2$ = (120 – 80)^2/6 = 44.44 and $\mathrm{cov}(b_0', b_1')$ = $\mathrm{cov}(b_1', b_2')$ = $\mathrm{cov}(b_0', b_2')$ = 0. Therefore, the prior variance-covariance matrix is

$$\mathbf{V}' = \begin{pmatrix} \sigma^2_{b'_0} & & 0 \\ & \sigma^2_{b'_1} & \\ 0 & & \sigma^2_{b'_2} \end{pmatrix} = \begin{pmatrix} 27777.78 & & 0 \\ & 44.44 & \\ 0 & & 44.44 \end{pmatrix}$$

2. *Posterior parameters at the time T = 3*:

- The least-squares estimates are calculated from the first $T = 3$ observations as

$$\hat{\mathbf{b}}^T = (\hat{b}_0, \hat{b}_1, \hat{b}_2) = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$$
$$= (45559.43,\ 15.5,\ 16.214).$$

- Using the formula (5), the posterior parameters of $\mathbf{b}$, given $\hat{\mathbf{b}}$, are calculated as

$$\overline{\mathbf{b}}'' = E(\mathbf{b}'') = \mathbf{G}''^{-1}(\mathbf{G}'\overline{\mathbf{b}}' + \mathbf{X}^T\mathbf{X}\hat{\mathbf{b}})$$
$$= (3225.031,\ 3.926,\ 4.255)$$

- The forecast for period $t = T + \tau$, for any lead time $\tau \geq 1$, we simply take expectation at origin T of

$$y_{T+\tau} = \overline{b}'' + \overline{b}_1'' x_{1,T+\tau} + \overline{b}_2'' x_{2,T+\tau} + u_{T+\tau}.$$

For $\tau = 1$

$$E(y_{T+1}) = \hat{y}_{T+1} = \overline{b}'' + \overline{b}_1'' x_{1,T+1} + \overline{b}_2'' x_{2,T+1}$$
$$\hat{y}_4 = \overline{b}'' + \overline{b}_1'' x_{1,4} + \overline{b}_2'' x_{2,4} = 4179.2$$

- Forecast for each future observation may be easily computed as a new observation becomes available through the three procedures above. Generally, we denote the current period by *T*. The forecast $\hat{y}_{T+\tau}$ for future observation $y_{T+\tau}$, for any lead time $\tau \geq 1$ is generated successively by setting the new origin of time equal to $T + \tau - 1$ and computing the new forecast for period $T + \tau$. These forecast are shown in Tab. 1 and illustrated in Fig. 1a.

4. ASSESSING THE PREDICTIVE ACCURACY OF THE ALTERNATIVE FORECASTING MODEL

In this section, we shall present the forecast results of the sales using double exponential smoothing approach [4]. The results of updating the forecasts of sales using double exponential smoothing are presented in Fig. 1b. The weighting factor α was equal to 0.10.

Fig. 1a illustrates a good quantitative model. This model closely follows all the actual data. The model based on the double exponential smoothing (Fig. 1b) fails to fit the actual swing at the period $t = 6$.

5. CONCLUSION

We have illustrated that the model based on Bayesian methodology can be useful tool for economic time series modelling and forecasting. The Bayesian technique is attractive for forecasting because it provides a framework within which the time series can be successful modelled by only little or no observations. Finally, Bayesian approach is also attractive from a strictly model building point of view because: the final model will, in general, be a parsimonious representation of the data, the appropriate noise structure is easily determined and has relatively simple procedures for revising the parameters.

The disadvantage of models based on Bayesian approach is that the variance of the disturbance term must be known. For unknown variance case see [6] Some difficulties arise by nonlinear and seasonal models see [1], [5].

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MONOPOLY DIFFUSION MODEL OF A NEW PRODUCT

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Abstract: This paper deals with the determination of optimal advertising strategies for new product diffusion models. We consider the introduction of a new consumer durable in a monopolistic market and the evolution of sales is modeled by a flexible diffusion model, discounting of future revenue and cost learning curve. This paper is a firs step in solving diffusion model for imperfect competition market. In our next work we will model the oligopoly competition.
Using standard methods of optimal control theory we characterize qualitatively the structure of an optimal advertising strategy of monopoly firm. We study model which combine elements of Bass demand growth model, the Vidale-Wolfe and Ozga advertising models. For our case is difficult to find continuous solution, therefore we used discrete form for finding the optimal advertising level.
We develop the monopolistic quadratic cost model, where the state variable represents the cumulative market share, which is based on Bass's demand growth model, and a learning curve production function. The two terms in the demand growth model are formally similar to the terms in the Nerlove-Arrow and the Ozga advertising models, respectively. We derive the optimal condition to the monopoly model form Hamiltonian function. For finding the solution we use algorithm, for which we made a Visual Basic Program in MS Excel sheet.

Key words: Diffusion model, Monopoly firm, Dynamic optimization, Theory of optimal control

1. ADVERTISING POLICES FOR DIFFUSION MODELS

Consider first purchases of a new consumer durable in a monopolistic market. We restrict the analysis to the case of a monopolist firm, which only manipulates its advertising expenditure over a fixed planning period. Let t denote the time $0 \leq t \leq T$ where the length of plan period is fixed, T.

Define:
$x = x(t)$ as a number of purchasers (demand) at time t out of a maximum customer pool x_M,
$u = u(t)$ as the firm's rate of advertising expenditure at time t.

A diffusion model is given by

$$x' = x'(t) = g(x(t), u(t)), \qquad x(0) = x_0 \geq 0 \qquad (1)$$

$x' = dx(t)/dt$ represents the current sales rate. Current sales rate is related to accumulate past sales and the current rate of advertising expenditure.
Now we introduce a cost learning curve by assuming that marginal costs, *C*, depend on cumulative sales such that marginal costs decrease with increasing cumulative output (experience).

$$C = C(x), \qquad dC(x)/dx = C'(x) \geq 0 \qquad (2)$$

We assume learning cost curve, which satisfy this condition.

$$C(t) = c_0 [x_0 / x(t)]^z \qquad (3)$$

Where $C(t)$ is the unit cost of production at time *t*, $x(t)$ is the accumulated sales volume at time *t*, c_0 and x_0 are the initial values of production cost and sales rate. Total unit cost of production of new products declines by a factor of from 10 to 50 percent each time the accumulated sales (production) volume doubles. That means $0{,}1 \leq z \leq 0{,}5$ [3]

2. SPECIFIC FUNCTIONAL FORM

Lets specify the form of current sales rate $x'(t)$ as a diffusion model where the parameters depending on the advertising rate.

$$x' = [\alpha + \beta u + (\gamma + \delta u)x](x_M - x) \qquad (4)$$

In this form all parameters are nonnegative and constant in time.
α is the innovation coefficient,
β is related to the effectiveness of advertising vis-à-vis the innovators,
γ is the imitation coefficient and
δ is related to the effectiveness of advertising toward the imitators.
Who are "innovators" and "imitators". "Innovators" are individuals, who accept a new product on their own independently of the decisions of others. "Imitators" are people who are influenced in their new product purchasing decisions by the fact that other people have already purchased it.
In the model (4) we can distinguish four components of the diffusion process:

$\alpha(x_M - x)$ - Adoption without interaction with previous adopters and without influence from advertising. It is like a term in Bass model.

$\beta u(x_M - x)$ - Adoption by interaction with previous adopters but stimulated by advertising. It is corresponding term in the Vidale-Wolfe model and represents effect of advertising on the innovators.

$\gamma x(x_M - x)$ - Adoption by interaction with previous adopters but without influence from advertising. It is like a term in Bass model.

$\delta ux(x_M - x)$ - Adoption by interaction with previous adopters and stimulated by advertising. It is like the corresponding term in Ogza´s advertising model and represents the effect of advertising on the imitators.

3. MODEL

Consider discount rate *r* and terminal time of planning horizon *T*. We can state the monopolistic optimal advertising problem as the following optimal control problem

$$\max J = \int_0^T e^{-rt}\left[(p - c_0(q_0/Q)^z)Q' - (a + bu + cu^2)\right]dt \qquad (5)$$

$$Q' = \left[\lambda + \beta u + (\gamma + \delta u)Q\right](1 - Q), \quad Q(0) = q_0 \qquad (6)$$

where $Q = x/x_M$, $a = a_1/x_M$, $b = b_1/x_M$, $c = c_1/x_M$ and
$Q = Q(t)$ can be interpreted as cumulative fraction sold of the market (market share) and $0 \le Q(t) \le 1$.
u advertising rate (advertising variable),
p constant price,

$(a + bu + cu^2)$ quadratic rate of advertising expenditure,
a, b, c nonnegative constant and
a represent the fix cost of advertising,
r discount (inflation) rate
The instantaneous profit stream to the firm is given by $P(Q,u) = (p - c_0(q_0/Q)^z)Q' - (a + bu + cu^2)$. We want to maximize the profit from advertising under diffusion constraint.

In the model we assume that sales are linearly increasing with advertising and that the firm has constant price, *p*, over the planning period. Ideally the optimization problem should include price as well as advertising. The interaction between optimal price and optimal advertising may be sensitive to the particular functional form of the diffusion model. Price reductions could possibly substitute increased advertising in order to increase instantaneous sales.

The simplest situations in which prices do not change and firms in general still advertise are:

a) Government regulates the firms and price competition is prohibited.

b) The firms have an agreement among themselves that sets the price for their product. In this point we can also assume, that the size of the potential market is not affected by the price.

4. SOLUTION

At first we try to solve the problem given in (5) by applying the maximum principle throw maximizing the Hamiltonian. So we compose it together with its derivations.

$$H=\left(p-C(Q)\right)\left[\alpha+\beta u+\left(\gamma+\delta u\right)Q\right]\left(1-Q\right)-\left(a+bu+cu^2\right)+m\left[\alpha+\beta u+\left(\gamma+\delta u\right)Q\right]\left(1-Q\right) \quad (7)$$

where *m=m(t)* is a current value adjoint variable and satisfy the following differential equations:

$$m'=rm-H_Q,\ \ m(T)=0$$

$$m'=rm+\left(p-c_0\left(\frac{q_0}{Q}\right)^z+m\right)\left[\alpha+\beta u+\left(\gamma+\delta u\right)\left(2Q-1\right)\right]-zc_0q_0^zQ^{-(z-1)}\left(1-Q\right)\left[\alpha+\beta u+\left(\gamma+\delta u\right)Q\right] \quad (8)$$

and transversality condition *m(T)=0*. Adjoint variable *m* has the interpretation. It represents the (dollar or finance) value (at time *t*) of a marginal increase in cumulative sales. The adjoint variable depends on three important dynamic factors: diffusion effect on the demand side, cost learning and discounting.

To find the optimal control we maximize *H* by differentiating *H* with respect to *u* (the control variable) and set the result to zero:

$$H_u=0$$

$$H_u=\left(p-C(Q)+m\right)\left(1-Q\right)\left(\beta+\delta Q\right)-b-2cu=0 \quad (9)$$

$$\Downarrow$$

$$u=\frac{\left(p-C(Q)+m\right)\left(1-Q\right)\left(\beta+\delta Q\right)-b}{2c} \quad (10)$$

If we substitute (10) into (6) and (7), we obtain a system of nonlinear differential equations. To solve these equations is not easy, therefore for finding the solution of our problem we use algorithm from Teng and Thompson [7] and we use it for monopoly case.

5. ALGORITHM

After specify the necessary condition for finding maximum of Hamiltonian, we have state equation (differential equation) for changing the market share in time, Q', with given initial condition $Q(0) = q_0$, adjoint equation *m* with their transversality conditions *m(T) = 0* and equation for control variable *u*. As we mentioned it could be difficult to solve this problem in continuous time, we formulate a problem as a discrete one.

The algorithm has two phases, the initial phase and computational phase.

Initial phase:
We assume the production cost is just c_0. With this assumption the profit of firm is

$$(p-c_0)Q' - (a + bu + cu^2) \quad (11)$$

for discrete instant of time. For discrete time we will use *k* instead continuous *t*. The starting control *u* will be

$$u = \max\begin{cases} \{(p-c_0)(1-Q)(\beta+\delta Q)-b\}/2c \\ 0 \end{cases} \quad (12)$$

which maximize (11) subject to $u \geq 0$.

Computational phase:
Step 1: Read the values of parameters: α, β, γ, δ, q_0, c_0, *r*, *p* and *T*.
Step 2: Calculate the starting values of *u* (*k*) and *Q* (*k+1*) from *k = 0* to *k = T* step by step as follows:
u (*k*) as in (12),

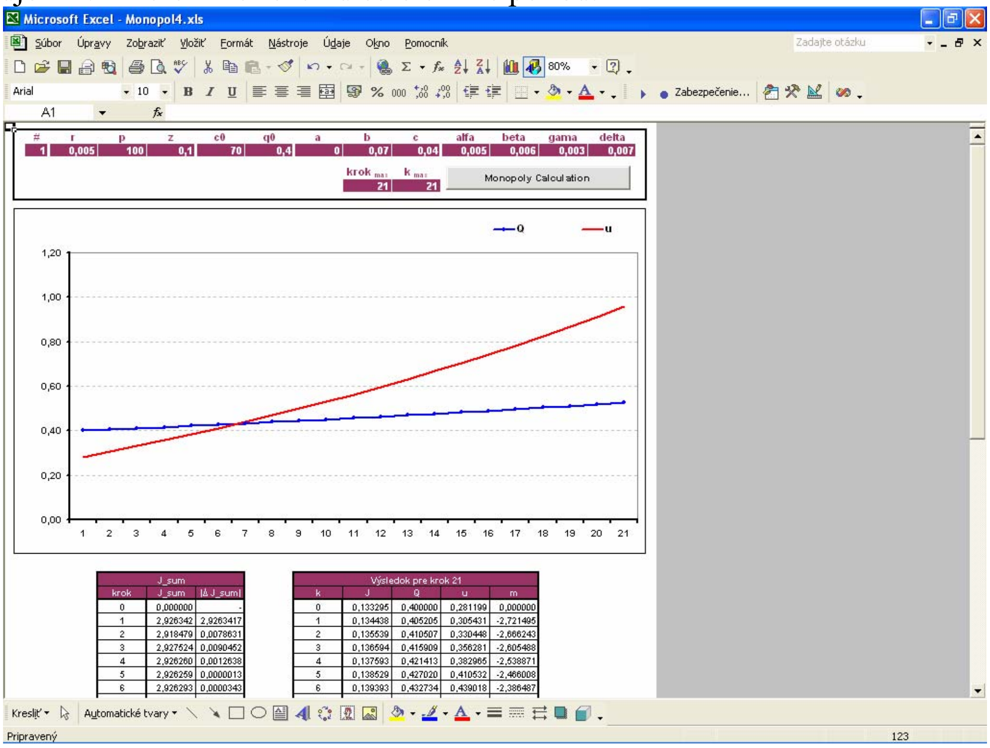
$$Q(k+1) = Q(k) + [\alpha + \beta u + (\gamma + \delta u)Q](1-Q) \quad (13)$$

and calculate the value of objective function *J* as in (5), then go to Step 3.
Step 3: Find the values of the adjoint variables backward in time by using the values of *u*(*k*), *Q*(*k*) and the terminal conditions on the adjoint variables, and substituting them in (10). Go to Step 4.
Step 4: Update the new values of *u*(*k*) and *Q*(*k*) forward in time step by step as follows:
u(*k*) as in (10),
Q(*k+1*) as in (13).
When $k + 1 = T$, calculate *J* as shown in (5) and go to Step 5.
Step 5: Check the difference between the new values of *J* (or *u (k)*) and the previously found values. If there is no significant difference then the algorithm is terminated. Otherwise, update *J,* and go to Step 3.

6. NUMERICAL RESULTS

For finding numerical results of algorithm from previous chapter we program this algorithm in VBA for excel. In this way it could be available for using to everybody. We can change in it all parameters: α, β, γ, δ, q_0, c_0, *r*, *p* and *T* (*T* is in the program replaced wit *k_max*) and we can also „say“ to the program how much iteration he has do.
We have 2 types of output from our program. The first one is a graphical one. We can see on chart how changes *Q* and *u* in the last iteration – so the optimal solution of market share, *Q,* and advertising rate, *u,* in time *k*.

The next output is arranged from two parts. One from number of steps from computational phase together with criteria of convergence. We used for it the absolute value of difference between the optimal values of objective function in two following steps. The second part of this output is a table with optimal values of objective function, *J,* market share, *Q,* advertising rate *u* and adjoint variable *m* for each discrete time period.



Picture 1: Output from VBA for monopoly diffusion model

7. EXAMPLE

For the first step we assume, the parameters like we could see on the Picture 1. That means

$$r = 0{,}005 \quad p = 100 \quad c_0 = 70 \quad a = 0 \quad \alpha = 0{,}005 \quad krok \ \max = 21$$

$$z = 0{,}1 \quad q_0 = 0{,}4 \quad b = 0{,}07 \quad \beta = 0{,}006 \quad k \ \max = 21$$

$$c = 0{,}04 \quad \gamma = 0{,}003$$

$$\delta = 0{,}007$$

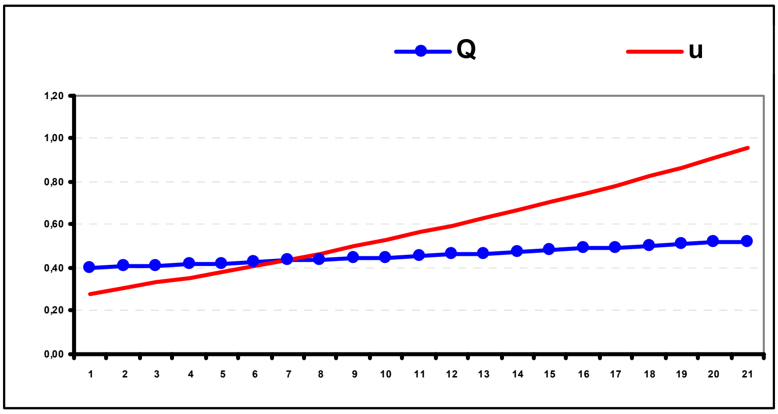
After letting to solve the program, we have result for convergence of algorithm and for optimal solution together with chart.

J_sum		
krok	**J_sum**	**|Δ J_sum|**
0	0,000000	-
1	2,926342	2,9263417
2	2,918479	0,0078631
3	2,927524	0,0090452
4	2,926260	0,0012638
5	2,926259	0,0000013
6	2,926293	0,0000343
7	2,926287	0,0000064
8	2,926287	0,0000003
9	2,926287	0,0000001
10	2,926287	0,0000000
11	2,926287	0,0000000
12	2,926287	0,0000000
13	2,926287	0,0000000
14	2,926287	0,0000000
15	2,926287	0,0000000
16	2,926287	0,0000000
17	2,926287	0,0000000
18	2,926287	0,0000000
19	2,926287	0,0000000
20	2,926287	0,0000000
21	2,926287	0,0000000

Výsledok pre krok 21				
k	**J**	**Q**	**U**	**M**
0	0,133295	0,400000	0,281199	0,000000
1	0,134438	0,405205	0,305431	-2,721495
2	0,135539	0,410507	0,330448	-2,666243
3	0,136594	0,415909	0,356281	-2,605488
4	0,137593	0,421413	0,382965	-2,538871
5	0,138529	0,427020	0,410532	-2,466008
6	0,139393	0,432734	0,439018	-2,386487
7	0,140175	0,438555	0,468459	-2,299868
8	0,140864	0,444486	0,498892	-2,205676
9	0,141447	0,450530	0,530357	-2,103407
10	0,141912	0,456687	0,562892	-1,992513
11	0,142245	0,462961	0,596538	-1,872413
12	0,142429	0,469352	0,631335	-1,742476
13	0,142447	0,475864	0,667328	-1,602027
14	0,142282	0,482496	0,704557	-1,450342
15	0,141911	0,489252	0,743067	-1,286636
16	0,141314	0,496132	0,782902	-1,110070
17	0,140467	0,503139	0,824105	-0,919737
18	0,139343	0,510272	0,866722	-0,714659
19	0,137916	0,517533	0,910797	-0,493783
20	0,136154	0,524923	0,956373	-0,255972
21	0,000000	0,000000	0,000000	0,000000

Picture 2: Convergence of algorithm and optimal solution

We could see, that the difference between two objective functions did not changed after 12^{th} step of algorithm. So we can say, it has a quite fast convergence. On the second part of Picture 2 we can see the optimal values of objective function, *J*, cumulative fraction sold of the market *Q*, advertising rate, *u* and adjoint variable, *m* for every discrete time. The same result for *Q* and *u* we can see also on Picture 3 graphically.



Picture 3: Optimal solution graphically

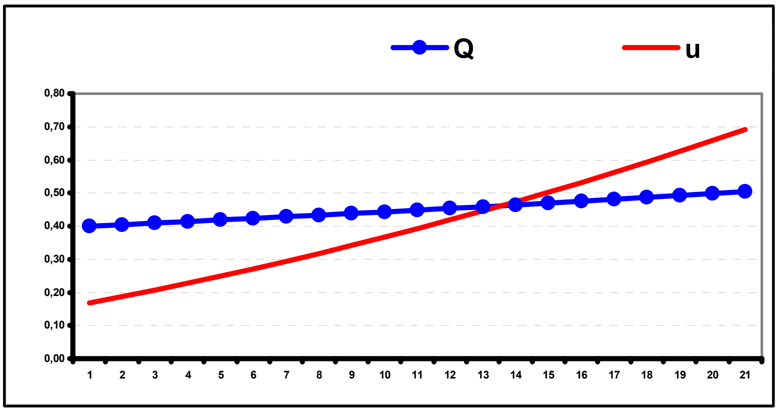
For the next example we consider, that the effect of advertising on innovators is higher, $\beta = 0{,}008$ and it affect only innovators. That means, $\delta = 0$. Now we know, that we do not need so much iteration, so we let *k_max = 11*.

J_sum		
krok	**J_sum**	**|Δ J_sum|**
0	0,000000	-
1	2,628358	2,6283584
2	2,628668	0,0003093
3	2,637271	0,0086038
4	2,635880	0,0013914
5	2,635800	0,0000802
6	2,635869	0,0000688
7	2,635860	0,0000091
8	2,635859	0,0000010
9	2,635859	0,0000005
10	2,635859	0,0000001
11	2,635859	0,0000000

Výsledok pre krok 11				
k	**J**	**Q**	**u**	**m**
0	0,122929	0,400000	0,168435	0,000000
1	0,123634	0,404528	0,187616	-2,911795
2	0,124300	0,409122	0,207476	-2,844707
3	0,124923	0,413783	0,228025	-2,771482
4	0,125495	0,418511	0,249277	-2,691836
5	0,126010	0,423308	0,271244	-2,605467
6	0,126462	0,428175	0,293935	-2,512062
7	0,126843	0,433113	0,317362	-2,411290
8	0,127144	0,438124	0,341534	-2,302807
9	0,127359	0,443207	0,366461	-2,186250
10	0,127477	0,448363	0,392150	-2,061239
11	0,127491	0,453594	0,418611	-1,927377
12	0,127390	0,458900	0,445848	-1,784248
13	0,127166	0,464280	0,473868	-1,631416
14	0,126807	0,469736	0,502677	-1,468425
15	0,126304	0,475267	0,532277	-1,294797
16	0,125645	0,480873	0,562672	-1,110032
17	0,124819	0,486554	0,593863	-0,913608
18	0,123815	0,492310	0,625851	-0,704978
19	0,122621	0,498140	0,658635	-0,483570
20	0,121225	0,504044	0,692212	-0,248786
21	0,000000	0,000000	0,000000	0,000000

Picture 4: Convergence of algorithm and optimal solution

Again we could see also the graphic solution.



Picture 5: Optimal solution graphically for no advertising affect on imitators

In this way we can change one or all parameters of model.

8. CONCLUSION

The problem of finding or characterizing an optimal advertising policy over time is one of the most important questions in the field of Marketing. In this paper, we first introduce the assumption that the production cost obey a learning curve and then develop a demand growth model combining elements of the Vidale-Wolfe and Ozga advertising models. For solving the model we used algorithm from Teng and Thompson [6] for which we also made a VBA program and we illustrate its work on an example.

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PORTFOLIO SELECTION IN THE PRESENCE OF THE STRUCTURED PRODUCTS

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Abstract: Structured products are financial innovation, which fill free space on financial market and they have become very favorite instrument. They represent combination of long and short position in derivatives and underlying assets. Objective of this submission is to find optimal allocation decision according to conflict objectives. Introducing of structured products to the portfolio improve investing opportunity of the investor and allows him to tailor the risk-premium profile of their portfolio in a more efficient way then simple linear exposure to the traditional asset classes.

Institutional investors and mutual funds are influenced by recent market downturns. Structured products are financial innovation, which fills free space on the financial market and they have become very favorite instrument. They represent combination of long and short position in derivatives and underlying assets. In general, structured products can be organized as a bonds or investment funds. The bond secures capital guarantee and the holder of the bond is entitled to the payment of the nominal value at the maturity. The holder of the structured product effectively holds portfolio of zero coupon bond and a derivative of the asset. It means, that one component of the payoff at the maturity is guaranteed and the second one depends on the price of the underlying asset. Disadvantage of the bond is limited lifetime, which is eliminated by mutual or pension funds. The next advantages are: the portfolio of the mutual fund is professionally managed, privileged market access, mutualization effect, efficient financing. As an underlying asset in the structured product can be used every asset, but this paper is concentrated on the stock index.

The structured product is packaged version of the dynamic trading in the cash and underlying asset. The portfolio that includes the structured product will effectively have time-varying weights. Investing the present value of a guaranteed sum in the zero coupon bond promise protection against price decrease and option position allows participation in price increase.

The model of the portfolio selection that is used in this contribution, divides funds between three groups of the assets: bonds, stocks and structured products. Hindsight option is used as a derivative of the underlying asset.

1. HINDSIGHT OPTION

A hindsight option is a specific kind of lookback option. It's a type of path-dependent option where the payoff is dependent on the maximum or minimum asset price over the life of the option. Maximum price is determined as a $S_0 + k(S_{max} - S_0)$.

To reach this payoff, it is needed to create portfolio P_0, which consists of the following elements: investment to zero coupon bond $S_0 e^{-rT}$, which give payoff S_0 at time T and investment to hindsight call option $k \times HC_0$, which ensure payoff $k(S_{max} - S_0)$ at time *T*.

According to Black-Scholes model, price of the hindsight option $HC(S_0, \sigma, r, T)$ is computed as follows [1]:

$$HC(S_0,\sigma,r,T) = S_0 e^{-rT}\left(N(b_1) - \frac{\sigma^2}{2r} e^{Y_2} N(-b_3)\right) + S_0 e^{-T}\frac{\sigma^2}{2r} N(-b_2) - S_0 e^{-T} N(b_2),$$

where

$$b_1 = \frac{\ln(S_{max}/S_0) + (-r + \sigma^2/2)T}{\sigma\sqrt{T}},$$

$$b_3 = \frac{\ln(S_{max}/S_0) + (r - \sigma^2/2)T}{\sigma\sqrt{T}},$$

$$b_2 = b_1 - \sigma\sqrt{T} \ ,$$

$$Y = \frac{2(r - \sigma^2/2)\ln(S_{max}/S_0)}{\sigma^2}.$$

As an explicit function for non-dividend paying stock we have

$$k = \frac{S_0 - S_0 e^{-Tr}}{HC(S_0, \sigma, r, T)}.$$

2. ASSUMPTION ON PREFERENCES

In the objective function it is used expected return of the portfolio $\frac{1}{T}\ln\frac{P_T}{P}$, where P_T is the portfolio value at time t and T is the time horizon. As a measure of the risk is applied Value–at-Risk (VaR) or conditional Value–at–Risk (CVaR), because heavy tails are present in asset returns.

The goal is to draw efficient frontier with respect to conflict objectives and show how introducing structured products influences the optimal allocation decision and tradeoff between risk and return. The following program is considered:

$$\underset{w}{Max}\frac{1}{T}\ln\frac{P_T}{P}$$

s.t.

$$CVaR(T)\leq K,$$

where $P_t = w_S S_t + w_B B_t + w_G G_t$ is the portfolio value at time t and $w=(w_{S,}w_B, w_G)$ is vector of portfolio weights. S represents allocation to stocks, B to bonds and G to guaranteed structured products. Than the whole effective frontier in a mean return - CVaR space is drawn by changing K.

3. ASSUMPTIONS ON ASSET RETURNS

The selection of the model is very crucial, because we want that the asset returns express fat tails in projected scenarios, which are seen in historical data. There is used model of the stochastic interest rate, which will be used in the process of simulating a bond portfolio price.

Process of generating data is given by the following model with the stochastic interest rate and stochastic volatility.

$$\frac{dS_t}{S_t} = \mu_t dt + \sqrt{V_t}dW_t^S,$$

$$dr_1 = a(b-r_t)dt + \sigma_r dW_t^B,$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_V\sqrt{V_t}dW_t^V,$$

where S_t is the value of the underlying asset (stock index) in the time t, r is the value of the short-term rate in the time t, V is variance of the log returns for the stock, μ is the time-varying expected return on equity, a is the mean-reversion parameter, W^S, W^B and W^V are correlated Brownian motions[10].

The process of short term rate allows for bond prices to be obtained in close form. In particular we have:

$$B(t,T) = \exp\left[-m(t,T)+\frac{1}{2}v(t,T)\right],$$

[10] For simplicity it is assumed, that W^B a W^V are independent Brownian motions.

$$m(t,T)=\beta(T-t)+(r_t-\beta)\frac{1-e^{-a(T-t)}}{a},$$
$$v(t,T)=\frac{\sigma_r^2}{2a^3}\left(1-e^{-a(T-t)}\right)^2+\frac{\sigma_r^2}{a^2}\left((T-t)-\frac{1-e^{-a(T-t)}}{a}\right),$$

where β is the risk adjusted long term value of the short term rate, given by $\beta=b-\frac{\sigma_r\lambda_B}{a}$,
and λ_B is the risk premium associated with interest rate risk. It should be noted, that the bond portfolio is global bond index, which can be used as a zero coupon bond with constant time-to-maturity τ:

$$B_t^\tau=\exp\left[-m_t^\tau+\frac{1}{2}v_t^\tau\right],$$
$$m_t^\tau=\beta\tau+(r_t-\beta)\frac{1-e^{-a\tau}}{a}, \qquad (1)$$
$$v_t^\tau=\frac{\sigma_r^2}{2a^3}\left(1-e^{-a\tau}\right)^2+\frac{\sigma_r^2}{a^2}\left(\tau-\frac{1-e^{-a\tau}}{a}\right)$$

Now we can specify the dynamics for the drift of the stock index. The market price of the risk vector is written as:

$$\begin{pmatrix}\lambda_S\\ \lambda_B\end{pmatrix}=\begin{pmatrix}\sigma_t^S & \operatorname{cov}_t\left(\frac{dB_t^\tau}{B_t^\tau},\frac{dS_t^\tau}{S_t^\tau}\right)\\ \operatorname{cov}_t\left(\frac{dB_t^\tau}{B_t^\tau},\frac{dS_t^\tau}{S_t^\tau}\right) & \sigma_t^B\end{pmatrix}^{-1}\begin{pmatrix}\mu_t^S-r_t\\ \mu_t^B-r_t\end{pmatrix},$$

where $\sigma_t^S=\sqrt{V_t}$ is the standard deviation or volatility of stock returns and σ_t^B is the volatility of the bond returns.
From the previous equation follows that

$$\begin{pmatrix}\mu_t^S-r_t\\ \mu_t^B-r_t\end{pmatrix}=\begin{pmatrix}\sigma_t^S & \operatorname{cov}_t\left(\frac{dB_t^\tau}{B_t^\tau},\frac{dS_t^\tau}{S_t^\tau}\right)\\ \operatorname{cov}_t\left(\frac{dB_t^\tau}{B_t^\tau},\frac{dS_t^\tau}{S_t^\tau}\right) & \sigma_t^B\end{pmatrix}\begin{pmatrix}\lambda_S\\ \lambda_B\end{pmatrix},$$

from which is known that

$$\mu_t^S = r_t + \sigma_t^S \lambda_S + \text{cov}_t\left(\frac{dB_t^\tau}{B_t^\tau}, \frac{dS_t^\tau}{S_t^\tau}\right)\lambda_B .$$

By using equation (1) the price of the bond can be written as the following exponential function of the short term rate

$$B_t^\tau = K_t^T e^{-L_t^T r_t},$$

where

$$L_t^T = \frac{1-e^{-a(T-t)}}{a}.$$

By using Ito's lemma we find that

$$\frac{dB_t^\tau}{B_t^\tau} = \left[\frac{\partial B}{\partial t} + \frac{\partial B}{\partial r} a(b - r_t) + \frac{1}{2}\sigma^2 \frac{\partial^2 B}{\partial r^2}\right]dt + \frac{\partial B}{\partial r}\sigma_r dW_t^B \equiv \mu_t^B dt + \sigma_t^B dW_t^B,$$

where

$$\sigma_t^B = \frac{\partial B}{\partial r}\sigma_r = -\frac{1-e^{-a(T-t)}}{a}\sigma_r .$$

Finally, we get

$$\mu_t^S - r_t = \sigma_t^S \lambda_S + \sigma_t^S \sigma_t^B \rho \lambda_B = \sigma_t^S\left(\lambda_S - \frac{1-e^{-a\tau}}{a}\sigma_r \rho \lambda_B\right).$$

Therefore, assuming constant market prices of the equity and bond price risk, we found out that the expected excess return on the equity index is given as a mean-reverting process because it is a linear function of the volatility process, which itself is assumed to be mean reverting.

4. OPTIMIZATION PROGRAM

The scenarios for the total returns on each asset class are insert to the following mean-CVaR optimization program [1]

$$\min \beta\left(\zeta + \frac{1}{N(1-\alpha)}\sum_{i=1}^{N} z_i\right) - (1-\beta)\left(\frac{1}{N}\sum_{i=1}^{N} r_{T,i}\right) \tag{2}$$

s.t

$$P_{T,i} = P_0\left(w_S \prod_{t=1}^{T} R_{t,i}^S + w_B \prod_{t=1}^{T} R_{t,i}^B + w_G \prod_{t=1}^{T} R_{t,i}^G\right), \qquad i = 1,\dots,N \tag{3}$$

$$r_{T,i}^{P} = \frac{P_{T,i}}{P_0} - 1, \qquad i = 1,\ldots,N \tag{4}$$

$$w_S + w_B + w_G = 1 \tag{5}$$

$$-r_{T,i}^{P} - \zeta \le z_i, \qquad i = 1,\ldots,N \tag{6}$$

$$z_i \ge 0, \qquad i = 1,\ldots,N \tag{7}$$

$$\zeta \text{ is free variable,} \tag{8}$$

$$w_S, w_B, w_G \ge 1 \tag{9}$$

For this optimization program following parameters and variables are defined:
Parameters:

T Number of time periods.
N Number of scenarios.
α CVaR confidence level (0,95 or 0,99).
P_0 Initial value of the portfolio.
β Weights (from 0 to 1) given to CVaR in objection function.
$R_{i,i}^{S}$ Total return on the stock index in period *t* under scenario *i.*
$R_{i,i}^{B}$ Total return on the bond index in period *t* under scenario *i.*
$R_{i,i}^{G}$ Total return on the guaranteed structured product in period *t* under scenario *i.*

Variables:

W_S Portfolio weight of stocks at the beginning.
W_B Portfolio weight of bonds at the beginning.
W_G Portfolio weight of guaranteed structured product at the beginning.
$P_{T,i}$ Value of the portfolio at the end of period *T* under scenario *i*.
$r_{T,i}^{P}$ Rate of return for *T* periods under scenario *i*.
ζ Dummy variable that approximates VaR in the optimal solution.
z_i Dummy variable connected with CVaR constraint under scenario *i*.

The objective of this optimization program is to construct effective frontier that is the combination of two conflict goals: expected return of the portfolio and CVaR of the portfolio. The strategy is based on the minimization of the convex combination of CVaR of the portfolio and negative expected return of the portfolio. Optimal solution is found by changing weights in objective function.

The objective function (2) is a convex combination of two goals: expected value of *T* period rates of return is weighted by $(1-\beta)$ and CVaR is weighted by β. In this model β is interpreted as a risk aversion. When $\beta = 1$, investor is interested in minimization of risk. When $\beta = 0$, investor's goal is to maximize return. Constraint (3) express definition of the ending portfolio value and constraint (4) represents the definition of T periods rate of return. In constraint (5) it is expressed that sum of portfolio weights is equal to one and in constraint (9) short sales are prohibited.

Constraints (6), (7) and (8) are needed to control goal CVaR in objective function. These constraints guarantee that optimal value of this expression in the objective function is CVaR and corresponding optimal value of ζ is equal to VaR. If there are many optimal values of ζ , than VaR is the point on the left end of the optimal interval.

5. CONCLUSION

Structured products allow to investor to profit from the risk premium of stocks without fully exposition to risk that is connected with the investment to stocks. The main benefit of structured products is that they allow for a non-linear exposure with respect to stock and bond returns in a passive way. The result is that the most institutional investors should optimally allocate a significant fraction of their portfolio to structured products. Beginning improvement in the transparency, liquidity and cost control of these products is already taking place and they caused massive inflows of cash from traditional investment to these alternatives.

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EXPERIMENTS WITH THE GENERAL EQUILIBRIUM MODEL OF A CLOSED ECONOMY

VERONIKA MIŤKOVÁ

Abstract: In this paper we are presenting the general equilibrium model of closed economy for the case of Slovakia. The data source of the model is a social accounting matrix, which is based on the system of national accounts. This matrix allows us to set the values of the parameters by procedure of calibration. The parameters are set in manner to replicate the base year dataset. The model is used for experiments where various changes in the values of exogenous variables, as capital stock, labor supply, are made.
Keywords: general equilibrium model, closed economy, primary factors change

INTRODUCTION

General equilibrium models describe by a system of equations the main idea that in economy resources equal to their uses. Model we are working with in this paper is based on social accounting matrix (SAM) which synthesizes two basic principles of economics: input-output idea according to purchase of one sector is a sale of another sector in the same time and principle of national accounts according to revenues equal to expenditures. The model presentation is followed by a social accounting matrix that includes the data needed to solve the model using calibration. The functional forms of the various relationships embodied in the model have been selected so as to assure that all parameters can be directly derived from the accompanying SAM. The calibration consists of finding numerical values for the model parameters so that for existing policy regime, the model reproduces the benchmark data as a model solution (Dawkins, C., Srinivasan, T., N., Whalley, J., 1998). The first task in general equilibrium analysis is not finding the equilibrium state but use the current equilibrium for finding parameter values.

The used social accounting matrix contains accounts of two households: urban and rural, two factors: labor and capital and two activities and associated commodities: agricultural and non-agricultural and savings-investment account. The explicit distinction between activities and commodities facilitates model calibration, but it is not necessary. The distinction is needed for models where at least one activity produces more than one commodity and/or at least one commodity is produced more than one activity. The savings-investment row receives payments from the household (the only saver) and the column shows spending on commodities used for investment. For savings-investment, assume the following:

a. household income is allocated in fixed shares to savings and consumption;
b. investment is savings-driven, that is the value of total investment spending is determined by the value of savings;
c. investment spending is allocated to the two commodities in a manner such that the ratio between the quantities is fixed.

Together, assumptions b. and c. mean that when savings values and/or the prices of investment commodities change, there is a proportional adjustment in the quantities of investment demand for each commodity, generating an investment value equal to the savings value.

The general equilibrium model is based on the microeconomic theory which assumes that producers maximize profits subject to production functions, with primary factors as arguments, while households maximize utility subject to budget constraints. The following model represented by equations (1) – (14) is based on the example model of H. Löfgren (2003). Cobb-Douglas functions are used both for producer technology (1) and the utility functions from which household consumption demands (9) are derived. Factors are mobile across activities, available in fixed supplies and demanded by producers at market-clearing prices. Intermediate demand is simply expressed as a product of the level of the activity and an intermediate input per unit of output (3). Value added per activity is given as the difference between price of the activity and the price of commodity (5). Prices of factors tend to be distorted in the real world in the broad sense that they differ across activities. On the basis of fixed shares, factor incomes are passed on to the households, providing them with their only income (7). The outputs are therefore demanded by the households at market-clearing prices. Thus it is assumed for the factor market that each activity pays a wage expressed as the product of a wage variable and a distortion factor (2). In each factor market, variations in the average wage clear the market. The outputs are demanded by the households at the market-clearing prices. The model is a simple economy model because it does not contain a government sector and a rest of the world sector.

The set of equilibrium conditions that is functionally dependent includes commodity (12) and factor market (11) equilibrium conditions, the savings-investment balance (13) and price normalization equation (14). According to the Walras law the model satisfies it would be possible to drop one of these equations. Instead of doing it, a variable called *WALRAS* is implemented to the savings-investment balance. The model still has an equal number of variables and equations, namely 34. If the model works correctly, the savings-investment balance should hold, that means value of *WALRAS* variable should be zero.

MATHEMATICAL MODEL STATEMENT

All endogenous variables are written in uppercase Latin letters, parameters (including variables with fixed or exogenous values) have lowercase Latin or

Greek letters. Subscripts refer to set indexes. Superscripts are part of the parameter name.

Sets:

activities: $\alpha \in A = \{AGR - A, NAGR - A\}$,

commodities: $c \in C = \{AGR - C, NAGR - C\}$, factors: $f \in F = \{LAB, CAP\}$,

households: $h \in H = \{U - HHD, R - HHD\}$,

where:

AGR-A agricultural activity,

NAGR-A nonagricultural activity,

AGR-C agricultural commodity,

NAGR-C nonagricultural commodity,

LAB labor, *CAP* capital,

U-HHD urban household,

R-HHD rural household.

Parameters: αd_α efficiency parameter in the production function for activity *a,*

cpi consumer price index (CPI),

$cwts_c$ weight of commodity *c* in the CPI,

$ica_{c\alpha}$ quantity of commodity *c* as intermediate input per unit of output in activity *a,*

$shry_{hf}$ share for household h in the income of factor *f,*

mps_h marginal (and average) propensity to save for household *h,*

qfs_f supply of factor *f,*

$\overline{qinv_c}$ base-year quantity of investment demand for commodity *c,*

$wfdist_{f\alpha}$ wage distortion factor for factor *f* in activity *a,*

$\alpha_{f\alpha}$ share of value-added for factor *f* in activity *a,*

β_{ch} share in household *h* consumption spending on commodity *c*,

$\theta_{\alpha c}$ yield of output *c* per unit of activity *a.*

Variables:

IADJ investment adjustment factor,

P_c market price of commodity *c,*

PA_α price of activity *a,*

PVA_α value-added (or net) price of activity *a* ,

Q_c output level in commodity *c,*

QA_α level of activity *a,*

$QF_{f\alpha}$ demand for factor *f* from activity *a*

QH_{ch} consumption of commodity *c* by household *h,*

$QINT_{c\alpha}$ quantity of commodity *c* as intermediate input in activity *a*,

$QINV_c$ quantity of investment demand for commodity *c*,

$WALRAS$ dummy variable (zero at equilibrium),
YF_{hf} income of household h from factor f,
YH_h income of household h,
WF_f price of factor f.

Equations:
Production and commodity block:
Production function (1)

$$QA_\alpha = \alpha d_\alpha \cdot \prod_{f \in F} QF_{f\alpha}^{\alpha_{f\alpha}} \quad \alpha \in A$$

Demand for factors (2)

$$WF_f \cdot wfdist_{f\alpha} = \frac{\alpha_{f\alpha} \cdot PVA_\alpha \cdot QA_\alpha}{QF_{f\alpha}} \quad f \in F,\ \alpha \in A$$

Intermadiate demand (3)

$$QINT_{c\alpha} = ica_{c\alpha} \cdot QA_\alpha \quad c \in C,\ \alpha \in A$$

Price of activity (4)

$$PA_\alpha = \sum_{c \in C} \theta_{\alpha c} \cdot P_c \quad \alpha \in A$$

Value added price for activity (5)

$$PVA_\alpha = PA_\alpha - \sum P_c \cdot ica_{c\alpha} \quad \alpha \in A$$

Output of commodity (6)

$$Q_c = \sum_{\alpha \in A} \theta_{\alpha c} \cdot QA_\alpha \quad c \in C$$

Institution sector:
Transfer of income from factor to household (7)

$$YH_{hf} = shry_{hf} \cdot WF_f \cdot wfdist_{f\alpha} \cdot \sum_{\alpha \in A} QF_{f\alpha} \quad h \in H,\ f \in F$$

Income of household (8)

$$YH_h = \sum_{f \in F} YH_{hf} \quad h \in H$$

Consumption demand for household and commodity (9)

$$QH_{ch} = \frac{\beta_{ch} \cdot (1 - mps_h) \cdot YH_h}{P_c} \quad c \in C,\ h \in H$$

Investment demand (10)

$$QINV_c = \overline{qinv_c} \cdot IADJ \quad c \in C$$

System constraint block:

Factor market equilibrium condition (11)

$$\sum_{\alpha \in A} QF_{f\alpha} = qfs_f \quad f \in F$$

Commodity market equilibrium condition (12)

$$Q_c = \sum_{h \in H} QH_{ch} + \sum_{\alpha \in A} QINT_{c\alpha} + QINV_c \quad c \in C$$

Savings-investment balance (13)

$$\sum_{c \in C} P_c \cdot QINV_c + WALRAS = \sum_{h \in H} mps_h \cdot YH_h$$

Price normalization equation (14)

$$\sum_{c \in C} cwts_c \cdot P_c = cpi$$

THE BASE SOCIAL ACCOUNTING MATRIX

The database for the analytical model is the social accounting matrix of Slovakia for year 2004, shown in the Table 1. This matrix is based on the data of the System of National Accounts of Slovakia (2006).

Table 1: Social accounting matrix for Slovakia in 2004, in millions SKK

	AGR-A	NAGR-A	AGR-C	NAGR-C	LAB	CAP	U-HHD	R-HHD	S-I	sum
AGR-A			618 130							618 130
NAGR-A				2 472 521						2 472 521
AGR-C	69 414	277 656					114 355	93 563	63 142	618 130
NAGR-C	277 656	1 110 625					457 418	374 252	252 570	2 472 521
LAB	103 274	413 095								516 369
CAP	167 786	671 145								838 931
U-HHD					284 003	461 412				745 415
R-HHD					232 366	377 519				609 885
S-I							173 642	142 070		315 712
sum	618 130	2 472 521	618 130	2 472 521	516 369	838 931	745 415	609 885	315 712	

THE EXPERIMENTS

This model can be used for modeling a policy changes whether in the primary factors supply (capital stock and labor) or in the inflation which results in the change of the new value of the consumer price index. After some modifications in the computer formulation of the model it can be also used for experiment with the value of the investments adjustment.

Let us assume a 10 percent increase in the capital stock. One value of the exogenous variable factor supply is changed. This change results in an increase of gross national product by 6.1% and similarly in increase of gross savings by 6.1%. The wage of factors has changed in opposite directions, it means that for primary factor labor the wage has increased by approximately 6% and for the capital factor the wage has decreased by 3.6%. The final social accounting matrix is presented in the Table 2. Results obtained for a 5% increase in the capital stock are similar; gross national product has increased by 3.1%, identical was the increase of gross savings. These changes have been caused by the increase of the investment adjustment factor.

Another experiment was made with a 10% increase in the value of the labor supply. This change resulted in the 3.7% increase in the gross national product as a direct consequence of the increase of the value of investment adjustment factor. The price of the labor has reduced by 5.7%.

Table 2: Social accounting matrix for Slovakia in 2004 after 10% increase in the capital stock, in millions SKK

	AGR-A	NAGR-A	AGR-C	NAGR-C	LAB	CAP	U-HHD	R-HHD	S-I	sum
AGR-A			655 695							655 695
NAGR-A				2 622 781						2 622 781
AGR-C	73 632	294 530					121 305	99 249	66 979	655 695
NAGR-C	294 530	1 178 120					485 216	396 996	267 919	2 622 871
LAB	109 550	438 200								547 750
CAP	177 983	711 932								889 915
U-HHD					301 262	489 453				790 715
R-HHD					246 487	400 462				646 949
S-I							184 195	150 704		334 898
sum	655 695	2 622 781	655 695	2 622 781	547 750	889 915	790 715	646 949	334 898	

CONCLUSION

In the suggested model we do not consider a sector of government and a sector of rest of the world. This macroeconomic model is based on the microeconomic theory according to consumers are maximizing their utility and firms are maximizing their profits. In the first step the social accounting matrix was created followed by the proceeding of the calibration where values of the model parameters were set. The model solution replicates the base year dataset.

The conclusions according the results of the policy changes are that the savings are not elastic subject to the change in the capital stock. The value of this elasticity is 0.615. The price elasticity of the labor supply is 0.57. It means that the labor supply is not elastic in the Slovak republic. This finding corresponds with the economic theory of the labor which claims that the elasticity of the labor supply is lower than 1.

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THE SLOVAK PENSION FUNDS PERFORMANCE AND RISK ANALYSIS

VLADIMÍR MLYNAROVIČ

Abstract: The paper presents results of the two methodological approaches application to the analysis of the performance and risk of the private pension funds in the Slovak Republic. At first the problem is formulated as a multiple criteria decision one and the Promethee methodology is used for outranking of the pension funds. The second approach uses the modern portfolio theory and analyses the pension funds in a risk – return space.

Keywords: pension funds, Promethee outranking, Black – Litterman portfolio optimization
JEL: G11

1. INTRODUCTION

After the reform the pension system in the Slovakia consists of the three pillars. The first pillar is the sate one and it is obligatory, the second one is also obligatory, but it is private, and the third one is private, but it is voluntary. The subject of the analysis is the second pillar of the system where each of the six companies (Aegon, Allianz, CSOB, VUB Generali, Winterthur and ING) runs three types of pension funds – conservative, balanced and growth – according to the relatively restrictive rules state in the legislature. The analyses are based on the weekly data that are published from April 1, 2005. The data reflects the performances of investment strategies and for each fund they consist of the following items:

- the current value of the income unit (VIU) - with starting value equals 1,
- the net asset value of the fund (NAV) ,
- the charge for asset management as percentage from the average monthly net asset value of the fund,

and data to the August 18, 2006 are as a illustration presented in the Table 1

Table 1: Results of pension funds to the August 18, 2006

	Conservative fund			Balanced fund			Growth fund		
	VIU	NAV (in EUR)	Charge	VIU	NAV (in EUR)	Charge	VIU	NAV (in EUR)	Charge
AEGON	1.0497	1 758 375	0.00%	1.0598	12 373 920	0.00%	1.0604	34 377 906	0.00%
Allianz	1.0486	8 002 572	0.07%	1.0542	49 882 658	0.07%	1.0584	99 894 830	0.07%
ČSOB	1.0418	946 193	0.00%	1.0594	9 324 558	0.07%	1.0605	19 918 666	0.07%
ING	1.0447	1 853 110	0.08%	1.0450	17 672 162	0.08%	1.0462	40 608 669	0.08%
VÚB Generali	1.0467	4 277 062	0.08%	1.0536	31 477 587	0.08%	1.0571	48 379 807	0.08%
Winterthur	1.0482	5 030 393	0.08%	1.0545	39 708 359	0.08%	1.0590	103 726 856	0.08%

2. PENSION FUNDS OUTRANKING

The problem of pension funds outranking can be formally written in the form of following multiple criteria decision making problem

$$\text{"max"} \quad \{y = (y_1, y_2, \dots y_k) \quad | \quad y \in Y\}$$

where elements of the set *Y* are assumed pension funds which of them is evaluated on the base of *k* selected criteria. Without a loss of universality it can be assumed that for each criterion holds "the more the better". The goal is to rank the funds in the form of preference structure (*P, S, I)*, or (*P, S, I, R*) where *P* means the strict preference, *S* means a weak preference, *I* denotes indifference and *R* denotes incomparability.

There are several classes of methods for solving of such kinds of problems. In the application the method PROMETHEE II was used (Brans – Mareschall – Vincek, 1986; Mlynarovič, 1998). The method is based on a construction of generalised criteria and indices of multiple criteria preferences. Intensity of one fund preference over the second is a function of the difference in performances according individual criteria and takes a value from 0 to 1. If *y* and *z* are two funds from the set *Y* which are to compare from the viewpoint of criterion *i*, then

$$d_i = y_i - z_i$$

and the value of preference function

$$F_i(y_i, z_i) = P(d_i) = 1 - e^{-\frac{d_i^2}{2\sigma^2}}, \; d_i \geq 0$$

where σ represents standard deviation, measures the contribution of criterion *i* to the total preference of *y* over *z*. Let us note that this Gaussian preference function is not only possible one. So called *usual criterion, quasi – criterion, criterion with linear preference level criterion* or *criterion with linear preference and indifference area* can also be used.

Let us suppose that for each criterion i a preference function F_i was defined and w_i expresses relative importance of criterion i . Then for all couples of pension funds y a z a following index of multiple criteria preferences is defined

$$\pi(y,z) = \frac{\sum_{i=1}^{k} w_i F_i(y,z)}{\sum_{i=1}^{k} w_i}$$

The index measures client preference intensity for fund y over fund z in such a way, where all criteria are taken into account simultaneously. From these calculations a matrix of indices can be developed. For each fund y, the mean of preference intensities over all other funds is defined in the form of outgoing flow

$$\Phi^+(y) = \frac{\sum_{z \in Y} \pi(y,z)}{n-1}$$

where n is the number of assumed funds.

I turn, the mean of preference intensities of all other funds over fund y is defined in the for so called incoming flow

$$\Phi^-(y) = \frac{\sum_{z \in Y} \pi(z,y)}{n-1}$$

Finally the net flow is defined as

$$\Phi(y) = \Phi^+(y) - \Phi^-(y)$$

and PROMETHEE II outranking relationships are defined as:

Fund y outranks fund z iff $\Phi(y) > \Phi(z)$.
Fund y is indifferent to fund z iff $\Phi(y) = \Phi(z)$.
The criteria that were used in the application of the methodology for pension funds outranking are presented in the Table 2. The weights of relative importance were stated as a result of consultations with some pension funds portfolio managers.

Table 2: Criteria for pension funds outranking

Criterion	Type	Weights
The current value of the income unit – average for the last four weeks	max	0.10
The weekly return in % - average for the last four weeks	max	0.15
The net asset value of the fund	max	0.15
The relative weekly change in the nest asset value of the fund	max	0.05
The charge for asset management in % of average monthly net asset	min	0.05
value of the fund	min	0.05
The historical weekly Value at Risk (95% confidence level)	min	0.05
The historical weekly Conditional Value at Risk (95% confidence	min	0.10
level)	max	0.10
The lower semi standard deviation of returns for the last 26 weeks		
The difference between the short run (the last 8 weeks) and long run	max	0.20
(the last 26 weeks) average weekly returns		
The difference between the return of the fund and the return of the		
market competition – average for the last four weeks		

An application of described methodology for outranking of conservative, balanced and growth pension funds on the base of weekly data provides three types of results:

- funds outranking on the base of the current week data,
- average results for the last ten weeks,
- results that present long-term tendencies of funds performance developments.

Table 3: Funds outranking in current week (August 18, 2006)

Company	Net flows of the funds		
	Growth	Balanced	Conservative
Aegon	-0.329892	-0.327461	-0.11608
Allianz	0.280520	0.216084	0.174763
CSOB	0.208482	0.243860	0.224584
ING	-0.425282	-0.395224	-0.23022
VUB	-0.025476	0.054430	-0.14696
Winterthur	0.291647	0.208311	0.093909

As an illustration results the current weekly results on August 18, 2006 are presented in Table 3. Corresponding funds outranking on the base the last ten weeks is presented in Table 4. The table contents together with values of average net flows for the last ten weeks also standard deviations of these values for the same period. These values measure volatilities of results for the period and provide a measure of risk. The combination of these two results leads to a construction of so called efficient funds boundary that consists of funds where a better average result can be achieved only with a higher risk. Such constructions create starting points for modern portfolio theory applications in decisions concern assumed investment opportunities space.

Table 4: Funds outranking on the base of the last ten weeks average (August 18, 2006)

Company	Average net flows of the funds and their standard deviations					
	Growth		Balanced		Conservative	
	Average	StDeviation	Average	StDeviation	Average	StDeviation
Aegon	-0.0162	0.1621	-0.0674	0.1667	0.1401	0.1894
Allianz	0.1954	0.0988	0.1504	0.0853	0.3090	0.1613
CSOB	0.0708	0.1554	0.1338	0.1550	-0.2292	0.1883
ING	-0.3574	0.2652	-0.2702	0.2712	-0.3705	0.1576
VUB	-0.0562	0.1283	-0.0361	0.1415	-0.0345	0.1166
Winterthur	0.1637	0.0717	0.0894	0.0804	0.1850	0.1349

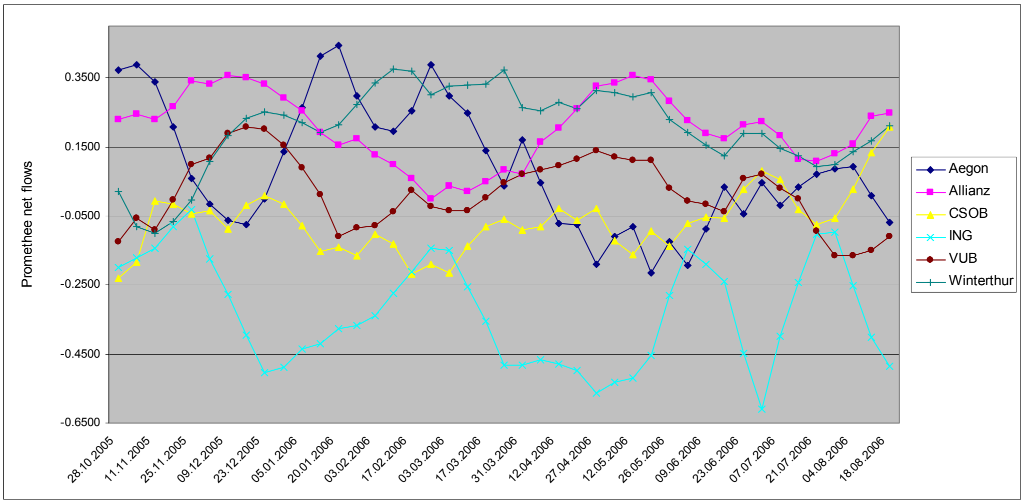

Figure 1: Average results for the last four weeks – growth funds

Funds performance from a long term view can be described with sequences of average four weeks results for the whole period of evaluation that starts on April 1, 2005. Such kind of results is illustrated for growth pension funds in Picture 1.

3. ANALYSIS OF PENSION FUNDS IN RISK – RETURN SPACE

On the base of published weekly data one can easy compute such risk - return characteristics of the funds investment strategies as average four weeks logarithmic return, standard deviation, historical VaR, historical Conditional VaR, lower semi standard deviation (SSD), lower semi absolute deviation (SAD), and kurtosis and skewness of returns distribution. Table 5, as an illustration, presents such characteristics for growth pension funds for the period from April 1, 2005 to August 18, 2006. In the table MC denotes market competition and simple one corresponds to the simple average of returns and weighted one corresponds to weighted average of returns, where weights are devoted from the level of the capitalization that is measured by the relative level of the fund net asset value.

Table 5: Growth funds risk - return characteristics on the base 4 weeks logarithmic returns

	Aegon	**Allianz**	**CSOB**	**ING**	**VUB**	**Winterthur**	**MC – simple**	**MC - weighted**
Return, p.a.	3.78%	3.43%	3.61%	2.75%	3.43%	3.49%	3.44%	3.43%
Stddev, p.a	1.39%	1.26%	1.38%	1.56%	1.25%	1.30%	1.26%	1.27%
Average return	0.29%	0.26%	0.27%	0.21%	0.26%	0.26%	0.26%	0.26%
Standard deviation	0.38%	0.35%	0.38%	0.43%	0.35%	0.36%	0.35%	0.35%
Kurtosis	0.93	1.14	0.77	1.62	1.18	1.02	1.41	1.35
Skewness	-0.60	-0.90	-0.65	-0.93	-1.11	-1.03	-1.17	-1.15
VaR (0.95)	0.47%	0.44%	0.41%	0.60%	0.41%	0.42%	0.51%	0.47%
CVaR (0.95)	0.66%	0.59%	0.61%	0.94%	0.59%	0.60%	0.63%	0.62%
Lower SSD	0.29%	0.28%	0.30%	0.34%	0.28%	0.29%	0.29%	0.29%
Lower SAD	0.14%	0.13%	0.14%	0.16%	0.13%	0.13%	0.13%	0.13%

Such characteristics enable us to compare pension funds in the selected risk – return space and derive conclusions about their efficiency on the Slovak markets of pension funds. This market is, of course, very young and all companies started to build up their investment strategies in a very conservative way. For example their investments into equities are still deeply under planned levels and also in growth funds at this time are below 20 percent. As it is stated in the legislature no fund can be worse as the market competition (simple) more then 5%. Such approaches are also reflected in corresponding correlation coefficients and Table 6 presents, as an illustration, the coefficients for growth funds.

Table 6: The correlation coefficients – growth funds

	Aegon	Allianz	CSOB	ING	VUB	Winterthur	MC-simple	MC-weighted
Aegon	1.000	0.792	0.625	0.570	0.630	0.652	0.790	0.708
Allianz	0.792	1.000	0.871	0.812	0.980	0.974	0.965	0.984
CSOB	0.758	0.871	1.000	0.830	0.885	0.877	0.932	0.907
ING	0.749	0.812	0.830	1.000	0.823	0.826	0.905	0.877
VUB	0.767	0.980	0.885	0.823	1.000	0.973	0.965	0.983
Winterthur	0.784	0.974	0.877	0.826	0.973	1.000	0.966	0.986
MC-simple	0.865	0.965	0.932	0.905	0.965	0.966	1.000	0.992
MC-weighted	0.820	0.984	0.907	0.877	0.983	0.986	0.992	1.000

Of course it is very simple to compare couples of funds, or investment strategies, and derive the conclusions about dominance relations. But in our opinion the much more interesting question is a one, what is the position of the fund in the risk – return space owing to the efficient frontier of the market. It is clear that we need to know the corresponding efficient frontier. Formally it is very simple because at now we are in the space of a generalized Markowitz portfolio selection problem that can be written in the form

$$eff\ \left\{ \mathbf{E}^T\mathbf{w}; \Omega(\mathbf{w}) \right\}$$

subject to

$\mathbf{e}^T\mathbf{w} = 1$

$\mathbf{w}^l \le \mathbf{w} \le \mathbf{w}^u$

where

Ω(w)- the risk measure scalar function**,**

w - the vector of portfolio weights,

E - the vector of expected returns,

$\mathbf{w}^l$ - the vector of lower bounds on portfolio weights,

$\mathbf{w}^u$ - the vector of upper bounds on portfolio weights,

e - the vector which elements equal 1,

and starting from Zeleny (1982) and Konno - Waki - Yuuki (2002) we can write

$$\Omega(\mathbf{w}) = \begin{cases} \Omega(\mathbf{w}, c, \alpha, \lambda) = \Omega(c, \alpha, \lambda) = \left[\sum_{r_k \le \lambda} \left| \mathbf{r}_k^T \mathbf{w} - c \right|^{\alpha} p_k \right]^{\frac{1}{\alpha}}, \quad \alpha > 0 \\ CVaR_{\beta}(\mathbf{w}, \beta) = \dfrac{1}{1-\beta} \mathrm{E}\left[-(\mathbf{r}^T\mathbf{w}) \middle| -(\mathbf{r}^T\mathbf{w}) \ge VaR_{\beta}(\mathbf{w}) \right] \end{cases}$$

where p_k is the probability of *k*th level $\mathbf{r}_k^T\mathbf{w}$ of portfolio return, *c* is a reference level of wealth from which deviations are measured. For example *c* could represent expected return of the asset, zero, the initial wealth level, the mode, the median, etc. Parameter α is the power to which deviations are raised, and thus α reflects the relative importance of large and small deviations. Parameter λ specifies what deviations are to be included in the risk measure if α > 0. Possible choices for parameter λ include ∞, *c*, a desired target level return, and some others. Conditional value at risk (*CVaR*) is an alternative measure of risk which maintains advantages of *VaR*, yet free from computational disadvantages of *VaR* , where β, $0 < \beta < 1$, is the confidence level.

Let us note that there exist *VBA procedures* (Jackson – Staunton, 2001; Mlynarovic, 2005) for an effective execution of the solution process in Excel environment which provide approximation of efficient frontier. From the view a practical application there are two main problems:

- how the select the particular risk measure function,
- how the estimate the expected return of assets

For $\alpha = 2$, $\lambda = \infty$ and $c = \mathbf{E}^T w$ we have the model of portfolio selection in the mean – variance space that is broadly used in fund management. It is used for allocation of assets for the purpose of setting fundamental fund management policy and also for the management of individual assets that form the portfolio, for risk management as well as for performance measurement, etc.

It is further used for specifying proportions of fund allocated to passive (index) management and for different types of active management. Its utility is determined by the following facts:

- if the rate of return has a normal distribution of probability, which was usually considered presumption fulfilled for common stock, then the model is consistent with "expected utility maximization" principle,
- quadratic programming problems, representing technical execution of the model, are solvable considering the existing knowledge of mathematical programming methodology.

Nevertheless, in recent years one can observe radical changes in investment environment. There are different financial instruments with asymmetric distribution of yield, such as options and bonds. Besides, recent statistical studies have shown that normal distribution of return is not recorded with all common stock. As a result, one can never rely on a standard model of portfolio selection.

In the past there were several risk measures proposed, different from the variance, including s*emi-standard deviation,* s*emi-absolute deviation* and below target *risk*. There are also models explicitly examining *skewness* of return distribution. Relatively new measure of lower partial *risk* comprises also *Value at Risk*, which is widely used for market risk measurement. This risk measure is very popular in conservative environment as probability of huge loss, larger than let's say $VaR_{0.99}$, is very low, provided that the portfolio's returns have normal distribution. However, considering the existing methodologies of non-linear programming it is impossible to find out a portfolio with the lowest *VaR*.

For this reason the *CVaR* (*conditional value at risk* or *expected loss*) becomes more and more attractive risk measure, namely with regard to its theoretical and computing features.

The second limitation of the mean – variance approach is that its recommended asset allocations are highly sensitive to small changes in inputs and, therefore, to estimation error. In its impact on the results of a mean – variance approach to asset allocation, estimation error in expected returns has been estimated to be roughly 10 times as important as estimation error in variances and 20 times as important as estimation error in covariances (Ziemba, 2003). Thus the most important inputs in mean - variance optimization are the expected returns.

Fisher Black and Robert Litterman (Black – Litterman, 1992) developed quantitative approach to dealing with the problem of estimation error. The goal of this model is to create stable, mean – variance efficient portfolios, which overcome the problem of input sensitivity. The Black - Litterman model uses “equilibrium” returns as a neutral starting point. Equilibrium returns are calculated using either CAPM or reverse optimization method in which the vector of implied expected equilibrium returns **P** is extracted from known information, where

$$\mathbf{P} = \delta \mathbf{C} \mathbf{w}$$

and **w** is in this case the vector of market capitalization weights, **C** is the covariance matrix, $n \times n$, where n is the number of assets, and δ is risk - aversion coefficient, which represents the market average risk tolerance. In general, the Black - Litterman approach consists of the following steps:

1. Define equilibrium market weights and covariance matrix for all asset classes.
2. Calculate the expected return implied from the market equilibrium portfolio.
3. Express market views and confidence for each view.
4. Calculate the view adjusted market equilibrium returns.
5. Run mean – variance optimization.

In our application we use this approach without market views expressions to describe efficient frontier of the Slovak pension funds market in the following way. Let vector $\mathbf{w}_c$ describe the capitalization on the market of the funds and E_c is the corresponding return of the weighted market competition for the current period. The risk adjusted return can be written in the form

$$E_c - \delta \mathbf{w}_c^T \mathbf{C} \mathbf{w}_c$$

and we assume that this return is for the weighted market competition zero. So we have

$$\delta_c = \frac{E_c}{\mathbf{w}_c^T \mathbf{C} \mathbf{w}_c}$$

and finally the vector

$$\mathbf{P}_c = \delta_c \mathbf{C} \mathbf{w}_c$$

is used as the vector of expected returns in mean – variance optimization.

Table 7: Black – Litterman equilibrium returns fro the growth pension funds

	Aegon	Allianz	CSOB	ING	VUB	Winterthur	MC-weighted	MC-simple
Market weights	7.09%	29.43%	5.79%	12.16%	14.69%	30.83%		
Equilibrium returns	0.86%	0.92%	0.94%	1.03%	0.91%	0.96%	0.94%	0.94%
Risk adjusted returns	-0.28%	-0.01%	-0.19%	-0.42%	0.00%	-0.04%	0.00%	-0.01%

The corresponding long - run equilibrium returns for the growth pension funds are presented together with the market weights for the last 4 weeks period to the August 18, 2006, and risk adjusted returns of the pension funds are presented in the Table 7. Let us no that (weighted) market return for this period equals 0.94% and the corresponding risk aversion coefficient was 768.66. Finally, the corresponding efficient frontier together with the positions of individual pension funds is presented in Picture 2.

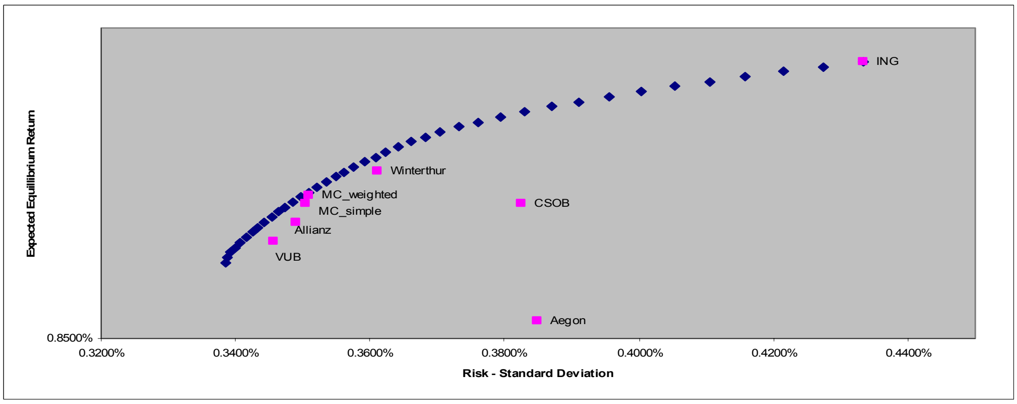

Figure 2: The efficient frontier of the Slovak growth pension funds

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STRATEGIC RESEARCH OF THE PRICE EFFECTS ON THE INDUSTRY'S ECONOMIC POSITION ON THE MARKET

SLÁVKA PAVLOVIČOVÁ

The economic system passed several changes since Slovak Republic's independence. The most serious one results from price changing (liberalization, autonomous price changing, etc). System of the national accounts ESA requires construction of input-output tables. These tables of interindustry relationship we will use in our analysis.

1. PROBLEM FORMULATION

The increase of the price in any industry caused by any factor creates several changes in other industries. This finally affects on their relative economic position on the market. We assume that an industry produces mostly products for intermediate consumption (stock, energy, services and etc.). Increase in price of products of given industry cause rise in price of products of industries using these products as an input. According to the market condition these industries will at least partial price increase transfer on their consumers, or decrease their profit by sharpen their economic position on the market. These are direct and indirect effects of primary or autonomous price increase. The final result will be new market equilibrium influencing the final consumption.

Autonomous price increase in one or several industries can be caused by establishment of the new taxes, or changing the tax system in the country. These are different situations on the market, which allow any industry to improve its position on the market. For analysis of different possible situations on the market we will construct several input-output models. We will concern on:

a) the effect of increase or establishment new type of taxes – in one or several industries – on the changes in price of products in other industry,
b) the effect of autonomous price increase in certain industries on the prices of products in other industries. These industries can transfer the price increase of its inputs on consumers without changing their relative economic position on the market.
c) The effect of autonomous price increase in certain industries on economic position of others. But because of price control or given market condition they can't transfer increase in price of inputs on their consumers.

2. INPUT-OUTPUT MODELS FOR ANALYSIS OF THE PRICE CHANGES

Three effects, listed above, require the specific form of models, input-output models know as Leontief's input-output models. Each column, industry, of the input-output table defines gross output of this industry X_j. Gross output consist of inputs according to the industry X_{ij} and elements of GDP H_j:

$$X_j = \sum_i X_{ij} + H_j, \quad i = 1, 2, 3, \ldots n \qquad (1)$$

where n is number of industries.

For simplicity value of the value added H_j will be a sum of its elements (amortization, wage, taxes, etc.). Using this formula (1) we can assume one of three possible situations on the market. Of course autonomous change in prices will cause induced effects of price (a change of the relative economic position of other industries). And finally this will create new equilibrium on the market with new values of gross output:

We also assume that the proportion of natural production of industry j and all its inputs are without any change. Just their values are changing. This type of change can be defined as price indexes or indexes of changing amount of value added:

$$\frac{X_j^{(1)}}{X_j} = c_j \quad \text{a} \quad \frac{H_j^{(1)}}{H_j} = z_j \qquad (2)$$

New structure of the gross output can be written as follows:

$$X_j c_j = \sum_i X_{ij} c_i + H_j z_j \quad i = 1, 2, 3, \ldots n \qquad (3)$$

At equation (3) on the right side c_{hi} is a price index of industry i, because X_{ij} presents the products of this industry and which industry j consume these products as intermediate products. Our analysis is based on this equation (3).

By modifications of this equation we obtain:

$$c_j = \sum a_{ij} c_i + h_j z_j, \quad i = 1, 2, 3, \ldots n \qquad (4)$$

The coefficient a_{ij} is known as a technical coefficient. It represent the necessity of products of industry i on a unit of production of industry j. An expression $h_j = H_j / X_j$ is coefficient of GDP industry j. H_j is value defines the necessity of GDP on a unit of production of industry j.

The change of the economic position of each industry we will measure by index z_j. Thus we can talk about increase or decrease of GDP on a unit of production of given industry.

3. HYPOTHETIC INPUT-OUTPUT TABLE FOR FIVE INDUSTRIES

For the analysis we will construct hypothetic input-output tables for 5 industries. The equation (4) for our hypothetic five industries will be written as follows:

$$
\begin{aligned}
c_1 &= a_{11}c_1 + a_{21}c_2 + a_{31}c_3 + a_{41}c_4 + a_{51}c_5 + h_1z_1 \\
c_2 &= a_{12}c_1 + a_{22}c_2 + a_{32}c_3 + a_{42}c_4 + a_{52}c_5 + h_2z_2 \\
c_3 &= a_{13}c_1 + a_{23}c_2 + a_{33}c_3 + a_{43}c_4 + a_{53}c_5 + h_3z_3 \\
c_4 &= a_{14}c_1 + a_{24}c_2 + a_{34}c_3 + a_{44}c_4 + a_{54}c_5 + h_4z_4 \\
c_5 &= a_{15}c_1 + a_{25}c_2 + a_{35}c_3 + a_{45}c_4 + a_{55}c_5 + h_5z_5
\end{aligned} \tag{5}
$$

In the matrix form can be written as follows:

$$\mathbf{c} = \mathbf{A}^T\mathbf{c} + \hat{\mathbf{h}}\mathbf{z} \tag{6}$$

where c is a column vector of price index, z is a column vector of GDP index, A^T is transposed matrix of technical coefficients and $\hat{h}$ is a diagonal coefficient matrix of GDP of industries.

Model (6) expresses how are the indexes of changing price and indexes of changing GDP interdependent. The initial assumption in our model is that technical coefficients will be similar also after any price change. They are constant. But accordingly to doing this raised also another problem, what will be the effect of the price change on the substitute relationships of inputs.

<u>First task</u>

We consider with establishment of new type of taxation, or with increase in rates already existing taxation. We have to expect that this tool will increase prices of products of these industries and also prices of other industries by reason of rise in prices of their inputs. Concerning the market condition we can expect the transfer of the price increase of inputs on their consumers. This tool (new type of taxation, increase or decrease rates of existing taxation) in our model is quantized by a percent of industry's GDP, thought indexes z_j of industries.

Using model (6) we obtain:

$$\mathbf{c} = \left(\mathbf{I} - \mathbf{A}^T\right)^{-1} . \hat{\mathbf{d}}\mathbf{z} \tag{7}$$

where I is a unit matrix.

Let's make this analysis on the hypothetic example of 5 industries, see tab. 1:

Table 1: Interindustry structure of hypothetic economy

	Industry					
		I	II	III	IV	V
Inputs on a unit of the production (**A**)	**I**	0	0,2	0,1	0,1	0,3
	II	0,1	0	0,3	0,2	0,2
	III	0,1	0,2	0	0,2	0,2
	IV	0,3	0,1	0,1	0	0,1
	V	0,1	0,3	0,1	0,2	0
GDP	h	0,4	0,2	0,4	0,3	0,2
Sum		1,0	1,0	1,0	1,0	1,0

Assess the new tax on products of industry 3. It will be 20% of the GDP of this industry, so z_3 = 1,2 with others ceteris paribus. GDP of others industries is unchanged because they transfer the price increase of their inputs on their consumers. Thus the prices of their products have to increase.
By substitution of the new vector *z* to equation 7 we will get new price vector *c*:

$$\mathbf{c} = \begin{pmatrix} 1{,}0320 \\ 1{,}0448 \\ 1{,}1052 \\ 1{,}0419 \\ 1{,}0438 \end{pmatrix}$$

Distributive effect of establishment new taxation on products of the third industry caused increase in the products prices of other industries. In the first industry by 3,20%, in the second one by 4,48%, in the third one by 10,52%, in the fourth one by 4,19% and in the fifth industry by 4,38%. New input-output table in the new prices is then:

Table 2: New interindustry structure of the hypothetic economy

	Industry					
		I	II	III	IV	V
Inputs on a unit of the production (**A**)	**I**	0	0,2064	0,1032	0,1032	0,3096
	II	0,1045	0	0,3134	0,2090	0,2090
	III	0,1105	0,2210	0	0,2210	0,2210
	IV	0,3126	0,1042	0,1042	0	0,1042
	V	0,1044	0,3131	0,1044	0,2088	0
	Sum	0,6320	0,8448	0,6252	0,7419	0,8438
GDP	h	0,4	0,2	0,48	0,3	0,2
Sum		1,0320	1,0448	1,1052	1,0419	1,0438

Second task

Now we will concern in autonomous price increase. This change can occur in one or in several industries. What will happen with prices of others industries, if e.g. due to the market condition they can transfer the increase in price of inputs on their consumers and their position on the market is unchanged?

Assume autonomous price increase in *s* industries *(s < n)*. It's necessary to expect that this increase in price will transfer also to others *r* industries *(r = n – s)*. It's result of interindustry relationships. Value of GDP of these industries is still unchanged. Thus the GDP indexes z_r are equal to 1.

To do this we have to change mathematical structure of the model. In the vectors and matrixes will be at first industries *r* (without autonomous change price) followed by industry *s* with autonomous price change.

$$\begin{pmatrix} \mathbf{c}_r \\ \mathbf{c}_s \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{rr}^T & \mathbf{A}_{rs}^T \\ \mathbf{A}_{sr}^T & \mathbf{A}_{ss}^T \end{pmatrix} * \begin{pmatrix} \mathbf{c}_r \\ \mathbf{c}_s \end{pmatrix} + \begin{pmatrix} \hat{\mathbf{h}}_r & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{h}}_s \end{pmatrix} * \begin{pmatrix} \mathbf{z}_r \\ \mathbf{z}_s \end{pmatrix} \quad (8)$$

As we already know, autonomous price change is in the vector *c*. Thus we have to calculate the column vector c_r. The elements of vector *z* are equals to 1. By mathematical modification of equation (8) and as vector z_r = [1] we find out, that

$$\mathbf{c}_r = (\mathbf{I} - \mathbf{A}_{rr}^T)^{-1} + (\mathbf{A}_{rs}^T \mathbf{c}_s + \hat{\mathbf{h}}_r) \quad (9)$$

Now we come to an answer how the prices of products change in the first industries *r* and how vector c_r depend from c_s.

We will apply this analysis on our example of 5 industries. Only in the third industry will be autonomous increase in price of products. Let the price increase by 10%, so $c_s = c_3$ = 1,1. How the prices of others industries will change?

$$\begin{pmatrix} c_1 \\ c_2 \\ c_4 \\ c_5 \end{pmatrix} = \begin{pmatrix} 1 & -0{,}1 & -0{,}3 & -0{,}1 \\ -0{,}2 & 1 & -0{,}1 & -0{,}3 \\ -0{,}1 & -0{,}2 & 1 & -0{,}2 \\ -0{,}3 & -0{,}2 & -0{,}1 & 1 \end{pmatrix}^{-1} * \left[\begin{pmatrix} 0{,}1 \\ 0{,}2 \\ 0{,}2 \\ 0{,}2 \end{pmatrix} * 1{,}1 + \begin{pmatrix} 0{,}4 \\ 0{,}2 \\ 0{,}3 \\ 0{,}2 \end{pmatrix} \right]$$

Prices of the first industry increase by 3,04%, prices of industry 2 increase o 4,25%, prices of industry 4 increase by cca 4% a prices of industry 5 increase by 4,16%.

Using this tool we can make another deeper analysis. Because the price increases in industry *r* the prices of inputs of industry *s* will rise. This initial price increase in this industry won't be the total increase of GDP of given industry. Hence is important to calculate also final effects on the value of GDP of industry *s*. This also stimulates the industry *s* to improve its economic position on the market.

By modification of the equation (8) we can write as follows:

$$\mathbf{z}_s = \hat{\mathbf{h}}_s^{-1}((\mathbf{I} - \mathbf{A}_{ss}^T)\mathbf{c}_s - \mathbf{A}_{sr}^T \mathbf{c}_r) \quad (10)$$

Let's verify this equation on our hypothetic economy.

$$z_s = \frac{1}{0,4}.\left[1.1,1 - \begin{pmatrix}0,1 & 0,3 & 0,1 & 0,1\end{pmatrix}.\begin{pmatrix}1,0304\\1,0425\\1.0399\\1,0416\end{pmatrix}\right] = 1,1901.$$

The value of the GDP of the industry 3 increase by 19,01%. The initial autonomous price increase of given industry was 10%. Value of the GDP of the third industry participate in its price 40%. If there wasn't be any induced increase in prices of inputs, the growth of GDP should be 25%. Rise in price of inputs occurs the effective growth just 19,01%.

Third task

Now we will be interested in problem of autonomous price increase. Because of a price control on the market these industries can't transfer rise in prices on their consumers. The economic position of these industries has to necessary change. Let the autonomous price increase is realized in industry *s*. Prices in industry *r* are unchanged. The result of induced reaction is decrease in the value of GDP in this industry. Thereby the economic position of this industry gets worse. Now assume the autonomous setting vector of price indexes c_s in industries *s*. We are interested in vector of indexes of change GDP z_r of industry *r*. Return to the model (8). After its mathematical modification we get:

$$\mathbf{z}_r = \hat{\mathbf{h}}_r^{-1}((\mathbf{I} - \mathbf{A}_{rr}^T)\mathbf{c}_\mathbf{r} - \mathbf{A}_{rs}^T\mathbf{c}_s) \qquad (11)$$

At the expression (11) we come to important economic (strategic) relation. Unknown vector of change indexes of GDP of the industry *r* is defined as a function of the autonomous setting vector of prices indexes in industry *s*. According to assumption the elements of the price vector of industry *r* (c_r) are equal to 1.

Assess the price increase in industry 3 by 10%, $c_s = c_3 = 1,1$. Substitute this parameter to the model (12). After calculating we achieve that $z_1 = 0,9750$, $z_2 = 0,9$, $z_4 = 0,933$ a $z_5 = 0,9$. The result is probably surprising. According to the price increase by 10% in third industry decrease GDP in industry 1 by 2,5%, in industry 2 by 10%, in industry 4 by 6,67% and in industry 5 by 10%. These industries couldn't transfer increase in price of inputs on their consumers by particular price increase of products. Autonomous price increase in the third industry cause total increase of its GDP by 25%. In other words the economic position of this industry on the market is better on the expense of industry 1, 2, 4 and 5. They couldn't change their prices, so the value of their GDP has to decrease.

Table 3: New interindustry structure of hypothetic economy

		Industry				
Inputs on a unit of the production (**A**)		I	II	III	IV	V
	I	0	0,2	0,1	0,1	0,3
	II	0,1	0	0,3	0,2	0,2
	III	0,11	0,22	0	0,22	0,22
	IV	0,3	0,1	0,1	0	0,1
	V	0,1	0,3	0,1	0,2	0
	Sum	0,61	0,82	0,6	0,72	0,82
GDP	h	0,39	0,18	0,5	0,28	0,18
Sum		1,0	1,0	1,1	1,0	1,0

CONCLUSION

In this paper we are concern on new types of economic models which can markedly help in the strategic calculation of possible effects of price changes on the economic position of industries on the market. For this we used input-output tables. We suggest three possible modifications of Leontief's input-output model. This model makes possible to thoroughly analyze the relation between prices and elements of gross production.

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TANDEM OF TWO LOSS QUEUES WITH LARGE *S*-TYPE ERLANG DISTRIBUTION

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Abstract: A loss queueing system with a tandem of servers is studied in the paper. Arrivals of customers are supposed to form a Poisson arrival stream. The service facility consists of two servers in series. Any accepted customer must be served by the first and second server respectively. The first server provides service in accordance with an exponentially distributed service time. The service time of the second server is Erlang-s distributed. The analysis of the behaviour of such a tandem queueing system is presented for large values of the parameter s representing the number of phases in the Erlangian service time distribution at the second server. The probabilities that measure the performance of the system are investigated for increasing values of s.

Keywords: loss tandem queueing system, Erlang service time distribution, large number of service phases, performance measures.

1. INTRODUCTION

Queues or queueing systems are service centres with appropriate facilities providing service to satisfy needs of customers. The structure of a service centre can be arranged in several ways including cases with parallel servers or those with servers in series as well as various combinations of serial and parallel servers forming a network of servers. A queueing system with two service stations in series is considered in the paper. There is not any waiting line in front of the first server. No waiting site is available in between the first and the second server but the customer, i.e. a requirement for service, remains in the first service station until the second one becomes empty. Such a queueing system is referred to as a tandem of two loss queues. Service time in the first service subsystem is an exponentially distributed random variable, which models situations with prevalence of short service duration representing so-called fast service. Some kinds of service cannot be modelled as the fast service but their nature allows to interpret them as composed of a number, say s, stages whose lengths are independent random variables each having an exponential distribution with the same mean. Then the s-type Erlang distribution models random service duration. An Erlang-s distributed random variable is supposed to represent the duration of service in the second subsystem of the tandem queueing system analysed in the paper when the value of s is large.

2. MATHEMATICAL MODEL AND ITS SOLUTION

The aforementioned queueing system composed of two service subsystems in series is supposed to serve customers that arrive at random according to a Poisson process with rate $\lambda > 0$ customers per unit time. Then the probability $\pi_r(t)$ of r arrivals in a t time units long period of time is given by the formula $\pi_r(t) = e^{-\lambda t}(\lambda t)^r / r!, t \geq 0, r = 0,1,\ldots$. An arriving customer is either given service if the first server is empty or rejected without service otherwise. The first service station can be occupied by a customer that is being served (an active customer) or by a customer that is waiting to obtain service in the second service station (a passive customer). Exponentially distributed service time T_1 of a customer in the first server is described by the probability density function $f_1(t) = \mu_1 e^{-\mu_1 t}, t \geq 0, \mu_1 > 0,$ with the corresponding mean $E[T_1] = 1/\mu_1$. Service time T_2 of a customer served in the second server consists of s phases with the lengths that are independent and identically distributed exponential random variables $T_{2,n}, n = 1,2,\ldots,s$. Denoting by $1/(s\mu_2)$ the mean length of each phase, the mean duration of service time T_2 is $E[T_2] = E[T_{2,1} + T_{2,2} + \cdots + T_{2,s}] = s[1/(s\mu_2)] = 1/\mu_2$. Thus, the probability density function of Erlang-s distributed service time T_2 is given by the prescription $f_2(t) = s\mu_2 e^{-s\mu_2 t}(s\mu_2 t)^{s-1} / (s-1)!, t \geq 0, \mu_2 > 0, s \in N = \{1,2,\ldots\}$. Any customer once accepted for service in this tandem queueing system gets both parts of service. The second part of service starts immediately after the first one has been finished in case the second server is free. Otherwise, the beginning of the second part is postponed until all remaining service phases of a previous customer in the second server are completed. The first server acts as a waiting room for a passive customer and the whole queueing system is blocked to accept any new customer in such a case.

The evolution of the queueing system described above can be represented by the random process $\{S(t), t \geq 0\}$, where $S(t) = (S_1(t), S_2(t))$ is the state of the system at time $t \geq 0$. The component $S_1(t)$ of the state variable $S(t)$ is the number of (active/passive) customers in the first service subsystem at time t. The component values are 0, 1, 1* with 1* used to reflect the situation when one passive customer is in the first subsystem. The component $S_2(t)$ is the number of phases remaining to be accomplished in order to complete service of a customer in the second subsystem. The set M of all possible values of the state variable is thus given by $M = M_1 \times M_2 - \{(1^*,0)\}$, where $M_1 = \{0,1,1^*\}$ and $M_2 = \{0,1,\ldots,s\}$. The concept of phases in service time of the second server together with the memoryless property of the exponential distribution leads to the fact that the process $\{S(t), t \geq 0\}$ is a Markov process for any time $t \geq 0$. To get information on the (long-run) operation of this tandem queueing system, a system of linear algebraic equations must be consequently built and solved for limiting state probabilities $p_{i,k} = \lim_{t\to\infty} p_{i,k}(t)$, $(i,k)\in M$, with $p_{i,k}(t) = P\{S(t) = (i,k)\}$, $t \geq 0$. The way how to derive and solve such equations is described in details in [2]. The equations are as follows:

$$\begin{aligned}
0 &= -\lambda p_{0,0} + s\mu_2 p_{0,1} \\
0 &= -(\lambda + s\mu_2)p_{0,k} + s\mu_2 p_{0,k+1}, \quad k = 1,2,\dots,s-1 \\
0 &= -(\lambda + s\mu_2)p_{0,s} + \mu_1 p_{1,0} + s\mu_2 p_{1^*,1} \\
0 &= \lambda p_{0,0} - \mu_1 p_{1,0} + s\mu_2 p_{1,1} \\
0 &= \lambda p_{0,k} - (\mu_1 + s\mu_2)p_{1,k} + s\mu_2 p_{1,k+1}, \quad k = 1,2,\dots,s-1 \\
0 &= \lambda p_{0,s} - (\mu_1 + s\mu_2)p_{1,s} \\
0 &= \mu_1 p_{1,k} - s\mu_2 p_{1^*,k} + s\mu_2 p_{1^*,k+1}, \quad k = 1,2,\dots,s-1 \\
0 &= \mu_1 p_{1,s} - s\mu_2 p_{1^*,s}
\end{aligned} \tag{1}$$

with the condition $\sum_{i=0}^{1}\sum_{k=0}^{s} p_{i,k} + \sum_{k=1}^{s} p_{1^*,k} = 1$.

Using notation $q_{i,k} = p_{i,k}/p_{0,0}$ for $(i,k)\in M$, we get from (1) the set of formulae for relative quantities $q_{i,k}$ as specified below:

$$\begin{aligned}
q_{0,0} &= 1 \\
q_{0,k} &= (\alpha_2/s)(1+\alpha_2/s)^{k-1}, \quad k = 1,2,\dots,s \\
q_{1,0} &= \alpha_1 + \{\alpha_1\alpha_2/[s(1+\alpha_2/(s\alpha_1))]\}S(1,1) \\
q_{1,k} &= \{\alpha_2^2/[s^2(1+\alpha_2/(s\alpha_1))]\}(1+\alpha_2/s)^{k-1}S(1,k), \quad k = 1,2,\dots,s \\
q_{1^*,k} &= \{\alpha_2^3/[s^3(\alpha_1+\alpha_2/s)]\}(1+\alpha_2/s)^{k-1}Q(1,k), \quad k = 1,2,\dots,s
\end{aligned} \tag{2}$$

where $\alpha_1 = \lambda/\mu_1$, $\alpha_2 = \lambda/\mu_2$,

$S(1,k) = \sum_{j=0}^{s-k}\beta^j$, $Q(1,k) = \sum_{j=0}^{s-k}\alpha^{s-k-j}S(1,s-j)$, $k = 1,2,\dots,s$,

and $\alpha = 1+\alpha_2/s$, $\beta = (\lambda + s\mu_2)/(\mu_1 + s\mu_2) = (1+\alpha_2/s)/[1+\alpha_2/(s\alpha_1)]$. The probability $p_{0,0}$ is then given by $p_{0,0} = \left[\sum_{i=0}^{1}\sum_{k=0}^{s} q_{i,k} + \sum_{k=1}^{s} q_{1^*,k}\right]^{-1}$, which follows from the condition accompanying system of equations (1). The other steady-state probabilities are obtainable from the equation $p_{i,k} = q_{i,k}p_{0,0}$ for any $(i,k)\in M$.

Steady-state probabilities enable us to calculate the probabilities that reflect the queueing system performance. The probability p_O of the immediate service of a customer correspond to the situations when the first subsystem is idle, that is

$$p_O = \sum_{k=0}^{s} p_{0,k} = p_{0,0} \sum_{k=0}^{s} q_{0,k} \ . \qquad (3)$$

The probability of blocking denoted by p_B describes the situations when the first subsystem is blocked by a passive customer that is waiting for service in the second subsystem. We have

$$p_B = \sum_{k=1}^{s} p_{1*,k} = p_{0,0} \sum_{k=1}^{s} q_{1*,k} \ . \qquad (4)$$

The loss probability p_L measures the chance of the situation when an arriving customer is lost as rejected because of the fact that the first subsystem is occupied by an active or a passive customer. The loss probability is given by

$$p_L = \sum_{k=0}^{s} p_{1,k} + \sum_{k=1}^{s} p_{1*,k} = p_{0,0}\left(\sum_{k=0}^{s} q_{1,k} + \sum_{k=1}^{s} q_{1*,k}\right). \qquad (5)$$

Probabilities p_O, p_B, p_L can be used to calculate other performance measures of the tandem queueing system as, for example, the average number of rejected customers, etc.

3. LIMITS OF PROBABILITIES

The steady-state probabilities $p_{i,k}, (i,k) \in M$, as well as the probabilities p_O, p_B, p_L depend not only on the queueing system parameters $\lambda > 0, \mu_1 > 0, \mu_2 > 0$, but also on the value of the number *s* of service phases in the second subsystem. The higher the value of *s*, the more complicated the service in the second subsystem. We are interested in the impact of the increasing value of *s* on the queueing system operation represented by probabilities p_O, p_B, p_L. The limits $p_O(+\infty), p_B(+\infty), p_L(+\infty)$ of probabilities p_O, p_B, p_L as $s \to +\infty$ are the boundary values for the course of the probabilities when *s* gradually increases. We sketch here the procedure of the calculation to find out the above limits. Using the notation $A_s = \sum_{k=0}^{s} q_{0,k}$, $B_s = \sum_{k=0}^{s} q_{1,k}$ and $C_s = \sum_{k=1}^{s} q_{1*,k}$, we need to determine the limits of sequences $\{A_s\}_{s=1}^{+\infty}$, $\{B_s\}_{s=1}^{+\infty}$ and $\{C_s\}_{s=1}^{+\infty}$. In case the sequences have finite limits, we can denote by *A*, *B*, *C* the respective limits $A = \lim_{s\to+\infty} A_s, B = \lim_{s\to+\infty} B_s, C = \lim_{s\to+\infty} C_s$. Then the limit $p_{0,0}(+\infty)$ of the probability $p_{0,0}$ as $s \to +\infty$ is given by $p_{0,0}(+\infty) = [A + B + C]^{-1}$ since

$$p_{0,0}^{-1}(+\infty) = \lim_{s\to+\infty} p_{0,0}^{-1} = \lim_{s\to+\infty} \sum_{k=0}^{s} q_{0,k} + \lim_{s\to+\infty} \sum_{k=0}^{s} q_{1,k} + \lim_{s\to+\infty} \sum_{k=1}^{s} q_{1*,k} = A + B + C$$

recalling that $p_{0,0} = \left[\sum_{i=0}^{1} \sum_{k=0}^{s} q_{i,k} + \sum_{k=1}^{s} q_{1*,k}\right]^{-1} = [A_s + B_s + C_s]^{-1}$.

The calculation of limits $\lim_{s\to+\infty}\sum_{k=0}^{s} q_{i,k}$ for $i = 0,1,1^*$ is a bit tedious, being based on the application of the formula $\lim_{s\to+\infty}(1 + k/s)^s = e^k$. It is necessary to treat two distinct cases associated with $\alpha_1 \neq 1$ and $\alpha_1 = 1$.

When the ratio $\alpha_1 = \lambda/\mu_1$ differs from 1 we get:

$$A = \lim_{s\to+\infty}\sum_{k=0}^{s} q_{0,k} = \lim_{s\to+\infty}\left(1 + \sum_{k=1}^{s} q_{0,k}\right) = 1 + \lim_{s\to+\infty}\sum_{k=1}^{s}\frac{\alpha_2}{s}(1+\alpha_2/s)^{k-1} = 1 + \lim_{s\to+\infty}\sum_{k=1}^{s}\frac{\alpha_2}{s}\alpha^{k-1}$$

$$= 1 + \lim_{s\to+\infty}\frac{\alpha_2}{s}\frac{\alpha^s - 1}{\alpha - 1} = 1 + \lim_{s\to+\infty}(\alpha^s - 1) = \lim_{s\to+\infty}\alpha^s = \lim_{s\to+\infty}(1+\alpha_2/s)^s = e^{\alpha_2}. \tag{6}$$

We obtain in a similar way that:

$$B = \lim_{s\to+\infty}\sum_{k=0}^{s} q_{1,k} = \frac{2\alpha_1^2 - \alpha_1}{\alpha_1 - 1}e^{\alpha_2} - \frac{\alpha_1^2}{\alpha_1 - 1}e^{\alpha_2 - \alpha_2/\alpha_1}, \tag{7}$$

$$C = \lim_{s\to+\infty}\sum_{k=1}^{s} q_{1^*,k} = (\alpha_2 - \alpha_1 - 1)e^{\alpha_2} + \frac{\alpha_1^2}{\alpha_1 - 1}e^{\alpha_2 - \alpha_2/\alpha_1} - \frac{1}{\alpha_1 - 1}. \tag{8}$$

Now we can find out the limit $\lim_{s\to+\infty} p_{0,0}^{-1} = A + B + C$, which after substituting (6) – (8) brings the final result of the form:

$$p_{0,0}(+\infty) = \lim_{s\to+\infty} p_{0,0} = [A + B + C]^{-1} = \frac{\alpha_1 - 1}{\left(\alpha_1^2 + \alpha_2(\alpha_1 - 1)\right)e^{\alpha_2} - 1}. \tag{9a}$$

The limit of the probability of immediate service is then:

$$p_O(+\infty) = \lim_{s\to+\infty}\sum_{k=0}^{s} p_{0,k} = \lim_{s\to+\infty}\sum_{k=0}^{s} q_{0,k}p_{0,0} = \lim_{s\to+\infty} p_{0,0}\lim_{s\to+\infty}\sum_{k=0}^{s} q_{0,k}$$

$$= p_{0,0}(+\infty)\,A = \frac{(\alpha_1 - 1)e^{\alpha_2}}{\left(\alpha_1^2 + \alpha_2(\alpha_1 - 1)\right)e^{\alpha_2} - 1}. \tag{10a}$$

The limit of the probability of blocking is:

$$p_B(+\infty) = \lim_{s\to+\infty}\sum_{k=1}^{s} p_{1^*,k} = \lim_{s\to+\infty}\sum_{k=1}^{s} q_{1^*,k}p_{0,0}$$

$$= p_{0,0}(+\infty)\,C = \frac{(\alpha_2 - \alpha_1 - 1)(\alpha_1 - 1)e^{\alpha_2} + \alpha_1 e^{\alpha_2 - \alpha_2/\alpha_1} - 1}{\left(\alpha_1^2 + \alpha_2(\alpha_1 - 1)\right)e^{\alpha_2} - 1}. \tag{11a}$$

The limit of the probability of losing a customer is:

$$p_L(+\infty) = \lim_{s\to+\infty} p_L = \lim_{s\to+\infty}\left(\sum_{k=0}^{s} p_{1,k} + \sum_{k=1}^{s} p_{1^*,k}\right) = \lim_{s\to+\infty}\sum_{k=0}^{s} q_{1,k}p_{0,0} + \lim_{s\to+\infty}\sum_{k=1}^{s} q_{1^*,k}p_{0,0}$$

$$= p_{0,0}(+\infty)(B + C) = \frac{\left((\alpha_1 - 1)(\alpha_1 + \alpha_2) + 1\right)e^{\alpha_2} - 1}{\left(\alpha_1^2 + \alpha_2(\alpha_1 - 1)\right)e^{\alpha_2} - 1}. \tag{12a}$$

When the ratio $\alpha_1 = \lambda/\mu_1$ equals 1 the formulae in (2) become easier because of $\alpha_1 = 1$ and

$S(1,k)=\sum_{j=0}^{s-k}\beta^j = \sum_{j=0}^{s-k}[(1+\alpha_2/s)/(1+\alpha_2/s)]^j = s-k+1$. The corresponding values of the limits *A*, *B*, *C* are then given by:

$A=\lim_{s\to+\infty}\sum_{k=0}^{s} q_{0,k} = e^{\alpha_2}$, $B=\lim_{s\to+\infty}\sum_{k=0}^{s} q_{1,k} = e^{\alpha_2}$,

$C=\lim_{s\to+\infty}\sum_{k=1}^{s} q_{1*,k} = (\alpha_2-1)^2 e^{\alpha_2} + 2e^{\alpha_2} - 2$,

which leads to the following limits of the probabilities for $\alpha_1 = 1$:

$$p_{0,0}(+\infty) = [A+B+C]^{-1} = \left[(\alpha_2-1)^2 e^{\alpha_2} + 4e^{\alpha_2} - 2\right]^{-1}, \qquad (9b)$$

$$p_O(+\infty) = p_{0,0}(+\infty)\,A = \frac{e^{\alpha_2}}{(\alpha_2-1)^2 e^{\alpha_2} + 4e^{\alpha_2} - 2}, \qquad (10b)$$

$$p_B(+\infty) = p_{0,0}(+\infty)\,C = \frac{(\alpha_2-1)^2 e^{\alpha_2} + 2e^{\alpha_2} - 2}{(\alpha_2-1)^2 e^{\alpha_2} + 4e^{\alpha_2} - 2}, \qquad (11b)$$

$$p_L(+\infty) = p_{0,0}(+\infty)\,(B+C) = \frac{(\alpha_2-1)^2 e^{\alpha_2} + 3e^{\alpha_2} - 2}{(\alpha_2-1)^2 e^{\alpha_2} + 4e^{\alpha_2} - 2}. \qquad (12b)$$

4. NUMERICAL EXPERIMENTS

The set of formulae in (2) enables one to calculate relative quantities $q_{i,k}$, $(i,k)\in M$, with respect to the tandem queueing system parameters α_1, α_2, s, which subsequently gives all of the $3s+2$ values of the steady-state probabilities $p_{i,k}$. A computer code can help with the vast process of calculation when there are, for example, 17 unknown probabilities for $s=5$, 32 probabilities for $s=10$, etc. We can fix the values of the parameters α_1, α_2 to follow how the probabilities are approaching the limits as the parameter *s* is tending to infinity. An illustrative example is given in the Table 1 and Figure 1 for the parameters $\alpha_1 = 0.05, \alpha_2 = 0.75$.

Table 1 *The changes of probabilities* $p_{0,0}, p_O, p_B, p_L$ *for* $\alpha_1 = 0.05, \alpha_2 = 0.75$

	$s=1$	$s=3$	$s=5$	$s=10$	$s=20$	$s=50$	$s=150$	$s=600$	$s\to+\infty$
$p_{0,0}$	0,4237	0,3981	0,3912	0,3856	0,3826	0,3801	0,3799	0,3796	0,3795
p_O	0,7415	0,7774	0,7869	0,7948	0,7990	0,8016	0,8028	0,8032	0,8035
p_B	0,2234	0,1858	0,1756	0,1675	0,1631	0,1603	0,1591	0,1586	0,1585
p_L	0,2585	0,2226	0,2131	0,2052	0,2010	0,1984	0,1972	0,1968	0,1965

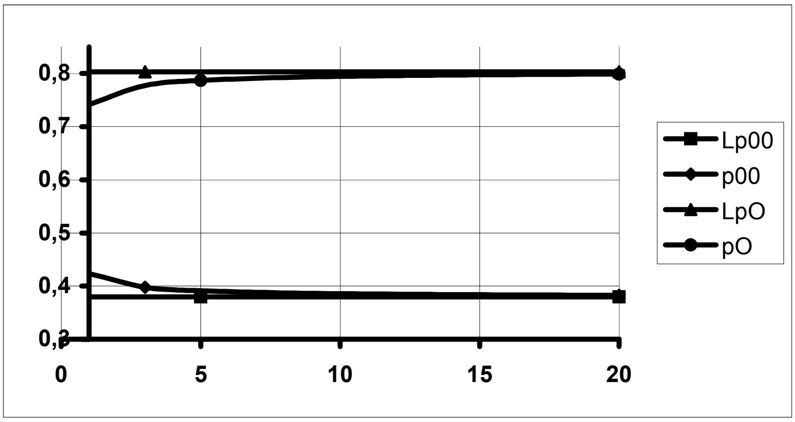

Figure 1 *The course of probabilities* $p_{0,0}, p_O$ *with respect to s*

The accuracy of calculations relatively strongly depends on the ratio α_2/α_1 as well as on whether or not the value of α_1 is close to 1 (for the case when $\alpha_1 \neq 1$). Numerical experiments reveal that there is a difference (even in the place of the tenths) between the values of limits and the values of numerical calculations if α_1 approaches 1.

5. CONCLUSION

The long-run behaviour of the tandem queueing system with two servers without waiting lines can be analysed using the set of steady-state probabilities, which depend on the number of phases as well as on the other system parameters. Therefore it is quite useful to have available both the set of formulae for numerical calculations of steady-state probabilities (with a variety of the system parameters values) and the expressions that give the limits of the probabilities for an increasing number of phases. If the number of phases is large, the limits can be used to analyse the system operation instead of doing exhaustive numerical calculations of exact values. Moreover, the expected duration of an Erlang-*s* distributed service time approximates deterministic service time *T* in case the number of phases, *s*, is high enough ($s \to \infty$) and the expected length of each phase equals T/s. That might help to avoid the use of a concept of embedded Markov chains in the analysis of a tandem queueing system with the structure mentioned above when service duration in the second subsystem is deterministic. The set of formulae (2) – (5) for numerical calculations with a large *s* and $\alpha_2 = \lambda(1/\mu_2) = \lambda T$ or the limits (9) – (12) would be employed to obtain performance measures of such a queueing system instead. If a numerical computational process were used, calculations should be repeated for an increasing *s* until the values of the performance measures become stabilize.

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SOME REMARKS ON RISK-SENSITIVE OPTIMALITY CRITERIA IN MARKOV DECISION PROCESSES

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Abstract: In this note we focus attention on discrete-time Markov decision processes with risk-sensitive optimality criteria (i.e. the case when the stream of rewards generated by the Markov processes is evaluated by an exponential utility function). It is shown that this problem can be studied as a classical Markov decision process on condition that the transition probabilities are replaced by general nonnegative matrices. Under some unichain conditions we suggest a value iteration method for finding optimal policies and show the connections between risk-sensitive and standard optimality criteria for Markov decision processes.

Keywords: Discrete-time Markov decision processes, finite state space, risk-sensitive optimality, value iterations, asymptotic behavior.

1. INTRODUCTION AND NOTATION

The usual optimization criteria for Markov decision processes (e.g. total discounted reward or mean reward) can be quite insufficient to fully capture the various aspects for a decision maker. It may be preferable to select more sophisticated criteria that also reflect variability-risk features of the problem. To this end, we focus attention on risk-sensitive optimality criteria (i.e. the case when the stream of rewards generated by the Markov processes is evaluated by an exponential utility function). The research of risk-sensitive optimality criteria in Markov decision processes was initiated in the seminal paper by Howard and Matheson [6] and followed by many other researchers in recent years (see e.g. [1,2,3,4,7,8,12]). In particular, this research was motivated mainly by [2,3] where algorithmic procedures for risk-sensitive Markov decision chains were studied.

In this note we consider a Markov decision chain $X = \{X_n, n = 0,1,...\}$ with finite state space $I = \{1,2,...,N\}$ and a finite set $A_i = \{1,2,...,K_i\}$ of possible decisions (actions) in state $i \in I$. Supposing that in state $i \in I$ action $k \in A_i$ is selected, then state j is reached in the next transition with a given probability p_{ij}^k and one-stage (nonnegative) transition reward r_{ij} will be accrued to such transition.

In what follows we shall suppose that the stream of transition rewards is evaluated by an exponential utility function, say $u^\gamma(\cdot)$, i.e. a utility function with constant risk sensitivity $\gamma \in \mathbb{R}$. Then the utility assigned to the (random) reward ξ is given by

$$u^\gamma(\xi) := \begin{cases} (\text{sign } \gamma)\exp(\gamma\xi), & \text{if } \gamma \neq 0 \\ \xi & \text{for } \gamma = 0. \end{cases} \qquad (1)$$

Obviously $u^\gamma(\cdot)$ is always continuous and strictly increasing and convex (resp. concave) for $\gamma > 0$ (resp. $\gamma < 0$). The symbol $Z^\gamma(\xi)$ denotes the corresponding certainty equivalent of the (random) variable ξ. So if ξ is a (bounded) random variable $u^\gamma(Z^\gamma(\xi)) = \mathrm{E}[u^\gamma(\xi)]$ (the symbol E is reserved for expectation). Hence we immediately can conclude that

$$Z^\gamma(\xi) = \begin{cases} \dfrac{1}{\gamma}\ln\{\mathrm{E}\,[\exp(\gamma\xi)]\}, & \text{if } \gamma \neq 0 \\ \mathrm{E}[\xi] & \text{for } \gamma = 0. \end{cases} \qquad (2)$$

Observe that if ξ is constant then $Z^\gamma(\xi) = \xi$, if ξ is nonconstant then by Jensen's inequality

$Z^\gamma(\xi) > \mathrm{E}\,\xi$ (if $\gamma > 0$ and the decision maker is risk averse)

$Z^\gamma(\xi) < \mathrm{E}\,\xi$ (if $\gamma < 0$ and the decision maker is risk seeking)

$Z^\gamma(\xi) = \mathrm{E}\,\xi$ (if $\gamma = 0$ and the decision maker is risk neutral)

A (Markovian) policy, say π, controlling the chain is a rule how to select actions in each state. We write $\pi = (f^0, f^1, \ldots)$ where $f^n \in A \equiv A_1 \times \ldots \times A_N$ for every $n = 0,1,2,\ldots$ and $f_i^n \in A_i$ is the decision at the nth transition when the chain X is in state i. A policy which takes at all times the same decision rule, i.e. selects actions only with respect to the current state and hence is fully identified by some decision vector f whose ith element $f_i \in A_i$, is called stationary. Stationary policy $\pi \sim (f)$ then completely identifies the transition probability matrix $P(f)$. Observe that the ith row of $P(f)$, denoted $p_i(f)$, has elements $p_{i1}^{f_i}, \ldots, p_{iN}^{f_i}$ and that $P^*(f) = \lim_{n\to\infty} n^{-1}\sum_{k=0}^{n-1}[P(f)]^k$ exists. Similarly R denotes the $N \times N$ nonnegative matrix of one stage rewards, i.e. the ijth element of R equals $r_{ij} \geq 0$.

In this note we make the following assumptions.

AS1. There exists a state, say N, such that for every decision vector $f \in A$ state N is accessible from any state $i \in I$.

AS2. For every decision vector $f \in A$ transition probability matrix $P(f)$ is aperiodic.

Observe that under assumptions AS1, AS2 for every stationary policy $\pi \sim (f)$ the transition probability matrix $P(f)$ contains a single class of recurrent states that is aperiodic, and possibly some transient states.

Let $\xi_n = \sum_{k=0}^{n-1} r_{X_k,X_{k+1}}$ be the stream of transition rewards received in the n next transitions of the considered Markov chain X, and similarly let $\xi^{(m,n)}$ be reserved for the total (random) reward obtained from the m th up to the n th transition (obviously, $\xi_n = r_{X_0,X_1} + \xi^{(1,n)}$). In case that $\gamma \neq 0$ then $u^{\gamma}(\xi_n) := (\operatorname{sign}\gamma) \mathrm{e}^{\gamma \sum_{k=0}^{n-1} r_{X_k,X_{k+1}}}$ is the (random) utility assigned to ξ_n, and $Z^{\gamma}(\xi) = \frac{1}{\gamma} \ln \{\mathrm{E}[\mathrm{e}^{\gamma \sum_{k=0}^{n-1} r_{X_k,X_{k+1}}}]\}$ is its certainty equivalent. Obviously, if $\gamma = 0$ then $u^{\gamma}(\xi_n) = \sum_{k=0}^{n-1} r_{X_k,X_{k+1}}$ and $Z^{\gamma}(\xi) = \mathrm{E}[\sum_{k=0}^{n-1} r_{X_k,X_{k+1}}]$.

Supposing that the chain starts in state $X_0 = i$ and policy $\pi = (f^n)$ is followed, then for the expected utility in the n next transitions and the corresponding certainty equivalent we have (E_i^{π} denotes expectation if policy $\pi = (f^n)$ is followed and $X_0 = i$)

$$U_i^{\pi}(\gamma, 0, n) := \mathrm{E}_i^{\pi}[u^{\gamma}(\xi_n)] = (\operatorname{sign}\gamma)\mathrm{E}_i^{\pi}[\exp(\gamma \sum_{k=0}^{n-1} r_{X_k,X_{k+1}})] \tag{3}$$

$$Z_i^{\pi}(\gamma, 0, n) := \frac{1}{\gamma} \ln \{\mathrm{E}_i^{\pi}[\exp(\gamma \sum_{k=0}^{n-1} r_{X_k,X_{k+1}})]\}. \tag{4}$$

Moreover, let

$$J_i^{\pi}(\gamma, 0) := \limsup_{n\to\infty} \frac{1}{n} Z_i^{\pi}(\gamma, 0, n) \tag{5}$$

be the mean value of the certainty equivalent.

In what follows we shall often abbreviate $U_i^\pi(\gamma,0,n)$, $Z_i^\pi(\gamma,0,n)$ and $J_i^\pi(\gamma,0)$ respectively by $U_i^\pi(\gamma,n)$, $Z_i^\pi(\gamma,n)$ and $J_i^\pi(\gamma)$ respectively. Similarly $U^\pi(\gamma,n)$ (resp. $Z^\pi(\gamma,n)$, resp. $J^\pi(\gamma)$) is reserved for the vector of expected utilities (resp. certainty equivalents, resp. mean values of certainty equivalents) whose i th element equals $U_i^\pi(\gamma,n)$ (resp. $Z_i^\pi(\gamma,n)$, resp. $J_i^\pi(\gamma)$).

In this note we focus attention on the asymptotic behavior of the expected utility and the corresponding certainty equivalents. In particular, we show that under the unichain assumptions on the underlying Markov chain there exists some $\gamma_0 > 0$ such that for any values of risk sensitivity $\gamma \in [0;\gamma_0)$ the mean value of the certainty equivalent is constant and hence does not depend on the starting state. In contrast to the recent paper [2] our analysis is based on the properties of nonnegative matrices arising the recursive formulas for the growth $\gamma \in [0;\gamma_0)$ in which the mean value of the certainty equivalent is constant.

2. ASYMPTOTIC BEHAVIOUR OF EXPECTED UTILITIES

Let $q_{ij}^{f_i} := p_{ij}^{f_i}\,\mathrm{e}^{\gamma r_{ij}}$. Conditioning in (3) on X_1 (since $u^\gamma(\xi_n) = \mathrm{E}[u^\gamma(r_{X_0,X_1})\cdot u^\gamma(\xi^{(1,n)}) \mid X_1 = j])$ from (3) we immediately get

$$U_i^\pi(\gamma,0,n) = \sum_{j\in I} q_{ij}^{f_i^0}\cdot U_j^\pi(\gamma,1,n) \qquad \text{with } U_i^\pi(\gamma,n,n) = 1 \tag{6}$$

or in vector notation

$$U^\pi(\gamma,0,n) = Q(f^0)\cdot U^\pi(\gamma,1,n) \qquad \text{with } U^\pi(\gamma,n,n) = e \tag{7}$$

where the ij th entry of the $N\times N$ matrix $Q(f)$ equals $q_{ij}^{f_i} = p_{ij}^{f_i}\mathrm{e}^{\gamma r_{ij}}$ and e is a unit (column) vector. Iterating (7) we get if policy $\pi = (f^n)$ is followed

$$U^\pi(\gamma,n) = Q(f^0)\cdot Q(f^1)\cdot\ldots\cdot Q(f^{n-1})\cdot e. \tag{8}$$

Observe that $Q(f)$ is a nonnegative matrix, and by the Perron–Frobenius theorem the spectral radius $\rho(f)$ of $Q(f)$ is equal to the maximum positive eigenvalue of $Q(f)$. Moreover, if $Q(f)$ is irreducible (i.e. if $P(f)$ is irreducible) the corresponding (right) eigenvector $v(f)$ can be selected strictly positive, i.e.

$$\rho(f)\,v(f) = Q(f)\cdot v(f) \qquad \text{with } \; v(f) > 0. \tag{9}$$

(In a vector inequality $a \ge b$ denotes that $a_i \ge b_i$ for all elements of the vectors a, b, and $a > b$ if and only if $a \ge b$ and $a_i > b_i$ at least for one, but not for all i's.)

Moreover, under the above irreducibility condition it can be shown (cf. e.g. [6], [8]) that there exists decision vector $f^* \in A$ such that

$$Q(f)\cdot v(f^*) \le \rho(f^*)\,v(f^*) = Q(f^*)\cdot v(f^*) \quad \text{with } v(f^*) > 0, \tag{10}$$

$$\rho(f) \le \rho(f^*) \equiv \rho^* \qquad \text{for all } f \in A. \tag{11}$$

In words, $\rho(f^*) \equiv \rho^*$ is the maximum possible eigenvalue of $Q(f)$ over all $f \in A$.

The above facts can be easily verified by policy iterations. In particular, we can construct a (finite) sequence of decision vectors $\{f^0, f^1, ..., f^M\}$, with $f^0 \in A$ selected arbitrarily, such that

$$\rho(f^m)v(f^m) = Q(f^m)\cdot v(f^m), \text{ for all } m = 0,1,...M \tag{12}$$

and select $f^{m+1} \in A$ such that (if possible)

$$Q(f^{m+1})\cdot v(f^m) > \rho(f^m)\,v(f^m) \tag{13}$$

(recall that the choice of actions in each state is independent of action selected in other states). Then the generated sequence of spectral radii $\{\rho(f^m), m = 0,1,...,M\}$ is increasing and, since there exists only a finite number of different matrices $Q(f)$'s, such a sequence must be finite. For more details, see [6] or [8].

Furthermore, (cf. [5,7,9,11,12]) (10), (11) still hold even for reducible matrices if for suitable labelling of states (i.e. for suitably permuting rows and corresponding columns regardless of the selected $f \in A$) it is possible to decompose $Q(f)$ such that

$$Q(f) = \begin{bmatrix} Q_{(00)}(f) & Q_{(01)}(f) & \cdots & Q_{(0r)}(f) \\ 0 & Q_{(11)}(f) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Q_{(rr)}(f) \end{bmatrix} \tag{14}$$

where the spectral radius $\rho_i(f)$ of every irreducible class of $Q_{(ii)}(f)$ (with *i=1,...,r*) is nongreater than $\rho(f^*)$ (with $\rho_i(f^*) = \rho(f^*)$) and the spectral radius of (possibly reducible) $Q_{(00)}(f)$ is less than $\rho(f^*)$ and at least some $Q_{(0j)}(f)$ is nonvanishing.
In particular, under Assumption AS1 for every policy $\pi \sim (f)$ the transition probability matrix $P(f)$ can be decomposed as

$$P(f) = \begin{bmatrix} P_{(00)}(f) & P_{(01)}(f) \\ 0 & P_{(11)}(f) \end{bmatrix} \tag{15}$$

and a similar decomposition also holds for the corresponding matrix $Q(f)$, in particular we have

$$Q(f) = \begin{bmatrix} Q_{(00)}(f) & Q_{(01)}(f) \\ 0 & Q_{(11)}(f) \end{bmatrix} \tag{16}$$

Now observe in virtue of AS1 for every policy $\pi \sim (f)$ the spectral radius of $P_{(00)}(f)$ is less than unity and, of course, the spectral radius of $P_{(00)}(f)$ equals one. Hence, as well known, the corresponding (right) eigenvector is the unit vector.

3. BOUNDS ON GROWTH RATE OF EXPECTED UTILITIES

In the rest of this note we focus attention on expected utilities given by (8) on condition that (possibly reducible) matrices $Q(f)$'s with elements $q_{ij}^{f_i} = p_{ij}^{f_i} \cdot \mathrm{e}^{\gamma r_{ij}}$ can be decomposed as in (16) and the corresponding $P(f)$'s fulfil assumptions AS1 and AS2. We show that if the risk aversion coefficient $\gamma > 0$ is sufficiently small the behavior of expected utilities and the corresponding certainty equivalents is quite similar to the irreducible case.
To this end recall that the spectral radius of any matrix is a continuous function of the matrix elements and if for some matrices A and B with nonnegative elements it holds $A \ge B$ then for their spectral radii we have $\rho(A) \ge \rho(B)$.

Then on inspecting the matrix $Q(f)$ given by (16) we can easily conclude that there exists some $\bar{\gamma}_0 > 0$ such that for every $\gamma \in [0; \bar{\gamma}_0)$ the spectral radius of any $Q_{(00)}(f)$ with $f \in A$ is nongreater than unity. Moreover, since for any $\gamma > 0$ (recall that we have assumed that all r_{ij}'s are nonnegative) it holds $Q(f) > P(f)$ for every $f \in A$, it may happen that the spectral radius of $Q_{(00)}(f)$ is greater than one, but still less than the spectral radius of $Q_{(11)}(f)$. In this case there exists a strictly positive eigenvector corresponding to the spectral radius of $Q(f)$ (or to the spectral radius of $Q_{(11)}(f)$) for all $\gamma \in [0;\gamma_0)$ with $\gamma_0 \geq \overline{\gamma}_0$). Here we tacitly assume that for γ greater than $\gamma_0 > 0$ the spectral radius of $Q_{(00)}(f^*)$ will greater than or equal to the spectral radius of $Q_{(11)}(f^*)$ and hence no $v(f^*) > 0$ exists.

Iterating (10) and using (11) we can immediate conclude that for any policy $\pi = (f^k)$

$$\prod_{k=0}^{n} Q(f^k) \cdot v(f^*) \leq (Q(f^*))^n \cdot v(f^*) = (\rho^*)^n v(f^*) \qquad (17)$$

and hence the asymptotic behaviour of $U^{\pi}(\gamma,n)$ (or of $U^{\pi}(\gamma,m,n)$ if m is fixed) heavily depends of on $\rho(f^*) \equiv \rho^*$, however the growth rate of all elements of $U^{\pi}(\gamma,n)$ need not be the same (recall that the growth rate of $U_i^{\pi}(\gamma,n)$ is given by $\limsup_{n\to\infty} [U_{i+1}^{\pi}(\gamma,n) / U_i^{\pi}(\gamma,n)]$).

Since in (10) $v(f^*) > 0$, on selecting $v(f^*) \geq e$, say we set $v_{\text{upb}}(f^*) := v(f^*) \geq e$, in virtue (17) we get an upper bound on the growth of $U^{\pi}(\gamma,n)$. In particular, we have

$$\prod_{k=0}^{n} Q(f^k) \cdot v_{\text{upb}}(f^*) \leq (Q(f^*))^n \cdot v_{\text{upb}}(f^*) = (\rho^*)^n v(f^*). \qquad (18)$$

Hence the growth rate of $U^{\pi}(\gamma,n)$ is bounded by ρ^* from above.

Similarly, on selecting $v(f^*) \leq e$, say we set $v_{\text{lowb}}(f^*) := v(f^*) \leq e$, we get a lower bound on the growth of $U^{\pi}(\gamma,n)$. In particular, this can be concluded by the following recursive relation if policy $\hat{\pi} = (\hat{f}^k)$ is selected such that $U^{\hat{\pi}}(\gamma,n) = \prod_{k=0}^{n} Q(\hat{f}^k) \cdot e \geq \prod_{k=0}^{n} Q(f^k) \cdot e$ for a given $n > 0$ and an arbitrary policy $\pi = (f^k)$. Then

$$U^{\hat{\pi}}(\gamma,n) = \prod_{k=0}^{n} Q(\hat{f}^k) \cdot e \geq (Q(f^*))^n \cdot e \geq (Q(f^*))^n \cdot v_{\text{lowb}}(f^*) \geq (\rho^*)^n v(f^*). \qquad (19)$$

Hence the growth rate of $U^{\hat{\pi}}(\gamma,n)$ is also bounded by ρ^* from below.

From (18) and (19) we can easily conclude that for any $\gamma \in [0;\gamma_0)$ there exists a stationary policy $\pi^* \sim (f^*)$ such that the maximum possible growth rate of every $U_i^{\pi^*}(\gamma,n)$ equals ρ^*.

Then for the corresponding values of certainty equivalents we get since $\gamma > 0$

$$Z_i^{\pi^*}(\gamma, n) = \frac{1}{\gamma} \cdot \ln[U_i^{\pi^*}(\gamma, n)] = \frac{1}{\gamma} \cdot [n.\ln(\rho^*) + w_i] \tag{20}$$

and for the mean value of certainty equivalents we have

$$J_i^{\pi^*}(\gamma) = \frac{1}{\gamma} \cdot \ln(\rho^*). \tag{21}$$

The above procedures also enables to generate upper and lower bounds of the maximum growth rate and the corresponding certainty equivalents. To this end, let us generate (backwards) a sequence of maximum possible expected utilities by the following dynamic programming recursion:

$$\hat{U}(n+1) = \max_{f \in A} Q(f) \cdot \hat{U}(n) := Q(\hat{f}^n)) \cdot \hat{U}(n), \quad n = 0,1,... \tag{22}$$

with $\hat{U}(0) = e$.

Then on employing elements of the sequence $\hat{U}(n)$ we can easily generate upper and lower bounds on the growth rate, denoted $\rho_{\max}(n)$ and $\rho_{\min}(n)$ respectively. In particular, it holds:

$$\rho_{\max}(n) := \max_{i \in I} \frac{\hat{U}_{i+1}(n)}{\hat{U}_i(n)}, \qquad \rho_{\min}(n) := \min_{i \in I} \frac{\hat{U}_{i+1}(n)}{\hat{U}_i(n)} \tag{23}$$

and the sequence $\{\rho_{\max}(n), n = 0,1,...\}$ is nonincreasing, the sequence $\{\rho_{\min}(n), n = 0,1,...\}$ is nondecreasing, and

$$\lim_{n \to \infty} \rho_{\max}(n) = \lim_{n \to \infty} \rho_{\min}(n) = \rho^* \tag{24}$$

where ρ^* is the maximum possible growth rate.

The same holds also for mean values of the corresponding certainty equivalents. In particular,

$$J_{\max}(\gamma, n) := \frac{1}{\gamma} \cdot \ln[\rho_{\max}(n)], \qquad J_{\min}(\gamma, n) := \frac{1}{\gamma} \cdot \ln[\rho_{\min}(n)] \tag{25}$$

are upper and lower bounds on the mean values of the corresponding certainty equivalents and the sequence $\{J_{\max}(\gamma, n), n = 0,1,...\}$ is nonincreasing, the sequence $\{J_{\min}(\gamma, n), n = 0,1,...\}$ is nondecreasing, and

$$\lim_{n \to \infty} J_{\max}(\gamma, n) = \lim_{n \to \infty} J_{\min}(\gamma, n) = \frac{1}{\gamma} \cdot \ln(\rho^*) \tag{26}$$

where $\frac{1}{\gamma} \cdot \ln(\rho^*)$ is the maximum value of the mean certainty equivalent.

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WILL EURO INCREASE THE PRICE LEVEL IN SLOVAKIA ?

JOZEF SOJKA

According to the data of Eurostat, where the money data are presented in Euro, the author analyses the behaviour of the prices in 12 countries before the entering the European Monetary Union and after it. It is interesting, that the prices, or, the Harmonized Indices of Consumer Prices (HICPs) were lower before the entry than after it. These facts declare not only the citizens, but also the data of the Eurostat. In 10 countries of the European Union there is a contradictious movement and that is the fall of the prices caused by the effort of these countries to come closer to the Maastricht's criteria. At present these criteria fulfill only the Czech Republic, Latvia and near to these criteria are Malta and Cyprus.

In the study the author tries to determine the factors, which influence the growth of the prices in the European Monetary Union. Between the basic factors common for all EMU states belong the growth of the GDP and the export-import prices, which are expressed by help of indices. These factors explain 56% of the inflation. The rest presents the influence of other factors. The author expanded these basic factors by the government debt, long-time interest rate and netto export. This explained the growth of the inflation to 80 % and 87%. Factors, which do not prove the statistical significance are energetic price indices, price indices of the government expenditures and price indices of investments.

After the introduction of the indices of the consumption of electric energy and gas in households the relevant variables became statistically nonsignificant. In EMU in the years 2001- 2005 the energetic factors did not influence or inluenced only a very little the growth of the prices.

The author tried to identify common factors, which influence the growth of the prices in 10 countries of the European Union. This attempt was not successful, because the solution was not statistically significant. The process of harmonizing of these countries is only at an early stage.

In spite of some positive results it can be expected that after entering the European Monetary Union, after the implementation of the Euro, we cannot avoid the growth of the price level, because the higher growth of the GDP and import-export is expected. Relatively to this growth the price level will grow too.

Understandably, this process will be influenced by the management work of our economic and control sphere and by the fact that while in the EMU the energetic factors nearly did not influence the price level, around the year 2010 this influence could be more significant. These are logical processes, which will appear, even if we do not wish them. We can affect them to a certain measure, but we can not eliminate them.

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DOES THE ENTRANCE OF THE COUNTRY INTO EUROPEAN UNION INFLUENCE AN INFLATION CONVERGENCE?

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1. METHODOLOGY

Inflation convergence within the European Union is a widely discussed topic. This study investigates the convergence process in EU countries. Do the inflation rates of the countries within the European Union (EU) converge at the common level? Does the entrance of the country into EU influence an inflation convergence? This paper examines these questions.

Every country before the entrance into EU and before set up common currency euro, aim one's attention to Maastricht's criterion. Main condition of the membership in eurozona is readiness (ability) of country on common currency and keeps up same level of economic as the other developed EU countries. We distinguish two categories of the convergence:

- real,
- nominal.

We know several Maastricht's criterions, for example, not only GDP per capita, but also stability of the price level. Nominal convergence of the price level is a formal condition for acceptance common currency.[11] Process of the real convergence does not represent only grow of the price level. This criterion can be fulfils with appreciation of the nominal exchange rate of the national currency.

The process we can express mathematically with following relation:

$$\varepsilon_t = \frac{e_t \cdot P_t}{P_t^*} \tag{1}$$

where ε_t is real exchange rate in time t, e_t is a nominal real exchange rate in time t and P_t, P_t^* are price levels of the home country and foreign country in time *t*. After the logarithm of the relation (1) we get

$$\ln(\varepsilon_t) = \ln(e_t) + \ln(P_t) - \ln(P_t^*) \quad . \tag{2}$$

[11] This process is call as execute of the Maastricht`s criterion.

On the basis of formula (2) we can defined the percentage change of the real exchange rate:

$$\dot{\varepsilon}_t = \dot{e}_t + \pi_t - \pi_t^*, \qquad (3)$$

where $\dot{\varepsilon}_t$ is percentage change of the real exchange rate, $\dot{e}_t$ represents percentage change of the nominal exchange rate, π_t and π_t^* are inflation rates the home country and in abroad.

The relation (3) express, that if the inflation will be equal the home country and in abroad, than pressure on change nominal of the exchange rate will not be exists. The countries, which have different rate of inflation, they have the pressure on real appreciation of the exchange rate. Real appreciation could be represent with nominal appreciation of the exchange rate or inflation differential.

Establishment of euro evocated the appreciation of the real exchange rate in consequence of price level increase. Following table contains data of harmonized indices of consumer prices (HICP)[12] of selected countries EU.

Table 1 HICP before and after come in euro

	1998	1999	2000	2001	2002	2003	2004
Austria*	0,9	1,5	4,5	4,4	3,0	2,8	2,3
Greece**	4,5	2,1	2,9	3,7	3,9	3,4	3,0
Ireland*	2,1	2,5	5,3	4,0	4,7	4,0	2,3
Spain*	1,8	2,2	3,5	2,8	3,6	3,1	3,1
Portugal*	2,2	2,2	2,8	4,4	3,7	3,3	2,5

Source: OECD

* come in euro 1.1.1999.

** come in euro 1.1.2001.

The article is focused on nominal convergence of the price level in EU country in literature exist lot of possibilities, how to measure convergence process, (see e. g. Barro and Sala-I-Martin (1995), Kočenda and Papell (1997) and Komínkova et al. (2005). We used for our analyses the methodology from Kočenda and Papell (1997).

Our analysis of measure convergence of inflation is based on combination of cross-sections of individual time-series (panel data). Then inflation for *i*- country in time *t* is defined as

$$\pi_{it} = \alpha + \phi\pi_{it-1} + u_{it}, \qquad (4)$$

where π_{it} is HICP for country *i* in time *t* and coefficient ϕ express coefficient of convergence. Inflation is described by AR(1) process. The average inflation

[12] Percentage change from previous year

of all countries European monetary Union (EMU) is possible to define by following equation:

$$\overline{\pi}_{EMU,t} = \alpha + \phi\overline{\pi}_{EMU,t-1} + u_{EMU,t}. \qquad (5)$$

The inflation differential (d_{it}) is defined as the difference between an individual inflation and the average HICP for the EMU in time *t*:

$$d_{it} = \pi_{it} - \overline{\pi}_{EMU,t} = \phi\left(\pi_{it-1} - \overline{\pi}_{EMU,t-1}\right) + v_{it}. \quad (6)$$

Convergence and divergence of inflation differentials is influenced by parameter ϕ (coefficient of convergence). If ϕ is less than one, this process is convergence one, on the other hand, if ϕ is greater than one, the process is divergence one. $\overline{\pi}_{EMU,t}$ is call a reference values.

Let the inflation process growing continuous process. Then rate of convergence (*r*) we can calculate following way

$$d_{it} = d_0 e^{-rt} \text{ or} \qquad (7)$$

$$r = -\ln(\phi). \qquad (8)$$

If we subtract from both side of relation (6) value d_{it-1}, we receive following model (equation for testing panel of the unit root test)

$$\Delta d_{it} = d_{it} - d_{i,t-1} = (\phi - 1)d_{i,t-1} + v_{it}. \qquad (9)$$

When we use unit root test, we receive coefficient of convergence. The null hypothesis are following

H_0: $(\phi - 1) = 0$,
H_1: $(\phi - 1) \neq 0$, that denote that ϕ is significantly different from one.

2. EMPIRICAL ANALYSIS

2.1. DATA

We used data HICP[13] of countries EU from the period from January 1997 - December 2005. Monthly data were obtained from Eurostat. Data were sectionalizing into four groups of panel data. The groups are described in Table 2.

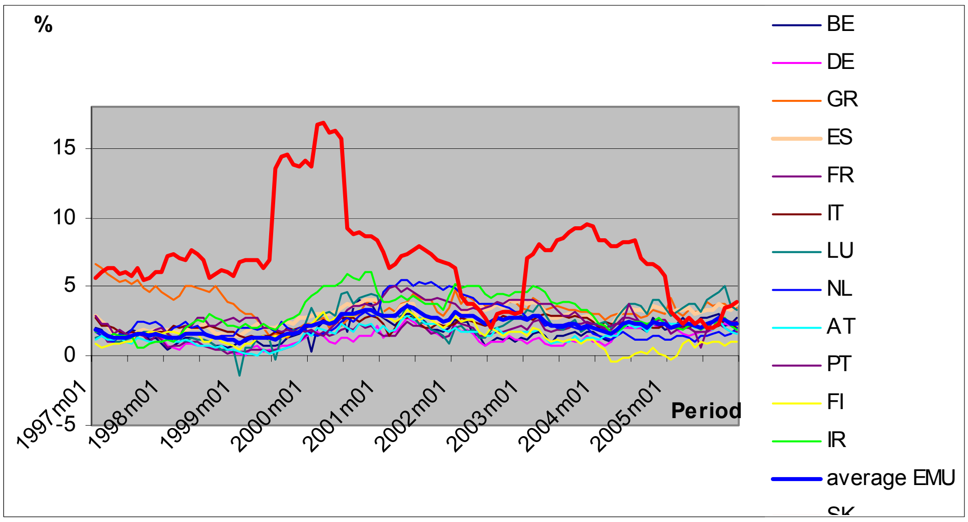
[13] Harmonized indices of consumer prices, monthly data, annual rate of change

Table 2 Overview of panel data

Name of Panel	V4	TP	EMU	EMUSK
Country	Slovakia Czech rep. Hungary Poland	Slovakia Czech rep. Hungary Poland Germany Italy Austria	Germany Belgium Spain France Italy Luxembourg Netherlands Austria Portugal Finland Ireland Greece	Germany Belgium Spain France Italy Luxembourg Netherlands Austria Portugal Finland Ireland Greece Slovakia

We analyzed by the left (separately) of panel data EMU and EMUSK (if integration the Slovak republic into panel EMU had any importance). We were expected, that the rate of convergence would be higher in panel data EMUSK, because Slovak republic has HICP significantly higher. Simultaneously we expected lower rate of convergence in panel EMU, because HICP countries EMU recording the stationary trend. The graph 1 denotes non-homogenous trend HICP in panel data EMUSK.

Graph 1 History annual rate of change HICP in EMU countries and in the Slovak republic



Source: www.europa.eu.int

2.2. RESULTS

For quantification the coefficients of convergence we used econometrics software Eviews 5.1. The data of HICP are in logarithmic expression. Common unit root test (Breitung test) were applied at the quantification coefficients of convergence.

Were analyzed more coefficients of convergence for each panel data, in accordance with entrance countries to the EU. The date of entrance of countries of V4 into EU (1.5.2004) was used like brake point in panel V4 and panel TP (Trade panel). More brake points (set up common currency euro, 1.1.1999 in 11 countries, set up euro in Greece 1.1.2001 and entrance Slovak republic into EU, 1.5.2004) were used in panel data EMU and EMUSK. Reference values (in the inflation differential) were defining as average values of HICP countries of EMU.

The results are concentrating in next tables.

Table 3 Panel V4

Period	Φ	t-stat (1-Φ)	Critical value	r
1997m01-2004m4	0,989	-2,379	-2,23**	1,106
2004m5-2005m12	0,966	-1,656	-1,61[10]	3,459

Table 4 Panel TP

Period	Φ	t-stat (1-Φ)	Critical value	r
1997m01-2004m4	0,990	-2,250	-2,23**	0,97
2004m5-2005m12	0,963	-1,872	-1,61[10]	3,76

Table 5 Panel EMU and EMUSK

Period	Panel EMU			Panel EMUSK		
	Φ	t-stat (1-Φ)	r	Φ	t-stat (1-Φ)	r
1997m1 - 1998m12	0,977[10]	-1,626	2,32	0,968	-1,734	3,16
1999m1 - 2005m12	0,976**	-2,253	2,43	0,971[10]	-2,713	2,95
1997m1 - 2000m12	0,979[10]	-1,693	2,12	0,992	-0,707	0,77
2001m1 - 2005m12	0,983[10]	-1,645	1,71	0,981[10]	-1,820	1,82
2005m1 - 2005m12	0,887	-2,556	11,99	0,942[10]	-1,789	5,87
1997m1 - 2004m4	0,987	-1,482	1,30	0,987[10]	-1,565	1,23
2004m5 - 2005m12	0,926	-2,054	7,68	0,961[10]	-1,910	3,92

[10] critical value on 10 % significant level
* critical value on 5 % significant level
** critical value on 2,5 % significant level
*** critical value on 1 % significant level

3. CONCLUSION

In accordance the obtained values the rate of convergence in panel V4 we note following:

1. It is possible to tell that before May 2004 is coming in countries V4 into convergence process in inflation rate (r = 1,106 %).
2. The same situation arises also after entrance of this country to EU, but the convergence process of inflation rate is faster (r = 3,459 %).

We enlarged panel V4 by countries Germany, Italy and Austria, the process of convergence was not influence.

From point of view establishment of common currency in 11 countries, is possible to notice, that there are no changes in the inflation convergence. The values of rate of convergence calculated before 1.1.1999 and after this time are not essentially changed (2,32 % and 2,43 %). After establishment euro in Greece the convergence process was confirmed, but the value of rate of converge was lower (1,71 %).

Our expectation mentioned in advance about faster convergence in panel EMUSK in comparison with panel EMU was not confirmed. The reason for this state may be the big differences in HICP in Slovak republic and countries EMU (especially in period July 1999 (13,6 %) – June 2000 (15,7 %)).

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MODELING OF SOME SYNCHRONIZATION PROBLEMS.*

KAROL ZIMMERMANN

Abstract: We consider an optimization problem consisting in the minimization of $\left\| x - \hat{x} \right\| \equiv \max_{j \in J} \left| x_j - \hat{x}_j \right|$, under two-sided $(\max,+)$-linear constraints of the form $\max_{j \in J} (a_{ij} + x_j) = \max_{j \in J} (b_{ij} + x_j), i \in I$, where I, J are given finite index sets, $\hat{x}_j, a_{ij}, b_{ij}$ for $i \in I, j \in J$ are given real numbers. the problems can be applied to some types of synchronization problems.

Keywords: optimization, non-convex optimization, operations research
MOS (AMS) Subject Classification: 90C26, 65K05

1. INTRODUCTION

The aim of this contribution is to present an algorithm for solving optimization problem consisting in minimizing the Tshebyshev distance from a given point in R^n under constraints, which are described as a finite system of so called two-sided $(\max,+)$-linear equations, which were studied in [1]. Such systems generalize "one-sided" systems, which were considered originally in [3] and further developed in [2]. The systems can be applied to some synchronization problems. We will present one of such problems as the following motivating example.

Example 1.1 Let us have n railway stations $S_j, j = 1,...,n,$ from which passengers are transporteded to m railway stations $C_i, i = 1,...,m.$ Let us assume that traveling times from S_j to C_i are equal to a_{ij}, so that if x_j denotes a departure time from S_j, the arrival time to C_i will be equal to $a_{ij} + x_j$. Under these assumptions the last train coming from stations $S_1, S_2,..., S_n$ will come to C_i at a time $a_i(x) \equiv \max_{1 \le j \le n} (a_{ij} + x_j)$. Let us assume that all passengers coming from $S_j, j = 1,...,n$ must have the possibility to change at C_i for other transport means. Let $T_1,...,T_p$ be another group of

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stations, from which passengers are transported to C_i by different transport means (e.g. by buses). Let traveling times from T_k to C_i be equal to b_{ik} for $i = 1,...,m, k = 1,..., p$ and let $y_1,..., y_p$ be departure times from stations $T_1,...,T_p$. Let $b_i(y) \equiv \max_{1\le k\le p}(b_{ik} + y_k)$. Then the earliest time for departure from station C_i is equal to $\max\{a_i(x), b_i(y)\}$. If $a_i(x) \ne b_i(y)$ for some i, it is evident that unnecessary waiting times occur. To minimize the waiting times at stations C_i we require to synchronize (if possible) the departure times in such a way that $a_i(x) = b_i(y)$ for all $i = 1,...,m$. We assume further that the departure times cannot be chosen arbitrarily, but within prescribed time intervals, i.e. that $\underline{x} \le x \le \overline{x}$, $\underline{y} \le y \le \overline{y}$ for given $\underline{x}, \overline{x}, \underline{y}, \overline{y}$. Therefore we have to find out whether such synchronized departure times exist and if the answer is positive, find a feasible (x, y), which lies within the prescribed bounds and satisfies the equalities $a_i(x) = b_i(y) \forall i \in \{1,...,m\}$. Let us note that if there is no connection between some stations C_i and S_j or T_k, or if it is not necessary to wait for the passengers from some station S_j at some station C_i, we can include formally such situations in the model by setting the corresponding travelling times a_{ij}, b_{ik} equal to $-\infty$ (note that for practical calculations, it is evidently possible to choose the corresponding coefficients as sufficiently low finite negative numbers). For similar reasons, we can assume w.l.o.g. that $n = p$. In some situations it may be given some recommended departure times $\hat{x}_j, \hat{x}_k$. It is required to find such x, y, which satisfy the given system $a_i(x) = b_i(y), \forall_i \in I, (\underline{x}, \underline{y}) \le (x, y) \le (\overline{x}, \overline{y})$ and minimize the Tshebyshev distance

$$\left\| (x, y) - \left(\hat{x}, \hat{y}\right) \right\| = \max_{j\in J}\left(\max\left(\left| x_j - \hat{x}_j \right|, \left| y_j - \hat{y}_j \right| \right) \right).$$

Let us note that the system $a_i(x) = b_i(y)$ in Example 1.1 can be considered as a system with the same variables x, y on both sides of the equations, since we can assume that variables y_k enter the left hand sides with sufficiently small coefficients a_{in+k} as $a_{in+k} + y_k$ and similarly that x_j enter the right hand sides with sufficiently small coefficients b_{in+j} as $b_{in+j} + x_j$. Therefore we will consider in the sequel the following simplified version of the optimisation problem with the same variables on both sides:

$$\left\| x - \hat{x} \right\| = \max_{j \in J} \left| x_j - \hat{x}_j \right| \to \min \qquad (1.1)$$

subject to

$$a_i(x) = b_i(x) \forall_i \in I \qquad (1.2)$$

$$\underline{x} \le x \le \overline{x} \qquad (1.3.)$$

where
$I = \{1,...,m\}, J = \{1,...,n\}, a_i(x) = \max_{1 \le j \le n} (a_{ij} + x_j), b_i(x) \equiv \max_{j \in J} (b_{ij} + x_j)$, and $\hat{x}, \underline{x}, \overline{x} \in R^n$ are given.

2. THE ALGORITHM

Let M denote the set of feasible solutions of problem (1.1) – (1.3). It was proved in [1] that if $M \ne \emptyset$, then set M contains always the maximum element $x^{\max}$ i.e. the element with the properties: $x^{\max} \in M$ and $x \le x^{\max} \forall x \in M$. In [1] is presented an $O(m^5 n^5)$ algorithm for finding $x^{\max}$. Therefore we will assume in the sequel that $M \ne \emptyset$ and we have element $x^{\max}$ at the disposal. The main idea of the following algorithm solving (1.1) – (1.3) is as follows? we begin with the element $x^{\max}$ and decrease the components of $x^{\max}$ in such a way that the value of the objective function is decreased and we stay at the same time in the set M, i.e. the idea is the same as in the feasible directions algorithms in convex optimization, but the set of the feasible solutions M is in general non-convex. Let us introduce the following notations:

$$f(x) \equiv \left\| x - \hat{x} \right\|,$$

$$F(x) \equiv \left\{ j \in J; f(x) = \left| x_j - \hat{x}_j \right| \right\},$$

$$J^+(x) \equiv \left\{ j \in J; x_j \rangle \hat{x}_j \right\},$$

$$J^-(x) \equiv J \setminus J^+(x),$$

$$L_i(x) \equiv \{ j \in J; a_i(x) = a_{ij} + x_j \} \forall_i \in I,$$

$$R_i(x) \equiv \{ j \in J; b_i(x) = b_{ij} + x_j \} \forall_i \in I,$$

$$V(x) \equiv \{ j \in J; x_j = \underline{x}_j \}.$$

ALGORITHM I.

1) $z := x^{\max}, D(z) := F(z)$;

2) $I_L(z) := \{i \in I; L_i(z) \not\subseteq D(z) \,\&\, R_i(z) \subseteq D(z) \}$;

3) $I_R(z) := \{i \in I; L_i(z) \subseteq D(z) \,\&\, \mathrm{R}_i(z) \not\subseteq D(z)\}$;

4) If $I_L(z) \cup I_R(z) = \varnothing$, go to 6;

5) $D(z) := U_{i \in I_L(z)} L_i(z) \cup U_{i \in I_R(z)} R_i(z) \cup D(z)$, go to 2;

6) $D(z)$ is the set of indexes of decreased variables at point $z = x^{\max}$, STOP.

ALGORITHM II.

1) $z := x^{\max}$;

2) Find $D(z)$ using **ALGORITHM I.;**

3) If $f(z) = 0$, go to 8;

4) If $J^-(z) \cap F(z) \neq \varnothing$, go to 8;

5) Set for $t \geq 0$:

$x_j(t) := z_j - t \forall_j \in D(z), x_j(t) := z_j \forall_j \in J \setminus D(z)$;

6) Increase *t* until for some $t \rangle 0$ one of the following situations occurs:

(a) $f(x(t)) = \max_{j \in J \setminus D(z)} \left| z_j - \hat{x}_j \right|$,

(b) $L_i(x(t)) \neq L_i(z)$ for some and $i \in I$,

(c) $R_i(x(t)) \neq R_i(z)$ for some and $i \in I$,

(d) $J^+(x(t)) \neq J^+(z)$,

(e) $f(x(t)) = \hat{x}_j - x_j(t)$ for some $j \in D(z) \cap J^-(z)$,

(f) $V(x(t)) \neq V(z)$

Denote the value $t \rangle 0$, at which one of the cases (a) – (e) occurs for the first time by τ;

7) $z := x(\tau)$, go to 2;

8) $x^{opt} := z$ (i.e. z is the optimal solution of (1.1) – (1.3)), STOP.

We will describe step 6 of ALGORITHM II in detail. First let us note that we have for all $t \in [0, \tau)$:

$f(x(t)) = f(z) - t, F(x(t)) = F(z), L_i(x(t)) = L_i(z), R_i(x(t)) = R_i(z),$
$J^+(x(t)) = J^+(z), D(x(t)) = D(z),$
$a_i(x(t)) = a_i(z) - t if L_i(z) \subseteq D(z), a_i(x(t)) = a_i(z)$ otherwise, and similarly
$b_i(x(t)) = b_i(z) - t if R_i(z) \subseteq D(z), b_b(x(t)) = b_i(z)$ otherwise.

(a) Let $\gamma \equiv \max_{j \in D(z)} \left| z_j - \hat{x}_j . \right|$ $F(x(t)) \neq F(z)$ if $f(x(t)) = \gamma$, i.e. if $f(z) - t = \gamma$, i.e. if $t = t^{(1)} \equiv f(z) - \gamma$.

(b) Let $\alpha_i \equiv \max_{j \notin D(z)} (a_{ij} + z_j)$. Then $L_i(x(t)) \neq L_i(z)$ if $i \in \widetilde{I} \equiv \{i \in I; L_i(z) \subseteq D(z)\}$, and $a_i(x(t)) = \alpha_i$, i.e. $a_i(z) - t = \alpha_i$, i.e. $t = t_i^{(2)} \equiv a_i(z) - \alpha_i$.
This situation occurs for the first time if $t = t^{(2)} \equiv \min_{i \in \widetilde{I}} t_i^{(2)}$.

(c) Let $\beta_i \equiv \max_{j \in D(z)} (b_{ij} + z_j)$. Then $R_i(x(t)) \neq R_i(z)$ if $i \in I^* \equiv \{i \in I; R_i(z) \subseteq D(z)\}$, and $b_i(x(t)) = \beta_{i,}$ i.e. $b_i(z) - t = \beta_i$, i.e. $t = t_i^{(3)} \equiv b_i(z) - \beta_i$. This situation occurs for the first time if $t = t^{(3)} \equiv \min_{i \in I^*} t_i^{(3)}$.

(d) $J^+(x(t)) \neq J^+(z)$ if for some $j \in (D(z) \cap J^+(z)), x_j(t) = \hat{x}_j$, i.e. if $z_j - t = \hat{x}_j$ so that $t = t_j^{(4)} \equiv (z_j - \hat{x}_j)$. This situation occurs for the first time if $t = t^{(4)} \equiv \min_{j \in (D(z) \cap J^+(z))} t_j^{(4)}$.

(e) $f(x(t)) = \hat{x}_j - x_j(t)$ for some $j \in D(z) \cap J^-(z)$ occurs if $f(z) - t = \hat{x}_j - z_j + t$, i.e. if $t = t_j^{(5)} \equiv (f(z) - \hat{x}_j + z_j)/2$. This situation occurs for the first time if $t = t^{(5)} \equiv \min_{j \in (J^-(z) \cap D(z))} t_j^{(5)}$.

(f) $V(x(t)) \neq V(z)$ if for some $j \in D(z)$ the equality $x_j(t) = \underline{x}_j$ holds. It means that $z_j - t = \underline{x}_j$, i.e. $t = t_j^{(6)} \equiv z_j - \hat{x}_j$. This situation occurs for the first time if $t = t^{(6)} \equiv \min_{j \in D(z)} t_j^{(6)}$.

Value τ is then defined as follows: $\tau \equiv \min_{1\le k\le 6} t^{(k)}$. We set as usual per definition minimum over the empty set equal to ∞ and maximum over the empty set equal to $-\infty$. Since always $t^{(6)}\rangle 0$ holds, it follows that $\tau\rangle 0$.

3. THEORETICAL BACKGROUND OF THE ALGORITHM

Lemm 3.1 Let z be a current upper bound in ALGORITHM II. Let $x \in M, x \le z, p \in D(z), x_p \rangle x_p(\tau)$. Then there exists an index $i_0 \in I$ such that either $p \in R_{i0}(z)$ and there exists $k \in D(z) \cap L_{i0}(z), k \ne p$ such that $x_k \rangle x_k(\tau)$, or $p \in L_{i0}(z)$ and there exists $k \in D(z) \cap R_{i0}(z), k \ne p$ such that $x_k \rangle x_k(\tau)$.

Proof:
We will prove only the first case, the second case can be proved by analogy.
Since $x_p \in (x_p(\tau), z_p\rfloor$, there exists $t_0 \in [0,\tau)$ such that
$x_p = z_p - t_0 = x_p(t_0)$ holds and we have:

$$a_{i0}(x(t_0)) = a_{i0}(z) - t_0 = b_{i0}(x(t_0)) = b_{i0}(z) - t_0 = b_{i0p} + z_p - t_0 = b_{i0p} + x_p \le b_{i0}(x).$$

Let us assume that $x \in M$, and $x_j \le x_j(\tau) \forall_j \in D(z) \setminus \{\mathrm{p}\}$ holds. Then we have:

$$a_{i0p} + x_p \le a_{i0p} + x_p(\tau) \langle a_{i0p} + x_p(t_0) \le a_{i0}(x(t_0)) = a_{i0}(z) - t_0$$

since $p \notin L_{i0}(z) = L_{i0}(x(t_0))$.
Further we have:

$$a_{i0j} + x_j \langle a_{i0j} + x_j(t_0) \le a_{i0}(z) - t_0 \forall_j \in D(z) \setminus \{\mathrm{p}\}$$

and

$$a_{i0j} + x_j = a_{i0j} + z_j \langle a_{i0}(z) - t_0 \forall_j \in J \setminus \mathrm{D(z)}.$$

Therefore $a_{i0}(x) \langle b_{i0}(x)$ so that $x \ne M$, which is a contradiction. This contradiction completes the proof.

Theorem 3.1 *The following implication holds:*

$$(x \not\le x(\tau) \,\&\, \mathrm{x} \in \mathrm{M}) \Rightarrow \mathrm{f(x(\tau))} \le \mathrm{f(x)}.$$

Proof:
Let $x_p \rangle x_p(\tau)$ for some $p \in J$. It follows from Lemma 3.1 that there exists an index $k_0 \in (D(z \setminus \{p\})$ such that $x_{k_0} \rangle x_{k_0}(\tau)$. Since k_0 was included into $D(z)$ by ALGORITHM I earlier than *p*, there exists an index $i_1 \in I$ such that either $k_0 \in R_{i_1}(z), k_0 \notin L_{i_1}(z)$ or $k_0 \in L_{i_1}(z)$ and there exists an index $k_1 \neq k_0, k_1 \in D(z)$ such that $x_{k_1} \rangle x_{k_1}(\tau)$, and so on. After at most $n-1$ steps we come to a situation that for some $r \leq (n-1)$ we have $x_{k_r} \rangle x_{k_r}(\tau), k_r \in (D(z) \setminus \{k_0,...,k_{r-1}\}) = D(z) \setminus F(z)$ so that it must be $k_r \in F(z), x_{k_r} \rangle x_{k_r}(\tau)$ and therefore $f(x) \rangle f(x(\tau))$. This completes the proof.
It follows that smaller values of *f* can be obtained for $x \in M$ only if $x \leq x(\tau)$ holds. Therefore we use after each iteration of ALGORITHM II $x(\tau)$ as a new upper bound and repeat the procedure.
We will estimate now the computational complexity of ALGORITHM I and II. On each iteration of ALGORITHM I we have to determine $2m$ sets $L_i(z), R_i(z)$. Determining of each of the sets needs O(n) operations, which makes together $O(mn)$ operations in each iteration. Since after each iteration at least one of variables $x_j \ \ j \in J$ enters the set D(z) and will never leave it until the end of ALGORITHM I, we obtain that the maximum number of operations of ALGORITHM I is $O(mn^2)$.
Let us investigate now the complexity of ALGORITHM II. In each iteration with $\tau = t^{(1)}$ at least one of the variables enters set $F(z)$ and remains in it until the end of calculations. Therefore we can have at most n such iterations. We will call such iterations macro-iterations. If on some iteration either $\tau = t^{(5)}$ or $\tau = t^{(6)}$, the calculations are finished and we obtained the optimal solution. Therefore we can have at most one such iteration. It remains to estimate the number of iterations with $\tau = t^{(k)}$, where $k \in \{2,3,4\}$, which can occur between any two successive macro-iterations. Each of at most $n-1$ variables in $D(z) \setminus F(z)$ can cause successively at most $m(n-2)$ iterations with either $\tau = t^{(2)}$ (i.e. a change of a set $L_i(z)$) or $\tau = t^{(3)}$ (i.e. a change of a set $R_i(z)$), and after that each of the variables can cause one iteration with $\tau = t^{(4)}$ (i.e. set $J^+(z)$ changes). This makes together at most $O(mn^2)$ iterations. Since each iteration uses in ste (2) ALGORITHM I which complexity $O(mn^2)$, we obtain finally that the complexity of ALGORITHM II is $O(m^2n^4)$.
Let us remark that ALGORITHM II needs at the beginning the maximum element of *M*, which can be found by making use of the algorithm from [1], which has complexity $O(m^5n^5)$.

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DECISION MAKING IN SLOVAC REPUBLIC ARMED FORCES CONDITIONS

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1. INTRODUCTION

Dynamical development of PC significantly affects military decision processes and above this the mathematical methods began to prevail in evaluation decision. According to above mentioned in our management department, especially in management operation group leaded by prof. Žák we are centered on decision support systems /DSS/ with available program package. In the present time we are working on our own DSS packages for practical army applications.

In the present time we are solving scientific problem, which is centered on military suitable DSS-s and their using for finding military decisions. We are assuming that it will be different moduls with using operation analysis methods. In the next text we are describing one of typical example queuing model from many others.

2. QUEUING MODELS

This waiting line module accommodates seven types of queuing models. In each of the seven models, the arrivel patern is assumed to be Poisson and queuing discipline is first come first serve. The models include: single and multiple servers; Constant, Erlang, and General service distribution; finite and infinite queue length; and finite calling population (combat techniques repair problem). The output consists of several performance measures such as average queue length, average wait in the queue, etc.

3. PROGRAM SOLUTION OF M/M/K FCFS/∞/∞ QUEUING MODELS

The practical application of queuing analysis models can be demonstrated in using optimisation of preparing processes, or repair combat techniques.

- Probability k- busy servers:

$$P_k = \frac{\frac{\alpha^k}{k!}}{\sum_{k=0}^{n-1} \frac{\alpha^k}{k!} + \frac{\alpha^n}{n!(1-\frac{\alpha}{n})}}$$

- Average number of requirements in the queuing system:

$$L_s = L_q + \alpha$$

- Average number of requirements waiting in line:

$$L_q = \frac{\lambda.\mu}{(n-1)!(n\mu - \lambda)^2}$$

- Average time of requirements in the queuing system:

$$W_s = W_q + \frac{1}{\mu}$$

- Awerage waiting time before served:
- $$W_q = \frac{L_q}{\lambda}$$

Now, when we consider the example regular revisions of combat techniques which arrives into repairment facility according to a Poisson process with arrival rate of 2 tank per day. We are suppose five servers with identical Exponential service distribution, mean service time is 0.5 tank per day. Let daily operating costs are 100 € for each server and breakdown costs are 4 500 € for one tank and day..

In order to solve the above problem we must load into PC our lerning program DSS. to obtain the „START-UP MENU“ screen first. Then we select „QUEUING PROGRAM“. Next , we move to the INPUT option and press <ENTER>. The program will ask for the model parameters as shown below.

≡ Input Edit Print File Solve Quit Setup

Problem title: REVISION of COMBAT TECHNIQUES

Is the calling population infinite: Y
Is the queue capacity infinite: Y
Enter number of servers: 5
Enter arrival rate (1/interarrival time): 2
Enter service rate (1/mean service time): 0.5

4. SOLVING AND RESULTS

In this stage we are prepared to the PC solving our task. Therefore, the pointer must be moved to the SOLVE option, in which we must select the Display Output sub-option. The PC will present the following report:

Problem title: REVISION of COMBAT TECHNIQUES

Analysis for model: M/M/S FCFS/infin/infin
Average number waiting in line : 2.22
Average number in service : 4.00
Average number in system : 6.22

Average time waiting in line : 1.11
Average time waiting in service : 2.00
Average time waiting in system : 3.11

Prob. Of no units in the system : 0.01299

Wish to see the probabilities of n units in the system ?: N
As shown in the output report, on average there are 6.22 combat systems in the repair system. Therefore, the fixed total cost because of breakdown combat techniques are 6.22 pcs . 4500 €/pcs = 27 990 €. Adding the cost of of 5 service servers (which amounts to 5x500€=2500€, the total costs are 30 490 €.
In order to determine the optimal number service servers, we must determine the total cost 6 servers, 7 servers, etc., and select the number of servers with the smallest total costs.
To solve the above problem, we must move the pointer to the EDIT option and after promt pressing of <ENTER>key several times to move the pointer to the „Enter number of servers. We need to change „5“ to „6“ and press <ENTER>. Next step is moving to the <SOLVE> option and pressing <ENTER>. After repeating a recording value „Average number in system“ for 7, 8, and 9 service servers we have following results:

No. Of service servers	Servers costs (€)	Average No. combat tech. in repair. system	breakdown costs	Total costs (€)
5	2500	6.22	27 990	30 490
6	3000	4.57	20 565	23 565
7	3500	4.18	18 810	22 310
8	4000	4.06	18 270	22 270
9	4500	4.02	18 090	22 590

As the number of service servers increases, increasing servers cost, average no. combat tech. in repair. system decreases which in turn decreases breakdown costs. The total cost therefore, decreases for a while then starts to increase.

5. CONCLUSIONS

- The optimal number of service servers is 8 with a total cost 22 270 €.and:

- The paper is example of contribution to practical using of DSS in decision making SR Armed forces.

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REPORTS

COMPARISON OF ECONOMETRICS TUITION AT UNIVERSITIES IN CZECH REPUBLIC AND SLOVAKIA[14]

MARTINA KUNCOVA - ANDREJ CHU

Abstract: University education is starting, especially latterly in conditions of European Union and international comparison possibility, to gain much more significance. The demand of European Union for the increase of the number of university students gives evidence of this. Another consequence is the increasing number of private universities and expansion of subjects and education at public universities. Wide spectra of specializations of the schools are often overlapping each other. The most remarkable is this trend in economic subjects, where it is possible to say that almost all public universities and heft of Czech private universities are offering at least one study program focused on economics. Within this trend it is often beyond power of educators to keep track on which schools are teaching particular subject and for which specializations. Nevertheless such a comparison could lead to consequential cooperation and education enhancement. Therefore we have decided to try, at least for guidance, to compare the tuition of quantitative methods at universities. In the first phase we were focusing on econometrics tuition. The principal aim of this paper is to state, where and at what level it is possible to study econometrics, and also which specializations have major differences. The research so far contains the comparison of data published on web pages of particular schools, faculties and/or departments. In next phase we assume to extend it by opinions of teachers and students, and also by comparison of another related subjects, such as operational research or statistics.

1. INTRODUCTION

Econometrics lies on the borderland of economics, mathematics and statistics. More plainly it is a quantitative economic discipline concerned with measuring and empiric testing of economic relations and dependencies. It is possible and necessary to use mathematical methods for modeling of economic regularities, predicting future development trends including estimation of absent information and for decision-making support within the scope of complex economic systems. Such methods are in especially probabilistic and mathematical statistics methods. Of the above it is resulting that this science discipline is connecting the knowledge of mathematics, statistics and economics. It is impossible to study econometrics without them – the tuition is therefore placed

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into later phases of university studies, i.e. master degree studies or later years of 5-year study programmes.

In term of tuition comparison we will target just the public universities, because it was discovered (by viewing the internet pages), that the relevant specializations are (in most cases) at private universities available only as bachelor degree, where the relevant subjects are not taught at such an advanced level (and in fact there was no success in search for a private university in Czech Republic offering tuition of econometrics).

First we will try to compare the situation of each country within its borders, i.e. at first in Czech Republic and then in Slovakia. At the end we will compare the tuition content and extent at schools in both countries.

2. TUITION OF ECONOMETRICS AT UNIVERSITIES IN CZECH REPUBLIC

According to information provided by Ministry of Education there are 26 public universities in Czech Republic at present time. Econometrics is taught at 17 of them (their names are in Czech because of the well known short titles):

ČVUT – České vysoké učení technické (Praha)
ČZU – Česká zemědělská univerzita (Praha)
MU – Masarykova univerzita (Brno)
MZLU – Mendelova zemědělská a lesnická univerzita (Brno)
OSU – Ostravská univerzita
SLU – Slezská univerzita (Opava)
TU – Technická univerzita v Liberci
UHK – Univerzita Hradec Králové
UJEP – Univerzita Jana Evangelisty Purkyně (Ústí nad Labem)
UK – Univerzita Karlova (Praha)
UPCE – Univerzita Pardubice
UPOL – Univerzita Palackého v Olomouci
UTB – Univerzita Tomáše Bati (Zlín)
VSB-TU – Vysoká škola báňská-Technická univerzita (Ostrava)
VŠE – Vysoká škola ekonomická v Praze
VUT – Vysoké učení technické (Brno)
ZČU – Západočeská univerzita v Plzni

In the most cases the subjects are actually concerned with specialization in later stages of university studies. Econometrics is obligatory at four universities already at bachelor degree – VŠE[15] Prague (specialization Economics), ČVUT

[15] Schools are named by their abbreviations

(Software Engineering in Economics), OSU (Mathematics Application in Economics) and VSB-TU Ostrava (Management and Decision Methods in Economics).
Already said above, econometrics is connected first of all to mathematics and statistics. In our comparison we have focused only on the subjects containing the word „econometrics" in their names or syllabi. It is probable that this science is also contained in other subjects (i.e. time series, advanced statistical methods, etc.), however the exploration of all such subjects would be beyond our powers.

Names of specializations, where the subject of econometrics is included as obligatory or obligatory optional, quite differ, so it is no possible to introduce all of them here (there are over 30 of them). Nevertheless it is possible to specify areas of orientations of these specializations: those are especially specializations in mathematics, statistics, economics, management, informatics, finance, insurance and economic policy. At ZČU in Pilsen and ČZU in Prague we can find most of the specializations where the econometrics subjects are obligatory (6 - both for the bachelor level and higher level). Five of such specializations can be found at VŠE in Prague. In term of specializations where the subjects are included in blocks of obligatory subjects, the largest section is at ZČU (9 specializations) and VŠB-TU Ostrava together with OSU (4 specializations).

In terms of numbers of econometrics subjects taught at all schools in total, we can find most of them at the UK (6 subjects), followed by VŠE (3 at one faculty) and ZČU (3 at different faculties). In term of tuition we have decided to pick some topics and to put together a table according to this fundamental. This table (Table 1) shows which topics are taught at particular schools in reffered-to subjects (according to syllabi). This comparison shows that the following elementals are taught at all schools:

1. The construction of standard linear regression model, its estimation using method of least squares (or method of generalized least squares)
2. The research of heteroscedasticity, autocorrelation and multicollinearity.
3. The econometric macromodels and their applications (usually connecting elementals 1, 2)

ČVUT, ČZU, UK, VSB-TU, VŠE and ZČU are extending the models by

1. The simultaneously dependant system, its identification and solving using the method of two- and three-degree least squares,
2. Reduced form of the system
3. Forecast error.

4. In subjects taught at UK, VSB-TU, VŠE and ZČU we can find special parts concerned with
5. Extended time series analysis
6. Advanced econometric marcomodels.

These universities can be recommended in case of deeper interest in econometrics studies.

3. TUITION OF ECONOMETRICS AT UNIVERSITIES IN SLOVAKIA

There are 20 public universities in Slovakia, according to information provided by Ministry of Education. Nevertheless there is lack of detailed and well-arranged information about tuition on the internet pages of some of them. Therefore we were able to obtain information about econometric subjects only from 5 of them (titles are in Slovak):

EU – Ekonomická univerzita in Bratislava
SPU – Slovenská poľnohospodárka univerzita (Nitra)
UK(B) – Univerzita Komenského Bratislava
UMB – Univerzita Mateja Bela (Banská Bystrica)
ŽU – Žilinská univerzita

Econometrics subjects are taught: at EU at bachelor level, specializations Accounting and Economic Informatics and Managerial Decisions and Information Technology; at UK(B) (Economic and Financial Mathematics) and SPU (Quantitative Methods in Economics). Specializations at later stages of university studies are similar to those in Czech Republic, maybe with little outweigh of specializations focused on finance, banking and economics (according to small number of schools and specializations). The leader in number of specializations, where econometrics subjects are obligatory, is EU (6 specializations). When considering obligatory optional subjects, the leader is UK(B) and UMB (both having 3 specializations). In general most econometrics subjects taught we can find at EU (4 subjects). In reference to the structure of studies and tuition of particular topics (see Table 2), all schools are covering certain wider econometric elementals with models applications. EU and UMB are offering in addition also extended time series analysis. EU in Bratislava appears to be best choice in terms of econometrics tuition in Slovakia.

4. CONCLUSION

The comparison of econometric tuition at the universities should certainly result from more detailed data than those available at web sites of the universities. It is questionable, however, if it would be possible to obtain such data otherwise than by direct participation in tuition –that is not actually accessible for us though. But after all we can claim that deep of tuition of econometric subjects is best at „major" or „significant" universities, i.e. UK, VŠE, VSB-TU and ZČU in Czech Republic and EU in Slovakia. It is hard to order these universities and say which is better and worse in teaching econometrics (it definitely depends on particular specialization or subject), but taking into consideration the tuition wideness (according to topics and specializations) and real applications, we are able to arrange following chart:

1. UK - Univerzita Karlova (Charles University in Prague)
2. ZČU – Západočeská univerzita v Plzni (University of West Bohemia in Pilsen)
3. EU – Ekonomická univerzita v Bratislavě (Economic University in Bratislava)
4. VŠE – Vysoká škola ekonomická v Praze (University of Economics in Prague)
5. VSB-TU – Vysoká škola báňská-Technická univerzita Ostrava (Technical university of Ostrava)

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Tables are available by e-mail on request.