QUANTITATIVE METHODS IN ECONOMICS

Multiple Criteria Decision Making XX

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CONTENT

MATHEMATICAL MODEL FOR THE RE-ALLOCATION OF PRODUCT LOCATIONS IN A SPECIFIC SPATIAL CONFIGURATION OF AN ORDER PICKING AREA ZONES

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Abstract
Order picking is the most labor intensive warehouse process that is taking place in a typical warehouse systems. Because of its significant contribution to the overall warehouse operation costs optimization of order picking processes is a permanent subject of researchers' interests. In this paper we are exploring a specific spatial configuration of an order picking area and historical data of realized order picking lists with the goal to re-optimize existing zone allocation of products over the order picking area regarding the minimization of distances required by order pickers to fulfill all picking tasks. We used frequency of joint ordering of pairs of products in the past as a parameter whose importance in re-allocation of products is tested. For that purpose, we developed an Integer Quadratic Programing mathematical model. Eventually, results of numerical tests showed that under some circumstances proposed parameter may influence the reduction of order-picker travelling distances.
Keywords: *Order-picking, Integer Quadratic Programing model, Warehouse*

JEL Classification: C61
AMS Classification: 90B06, 90C10, 90C20.

1 INTRODUCTION

Order-picking (OP) is the process of retrieving products from storage (or buffer areas) in response to a specific customer request (de Koster et al., 2007). Depending on the implemented storage technology OP can be realized in numerous ways (man-to-goods, goods-to-man, batch OP, zoned OP, etc. For more details we refer interested reader to Djurdjevic, 2019; Gu et al, 2010; Dallari et al., 2009; Hompel and Schmidt, 2007; Wascher, 2004) Nevertheless, due to its simplicity and robustness probably the most widely spread is sequential men-to-goods OP strategy which means that for every customer demand order-picker goes from one to another location of products within a warehouse and picks up a required quantity of products. Order picker starts from the starting location and finishes and the end location which could be spatially separated. At the end location shipment is prepared (checked, consolidated, labeled etc.) for the next step in the delivery process. Routing of pickers through warehouse aisles depends on the selected routing strategy (S-shape, largest gap, return, etc.) and it can significantly influence order pickers' travelling distances.

Because OP is the most demanding, the most labor intensive and accordingly the costliest activity in a warehouse, with the contribution of around 50% in the warehouse operating costs (Frazelle, 2001), the OP process has been potential for the optimization from different points of view at different decision-making levels. However, due to the limited space and numerous papers dealing with order picking related problems, for detailed overview of considered problems, used parameters and implemented solution techniques we suggest interested reader to the paper van Gils et al (2018).

In this research our goal was to examine a possible rationalization potential of order pickers' movements by utilizing historical customers' demand data, existing layout and positions of starting and ending locations within a warehouse. In the following section we give more details about the considered problem, as well as mathematical model for solving it. In Section 3 we give settings and results of conducted numerical experiments. Eventually, in Section 4 we analyze results from numerical experiments and give conclusions.

2 PROBLEM DESCRIPTION AND A SOLUTION MODEL

In this research we tested the potential for reducing order pickers' movement by re-allocating positions of products in the existing warehouse layout. Namely, depending on the historical data about customer orders, and the forecasted demand, storage locations of products are changed whenever it is evident that current allocation induces unnecessary movement. Usually, the allocation is realized according to an expected frequency of products on customers' order lists in the following period. However, product weight, packing suitability or some other product feature may also be considered in defining its position. In this research beside the order frequency we utilized the frequency of joint occurrence of pairs of all products in the previous period.

The layout of warehouse aisles and the positions of the starting location and two considered end locations are given at the figure 1. Beside that, we supposed that the S-shape order picker routing strategy is applied. Accordingly, in the case of end location 1 an order picker ends its order requirement by reaching either the point 4 (when all ordered products' locations are within aisles I and II) or the point 8 (when at least one location of ordered products is located within aisles III or IV). Similarly, in the case of the end location 2 we assumed that order pickers end order picking lists by reaching points 6 or 8. Because we assumed points 4 or 6 as first possible point on which order pickers end their order lists, we refer a part of warehouse before that point as the zone I, while the part of the warehouse after that point is referred as the zone II.

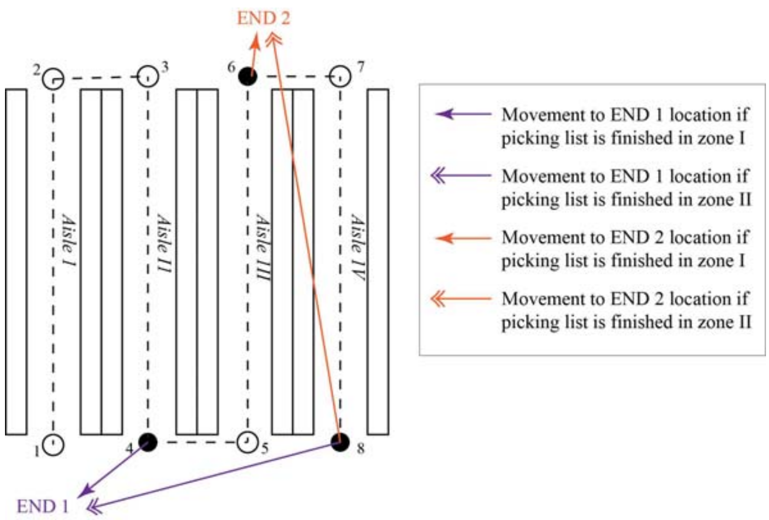

Figure 1 Considered warehouse layout and positions of starting and ending locations

Based on previously said it is obvious that allocating frequent product in the zone II will cause an excessive movement of order pickers. Like we said, usually the products are located along an imaginary S-shaped line starting from the starting location with most frequent product and ending near the point 8 with the least frequent product. However, respecting only frequencies of products might induce unnecessary movement since some of low frequency products may always be ordered with some high frequency products, meaning that order pickers will have to enter the zone II. Therefore, the goal of our research is to get an insight of the usefulness of the joint order frequency as a parameter in allocating products to considered zones.

In order to achieve the defined goal, we used a mathematical model with the idea to separate all products to zones I and II in such a way that the sum of all joint order frequencies of products in different zones is minimized. In that way we minimize the necessity of order pickers to go to zone II after reaching the end point of zone I. The mathematical model is developed in the form of Integer Quadratic Programming (IQP) model as:

$$\min \sum_{(i,j)\in\{(i,j):f_{ij}>0\}} \left(I_i \cdot II_j + I_j \cdot II_i\right)\cdot f_{ij} \qquad (1)$$

subject to

$$I_i + II_j = 1 \qquad \forall i \in A \qquad (2)$$

$$\sum_{i\in A} I_i = Q^I \qquad (3)$$

$$\sum_{i\in A} II_i = Q^{II} \qquad (4)$$

$$I_i, II_i \in \{0,1\} \qquad (5)$$

Where *A* represents a set of products in a warehouse in the following period and where we use the following parameters and variables:

Parameters

f_{ij} - frequency of joint occurrences of products *i* and *j*, $i,j\in A$, on the same order list in historical data

Q^I - capacity of the zone I (as a number of keeping units)

Q^{II} - capacity of the zone II (as a number of keeping units)

Variables

$$I_i = \begin{cases} 1 \text{ - if product } i \text{ is allocated to zone I} \\ 0 \text{ - otherwise} \end{cases}$$

$$II_i = \begin{cases} 1 \text{ - if product } i \text{ is allocated to zone II} \\ 0 \text{ - otherwise} \end{cases}$$

Objective function (1) minimizes the sum of all frequencies of joint occurrences of product pairs on order picking lists. In that way number of entries in zone II is minimized as well. Product of two binary variables provides that only situations when products *i* and *j* are located in the zone I and the zone II, respectively, are respected. Moreover, the sum of two products in brackets is used to cover all situations of placing products in different zones. It should be also noted that not all pairs of products are respected because some pairs of products never occur jointly on the same order picking list and because of the nature of objective function that pair will be selected to be placed in different zones. By summing only pairs (*i,j*) where $f_{ij} > 0$ beforementioned shortcoming is avoided.

Constraints (2) forces products to be allocated either to the zone I or the zone II. Constraints (3) and (4) limit numbers of products allocated to zones I and II to their capacities Q^I and Q^{II}. Eventually, constraints (5) define binary nature of variables I_i and II_i.

3 NUMERICAL EXPERIMENTS

As it has been said earlier, the goal of this research was to investigate possible effects of including the frequency of joint occurrences of products at the same order lists in the cases of using S-shape moving strategy and warehouse layout that can be considered as the one at the figure 1. In order to fulfill that goal, we chose a performance measure, called number of zone II entries, as the most relevant. Selected measure represents a number of order list for whose fulfillment order picker had to enter the zone II and therefore needed to pass the longest path, i.e. to finish that route at the point 8.

In order to compare allocation strategy with included frequencies of joint ordering of products (referred from this point as *Modified allocation* strategy) and the allocation strategy based only on products' ordering frequencies (referred as *Frequency based allocation* strategy) we used a set of data of a Serbian food distribution company with 220 storage locations, i.e. 55 locations per aisle, and the same number of products. We selected all generated ordering lists from three months and analyzed them in order to get values for f_{ij}, as well as to get ordering frequencies (F) of product in given period.

Moreover, in order to investigate the influence of zones sizes ratio we considered both end location positions because the end zone location 1 implies zones of equal sizes (Figure 2a), and the end zone location 2 implies that the zone I is three times larger than the zone II (Figure 2b). In the case of ending location 1 and the ordering frequency-based allocation the first 110 products with the largest F values are located to the zone I, and the rest to the zone II. Similarly, in the case of the ending location 2 and the ordering frequency-based allocation, the first 165 the most frequent products is located to the zone I, and the rest to the zone II.

For the case of modified allocation strategy, we used the presented IQP model for allocating products to zones. Model is developed in Python 3.6 and solved by using GUROBI 8.1 API on Intel i7-4712MQ@2.3GHz CPU with 12GB of RAM under Windows 8.1 OS. It should be noted that given model could not be solved to optimality even after 24h of CPU's work. However, the incumbent solution did not change after the first 10 minutes which is the reason why we used solutions after 10 minutes as the model solutions.

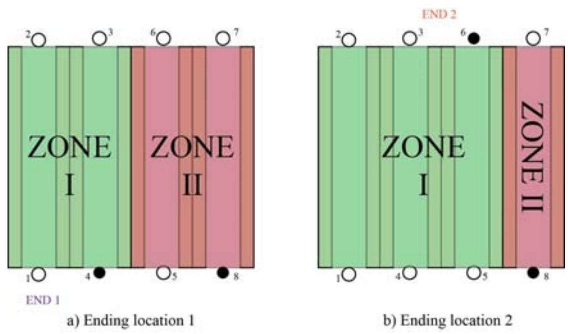

Figure 2. Alternative sizes of zones I and II

After the allocation of products in modified strategy was realized for every order list in history data we checked, based on the set of products in a list, if an order picker must enter the zone II for picking up at least one product. Number of such entries is given in the table 1.

Table 1. Number of entries in the zone II for selected three months

Month	Number of OP lists	Ending location 1		Ending Location 2	
		Frequency based allocation	Modified allocation	Frequency based allocation	Modified allocation
1	8290	2042	1974	210	145
2	8249	1603	2362	134	117
3	9796	1784	2571	198	107
Total	26335	5429	6907	542	369

According to the results in table 1 it is obvious that the ratio of zones' sizes has an important role in the possible reduction of order picker movements. In the case of equal zone sizes, i.e. for the end location 1, the frequency-based allocation strategy had fewer zone II entrances. Nevertheless, for one considered moth Modified allocation strategy turn out to be with better performance. On the other side, in the case when the zone I is larger Modified allocation strategy gave better results for all considered months. We believe that the reason for that lies in the fact that was our initial assumption, i.e. that some low frequency (F) products are usually ordered jointly with some high frequency product and therefore their placement in zone I is preferable.

Eventually, it should be stated that based on the obtained results it could be said that joint ordering frequency has a potential of improving order pickers' performances in considered circumstances. However, due to simplified view to the allocation process, a deeper analysis which must include other products' features related to allocation, must be realized prior to including mentioned parameter as allocation criterion.

4 CONCLUSIONS

In this paper we give a report about the research conducted with the goal to investigate the possibility of including join ordering frequency of products on order lists as a relevant parameter in defining the allocation of products within the order picking area divided in two zones with S-shape pickers' moving strategy. For determining allocation of products in such warehouse organization we developed a Quadratic Integer Programming mathematical model. Comparison with an allocation strategy which implies that only frequency of occurrence of a product is considered, proved that joint ordering frequency has a potential to reduce order pickers' movements. However, it should be mentioned that the next step in the allocation process implies allocation of products to storage places, and not to zones, and that it includes respect of other important features of products, such as wight, stacking suitability etc.. Accordingly, their involvement in the zone allocation would be one direction for the further research in solving the considered problem.

Other potential for improving the quality of zone allocation decisions based on the criterion considered in this research is generalization of number of considered zones for allocation of problems, and the linearization of proposed objective function of mathematical model.

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STOCK PORTFOLIO SELECTION VIA MEAN-SEMIVARIANCE MODEL

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Abstract

This article deals with an increasingly popular topic – portfolio selection. Thousands and thousands of people face this problem all over the world. Range of investment instruments is increasingly wider. Deciding on the composition of the portfolio thus becomes more complex. Well-chosen methodological support in such a decision-making process can play a very positive, and of course, important role. Well-known mean-variance model can undoubtedly be one of them. However, this approach has some drawbacks, especially from the perspective of the implemented risk measure. This is the main reason why it was modified to the mean-semivariance form. The specifics of this risk measure, and the whole model, are declared. The mathematical model can be generally modified for practical purposes. Such a mean-semivariance model is applied for selecting an investment portfolio from the stocks traded on the Czech capital market. The results are analyzed and confronted with the output of more commonly used mean-variance approach from the algorithmic-application perspective.

Keywords: *Mean-semivariance, mean-variance, portfolio selection, return, risk, stock.*

JEL Classification: C44, C61, G11
AMS Classification: 90B50, 90C90

1 INTRODUCTION

A portfolio selection problem is still very life in society. Many people must make an investment decision over the whole world. Such a decision can significantly affect their live. Therefore, a decision making process should be reasonably sophisticated. To make this, the application of one of the mean-risk models seems very appropriate.

It is no doubt, that Markowitz mean-variance framework is widely adopted by practitioners. However, there is a criticism how its risk is measured. The variance concept assumes the same investor's attitude to upside and downside return volatility. This proves too simplistic which is not only discussed by Swisher and Kasten (2005). The second non-negligible limit for a proper application of mean-semivariance concept is an assumption of the return normality which can be very complicated in a portfolio making practice full of distribution asymmetry and skewness, as deeply discussed by (e.g.) Jansen and Vries (1991). If these two essential requirements are not met the results can be misleading. To eliminate these basic drawbacks, a principle of semi-deviation risk measure penalizing only a downside variability has been studied. One of them is semivariance concept which has already discussed by Markowitz (1952). This concept is more sensible to the distribution asymmetry and penalization of two-sided deviation.

The aim of this article is to demonstrate a strong application power of the mean-semivariance concept against to the basic mean-variance model. The differences among both concepts are discussed from the practical and also theoretical perspective. For analysis, a portfolio from the stocks traded on the Czech exchange RM-SYSTEM is made. This segment of Czech capital market is suitable for "smaller" investors. The trading is supported by a user-friendly online trading system. Then, the article can be beneficial to a wide range of users which is certainly a commendable research intent. In addition, the paper should serve as a methodological support for a real-life portfolio selection. Finally, besides applying mean-risk models to the Prague Stock Exchange (e.g. Janková, 2019), the RM-SYSTEM is neglected. This makes the analysis unique and beneficial.

The article is structured as follows. After the introduction, mean-variance and mean-semivariance models are described. Their differences are discussed. In the application part, the stock portfolios are made via both optimization models. The results are compared. Summary of the article contribution is concluded by the outline of the interesting ideas for further research.

2 MEAN-RISK MODELS FOR A PORTFOLIO SELECTION

After the introduction, two selected mean-risk models are necessarily described. Firstly, a notoriously known Markowitz mean-variance model is briefly introduced. Then, another possible risk measure is discussed through a mean-semivariance model.

2.1 Mean-variance model

Mean-variance model is developed by professor H. Markowitz (Markowitz, 1952; Markowitz, 1959). Risk is expressed by a well-known statistical concept, i.e. variance, or standard deviation. The model can be formulated as follows

$$\begin{aligned} &\min \quad \mathbf{x}^{\mathbf{T}}\mathbf{V}\mathbf{x} \\ &\mathbf{x}^{\mathbf{T}}\mathbf{r} \geq M \\ &\mathbf{e}^{\mathbf{T}}\mathbf{x} = 1 \\ &\mathbf{x} \geq \mathbf{0} \end{aligned} , \qquad (1)$$

where $\mathbf{e}^{\mathbf{T}} = (1,1,...,1)$, $\mathbf{x} = (x_1, x_2, ..., x_n)^T$ including variables $x_i, i = 1,2,...,n,$ representing the weight (share) of the *i*-th asset in portfolio, $\mathbf{r} = (r_1, r_2, ..., r_n)^T$ is a vector with the elements r_i, $i = 1,2,...,n,$ expressing an expected (mean) return of the *i*-th asset, $\mathbf{V}$ is the $n \times n$ matrix including the covariance between the return of *i*-th and *j*-th asset, *M* denoting the required expected return. The model minimizes a portfolio risk under a lower limit for the required return. Condition $\mathbf{e}^{\mathbf{T}}\mathbf{x} = 1$ defines the vector **x** as a portfolio, the non-negativity conditions $\mathbf{x} \geq \mathbf{0}$ prohibit a short selling.

Quadratic programming model (1) is quite easily solved. Thus, the portfolio can be effectively made for the investor. However, this concept is burdened by some (unfortunate) assumptions. One of them says that the positive deviation is penalized which is naturally considered undesirable. Further, a quadratic utility function of the investor is assumed. Moreover, the asset returns do not usually follow a normal distribution. Many researches have declared an asymmetry and more significant skewness of the return distribution (e.g. Tobin, 1958; Prakash et al., 2003). To eliminate drawbacks of the variance concept, the downside (semi-deviation) risk measures were developed (semivariance or conditional value at risk). Below, we focus only on the semivariance risk measurement approach because it is often considered as a more plausible measure of risk. This study was ignited by the revised Markowitz's book (Markowitz, 1991).

2.2 Mean-semivariance model

As mentioned above, semivariance is alternative risk measure to a standard deviation already discussed by Markowitz (1959). The model of quadratic programming, using semivariance above the mean reducing the (cova)variance, is formulated by Ballestero (2005)

$$\begin{aligned} &\min \quad \mathbf{x^T V_S x} = \mathbf{x^T (V - B) x} \\ &\mathbf{x^T r} \geq M \\ &\mathbf{e^T x} = 1 \\ &\mathbf{x} \geq \mathbf{0} \end{aligned} , \tag{2}$$

where $\mathbf{V_S}$ denotes a semicovariance matrix. Let $v(\mathbf{r_M} > E_M)$ be a semivariance above the mean return for a market portfolio whose p historical returns are arranged to the vector $\mathbf{r_M} = (r_{M_1}, r_{M_2}, ..., r_{M_p})^T$, and expected return is denoted as $E_M = \frac{1}{p}\sum_{j=1}^{p} r_{M_j}$. One way to make a market portfolio is to use a naive strategy. Such a portfolio contains all n assets with the same weights. Then it must hold

$$r_{M_j} = \frac{1}{n}\sum_{i=1}^{n} r_{ij} \quad j = 1, 2, ..., p, \tag{3}$$

where $r_{ij}, i = 1, 2, ..., n, j = 1, 2, ..., p,$ represents the j-th historical return of i-th asset. Finally, the semivariance is calculated as

$$v(\mathbf{r_M} > E_M) = \frac{1}{p}\sum_{j=1}^{p}\left[\max(r_{M_j} - E_M, 0)\right]^2 . \tag{4}$$

Then we can define the $n \times n$ matrix $\mathbf{B}$ with the generic elements calculated as $b_{ij} = \beta_i \beta_j v(\mathbf{r_M} > E_M),\ i, j = 1, 2, ..., n,$, where Sharpe's β_i, or β_j, is computed as follows

$$\beta_i = \frac{\mathrm{cov}(\mathbf{r_i}, \mathbf{r_M})}{\mathrm{var}(\mathbf{r_M})} \quad i = 1, 2, ..., n, \text{ or } \quad \beta_j = \frac{\mathrm{cov}(\mathbf{r_j}, \mathbf{r_M})}{\mathrm{var}(\mathbf{r_M})} \quad j = 1, 2, ..., n, \tag{5}$$

where $\mathbf{r_i} = (r_{i_1}, r_{i_2}, ..., r_{i_p})^T$, or $\mathbf{r_j} = (r_{j_1}, r_{j_2}, ..., r_{j_p})^T$ is a vector of m historical returns of the i-th, or j-th asset, $\mathrm{cov}(\mathbf{r_i}, \mathbf{r_M})$, or $\mathrm{cov}(\mathbf{r_j}, \mathbf{r_M})$ is a covariance of returns of the i-th, or j-th asset and return of the market portfolio.

This model was proved by its author Ballestero (2005) and practically used by many authors (e.g. Pla-Santamaria and Bravo, 2013). As is usually the case, each concept suffers from certain shortcomings. From my point of view, this drawback can lay in a determination of the market portfolio. It is subject external information (from the investor, analytics, etc.) affecting a portfolio composition. In the aforementioned specification of the model (2), the market portfolio is strictly determined. Then the user may not specify this reference point which greatly facilitates his/her role.

3 PORTFOLIO SELECTION VIA MEAN-SEMIVARIANCE MODEL

In the practical part of the paper, the investment portfolio is made via a mean-semivariance optimization model in the field of the Czech exchange RM-SYSTEM. The result is analyzed and confronted with output of the basic mean-variance model.

3.1 Investment strategy and data

The analysis should be applicable to the widest possible spectrum of investors. Therefore, stocks traded on the Czech exchange RM-SYSTEM are chosen. This market is closer to the "smaller" investors (unlike Prague Stock Exchange) who have not any huge amount of free financial resources. Standardized trading units are considerably smaller on this market. The offer of companies issuing shares is wide. Moreover, the investors can use a user-friendly online trading system EasyClick (see more RM-SYSTEM 2020a).

Especially “smaller” investors usually prefer a longer-time investment. Many of them try to save their free financial funds with some additional benefit for their pension age. Then they usually follow a less risky investment. It is no doubt that two main factors affecting the investment behavior are return and risk (connected with a possible loss). Both factors may have different importance based on the investor’s preferences. The aim is to make a portfolio with the most favorable combination of return and risk values for the investor.

Return is comprehended as a random variable whose values are approximately normally distributed. From the historical observations of returns, the basic statistical characteristics for the stocks can be calculated – mean, variance (standard deviation) and covariance. The historical period should be representative for a longer-time price development. Therefore, ten-year period with some rises, fall and “calm” subperiods was chosen, i.e. from February 2010 to February 2020. The statistical characteristics are calculated from the monthly returns. We can see mean monthly return and standard deviation (representing an individual risk) of the selected 16 stocks actively traded on the Czech exchange RM-SYSTEM for a longer-time period in the following table (Tab. 1).

Tab. 1 Stock data (in %)

Stock	**Return**	**St. dev.**	**Stock**	**Return**	**St. dev.**
CETV	0.07	0.01	Microsoft	1.21	4.68
ČEZ	-0.52	13.30	Nokia	1.73	5.81
Deutsche Telecom	-0.46	5.47	Orco	-0.33	11.12
Erste Group Bank	0.44	4.69	O2 C.R.	-0.73	21.70
Exxon Mobil	0.44	8.76	Pegas Nonwovens	-0.14	9.65
Intel	0.09	5.24	Philip Morris	0.35	4.58
Komerční banka	1.19	6.54	VIG	0.37	4.35
McDonald’s	-0.59	8.81	Volkswagen	-0.26	5.81

The stock prices are downloaded from RM-SYSTEM web site (2020b). Then the values of all characteristics are calculated in the MS Excel environment.

3.2 Making portfolio from the stocks traded on the Czech market

To make a portfolio from the selected stocks, the model (2) in the following form is applied

$$\begin{aligned} &\min \quad \mathbf{x}^{\mathrm{T}}\mathbf{V}_{\mathrm{S}}\mathbf{x} \\ &\mathbf{x}^{\mathrm{T}}\mathbf{r} \geq M \\ &\mathbf{e}^{\mathrm{T}}\mathbf{x} = 1 \\ &\mathbf{x} \geq \mathbf{0} \end{aligned} \quad , \qquad (6)$$

where $\mathbf{x} = (x_1, x_2, ..., x_{16})^T$ is a vector of variables expressing a share of the stocks in order from right in Table 1. Vector $\mathbf{r} = (r_1, r_2, ..., r_{16})^T$ contains a mean return of the assets in the same order from the aforementioned table. The elements (semicovariances) of the matrix $\mathbf{V}_{\mathrm{S}}$ are calculated through the formulas (3)-(5). The M value is derived from the possible minimum and maximum level of portfolio return which are stated through the following optimization models

$$\begin{array}{ll} \min \quad \mathbf{x}^{\mathbf{T}}\mathbf{r} & \max \quad \mathbf{x}^{\mathbf{T}}\mathbf{r} \\ \mathbf{e}^{\mathbf{T}}\mathbf{x} = 1 \quad , & \mathbf{e}^{\mathbf{T}}\mathbf{x} = 1 \quad . \\ \mathbf{x} \geq \mathbf{0} & \mathbf{x} \geq \mathbf{0} \end{array} \tag{7}$$

Based on the results of both models (7), a portfolio return is moving in $\langle$-0.727%,1.728%$\rangle$. Of course, the result is intuitive. It is a "portfolio" of one asset with the worst, or the best return.

By gradually setting the (positive) value of *M* we get combinations with a particular level of risk, or a drawn effective frontier, by solving the model (6) through the LINGO optimization support system. Then the efficient frontiers can be displayed in the following graph (Fig. 1).

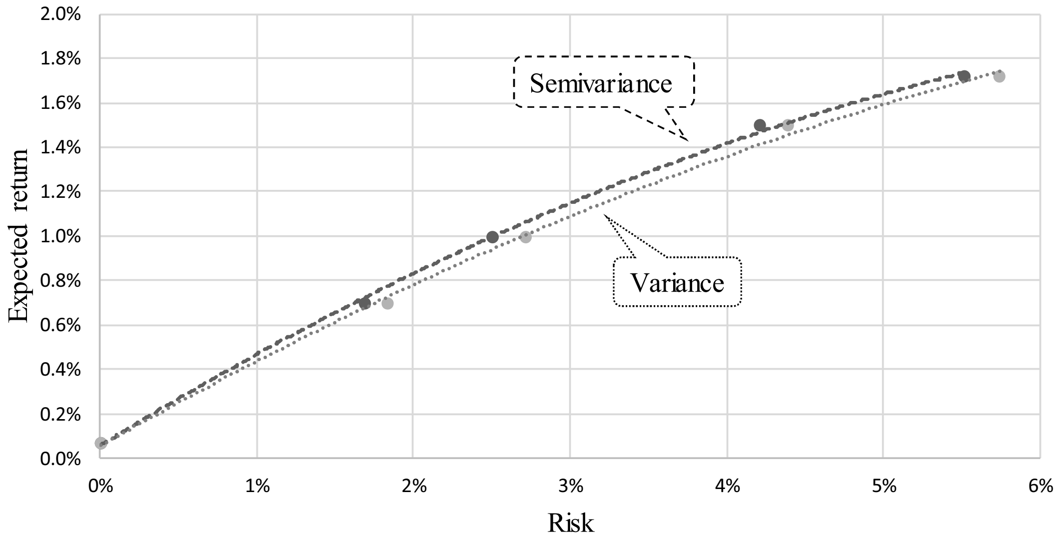

Fig. 1 Mean-variance and mean-semivariance efficient frontier

To compare the results with more usual mean-risk approach, a mean-variance efficient frontier is also depicted. It is obvious that the "extreme" combinations (with the lowest, or highest values of both characteristics) are almost the same. The biggest differences are approximately in the middle of the curves. The mean-semivariance efficient frontier is above the mean-variance curve. Different portfolios for the same level of return, or risk, correspond to this fact which can be seen in the following table (Tab. 2).

Tab. 2 Portfolios and their characteristics (in %)

Stock	**M-SeV**	**M-Var**	**M-SeV**	**M-Var**	**M-SeV**	**M-Var**
Return	0.7		1		1.5	
Risk	1.7	1.84	2.51	2.72	4.22	4.4
CETV	36.08	41.4	5.56	13.41	-	-
Erste Group Bank	8.92	5.98	13.17	8.84	-	-
Microsoft	18.19	19.8	26.87	29.25	37.06	40.18
Nokia	21.26	20.92	31.42	30.91	60.27	58.32
O2 C.R.	0.81	-	1.2	-	-	-
Philip Morris	12.05	10.07	17.81	14.89	2.67	1.5
VIG	2.69	1.83	3.97	2.7	-	-

Notes: M-SeV – *mean-semivariance model*, M-Var – *mean-variance model*

As we can see, the stocks with a zero share in all portfolios are not included in the table. These stocks have not such risky diversification abilities and return feature. If the return is set at the worst possible level (see in the interval mentioned above), then the portfolio is practically made by one asset – CETV stock. This stock has a very small but positive return. A standard deviation (or variance) of its return is the smallest. The covariance among its return and returns of other stocks are almost zero. The covariances among other stocks' returns are positive in most cases, albeit rather moderately. This property on a smaller stock market was expected. With raising level of expected return, the CETV share is falling. On the other side, the share of stock with the highest expected return (Nokia) increases. This movement is followed by the stock with the second highest return (Microsoft). O2 C.R. and VIG stocks then subtly complement some investment portfolios. At the end of the process of expected return rise, a single component "portfolio" is formed by, of course, Nokia stock.

It is evident that a development of the components' shares during changing a return reference level is quite similar for both risk measures. Portfolios are almost identical at the edges of the curves. In the middle, there are the most significant differences. However, the difference is very weak which is proved through the portfolio compositions seen in Tab. 2. At the same level of risk (measured via different concept!), the portfolio return made by mean-semivariance concept is slightly better. Measured by the Sortino (Sortino, 1996), or Sharpe (Sharpe, 1966) ratio, the selected portfolios are more efficient made by a mean-semivariance, or mean-variance model. In addition, a relation of the portfolio risk to its worst level (under a particular level of return) seems to be slightly better for a mean-variance portfolio. In other words, the results are not dramatically different. What do the results mean for economic, decision-making practice?

3.3 Impact of the analysis results on a real-life investment decision making

One advantage of these mean-risk models is a form of the result. The investor has available set of possible portfolios with an efficient mean-risk combination. The investor can rationally select the portfolio according to his/her preferences about an acceptable level of return and risk. For instance, a risk-averse investor selects a portfolio with risk at a lower level (e.g. up to 2%). The application advantage of the mean-semivariance concept is that it is more useful under the asymmetry of return distribution. Of course, it can be also applied for a symmetric case (Estrada, 2007). In our analysis, the distribution of the stock returns is not strictly normal. However, a deviation from this symmetric distribution is not too significant as confirmed a Kolmogorov-Smirnov statistical test (performed in Crystal Ball in the MS Excel environment). This fact is also proved by the similar results of both mean-risk models. In the end, this was expected because the historical data is not a high-frequency that is more susceptible to the non-normality. Thus, this benefit of mean-semivariance model may be more significant in the case of another investment instruments or selected historical period. Unlike variance, or standard deviation, a mean-semivariance model does not penalize upside volatility. This is very reasonable from a practical point of view. Markowitz et al. (1993) also claims that investor worries about underperformance rather than overperformance of the investment portfolio. Then a semi-deviation concept seems to be more appropriate measure of investor's risk than variance.

4 CONCLUSION

The focus of this article is on the mean-semivariance model and its application power. This model is naturally confronted with the basic well-known mean-variance (optimization) principle. The application power is demonstrated on a selection of the portfolio with stocks traded on the Czech exchange RM-SYSTEM using a user-friendly trading system for a wider range of investors. The mean-risk models seem to be very useful in practice. They provide a set of portfolios with various efficient combinations of two most important investment characteristics – return and risk. Based on the additional investor's preferences, or accompanying indicator(s), the most suitable portfolio then can be compiled. Implementation of the models is not too difficult. Even MS Excel Solver is sufficient which enhances wide use in investment practice. However, a more sophisticated optimization system for support modeling is more convenient, especially for a larger portfolio selection problem (i.e. a higher number of assets considered for investment). Moreover, the benefit of the semivariance model (compared to a mean-variance concept) is also proved. No assumption of normality of a return distribution can be useful in many portfolio selection problems because of its asymmetry, or skewness. Penalization of only downside volatility also seems to be very meaningful because the investors naturally are not afraid of portfolio overperformance. For future research, an unstable level of return, or risk, can be discussed, or integrated to the optimization model, using the instruments of a fuzzy set theory.

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SIMULATION AS A TOOL TO MODEL SERVICE QUALITY OF ELECTRIC VEHICLE CHARGING STATIONS

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Abstract
The implementation of green technology in transportation is a new trend in which we expect a significant shift shortly. The use of electric vehicles represents a significant reduction of noise and emissions from transport with positive impacts on the quality of life. The progress of electric mobility in the automotive industry is related to the emergence of new technologies and the necessity for new infrastructure of charging stations. The increase of electric vehicles on the road, considering the limited driving range of electric vehicles, and lack of adequate charging infrastructure could be a barrier for the usage of electric vehicles. The paper aims to introduce simulation as a tool to evaluate the level of provided services of charging stations infrastructure.

Keywords: *Simulation Modeling, Queuing Theory, Charging Station*

JEL Classification: C44, C63
AMS Classification: 00A72, 37M05

1 INTRODUCTION

Green technologies are currently booming in various areas. One of the areas where green technology begins to be implemented is electromobility. It is transport and the automotive industry in which cars with different alternative drives are developed, manufactured, and used. The use of electric vehicles represents a significant reduction in noise and emissions from transport, with a positive impact on improving the quality of life. The advent of electromobility is causing revolutionary changes in the automotive industry related to the advent of new technologies. While the internal combustion engine will continue to be used shortly, it is necessary to create the right conditions and to start a gradual transition from internal combustion engines to new and more efficient technologies. Slovakia, as a global power of car production per capita, should keep pace with international competition and develop its position in the world of electromobility so that it maintains its position in car production and strengthens its scientific research and innovation potential. In order to encourage electromobility in Slovakia, it is necessary to take some action, for example, to build sufficient infrastructure of charging stations.

The deployment of public charging infrastructure networks has been a significant factor in enabling electric vehicle technology transition and must continue to support the adoption of this technology. Fast charging stations increase customer convenience by lowering charging time and enables long-distance electric vehicle travel. High capital costs and uneven power demand may be an issue for the development of fast-charging stations. Therefore there is a need better to understand the system from the queuing theory perspective.

The paper provides the quantitative tool, the simulation to analyze and support decision making in order to ensure a sufficient level of charging stations infrastructure.

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2 LITERATURE REVIEW

Experimenting with a real system is based on exploring the behavior of a real existing system. The advantage of such an approach is that we do not accept any simplifications. Its disadvantage is that the experiment can be expensive and can lead to irreversible losses, in extreme cases, to the destruction of the system itself. In contrast, experimenting with a system model - simulation model is based on the creation of the model that is a representation of the system under investigation for its subsequent analysis of various scenarios. (Dlouhý et al., 2011). In the following section, we provide a short literate review of simulation models dealing with electric car charging operations and infrastructure.

Battery-charging operations constitute one of the most critical obstacles toward a large-scale uptake of electric mobility, due to performance issues and implementation complexities. (Bedogni et al., 2015) proposed a simulation platform that can assist in the redeployment of charging infrastructures and services on realistic large-scale electric mobility scenarios.

(Lewandowski et al., 2010) in their paper are analyzing a problem of blackouts. In case of a high simultaneity factor for power demand, the introduction of an uncoordinated fast recharge infrastructure would inevitably increase the risk for local substation blackouts.

(Brenna et al., 2017) electric cars require fast-charging station networks to allowing owners to charge their batteries when they drive relatively long routes rapidly. They investigate opportunities introduced by the use of railway infrastructures for the power supply of fast charging stations located in highways. Specifically, this work concentrates on fast-charging electric cars in motorway service areas by using high-speed lines for supplying the required power.

(Canizes et al., 2019) present a research study on the impact of the variation of the electricity prices on the behavior of electric vehicle users. The study compared the benefits when using the variable and fixed charging prices. The variable prices are determined based on the calculation of distribution locational marginal pricing, which are recalculated and adapted continuously according to the users' trips and behavior.

(Fan et al., 2015) model the fast charging station operation as an M/G/S/K queue and incorporate the dc fast charging model into the queuing analysis, as well as the revenue model of the fast charging station.

(Ucer et al., 2019) carried out Monte Carlo analysis in three types of areas-urban, suburban, and rural-to quantify the effect of uncertain parameters on charging station loading and service quality. Additional simulations based on a homogeneous vehicle population are carried out, and closed-form equations are derived from there to estimate charging duration and waiting time in the queue.

Other quantitative approaches are also used to optimize charging infrastructure. (Kohání et al., 2017a), (Kohání et al., 2017b) proposed optimization model to design a charging infrastructure for a fleet of electric vehicles. They design a private network of charging stations that is specifically adjusted to the fleet operation needs, for example fleet of vans used in the city logistics, a fleet of taxicabs or a fleet of shared vehicles operating in urban areas. Other example charging infrastructure optimization model is paper (Pekár et al., 2016) proposing a method for constructing a network of charging stations to ensure two main objectives, namely the availability of rapid charging stations for the highest possible number of citizens, and accessibility for population from the entire analyzed area.

3 SIMULATION CASE STUDY

Simulation, in the broader sense, means to imitate, mimic the possible states of a particular system. The term system refers to a part of the real world that is the object of interest. The subject of the simulation can be any real or abstract system. In this paper, the simulation model was created to model the classical queuing theory problem. Specifically, the multiple server queuing model applied to model service quality of electric vehicle charging stations (Taha, 2013). The generalization form of multiple server model is shown in Figure 1.

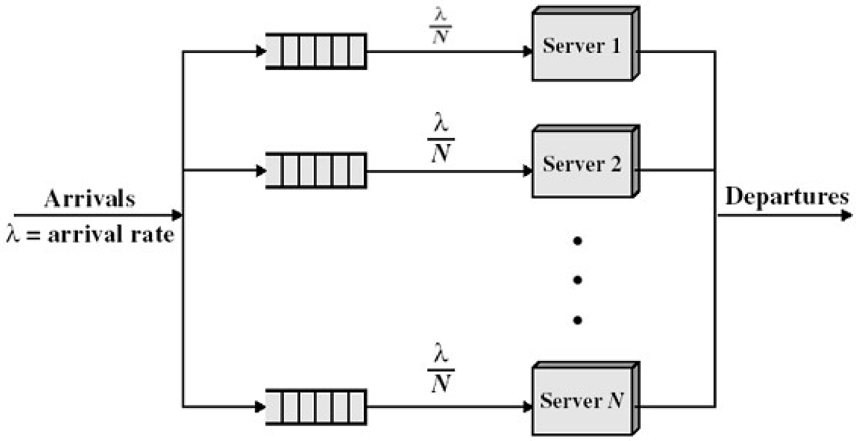

Figure 1: Generalization form of multiple server model

In the paper, we model the charging station system in the area of Bratislava city. The charging stations, servers are displayed on the map in Figure 1. The simulation was performed in simulation software Simul8. Simul8 is a discrete event simulation software, which can use real-world constraints and capacities. Because of the lack of data, the arrival rate of an electric car to the charging stations is modeled based on qualitative information. The triangular probability distribution was chosen. Also, the charging times for the lack of historical data are modeled with a uniform probability distribution. The set up numerical values in the simulation model are displayed in Table 1. Selected variables of performance measure are listed in Table 2. Based on measures, average queue size, and average queuing time at the queue for charging stations, the average time in the system and utilization of resources, it is possible to conclude whether that the quality of service is not sufficient or not and needs to be improved. The simulation software enables us to measure more variables of a performance measure, in the case study are selected only the basic ones. We can conclude, based on information from Table 2, that our modeled system provides a sufficient level of customer service. Waiting times and queue length are reasonably short, utilization of the system is only around 12%, and quality measure average time in the system is 56.26 minutes is influenced by the technology process itself. Relatively long charging times are required to charge the battery. In the case study, charging times are modeled by uniform distribution with minimum charging time 60 minutes and maximum charging time 120 minutes and for the faster type of charging stations are modeled by uniform distribution with minimum charging time 20 minutes and maximum charging time 60 minutes (see the Table 1).

Table1: Probability distribution for customer's arrival and service time

Distributions	
Arrival	[Triangular: 5, 20, 60]
Charging	[Uniform: 20, 60]
Charging long	[Uniform: 60, 120]

Source: author's calculations

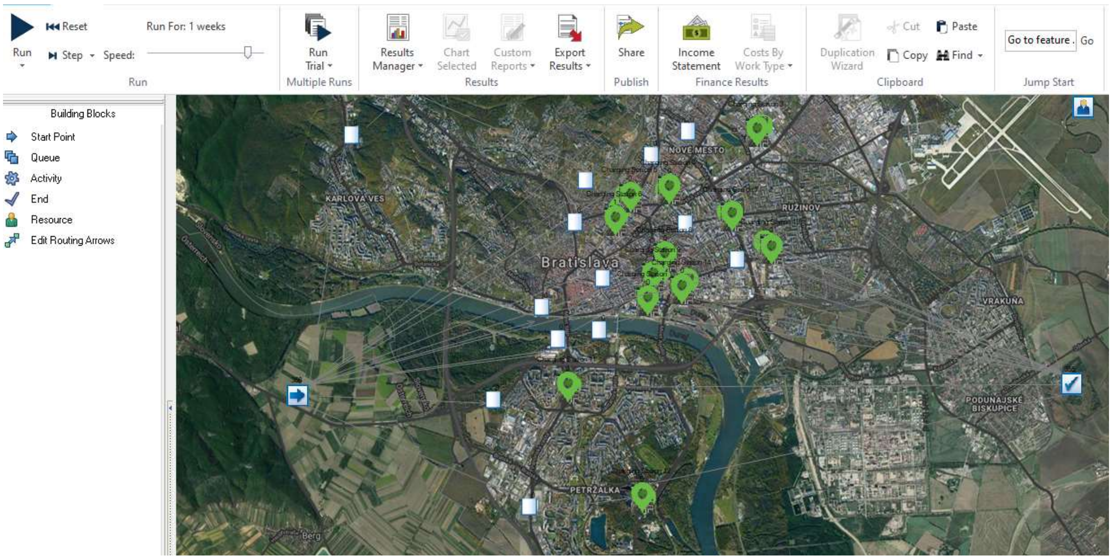

Figure 2: Simulation model in software Simul8

Table 2: Performance measure of the system Source: own processing in Simul8

		Low 95% range	Average Result	High 95% range
Queue for Charging Station 1	Average Queuing Time	1.15041452975973	1.25971335872712	1.36901218769451
Queue for Charging Station 1	Average Queue Size	0.0030275443478525	0.00333548294395042	0.00364342154004834
Queue for Charging Station 2	Average Queuing Time	1.14334603572297	1.25089466999585	1.35844330426874
Queue for Charging Station 2	Average Queue Size	0.00292706362671906	0.00321722306877547	0.00350738251083187
Queue for Charging Station 3	Average Queuing Time	1.08499479400679	1.18835883201113	1.29172287001548
Queue for Charging Station 3	Average Queue Size	0.00285762706359361	0.00314985863534757	0.00344209020710153
Queue for Charging Station 4	Average Queuing Time	1.12021287315755	1.23158614293064	1.34295941270373
Queue for Charging Station 4	Average Queue Size	0.00292383628267734	0.00322337420728582	0.003522912131894З
Queue for Charging Station 5	Average Queue Size	0.00270806193359379	0.00296631778328605	0.00322457363297831
Queue for Charging Station 5	Average Queuing Time	1.02224271057232	1.11590317859103	1.20956364660975
Queue for Charging Station 6	Average Queue Size	0.00317403161531053	0.00349709808519369	0.00382016455507685
Queue for Charging Station 6	Average Queuing Time	1.20566706401384	1.32395286853697	1.44223867306009
Queue for Charging Station 7	Average Queue Size	0.0253987574882712	0.0270964830110272	0.0287942085337831
Queue for Charging Station 7	Average Queuing Time	9.58703598693772	10.14667438819	10.7063127894423
Queue for Charging Station 8	Average Queue Size	0.00301911873794799	0.00331633510904643	0.00361355148014488
Queue for Charging Station 8	Average Queuing Time	1.16299238726555	1.27287374747265	1.38275510767975
Queue for Charging Station 9	Average Queue Size	0.0235887127674004	0.025283198465035	0.0269776841626695
Queue for Charging Station 9	Average Queuing Time	9.10796934190446	9.69335666214254	10.2787439823806
Queue for Charging Station 10	Average Queue Size	0.00311435027268672	0.00341455426771719	0.00371475826274766
Queue for Charging Station 10	Average Queuing Time	1.2166661334454	1.3271039864296	1.4375418394138
Queue for Charging Station 11	Average Queue Size	0.00326194776424528	0.00357855055884271	0.00389515335344013
Queue for Charging Station 11	Average Queuing Time	1.24468949238578	1.36338527751572	1.48208106264567
Queue for Charging Station 12	Average Queue Size	0.00274702895579461	0.00302485243340297	0.00330267591101133
Queue for Charging Station 12	Average Queuing Time	1.06504015260423	1.17264148481659	1.28024281702895
Queue for Charging Station 13	Average Queue Size	0.00303366039388799	0.00334232850183371	0.00365099660977942
Queue for Charging Station 13	Average Queuing Time	1.14911197052575	1.25613955109411	1.36316713166247
Queue for Charging Station 14	Average Queue Size	0.00305871621482169	0.00334629695268909	0.00363387769055649
Queue for Charging Station 14	Average Queuing Time	1.19102922715817	1.29811057036526	1.40519191357234
Time in the system	Average Time in System	56.1226681727101	56.2576673209675	56.3926664692248
Resource	Utilization %	11.8735712058284	11.9061330134398	11.9386948210512

Source: author's calculations

4 CONCLUSIONS

Simulation is one of the most commonly used quantitative decision-making approaches, it enables to study real system on a computer simulation model. The simulation model, which represents a real system contains logical relationships and mathematical expressions that describe how to calculate the output values of the given inputs. The paper utilizes a simulation approach to model the arrival of electric cars to the charging stations and, based on characteristics of the queuing system, evaluates its quality. The software Simul8 was used to create a simulation model.

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DUOPOLY PRICING IN SPATIAL COMPETITION

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Abstract
Analysis of oligopolistic market in space is a relatively discussed topic. Spatial games focused on imperfect competition from a spatial perspective is a specific field of game theory, analysing behaviour of subjects on the market competing to attract their customers with a purpose to find the best location for their branch. Each firm takes its own pricing policy, which affects its market share. This paper focuses on solving specific situation of two firms making decisions about their location with aim to maximize their profits, while the result is information about price of the first producer affected by a pre-known price of the second one. Customers are choosing one of the producers based on their total costs consisting of the price of the product and transport costs. This situation is presented on the illustrative example.

Keywords: *duopoly, game theory, pricing policy, spatial competition*

JEL Classification: C62, C72, D43
AMS Classification: 91A05, 91A10, 91A24

INTRODUCTION

The goal of every company on the market is to improve its position, grow, attract as many customers as possible and maximize profits. The way a company behaves on the market is also influenced by the type of market structure to which the firms belong. We distinguish between perfect and imperfect competition on the market, which both have their characteristic features. Imperfect competition is characterized by two basic assumptions - an identifiable product and the ability to set and control product prices by firms or manufacturers. We know imperfect competition on the supply side in three forms: monopolistic competition, monopoly and oligopoly, which can be differentiated or concentrated (Goga, 2013). These market structures differ from each other in the manner in which market operators behave.

Oligopoly is a complex market structure in which companies try to maximize their profits and improve their market position, what is the reason for the conflict with other companies, even though the company tries to avoid conflicts (Goga, 2013). The behaviour of firms on the market of imperfect competition must take into account the decisions of other entities (competing firms but also consumers) and game theory can be used to analyse this type of market.

When deciding on the quantity and price of the offered product, the company (the player) must consider the actions of other companies on the market. We can assume that if the market position is the same, the actions of companies will be the same (Goga, 2013). Their decisions can then lead to market equilibrium. From the game theory point of view, the decision-making situation of individual oligopolists can then be considered as a game. Players strive to maximize their expected payment through their strategic behaviour. The basic concept of solving such games is Nash's equilibrium - strategy profile, in which each strategy is the best answer to the rest of the profiles.

1 MODELS OF SPATIAL COMPETITION

Analysis of oligopolistic market in space is a relatively discussed topic. One of the first ones, who began to address this issue was the mathematician and economist Harold Hotelling (1929), who introduced a model whose concept consists of the presence of two companies seeking the most advantageous position in the linear market. The model is the basis of a number of product differentiation and location theories, but despite its usefulness, it has undergone many criticisms. C. D'Aspremont, J. Jaskold Gabszewicz and J.-F. Thesese (1979) point to its flaws and show that it is not possible to find the equilibrium if firms are close to each other. The modified model results in a model that addresses the existence of equilibrium at any point in the market (C. D'Aspremont et al., 1979).

Melvin L. Greenhut, George Norman and Chao-Shun Hung in their publication from 1987 took a different approach to traditional pricing theory and imperfect competition analysis, through a spatial perspective that they apply to the whole a number of non-spatial problems associated with imperfect competition.

Fetter (1924) published a work that had a significant impact on network competition theory. Its further extension can be found in the publications of many other authors such as (Hamoudi.and Risueño, 2012) and (Hamoudi and Bustamante, 2011).

The publication *The Economic Theory of Product Differentiation* (Beath and Katsoulacos, 1991) was also based on the Hotelling model. Among other things, the authors focus on the price competition of the spatial duopoly. Customers located along the linear market, forced to travel to purchase the products they offer, are the only ones who bear the transport costs. For producers, the location of their products is an exogenous parameter, so the price is their only decisive variable (Beath and Katsoulacos, 1991).

In this paper we will present the specific problem of spatial game in the case of duopoly. This is a specific type of oligopoly where only two subjects operate on the supply side of the market. Duopolistic market will be presented in a space that can be characterized in the form of a graph. Players make their decision to place their branch and the outcome of the game is determined by their pricing that will affect their respective market share. Customers choose one of the two duopolists on the basis of their lower costs, which include both the price of the goods and the transport costs. We will present an original mathematical model that enables to set a price for one of the duopolists on the basis of the already known price of the opponent, so that his sales were as high as possible.

2 DETERMINING THE PRICE OF THE DUOPOLIST BASED ON THE BEST RESPONSE

In this section, we will present an original mathematical model that allows you to determine the price of a duopolist on the basis of the price of an opponent in a specific spatial game.

In the model, as in Hotelling's basic model, we apply the basic assumptions: product homogeneity (both companies on the market offer a very similar product), zero production costs of companies, inelasticity of demand (consumption of one unit by customers at each market point), indifference of customers (due to choosing the producer).

The idea of spatial play is based on an article (Lopez and Čičková, 2018). We assume the following assumptions: Let $V = \{1,2, \dots n\}, n \in Z^{+}$ be a set of customers and if $G = (V, H)$, where V represents graph nodes and $H \subset VxV$ represents a set of edges $h_{ij} = (v_i, v_j)$ from node v_i to node v_j, while each oriented edge h_{ij} has assigned a real number $o(h_{ij})$, referred to as valuation or value h_{ij}. Spatial game was formulated in the so-called full-valued graph $\bar{G} = (V, \bar{H})$ with the same set of nodes as graph G, where $\bar{H}$ is the set of edges between each pair of nodes v_i and v_j, where their valuation is equal to the minimum price between nodes v_i a v_j in the original graph, $i, j \in V$. It is often assumed that $o(h_{ij}) = d_{ij}$ where d_{ij} represents the minimum distance (shortest path length) between nodes v_i a v_j, then the matrix $\mathbf{D}_{nxn} = \{d_{ij}\}$ is the matrix of shortest distances between nodes v_i a v_j.

In the article (Lopez and Čičková, 2018) it was assumed that there are two companies (players) $P = \{1,2\}$, that offer a homogeneous product (goods or service) and these companies have the possibility to place their branches in one of the nodes, meaning in any element of the set $V = \{1,2, \dots n\}$, which are also the locations of customers. Constant (unit) demand at each node was considered. Although both players offer an identical product in unlimited quantities, the price of the products may be different. Let $p^{(1)}$ be the price of player 1 and $p^{(2)}$ the price of player 2. No capacity restrictions were considered, and each customer can purchase from any company (the customer always makes a purchase). In making their decision, customers considered the total cost of purchasing the product, which consisted of the price of the product and the cost of transport to the selected company. Transport costs are expressed as t per unit of distance, so we assume that the customer has to pay the product price and transport costs to each company. If player 1 places his branch at the $i-$ node ($i \in V$) and player 2 places his branch at the $j-$ node ($j \in V$), player 1 gets the customer from the $k-$node ($k \in V$) only in case if $t * d_{ki} + p^{(1)} < t * d_{kj} + p^{(2)}$, otherwise the customer from the $i-$ node is served by player 2. If $t * d_{ki} + p^{(1)} = t * d_{kj} + p^{(2)}$, players share the demand equally.

The article (Lopez and Čičková, 2018) considered a situation where the prices of products were known in advance and based on the above assumptions allowed to explicitly calculate the elements of the payoff matrix of the player 1 $\mathbf{A} = (a_{ij}), i, j \in V$ (where a_{ij} represents the number of nodes served player 1 if player 1 builds branch on *i*-th node and opponent in *j*-th node). Then, the equilibrium strategies of players deciding to place their branches by a constant-sum game can be determined. In this case, the equilibrium strategies of the players can be obtained through the linear programming problem.

Parameters:
- $a_{ij}, i, j \in V$ – payment of a player in his *i*-th strategy and *j*-th opponent strategy

Variables:
- w – final payment of the player
- $x_i \geq 0$, $i \in V$ – *i*-th mixed strategy of the player

Equilibrium strategies can then be determined as follows:

$$w \to \max \tag{1}$$

$$\sum_{i \in V} a_{ij} x_i \geq w, j \in V \tag{2}$$

$$\sum_{i \in V} x_i = 1 \tag{3}$$

Let's now consider the problem of determining the player's equilibrium price in response to the opponent's fixed price. Let the price of the second player's goods $p^{(2)}$ be known in advance. However, we consider the first player's price $p^{(1)}$ to be a variable, and the player would like to set it to maximize its revenue. Obviously, with such assumptions, the elements of the player 1 payoff matrix will depend on the value of $p^{(1)}$. The relationship between the values of the payoff matrix elements and the price $p^{(1)}$ is:

$$a_{ij} = \sum_{i \in V} (sgn(t * d_{kj} + p^{(2)} - (t * d_{ki} + p^{(1)})) + 1)/2 \tag{4}$$

Signum discontinuity is a problem here, but now let's show how this function can be integrated into the mathematical programming problem with binary variables. We consider the following variables and parameters:

Parameters:

- $a_{ij}, i, j \in V$ – payment of a player in his *i*-th strategy and *j*-th opponent strategy,
- $d_{ij}, i, j \in V$ – the shortest distance from node *i* to node *j*,
- t – cost per unit of distance,
- $p^{(2)}$- the price of the opponent's product (player 2).
- M – big positive number.

Variables:

- w – final number of nodes served,
- $x_i \geq 0,\ i \in V$ – *i*-th mixed strategy of the player,
- $p^{(1)} > 0$ – product price of player 1,
- $b_{kij}^{(1)} \in \{0,1\}; k, i, j \in V$ – binary variable,
- $b_{kij}^{(2)} \in \{0,1\}; k, i, j \in V$– binary variable,
- $b_{kij} \in \langle -1,1 \rangle;\ k, i, j \in$ V – bounded variable.

This situation can be described by the following mathematical model:

$$w * p^{(1)} \to max \tag{5}$$

$$t * d_{kj} + p^{(2)} - (t * d_{ki} + p^{(1)}) \leq M * b_{kij}^{(1)}; k, i, j \in V \tag{6}$$

$$t * d_{kj} + p^{(2)} - \left(t * d_{ki} + p^{(1)}\right) \geq -M * b_{kij}^{(2)}; k, i, j \in V \tag{7}$$

$$b_{kij}^{(1)} + b_{kij}^{(2)} \leq 1; k, i, j \in V \tag{8}$$

$$b_{kij} = b_{kij}^{(1)} - b_{kij}^{(2)}; k, i, j \in V \tag{9}$$

$$b_{kij} * (t * d_{kj} + p^{(2)} - \left(t * d_{ki} + p^{(1)}\right)) \geq \varepsilon * b_{kij}; k, i, j \in V \tag{10}$$

$$a_{ij} = \sum_{k \in V} (b_{kij} + 1)/2; i, j \in V \tag{11}$$

$$w \leq \sum_{i \in V} a_{ij} * x_i \,; i, j \in V \tag{12}$$

$$\sum_{i \in V} x_i = 1 \tag{13}$$

Objective function (5) represents the profit function of player 1. Equations (6) to (10) serve to determine the payment matrix of player 1. Equations (6) and (7) allow to determine the assignment of consumers from the k-th node to position (i,j), $k, i, j \in V$ in case of sharp inequality of costs (if k-th node is assigned to player 1, variable $b_{kij}^{(1)}$ = 1, if this node is assigned to player 2, variable $b_{kij}^{(2)}$ = 1). The variables b_{kij} calculate the served nodes for player 1 (if k-th node is assigned to player 1, variable b_{kij} = 1, if this node is assigned to player 2, variable b_{kij} = -1). In case of equality of costs, equations (8) to (10) allow to set the value of b_{kij} = 0. The final calculations of the elements a_{ij} are provided by equations (11) (if b_{kij} = 1, element a_{ij} is increased by 1; if b_{kij} = 0 element a_{ij} is increased by 0.5 and if b_{kij} = -1, element a_{ij} remains unchanged). Relationships (12) and (13) make it possible to establish a balanced mixed strategy of player 1.

3 ILLUSTRATIVE EXAMPLE

We consider set V={1,2,…,4}, which represents a closed set of customers. Each of these customers is situated in one of the four nodes of graph G. Set P={1,2} represents set of two players (two companies) offering their products. Although these products are homogeneous, their prices are not the same. Unlike the basic location model, where companies set prices for their products simultaneously, we will consider a situation where producer 2 sets its price in advance, which means, it is a known variable in the model. Based on this information, the producer 1 solves the problem of pricing its product in order to maximize its profit.

We have a situation in which the known price of the second player's product is $p^{(2)} = 10€$. Shipping costs will be $t = 1€$. To determine the total transport cost, it is necessary to define a matrix of shortest distances $\mathbf{D} = d(i, j)$ between nodes. Let the matrix $\mathbf{D} = d(i, j)$, be as follows:

$$\mathbf{D}=\begin{bmatrix} 0 & 42 & 49 & 63 \\ 42 & 0 & 84 & 105 \\ 49 & 84 & 0 & 77 \\ 63 & 105 & 77 & 0 \end{bmatrix}$$

Based on model (5) and (13) we will obtain these results (the combined integer nonlinear programming problem (MINLP), solved by GAMS and its solver Bonmin, which is an area of optimization solving nonlinear problems with continuous and integer variables):

Payment matrix **A**:

$$\mathbf{A}=\begin{bmatrix} 4 & 3 & 3 & 3 \\ 1 & 4 & 2 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 2 & 1 & 4 \end{bmatrix}$$

The first player's strategy is given by the vector $\mathbf{x}^T = (1; 0; 0; 0)$. This means that the first player should invest 100% of his capital in the 1st node. The calculated product price of first player at a known price $p^{(2)}$ increased to 10€ fell just below its level to $p^{(1)} = 9.999€$. At this price, the player achieves a maximum revenue of 29.997€ (based on (5)), serving demand from three nodes (w=3).

For comparison, consider the situation where $p^{(2)} = 9€$. Based on model (5) and (13) we obtain the following results:

Payment matrix **A**:

$$\mathbf{A}=\begin{bmatrix} 0 & 2 & 2 & 2 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

The first player's strategy is given by the vector $\mathbf{x}^{\boldsymbol{T}} = (0.143; 0.286; 0.286; 0.286)$. This means that the first player should invest 14.3% of his capital in the first node and distribute the rest of his capital equally by 28.6% to the other three nodes. The calculated price of the first player's product will be significantly higher than the known price of $p^{(2)} = 9€$, specifically $p^{(1)} = 50.999€$. At this price, the first player achieves a maximum revenue of 43.713€ (based on (5)) while the average value of the game will be 0.857, which means that while on average serving less than 1 node (w=0.857), its revenue would in this case significantly increase.

4 CONCLUSIONS

This paper is focused on analysis of oligopolistic, specifically duopolistic, market in space. Two firms, making decisions about their location with aim to maximize their profits, represent situation with a competitive character. Therefore, we were able to use game theory to solve the problem. Duopolists compete to attract their customers, who are choosing one of the producers based on their total costs consisting of the price of the product and transport costs. Our paper solves the problem assuming the known price of the second producer in advance, which means the price is variable in the model. Based on this information, firm 1 takes pricing policy in order to maximize its profit. Information about final price $p^{(1)}$ is part of the result of the solved problem, together with information about its final profit. In illustrative examples we present comparation of two situations, where the difference is in pre-known price $p^{(2)}$ affecting the final $p^{(1)}$, profit and number of served nodes. We used software GAMS, effective software for optimization computing, to solve presented problem.

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MULTIPLE EQUILIBRIA IN THE SPANISH LABOR MARKET

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Abstract
The aim of the paper is to incorporate the Keynesian principle of weak aggregate demand into the basic search-matching model of unemployment. Multiple equilibrium unemployment rates emerge as a result of this modification. Output demand thus plays not only short-term role but might be essential in the long run as well. This is because the initial fall in aggregate demand may cause unemployment rate to converge to a higher (long-run) equilibrium. All these aspects are illustrated for the Spanish labor market.

Keywords: *search-matching model, unemployment rate, output demand, multiple equilibria*

JEL Classification: E24, J23, J64
AMS Classification: 91B40, 91B84

1. INTRODUCTION

Aggregate output demand plays only a minor role in the standard search-matching Diamond-Mortensen-Pissarides[1] (DMP) model. However, there is empirical evidence favoring demand-oriented theories of job creation to the model of job creation proposed by DMP modeling framework (Carlsson et al. 2006). The presented paper incorporates the traditional Keynesian concept of weak aggregate demand into the basic search-matching model. The important consequence is that multiple (long-run) equilibrium unemployment rates might emerge which is illustrated for the Spanish labor market.

The structure of the paper is as follows. Chapter 2 briefly summarizes a well-known stochastic discrete-time version of the basic DMP model with aggregate uncertainty (Hagedorn, Manovskii, 2008) (HM model, hereafter). The concept of weak aggregate output demand is then incorporated into this basic search-matching model in chapter 3. Data and econometric estimation is described in chapters 4 and 5. Multiplicity of equilibria of the weak demand model is analyzed in greater detail in chapter 6. The final chapter summarizes main findings.

2. HAGEDORN-MANOVSKII MODEL

Infinitely lived workers maximize their expected lifetime utility, $E\sum_{t=0}^{\infty}\delta^t y_t$, where y_t represents income in period t and $\delta \in (0,1)$ is a discount factor.

Output per worker is denoted by p_t and follows the first-order autoregressive process:

$$\log(p_t) = \rho^p \cdot \log(p_{t-1}) + \varepsilon_t^1, \tag{1}$$

where $\rho^p \in (0;1)$ and $\varepsilon_t^1 \sim N(0, \sigma_1^2)$ is i.i.d. productivity shock.

Flow cost c_t of posting a vacancy is assumed to change over the business cycle according to

$$c_t = c_K \cdot p_t + c_W \cdot p_t^{\xi}, \tag{2}$$

where c_K, c_W and ξ are parameters.

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[1] The work of these Nobel Price winners is summarized by (Pissarides, 2000).

Workers and firms separate with a constant[2] probability s per period. Employed workers are paid a wage w_t and unemployed get a flow utility z from leisure/non-market activity. Wages are determined by the generalized Nash bargaining solution. The bargaining power of workers is $\beta \in (0;1)$.

Let u_t denote the unemployment rate, $n_t = 1 - u_t$ the employment rate, v_t the number of vacancies and $\theta_t = v_t / u_t$ the market tightness. The number of new matches (starting to produce output at $t+1$) is given by[3]

$$m(u_t, v_t) = m_0 \cdot u_t^{\eta} \cdot v_t^{1-\eta} \cdot \exp(\varepsilon_t^2), \tag{3}$$

where m_0 is parameter representing matching efficiency and $\eta \in (0;1)$.

The shock to matching efficiency ε_t^2 is supposed to be persistent:

$$\varepsilon_t^2 = \rho^m \cdot \varepsilon_{t-1}^2 + \tilde{\varepsilon}_t^2 , \tag{4}$$

where $\rho^m \in (0;1)$ and $\tilde{\varepsilon}_t^2 \sim N(0, \sigma_2^2)$ is i.i.d. random error.

Probability for an unemployed worker to be matched with a vacancy equals

$$f_t \equiv f(\theta_t) \equiv \frac{m(u_t, v_t)}{u_t} = m_0 \cdot \theta_t^{1-\eta} \cdot \exp(\varepsilon_t^2) \tag{5}$$

and the probability for a vacancy to be filled is

$$q_t \equiv q(\theta_t) \equiv \frac{m(u_t, v_t)}{v_t} = m_0 \cdot \theta_t^{-\eta} \cdot \exp(\varepsilon_t^2). \tag{6}$$

Evolution of employment rate is given by

$$n_{t+1} = (1-s) \cdot n_t + f_t \cdot u_t + \varepsilon_{t+1}^3, \tag{7}$$

where $\varepsilon_{t+1}^3 \sim N(0, \sigma_3^2)$ is i.i.d. shock to the process of unemployment.

It can be shown by standard methods that the first-order conditions of the optimization problem lead to the following equilibrium condition:

$$\frac{c_t}{\delta \cdot q(\theta_t)} = E_t\left[(1-\beta) \cdot (p_{t+1} - z) - c_{t+1} \cdot \beta \cdot \theta_{t+1} + \frac{(1-s) \cdot c_{t+1}}{q(\theta_{t+1})}\right]. \tag{8}$$

It is also easy to show by standard methods that wages are given by

$$w_t = \beta \cdot p_t + (1-\beta) \cdot z + c_t \cdot \beta \cdot \theta_t + \varepsilon_t^4 , \tag{9}$$

where $\varepsilon_t^4 \sim N(0, \sigma_4^2)$ is i.i.d. shock added to the wage equation for the purpose of econometric estimation.

3. WEAK DEMAND MODEL

The basic DMP model of the previous subchapter implicitly assumes that output produced by a worker p_t will also be sold. The principle of weak output demand is incorporated into this

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[2] There is empirical evidence that fluctuations in job finding probability during business cycle frequencies are substantial, while separation probability is nearly acyclic (Hall, 2005; Shimer, 2012).

[3] Hagedorn, Manovskii (2008) applied another form of matching function. Specifically, they used a matching function of the form $m(u_t, v_t) = u_t \cdot v_t / (u_t^l + v_t^l)^{1/l}$ which was proposed by den Haan, Ramey, Watson (2000). Nonetheless, standard Cobb-Douglas matching function performed better from an empirical point of view.

model by assuming that the output actually sold p_t^s depends positively on purchasing power of customers which is given by

$$\kappa_t = (1-u_t)\cdot w_t + u_t \cdot z, \tag{10}$$

where κ_t is purchasing power of customers, u_t is unemployment rate, w_t represents wage and z stands for unemployment benefits.

The process describing the output actually sold is modeled by a generalization of a simple autoregressive process (1) as follows

$$\log(p_t^s) = \rho^p \cdot \log(p_{t-1}^s) + \gamma \cdot (\kappa_t - \bar{\kappa}) + \varepsilon_t^1, \tag{11}$$

where p_t^s is the output actually sold and $\bar{\kappa}$ is an arithmetic mean of the variable κ_t.

Alternatively, the output actually sold p_t^s is modeled as a decreasing function of unemployment as unemployment is negatively correlated with purchasing power of customers[4]

$$\log(p_t^s) = \rho^p \cdot \log(p_{t-1}^s) - \gamma \cdot (u_t - \bar{u}) + \varepsilon_t^1, \tag{12}$$

where $\bar{u}$ is an arithmetic mean of the unemployment rate u_t.

It can be shown by standard methods that all the equations from the previous chapter 2 remains the same except the equations (8) and (9) which are slightly modified in the following manner:

$$\frac{c_t}{\delta \cdot q(\theta_t)} = E_t\left[(1-\beta)\cdot(p_{t+1}^s - z) - c_{t+1}\cdot\beta\cdot\theta_{t+1} + \frac{(1-s)\cdot c_{t+1}}{q(\theta_{t+1})}\right], \tag{13}$$

$$w_t = \beta \cdot p_t^s + (1-\beta)\cdot z + c_t \cdot \beta \cdot \theta_t + \varepsilon_t^4, \tag{14}$$

where $c_t = c_K \cdot p_t^s + c_W \cdot (p_t^s)^{\xi}$.

4. DATA

The source of the data is OECD database. All data is seasonally adjusted. The first observable variable is the standardized unemployment rate in Spain u_t (relating to all ages of workers) which is measured monthly from 1986 M4 to 2019 M8. The second variable market tightness θ_t calculated as a ratio of number of unfilled vacancies to number of unemployed persons from 1986 M4 to 2005 M4. The third observable variable is productivity p_t which is measured as a relative deviation from a linear trend of an industrial production index in manufacturing. It is also measured in monthly frequency from 1986 M4 to 2019 M8. The last observable variable relates to wages w_t. The variable w_t is measured as a relative deviation from a linear trend of an index of (real) hourly earnings in manufacturing. This measure of w_t implies that its mean value equals approximately to one as in the case of p_t. Therefore, monthly rate of change $(w_t - w_{t-1})/w_{t-1}$ is used as an observable variable. Index of hourly earnings in manufacturing was transformed from quarterly frequency into a monthly frequency by cubic spline. Data ranges from 1986 M4 to 2019 M4 after this transformation.

[4] Similar assumption is common in literature. Aggregate purchasing power is modeled by the number of unemployed workers in the famous model formulated by Diamond (1982).

5. ECONOMETRIC ESTIMATION

The results of econometric estimation performed in Dynare (together with priors) are summarized in the following table:

Parameter	Prior density			Posterior mean	90% confidence interval
	Density	Mean	Std. Dev.		
ρ^p	beta	0,80	0,20	0,814	(0,811; 0,818)
ρ^m	beta	0,80	0,20	0,976	(0,967; 0,985)
c_K	beta	0,47	0,20	0,534	(0,528; 0,541)
c_W	beta	0,11	0,20	0,1113	(0,1112; 0,1114)
ξ	beta	0,45	0,20	0,669	(0,665; 0,673)
β	beta	0,50	0,20	0,635	(0,629; 0,641)
η	beta	0,50	0,20	0,623	(0,620; 0,625)
z	beta	0,40	0,20	0,9837	(0,9829; 0,9844)
γ	beta	0,20	0,20	0,268	(0,264; 0,272)
σ_1	inv.gamma	0,01	1	0,0188	(0,0188; 0,0189)
σ_2	inv.gamma	0,01	1	0,023	(0,019; 0,028)
σ_3	inv.gamma	0,01	1	0,0023	(0,0022; 0,0024)
σ_4	inv.gamma	0,01	1	0,099	(0,098; 0,099)

Table 1: Parameter estimates of the weak demand model

These parameter estimates imply a situation very close to the case of multiple equilibria as will be shown in the next chapter. The important fact is that this result is robust to the choice of prior mean of γ. The result that the estimated weak demand model is close to the case of multiple equilibria was detected for all the values 0,1, 0,2, 0,3, 0,4 and 0,5 used as a prior mean of γ. Another robust result also is that there were technical problems in Dynare when using posterior means as parameter values in stochastic simulations. The simulation was either not performed at all or the dynamics was explosive which happened due to approximations stemming from a linearization around a steady state. All these problems demonstrate that multiple equilibria models can be estimated by standard algorithms implemented in Dynare with extreme difficulty as these algorithms are based on linearization around a uniquely determined steady state.

6. ANALYSIS OF STEADY STATES

Steady state values of market tightness $\theta(u)$ is implicitly defined by relation (13) as follows:

$$\frac{c(u)}{\delta \cdot q(\theta(u))} = E\left[(1-\beta)\cdot\left(p^s(u)-z\right)-c(u)\cdot\beta\cdot\theta(u)+\frac{(1-s)\cdot c(u)}{q(\theta(u))}\right]. \qquad (15)$$

from which it follows that market tightness $\theta(u)$ is a decreasing function of unemployment.

Dependence of market tightness on unemployment rate implies that a job-finding probability is also a function of unemployment rate

$$f(u) = 0{,}287\cdot\theta(u)^{1-0{,}623}, \qquad (16)$$

where the value of 0,623 is the posterior mean of η and $m0 = 0.287$ was calculated according to $m_0 = mean\left(f_t / \theta_t^{1-0.623}\right)$.

Stationary unemployment rate $u_t = u$ is given by

$$u = \frac{s}{s + f(u)}, \tag{17}$$

where s is separation rate and $f(u)$ is a steady-state value job-finding rate as a function of steady-state unemployment rate.

The function $s / (s + f(u))$ together with a 45° line representing variable u on the left-hand side of the equation (17) is depicted at the following figure.

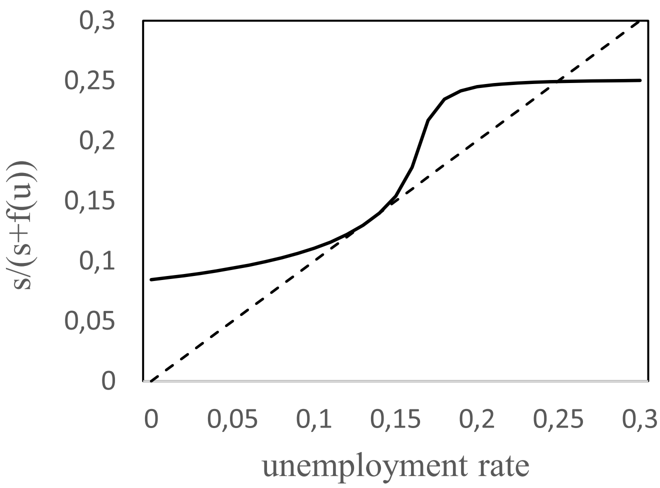

Figure 3: Equilibrium unemployment rates for the weak demand model

There are two equilibrium unemployment rates $u^1 = 0{,}14$ and $u^2 = 0{,}25$. The equilibrium point u^2 is stable, while the equilibrium u^1 is semistable. We know that the right-hand side of this equation corresponds to change in unemployment rate Δu. Also observe that $\Delta u > 0 \Leftrightarrow s / (s + f(u)) > u$. Therefore, unemployment rate u is rising whenever $s / (s + f(u))$ is above the 45° line.

The line $s / (s + f(u))$ is upward-sloping in the WD model because of the endogenization of the job finding probability $f(u)$ [5] which makes it possible to cross the 45° line in more than one point.

The reason for the existence of multiple equilibriums in the WD model is that the labor market is “less effective” when unemployment is high. This is modeled by making job finding probability $f(u)$ endogenous. The probability $f(u)$ is low (high) when unemployment rate u is high (low). But low (high) value of $f(u)$ keeps unemployment at high (low) levels because it is hard (easy) to find a job. Similar results were obtained by other authors (Diamond, 1982; Kaplan, Menzio, 2015).

[5] Separation rate was not made endogenous as there is empirical evidence that it is nearly acyclic (Shimer, 2012).

7. CONCLUSION

This paper contributes to the existing literature by introducing the concept of weak demand into the basic search-matching framework of unemployment. The significant finding is that incorporating this principle gives rise to a multiplicity of equilibrium unemployment rates which makes it very hard to econometrically estimate such a model by standard algorithms implemented in Dynare as these algorithms are based on a linearization around a steady state.

An empirical labor market analysis performed in this paper could be extended in several dimensions. Unemployment could be disaggregated for different groups of workers - high and low skilled (Hagedorn, Manovskii, Stetsenko, 2016), young and old (Hahn, 2009; Janíčko, 2012), long-term and short-term unemployed (Hynninen, 2009). Spatial econometric analysis of unemployment could also be performed (Addario, 2011; Formánek, Hušek, 2015). Empirical performance of the weak demand model could be compared to the baseline search and matching model not only for Spain but also for other economies. Comparison of the formulated model with DMP-DSGE or RBC approach is also an interesting topic for a future research (Němec, 2013; Szomolanyi et al. 2014).

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KUHN-TUCKER OPTIMALITY CONDITIONS IN MODEL OF A MONOPOLY PRODUCTION PRICE DIFFERENTIATION

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Abstract

One of the effective tools to exercise monopoly's market position or economic strength to define – in accordance with the monopoly's interests – a market price on the level guaranteeing maximum profit is a price differentiation. We can speak of a price differentiation of a monopoly production when a monopoly uses its market position or economic power to set and enforce a market price in accordance with its interests and thereby ensures maximum profit on a relevant market. We can say that a monopoly uses its position to reach a monopoly profit which exceeds profit on a perfect competition market. We will analyze a role of consumer's behavior optimization when the consumer's willingness to spend his funds on goods with differentiated prices is related to maximization of his total utility. For the optimization problems we will formulate the Kuhn-Tucker optimality conditions and we will study their interpretation options.

Keywords: *Perfect price differentiation, market position, profit maximization model, non-linear price setting, Kuhn-Tucker optimality conditions*

JEL Classification: D11, D43, L1144
AMS Classification: 49M05, 90C30, 90B30

1 INTRODUCTION

Due to a specific position which the subjects on a supply side have on an imperfect competition market, the producers can promote their interests without immediate danger of a competitor producing the same or similar product entering a relevant market.

One of the effective tools to use monopoly's market position or economic power to, according to its interests, set a market price to the level which guarantees maximum profit, is a price differentiation. We can speak of a price differentiation in a situation when the identical products are being sold at different prices while this inequality is not due to different production costs.

In literature price discrimination is often used as a synonym to price differentiation. We think, however, that this collocation has a slightly negativistic tone which does not correspond to its factual technical meaning, therefore following we will prefer price differentiation to indicate this microeconomic attribute.

In this article we will analyze a role of a consumer behavior optimization, whose willingness to spend his funds to purchase a good with a differentiated price is related to maximization of his total utility.

A problem of monopoly's profit optimization is built on a specific hypothesis that a monopoly gives a consumer an „all or nothing“ offer, meaning that a differentiated price only applies in a case that a consumer buys the whole amount of a good offered with a differentiated price and a corresponding revenue then guarantees a monopoly's maximum profit.

We will analyze a role of consumer's behavior optimization when the consumer's willingness to spend his funds on goods with differentiated prices is related to maximization of his total utility. For the optimization problems we will formulate the Kuhn-Tucker optimality conditions and we will study their interpretation options.

2 CONDITIONS FOR PRICE DIFFERENTIATION

Price differentiation is of course possible only under an assumption that a consumer is willing to pay different prices for different amounts of goods. That is, for example, at a lower price he is willing to purchase more – the price-demand function is decreasing. Besides, an effective price differentiation is based on the assumption that a consumer who purchased a product at a lower, favored price would not resell it at a slightly higher price to another consumer, who would otherwise paid a higher price. When such arbitrage deals cannot be ruled out price differentiation cannot be applied.

If a price differentiation can be applied then in fact a monopoly uses its monopolistic position to gain monopolistic profit. A tool for this is a market price while a monopoly can use price differentiation in following areas:

• to set a different price when purchasing different amounts of a same good,

• to set a different price for different consumers or consumer groups.

In a first case it is relatively simple to use a price differentiation. When meeting the conditions of purchasing volume an agreed discount is given. To successfully apply the second method of a price differentiation a mechanism must exist to identify a consumer belonging to a certain consumer group. In literature, we can find three types of price differentiation (Carlton, D.W.- Perloff, J. M., 2006) according to traditional classification:

(a) First degree price differentiation. This type of price differentiation of a monopolistic company is sometimes called a perfect price differentiation. A monopoly uses its privileged position on a market to set different prices for different volumes of a same product as well as for different consumer groups. Meaning that a seller in fact sets an individual price for each unit of a product and a price of a certain unit corresponds with a willingness to pay a maximum price by a certain consumer at a certain conditions.

(b) Second degree price differentiation represents a situation when the product prices depend on the purchasing volume of the products but do not depend on any characteristic a consumer may have. This phenomenon of pricing is also interpreted as a nonlinear pricing. Identical pricelists apply to all consumers but the pricelists vary for different purchasing volumes. A monopoly thus does not differentiate prices for particular consumers or consumer groups. The differentiation applies to varying amounts of purchased goods. An example of this approach to price differentiation is a volume rebate.

(c) Third degree price differentiation. With this type of price differentiation a monopolistic company sells any amount of goods at a same unit price. The price varies however for a specifically defined consumer groups. Third degree price differentiation represents probably the most common form of price differentiation. These are for example various types of discounts for students, child or pensioner's travel tickets, different prices on a different days (weekend discounts) and so on.

3 MODEL OF OPTIMAL CONSUMER BEHAVIOR IN THE CONDITIONS OF PRICE DIFFERENTIATION

Let's now examine a simplified analytical model of price differentiation of two potential consumers S_1, S_2, who both have a price-differentiated product in their market basket. Variable x represents its purchase in units and w is the total value of purchase of the other goods in monetary units (O'Sullivan, A. - Sheffrin, S. - Perez, P., 2006). A consumer can possibly decide:

- to exclusively purchase the studied product with a differentiated price and not to purchase the other goods at all,

- to purchase only the other goods from his market basket and not to purchase the studied product with a differentiated price,

- to purchase a full market basket represented by the studied product with a differentiated price as well as other goods.

Total utility expressed in monetary units, which a consumer feels when purchasing all the goods from the market basket is represented in a form of utility function:

$$f_i(x_i, w_i) = u_i(x_i) + w_i \qquad i = 1,2 \tag{1}$$

where

$u_i(x_i): R \to R$, $i = 1,2$ - is a concave, continuous and differentiable utility function for a product with differentiated price and represents a feeling of utility of a consumer in monetary units corresponding with a purchase of x units of the product,

w_i – expenses on other goods in the market basket,
$f_i(x_i, w_i): R^2 \to R$, $i = 1,2$ - consumer's total utility function.

To simplify, we will assume that utility for zero purchase of the studied product has a standardized zero value $u_i(0) = 0$. Maximum willingness of a consumer i to pay a certain price in monetary units for a purchase of x_i units of the product is represented by a function $r_i(x_i)$. This function is a solution to an equation:

$$u_i(0) + w_i = u_i(x_i) - r_i(x_i) + w_i \qquad i = 1, 2 \tag{2}$$

where on the left side is utility of a purchase of zero units of the product plus a value of the other goods from the market basket and on the right side is utility of a purchase of x units of the product reduced by a payment for their purchase plus a value of the other goods from the market basket. When a condition of a standardized zero value of utility at a zero consumption of the product from (2) validates, we get

$$u_i(x_i) \equiv r_i(x_i) \qquad i = 1,2 \tag{3}$$

In other words, a consumer S_i is willing to pay for x units of the product a maximum price which correspondents to his feeling of satisfaction from the product purchase represented in monetary units (Fendekova, E. - Fendek, M., 2018). The function of utility can be thus, with a certain degree of approximation, perceived as a function representing a willingness of a consumer to pay for the studied product the maximum price $r_i(x_i)$.

Function $r_i(x_i)$ has after all one more interesting economic interpretation. Its first derivation function $r_i'(x_i)$, which is a function of a marginal willingness of a consumer to pay a relevant price for a certain demand volume, in fact represents how much a consumer is willing to pay for the last unit purchased. So the value of the function of marginal willingness to pay represents the price p for which a consumer is willing to buy the whole amount of x units of goods:

$$r_i'(x_i) = p \qquad i = 1,2 \tag{4}$$

The function $r_i'(x_i)$ of marginal willingness of a consumer to pay an amount $r_i(x_i)$ de facto ultimately represents an inverse demand function of an i-th consumer

$$x_i = (r_i')^{-1}\,(p) \qquad i = 1,2 \tag{5}$$

Let's now examine a consumer's behavior optimization problem or a total utility maximization problem of a consumer who has the funds m_i at his disposal. He uses these funds to purchase a market basket (x_i, w_i), while he purchases x_i units of the studied product at a market price p and the variable w_i represents the total expenses related to the purchase of other goods from the market basket.

The problem is to find the values of variables x_i and w_i so that the value of the utility function (1) would be maximal while respecting a budget constraint of a consumer. Mathematical programming problem for i-th consumer is analytically represented by:

$$f_i(x_i, w_i) = u_i(x_i) + w_i \rightarrow max$$

subject to (6)

$$px_i + w_i = m_i$$

$$x_i, w_i \geq 0$$

This optimization problem of mathematical programming represents maximization constrained extreme problem. Let's modify this problem to a standard form, which is a form of a minimization problem:

$$-f_i(x_i, w_i) = -u_i(x_i) - w_i \rightarrow min$$

subject to (7)

$$px_i + w_i = m_i$$

$$x_i, w_i \geq 0$$

Let's formulate a generalized Lagrangian function for this problem. Let us mention that a generalized Lagrangian function does not explicitly include the conditions of non-negative variables, these are accounted for implicitly in a Kuhn-Tucker optimality conditions. A generalized Lagrangian function of a mathematical programming problem (7) is:

$$\mathcal{L}_i(x_i, w_i, \lambda_i) = -u_i(x_i) - w_i + \lambda_i(px_i + w_i - m_i) \tag{8}$$

Kuhn-Tucker optimality conditions (Avriel, 2003) for Lagrangian function (8) of the i-th consumer S_i are translated in a form:

$$\frac{\partial \mathcal{L}_i(x_i, w_i, \lambda_i)}{\partial x_i} \geq 0 \qquad \frac{\partial \mathcal{L}_i(x_i, w_i, \lambda_i)}{\partial w_i} \geq 0 \qquad \frac{\partial \mathcal{L}_i(x_i, w_i, \lambda_i)}{\partial \lambda_i} = 0$$

$$x_i \frac{\partial \mathcal{L}_i(x_i, w_i, \lambda_i)}{\partial x_i} = 0 \qquad\qquad w_i \frac{\partial \mathcal{L}_i(x_i, w_i, \lambda_i)}{\partial w_i} = 0 \tag{9}$$

$$x_i \geq 0 \qquad\qquad w_i \geq 0$$

After we substitute an analytical form of the Lagrangian function (8) we can restate the Kuhn-Tucker optimality conditions (9) of the total utility maximization problem as follows:

$$-u_i'(x) + \lambda p \geq 0 \quad (9.1) \qquad -1 + \lambda \geq 0 \quad (9.4) \qquad px + w - m = 0 \quad (9.7)$$

$$x(-u_i'(x) + \lambda p) = 0 \quad (9.2) \qquad w(-1 + \lambda) = 0 \quad (9.5)$$

$$x \geq 0 \quad (9.3) \qquad w \geq 0 \quad (9.6)$$

In other words, if a consumer aspires to identify an optimal consumer strategy $(x_i^\star, w_i^\star)$, meaning that a consumption of $x_i^\star$ units of the product with a differentiated price p and the expenses $w_i^\star$ of consumption of the other goods in a market basket maximize his total utility $f_i(x_i, w_i) = u_i(x_i) + w_i$, then such a Lagrange multiplier λ_i^* must exist, for which the Kuhn-Tucker optimality conditions (9) are met, i.e. the variables vector $(x_i^\star, w_i^\star, \lambda_i^*)$ is a solution to the system of equations and inequalities (9.1),..., (9.7).

We can derive some interesting consequences for the optimal combination of a supply and a price of a price-differentiated product from validation of the Kuhn-Tucker optimality conditions (Fendekova, E. - Fendek, M., 2012) for the total utility maximization problem (6).

a) Validity of the condition (9.7) guarantees that a consumer has precisely such an optimal consumer strategy $(x_i^\star, w_i^\star)$,that means such an optimal consumption of $x_i^\star$ units of the product with a differentiated price and on optimal expenses $w_i^\star$ of consumption of the other goods in a market basket, which he can implement using available financial resources m_i. In other words the optimal consumer strategy $(x_i^\star, w_i^\star)$ meets the condition of a budget constraint.

b) Let's now analyze a structure of an optimal market basket meeting the obvious assumption that a consumer has some other goods in his market basket than the studied product. That means that a variable representing the expenses $w_i^\star$ of consumption of the other goods in the market basket is positive $w_i^\star > 0$.

Validity of (9.5) results in the optimal value of Lagrange multiplier $\lambda_i^* = 1$. However if $\lambda_i^* = 1$ and (9.2) and (9.1) validate, then at the same time for the optimal positive volume of consumption $x_i^\star > 0$ stands

$$-u_i'(x_i^\star) + \lambda_i^* p = 0 \quad \wedge \quad \lambda_i^* = 1 \Longrightarrow$$

$$p = u_i'(x_i^\star) \tag{10}$$

Relation (10) represents a significant phenomenon of consumer behavior in the conditions of differentiated price. Above all we realize that (10) in fact represents an inverse demand function or a price-demand function and

$$p = p_i(x) = u_i'(x) \tag{11}$$

Price-demand function of the i-th consumer then determines the price $p = p_i(x_i)$, for which a consumer is willing to buy x_i units of goods, while based on (10) this price is equal to marginal utility $u_i'(x_i)$ corresponding to purchase of x_i units of the good.

4 CONCLUSION

Based on the formalized analytical tools we showed that if a producer has enough market power to not only accept the market price but to be able to significantly influence and create it, he can quite effectively use his knowledge of consumer behavior to optimize a combination of supply and price of his product.

As a matter of fact, it is a rational use of the information complex about the behavior of a consumer with specifically structured market basket, where they separately analyze consumer's utility regarding purchase of optimal volume of a price-differentiated product and this utility is represented in monetary units. Other goods in the market basket are being studied without any further specification of their volumes or range as one "aggregated good" and utility regarding purchase of these other goods is represented in monetary units as a simple sum of expenses spent on the purchase.

Significant is a fact that a company with a substantial market position in order to optimize its behavior at determining a combination of supply and differentiated price of a product, derives from a thorough analysis of consumers behavior while using analytical tools – demand functions and utility functions.

A mathematical programming problem which maximizes utility function of a consumer at budgetary restraints was examined in this article as it is a relevant tool for consumer behavior analysis. We showed that Kuhn-Tucker optimality conditions formulated for this optimization problem confirm the validity of consumer decision making schemes at optimization of his demand in the conditions of differentiated prices.

In a similar way we examined a monopoly profit maximization problem in the conditions of differentiated prices and we showed the fundamental schemes of price differentiation which monopoly can effectively use as a result of its market position to maximize its revenues as well as its profits from selling the products with differentiated price.

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SOLUTIONS TO CURRENT PROBLEMS ARISING FROM ICEV USE ENVIRONMENTS

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Abstract
With a template consequential from $M|G|\infty$ queue system, we create a model where *ICEV*-**I**nternal **C**ombustion **E**ngine **V**ehicles, commonly cars but not only, get idle, in a rare conventional energy ambience, and are either recycled, turning either *EV*-**E**lectric **V**ehicles or *HEV*-**H**ybrid **E**lectric **V**ehicles or *FCEV*- **F**uel **C**ell **E**lectric **V**ehicles , or dismantled becoming *DV*-**D**ismantled **V**ehicles. The model permits concluding that when the rhythm *ICEV* become *EV*, *HEV*, *FCEV*, *DV* is greater than the rate at which they become idle the system has a tendency to balance. Moreover, it is performed a cost-benefit analysis.

Keywords: *ICEV, EV, HEV, FCEV, DV,* $M|G|\infty$*, hazard rate function.*

JEL Classification: C18
AMS Classification: 60G99

1 INTRODUCTION

Worldwide many of non-renewable resources seem to be over-exploited and conventional sources of energy such as oil, gas and coal will collapse, very likely, in a near future. Maybe firstly oil, then gas and lastly coal. New prospects for business become obvious and practiced all over the world related to new sources of energy.

The objective of this paper is to show that *ICEV*, which work based on oil, may have an economically viable alternative use when this conventional source of energy collapses; or they may become dismantled, giving also conditions so that this action is economically viable. Indeed, in the showing model, using infinite servers' queues -see for instance (Ferreira and Filipe, 2017) and (Filipe and Ferreira, 2019)- we state that too many *ICEV* will become idle if either conventional energy misses or conventional energy becomes replaced by a renewable one. We accept as true that *ICEV* dismantle or recycling will become very usual because there will not be a way to get them functional with conventional oil, since the moment it is depleted.

We shall state that it is essentially relevant the cadence at which we perform the recycling and dismantling actions, being important in this analysis the hazard rate function of the service time.

Note that the hazard rate function of the service time depends essentially on the technology and protocols used to recycle and dismantle *ICEV*. Therefore, your choice, among those available now is of crucial importance.

It is imperative to draw attention here to another problem related to the future of cars, in this case in a very short time. This is the decision of major producers, in a very short time, possibly after 2030, for ethical and environmental reasons to stop making diesel cars, and to manufacture only gasoline, electric, hybrid and fuel cell cars.

Suppose the whole car manufacturers join this idea. Depending on the time it will take for the shortage of conventional energy, the case that the urgency to dismantle or recycle *ICEV* is not so pressing because it has occurred a preliminary replacement of diesel cars with either electric or hybrid or fuel cell cars. From the economic point of view, it is difficult to predict what will happen because there are several variables to consider, for example:

-Diesel cars will lose value due to this "decreed" obsolescence. Alternatively, will their value increase due to their increasing rarity?

-What will be the *cost* of dismantling or converting the existing diesel car manufacturing plants?

-What happen with the diesel cars in the meantime as they get idle: **dismantling**, **recycling**? Moreover, at what *cost*?

-Decreasing the number of diesel cars maybe will lead to a reduction in oil consumption, delaying the appearance of traditional energy shortage situation.

This work is commited to a goal that we think of primary importance: to contribute to the sustainability of humankind standard of living, compatible with a properly preserved natural environment.Here are some more in this path: (Andrade and Ferreira, 2007, 2009, 2010), (Andrade et al., 2012), Ferreira(2014), (Ferreira and Matos, 2018), (Ferreira et al., 2008, 2012, 2014, 2014a, 2016), (Filipe et al., 2012), (Matos and Ferreira, 2005, 2006), (Matos et al., 2018) and (Selvarasu et al, 2009).

Some of this material is presented in (Ferreira and Filipe, 2019).

2 EVOLVING THE MODEL

In $M|G|\infty$ queue, customers arrive according to a Poisson process at rate λ, receive a service which length is a positive random variable with *distribution function* $G(\cdot)$ and mean value α. There are infinite servers, that is: when a customer arrives, it always finds an available server, see for instance (Ferreira and Filipe, 2017) and (Filipe and Ferreira, 2019). The service length of a customer is independent from the other customers' service length and from the arrivals process. An important parameter is the *traffic intensity*, called $\rho=\lambda\alpha$. $M|G|\infty$ queue has neither losses nor waiting.

Concerning the present study, the costumers are the *ICEV* that become idle. The arrivals rate is the rate at which the *ICEV* become idle. The service length for each one is the time that goes from the instant they get idle until the instant they are either recycled or dismantled.

Define for the service length, the *hazard rate function* as:

$$h(t) = \frac{g(t)}{1 - G(t)} \quad (1)$$

At expression (1), $g(t)$ is the *probability density function* associated to $G(t)$. The *hazard rate function* of the service length is the rate at which the services end. For the situation under study here, is the rate at which the motorcars are either recycled, turning either *EV* or *HEV* or *FCEV*, or dismantled, turning *DV*.

Note that the *hazard rate function* of the service length depends essentially on the technology and protocols used to recycle and dismantle *ICEV*. Therefore, your choice, among those available now is of crucial importance.

Denoting $p_{1'0}(t) = G(t)e^{-\lambda\int_0^t [1-G(v)]dv}$, the probability the $M|G|\infty$ queue has no costumers at instant *t*, being the time origin an instant at which a costumer arrives at the system finding it empty (symbol $1'$), see, for instance, Ferreira (1991)

Proposition1
If $G(t) < 1, t > 0$ continuous and differentiable and

$$h(t) \geq \lambda, t > 0 \quad (2)$$

$p_{1'0}(t)$ is non- decreasing.

Dem.: It is enough to note that $\frac{d}{dt}\, p_{1'0}(t) = e^{-\lambda\int_0^t [1-G(v)]dv}\big(1 - G(t)\big)\big(h(t) - \lambda G(t)\big)$.
Obs.:
-If the rate at which the services end is greater or equal than the costumers 'arrivals rate $p_{1'0}(t)$ is non-decreasing.
-For the $M|M|\infty$ queue system, exponential service times, $h(t) = {}^{1}/_{\alpha}$ and equation (2) is equivalent to

$$\rho \leq 1 \text{ (3). } \blacksquare$$

Either equation (2) evidences that if the recycling or the dismantling rate is greater or equal than the rate at which the motorcars become idle, the probability that the system is empty at instant *t*, meaning it that there is no idle *ICEV*, does not decrease with *t*. Therefore, the system tends to balance as far as time goes on.

3 ACHIEVING COST-BENEFIT ANALYSIS

In former section, we saw how important were the roles of *h (t)* and λ , in monitoring the *ICEV* recycling and dismantling management.

To perform an economic analysis, based on the model presented behind, consider additionally *p* as the probability, or percentage, of the *ICEV* arrivals designed to the recycling being consequently *1-p* the same to the dismantling. In addition, be *q* the percentage or probability of *ICEV* designed for recycling that turn *EV*; *r* will be the same for *ICEV* designed for recycling that turn *HEV*; and *1-q-r* will be the same for *ICEV* designed for recycling that turn *FCEV*. Call $h_i(t), c_i(t)$ and $b_i(t), i = EV, HEV, FCEV, DV$ the *hazard rate function*, the mean *cost* and the mean *benefit*, respectively for an *ICEV* turn either *EV* or *HEV* or *FCEV* or *DV*. Therefore, the total *cost* per unit of time for motor cars recycling and dismantling is:

$$C(t) = \lambda[pqc_{EV}(t) + prc_{HEV}(t) + p(1 - q - r)c_{FCEV}(t) + (1 - p)c_{DV}(t)] \quad (4)$$

and the *benefit* per unit of time resulting from recycling and dismantling

$$B(t) = b_{EV}(t)h_{EV}(t) + b_{HEV}(t)h_{HEV}(t) + b_{FCEV}(t)h_{FCEV}(t) + b_{DV}(t)h_{DV}(t) \quad (5).$$

So, consider a period *T*. It must be $\int_0^T B(t)dt > \int_0^T C(t)dt$. It is not a simple matter to deal analytically with this expression. But, considering $b_{EV}(t), b_{HEV}(t), b_{FCEV}(t), b_{DV}(t)$ are all constant in $[0, T]$ with values $b_{EV}, b_{HEV}, b_{FCEV}, b_{DV}$, respectively. If moreover $G_{EV}(t), G_{HEV}(t), G_{FCEV}(t), G_{DV}(t)$ are all exponential, with means $\alpha_{EV}, \alpha_{HEV}, \alpha_{FCEV}, \alpha_{DV}$, respectively, and calling $C_i^T = \int_0^T c_i(t)dt, i = EV, HEV, FCEV, DV$:

- Recycling, turning *ICEV* in *EV*, is interesting, if

$$b_{EV} > \max\left\{\frac{\rho_{EV}[pqC_{EV}^T + prC_{HEV}^T + p(1-q-r)C_{FCEV}^T + (1-p)C_{DV}^T]}{T} - b_{HEV}\frac{\alpha_{EV}}{\alpha_{HEV}} - b_{FCEV}\frac{\alpha_{EV}}{\alpha_{FCEV}} - b_{DV}\frac{\alpha_{EV}}{\alpha_{DV}}, 0\right\} \quad (6)$$

with $\rho_{EV} = \lambda\alpha_{EV}$

- Recycling, turning *ICEV* in *HEV*, is interesting if

$$b_{HEV} > \max\left\{\frac{\rho_{HEV}[pqC_{EV}^T + prC_{HEV}^T + p(1-q-r)C_{FCEV}^T + (1-p)C_{DV}^T]}{T} - b_{EV}\frac{\alpha_{HEV}}{\alpha_{EV}} - b_{FCEV}\frac{\alpha_{HEV}}{\alpha_{FCEV}} - b_{DV}\frac{\alpha_{HEV}}{\alpha_{DV}}, 0\right\} \quad (7)$$

with $\rho_{HEV} = \lambda\alpha_{HEV}$

- Recycling, turning *ICEV* in *FCEV*, is interesting, if

$$b_{FCEV} > \max\left\{\frac{\rho_{FCEV}[pqC_{EV}^T + prC_{HEV}^T + p(1-q-r)C_{FCEV}^T + (1-p)C_{DV}^T]}{T} - b_{EV}\frac{\alpha_{FCEV}}{\alpha_{EV}} - b_{HEV}\frac{\alpha_{FCEV}}{\alpha_{HEV}} - b_{DV}\frac{\alpha_{FCEV}}{\alpha_{DV}}, 0\right\} \quad (8)$$

with $\rho_{FCEV} = \lambda\alpha_{FCEV}$

- Dismantling is interesting if

$$b_{DV} > \max\left\{\frac{\rho_{DV}[pqC_{EV}^T + p(1-q)C_{HEV}^T + p(1-q-r)C_{FCEV}^T + (1-p)C_{DV}^T]}{T} - b_{EV}\frac{\alpha_{DV}}{\alpha_{EV}} - b_{HEV}\frac{\alpha_{DV}}{\alpha_{HEV}} - b_{FCEV}\frac{\alpha_{DV}}{\alpha_{FCEV}}, 0\right\} \quad (9)$$

with $\rho_{DV} = \lambda\alpha_{DV}$.

This model is also be applicable to the dismantling and recycling situation resulting from the universal abandonment of the construction of diesel cars described in section 1. Now the option of recycling should consider a fourth option, the conversion of diesel cars to gasoline cars.

4 CONCLUSIONS

To apply this model, and get reliable conclusions, it is important to check customers' arrivals occur according to a Poisson process. For this kind of problem, this is pacific, in general, due to the huge quantity of motorcars, which owners certainly will look for these services. It is essential to estimate λ and *h(t)* to get conclusive results about the system after the available data. A correct estimate of λ will depend also on the arrivals process to be Poisson in real. Moreover, certainly the way is to decide for a mean λ estimate for a given period since it is easy to admit that the arrivals rate will depend on time. Also, it is correct to acknowledge that with very large populations, such as the ones dealt in these situations, the estimation of *h(t)* is in general technically complicated. Then the best is to directly estimate *h(t)* instead of estimating first the service time distribution followed by the consequent computation of *h(t)*. For exponential service times all this is particularly easy since in this case *h(t)* does not depend on *t*.

From *Cost-Benefit* analysis performed standing on this model, it is concluded that there are minimum *benefits* above which both dismantling and recycling are interesting. Moreover, the most interesting option is the one for which this minimum *benefit* is the least.

The model here presented contributes for a better understanding of this kind of problems. With eventual modifications, to study for example some other social economic and financial problems such as unemployment, healthcare, pensions' funds, investment projects or repair systems the model is also applicable.

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COMPETITION BETWEEN INTERCONNECTED NETWORKS

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Abstract
The motivation for this paper was to create a theoretical framework that would capture model tools for analyzing competition between interconnected networks providing services of new network industries. The formal framework is based on game and negotiation model approaches. The models are devoted to analysis of specific issues. Critical elements of competition are terms and conditions of the access arrangements between networks. Access fees are determined either by a regulator or by competition. The models analyze when network competition is sustainable without regulation, and how regulation can promote sustainable network competition when it is not. The sustainability of competition between networks is influenced by initial market share allocations, propensity of subscribers to switch networks, fixed costs of operations, and a market's price. The impact of competition between networks on negotiation and integration within a network is analyzed also.

Keywords: *Networks, Competition, Regulation, Games*

JEL Classification: L14, L22, C70
AMS Classification: 93A30

1 INTRODUCTION

Many modern industries involve networks (Shapiro and Varian 1999, Shy 2001, Fiala 2016a). In some cases, these networks are interconnected with others and engaged in competition for subscribers. The network industry typically requires access to rival networks to provide services or to satisfy its customers. Such examples include networks for communication services, electricity transmission, gas transportation, banks' ATM networks, etc. This feature is what distinguishes the network industries from others in that interconnected firms try to take dominant position not only by competing in prices but also by deteriorating competing network by charging excessive access fees. There are still many open research questions in the network competition analysis. Further development of network competition theory can help resolve these issues and improve policy (Knieps 2016). The traditional network industries are in the areas of transport (Laroche et al. 2017), (rail, road, air, maritime, urban), communications (mobile, postal services), (Houpis et al. 2016), energy (electricity, gas), water and wastewater. Profound technological transformations, especially digitalization has furthermore transformed these traditional network industries and has given rise to a new type of network industries, namely digital platforms. These call for new and innovative ways of regulation. The motivation for this paper was to create a theoretical framework that would capture model tools for analyzing competition and regulation of both traditional and new network industries. Specific models analyze interconnections of network competition, regulation and integration.

2 BASIC THEORETICAL FRAMEWORK

The theoretical framework is based on traditional network competition literature e.g. (Armstrong 1998), (Laffont-Rey-Tirole 1998a, 1998b), and (Dessein 2003) and is modified to meet the needs of specific models. The framework serves as a supply of methods and models from the areas of game, oligopoly, and negotiation theories. Laffont, Rey and Tirole (LRT) have analyzed a model of two local network companies that possess different attributes for consumers. In their model, the two companies, given access charges, set the local prices competitively. The customers of a network are charged the same price independent of the network which completes their call. The networks compete only in prices since the other attributes are assumed to be fixed. They use the standard Hotelling (1929) location model. LRT model have provided a basic theoretical framework to analyze the network competition and interconnection issues.

The simplicity of the framework suggests that it should be possible to extend it in a number of different directions. The framework can be extended in terms of the number of networks, economic instruments, cost structures, price discrimination, asymmetric structures, etc. There is a vast literature about the possible extensions, e.g. (Chemla 2003), (De Fontenay, Gans 2004). The general framework contains the extension possibilities and modeling instruments.

The modeling instruments for analyses of network competition are:

- Game theory models
- Oligopoly models
- Negotiation models

Game theory is the basis for development of network competition models, non-cooperative and cooperative models as well. The Cournot, Stackelberg and Bertrand models are representations of oligopolistic behavior. Nash equilibrium concept is used for solution. Cartel models are representations of cooperative behavior. Negotiations take place in cooperative solutions of competition problems. There are approaches based on game theory and other approaches including ones based on multicriteria evaluations.

The modeling framework serves as a common basis for developing special models for analyzing specific features in network competition. In the next two sessions there are presented two simple models for analyzing relations of network competition and regulation and network competition and integration.

3 MODEL OF COMPETITION AND REGULATION

3.1 Basic model

The model is based on simplified assumptions. There are two interconnected symmetric networks in the market. The firms provide network services which are close substitutes. The Bertrand model is a base for the situation. Networks compete in prices (p_1, p_2) and are assumed to have the following linear demands

$$q_i(p_i, p_j) = b_0 - b_1 p_i - b_2 p_j \, , \; i, j = 1, 2, \tag{1}$$

where b_0, b_1, b_2 are parameters of the demand function. The number of calls originating on a network and completed on the network is equal to the demand for the service.

Networks have the same cost structure. There is a fixed cost f of providing services. A network incurs a marginal cost c_0 per payment at the originating and terminating ends of the payment and marginal cost c_1 in between. The total marginal cost of a payment is thus $c = 2c_0 + c_1$. To provide interconnection services, it is necessary to provide essential input services to its competing network. Networks charge fees for this service called access charges (a_1, a_2).

Networks may set their access fees non-cooperatively and therefore possibly asymmetrically in nonreciprocal access pricing.

Objectives of networks are to maximize their profits. The network profit is given by

$$z_i(p_i, p_j, a_i, a_j) = [(p_i - c)q_i - f] + [(a_i - c_0)q_j - (a_j - c_0)q_i],\ i, j = 1, 2. \tag{2}$$

Two different market environments are analyzed:

- Access fees are determined by a regulator and networks compete non-cooperatively in prices
- In the first stage networks compete in access fees and in the second stage they compete in prices.

3.2 Regulation of access fees

A regulator sets a reciprocal access charge a_r for both networks. The networks compete in prices by given access charge. The network profit is given by

$$z_i(p_i, p_j) = [(p_i - c)q_i - f] + [(a_r - c_0)q_j - (a_r - c_0)q_i],\ i, j = 1, 2. \tag{3}$$

Nash equilibrium is computed by solving following equations

$$\frac{\partial z_i}{\partial p_i} = 0,\ i = 1, 2. \tag{4}$$

There is a symmetric Nash equilibrium

$$p_r = p_1 = p_2 = \frac{b_0 + b_1 c}{2b_1 - b_2} + \frac{(b_1 + b_2)(a_r - c_0)}{2b_1 - b_2}. \tag{5}$$

The final prices consist from two parts. The first part is determined by price competition and the second part is determined by access fees. The access charge depends on an objective of the regulator. If the access charge is set $a_r = c_0$, then the final price is the same as the one under the typical Bertrand competition

$$p^* = \frac{b_0 + b_1 c}{2b_1 - b_2}. \tag{6}$$

If the objective is to maximize consumer welfare under constraints that network profit is zero, then we get the access charge a_0 and the price p_0.
It can be shown that it holds

$$p_0 < p^* \text{ and } a_0 < c_0. \tag{7}$$

The access fee lower than marginal costs ($a_0 < c_0$) does not imply that networks face deficits of providing access. At equilibrium, flows in and out of each network are balanced, and thus there is no deficit in access revenue. The regulator can effectively control the price by regulating the access fee.

3.3 Competition in access fees

The networks set access fees non-cooperatively. The model is based on two stage game where the networks first set access fees and second compete in prices. The analysis starts with the second price competition stage, taking nonreciprocal access charges as given, and then continues with the first access charge competition stage backward.
For given access fees, networks' profit maximizing first order conditions are given by

$$\frac{\partial z_i}{\partial p_i} = 0, \; i = 1, 2. \tag{8}$$

From these conditions the second stage Nash equilibrium given access charges can be derived. Using backward induction, it can be expressed each network's profit in terms of access fees. Then it can be easily to find the Nash equilibrium for this two-stage game

$$\frac{\partial z_i}{\partial a_i} = 0, \; i = 1, 2. \tag{9}$$

There is a symmetric equilibrium where the access fee $a_c = a_1 = a_2$ and the final price is determined by

$$p_c = p_1 = p_2 = \frac{b_0 + b_1 c}{2b_1 - b_2} + \frac{(b_1 + b_2)(a_c - c_0)}{2b_1 - b_2}. \tag{10}$$

Difference between p_r and p_c differs only from the way how access charges are determined, if they are determined by a regulator or by the market. The equilibrium price increases as the equilibrium access charge increases.

It can be shown that it holds

$$p_0 < p^* < p_c \text{ and } a_0 < c_0 < a_c. \tag{11}$$

The price is higher when the networks compete in both access charges and prices than when access fees are regulated, since the regulator sets access charges below the marginal costs. The access charges determined by market forces are above the marginal costs. The networks have an incentive to lower the price in order to attract more consumers when markets are more competitive, but at the same time has an incentive to increase rival's prices by increasing access charges. The networks are not only competitors in the final product market but also input suppliers as providers of facilities. The regulator's intervention in the access pricing can facilitate a tacit collusion.

4 MODEL OF COMPETITION AND INTEGRATION

4.1 Basic model

The impact of competition between networks on negotiation and integration within a network is analyzed. The model is based on simplified assumptions which make possible to analyze investigated effects. There is a case of m oligopolists O_i, $i = 1, 2, \ldots, m$, each of whom sells an input to n_i downstream units, r_i of them are integrated with the oligopolist. The hole system consists of m networks N_i, $i = 1, 2, \ldots, m$. The network $N_i = (O_i, n_i, r_i)$ composed from the oligopolist O_i, n_i downstream units, r_i of them are integrated with the oligopolist.

For simplicity, we assume that the oligopolist provides an input a unit of which can be converted by into a unit of the final product. Downstream units have limited capacity; they can produce at most one unit of the final product. The products are perfect substitutes.

Downstream units produce a total quantity $n = \sum_{i=1}^{m} n_i$, this results in a market price of $p(n)$. The final product price linearly depends on the total number of products

$$p(n) = b - n\,, \tag{12}$$

where b is a parameter of the demand function. The system has following cost structure. Each downstream unit has fixed costs f. There are no additional marginal costs associated with producing the unit other than those arising from payments upstream. A vertically integrated downstream unit has an additional fixed cost g.

The gross profit of non-integrated unit and for of integrated unit equals $z = p - f$, $z = p - f - g$, respectively. The profit accruing to the network N_i, $i = 1, 2, \ldots, m$, is

$$Z_i = p(n)\, n_i - fn_i - gr_i. \tag{13}$$

For comparison, we take the classical Cournot oligopoly model. Nash equilibrium is computed by solving following equations

$$\frac{\partial Z_i}{\partial n_i} = 0\,,\ i = 1, 2, \ldots, m. \tag{14}$$

There is a symmetric Nash equilibrium

$$n_i^* = \frac{b - f}{m + 1},\ i = 1, 2, \ldots, m. \tag{15}$$

The modified negotiation model is based on a negotiation game in each network. The negotiation game has the following stages:

- System structure
- Negotiations
- Competition

4.2 System structure

Each oligopolist O_i chooses the number of integrated units r_i in its network and the number (n_i – r_i) of independent downstream units enter to be supplied by O_i. The key assumption here is that these decisions happen simultaneously. The oligopolist integrates for strategic reasons. The initial set of units cannot be replaced.

4.3 Negotiations

Oligopolists negotiate with their independent downstream units over a supply contract. The negotiations are about quantities n_i and transfer supply prices $a_i(n_i, r_i)$. The supply prices can be interpreted also as access fees for downstream units to be members of the network. For negotiation process can be used various negotiation models. When negotiations with an individual firm break down the oligopolist must also renegotiate pricing arrangements with other firms and may face a competitive response from rival networks.

There is a symmetric Nash equilibrium for negotiation model without integration

$$n_i^0 = \frac{3(b-f)}{3m+1} > n_i^* = \frac{b-f}{m+1}, \quad i = 1, 2, \ldots, m. \tag{16}$$

The resulting supply prices for independent units in case without integration $a_i^0 = a_i\,(n_i, 0)$.

The incentive to integrate is given by this schema:

- for $g = 0$, there is complete integration, $r_i = n_i = n_i^*$,
- for $g < a_i^0$, there is partial integration, $r_i < n_i$,
- for $g \geq a_i^0$, there is no integration, $r_i = 0$.

Integration will occur if it is relatively cheap, and it will result in a reduction in network and total output. Integration allows the oligopolist to negotiate higher supply prices to independent downstream units.

There is a symmetric Nash equilibrium for negotiation model with integration

$$n_i^I = \min\left(\frac{b-f+2g}{m+1}, \frac{3(b-f)}{3m+1}\right), \tag{17}$$

$$r_i^I = \max\left(0, \frac{b-f-(3m+1)g}{m+1}\right). \tag{18}$$

4.4 Competition

Competition of downstream units occurs and payoffs are realized. Important managerial implications result from the model. Issues of competition and its impact on integration are considered. The increased inter-network competition can mitigate incentives for inefficient integration. As the level of upstream competition increases, integration is less likely to occur. Increased network competition both improves competitive outcomes and reduces inefficiencies that might arise from inefficient strategic vertical integration.

5 CONCLUSION

Network competition is the important subject of an intensive economic research. The paper presents a basic modeling framework for analyses of network competition. The framework makes possible to develop simple models for analyzing specific features in network competition. Two simple models are presented. The simple models have very important managerial implications indeed. More sophisticated models can be designed for deeper analysis of competition, which will include common effects of competition and cooperation using biform games, which combine the tools of non-cooperative and cooperative games (Fiala, 2016b). Negotiation models could include multiple evaluation criteria (Fiala, 1999) and duality theory could be used to analyze these multi-criteria models more closely (Fiala, 1981). The combination of such models can give more complex views on the problem of network competition. Presented models are being prepared for application in both traditional (mobile) and new (digital platforms) network industries.

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IMPLEMENTATION OF THE CONDITION OF INDIVIDUAL STABILITY IN THE COOPERATIVE VEHICLE ROUTING PROBLEM

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Abstract
In cooperative games, we assume that players have the possibility to enter into a binding agreement with other players and can cooperate with each other. They will cooperate when it is beneficial to them. It means that they achieve more winnings in coalitions than if they do not cooperate. Coalition behaviour of players must therefore be subject to agreements on the redistribution of winnings. Such agreements must respect the axioms of rationality. In the following paper, we present the implementation of the condition of individual stability into the cooperative vehicle routing problem with homogeneous fleet. The condition of individual stability ensures that each player in coalition will improve their situation in comparison with non-cooperative behaviour. We compare the results of two situations. Firstly, vehicles have to return to their initial warehouse after serving the customers. In the second situation are vehicles reloaded in a non-initial warehouse after serving which leads to an additional cost savings.

Keywords: *cooperative game, individual stability, cooperative vehicle routing problem*

JEL Classification: JEL C61, JEL C71
AMS Classification: AMS 91A12, AMS 91A80

1 INTRODUCTION

In cooperative conflicts, we assume that players have the possibility to enter into a binding agreement and can cooperate with each other. They will do this when cooperation is beneficial to them which means that all participants of the conflict situation achieve more winnings compared to the non-cooperative situation. The necessary of cooperation in the market may be different for their actors (players), but the main objectives are usually to expand markets, gain competitive advantage or achieve know-how. In general, the cooperative games can be divided into two groups according to their distribution of pay-off. In cooperative game theory, the best theoretically developed group of games are games with a free disjunct coalition structure and transferable payments (Aumann, 1964). Such games consist of forming a coalition structure where members of each coalition agree to choose a common strategy that will ensure the highest possible coalition payment. After that they distribute the payment according to a pre-agreed binding agreement. On the other hand, there are games with non-transferable utility (pay-off). It is characteristic for games with non-transferable utility that the pay-offs are tied to the individual players and their benefits cannot be transferred to other teammates. The Non-Transferable Utility (NTU) value is a solution concept for multi-person cooperative games in which utility is not "transferable" (games "without side payments"). An example of such a game is supporting a candidate in a contest that may have a limited number of players (for example an election of a president). Many economic contexts are more naturally modelled by NTU than by TU games and indeed, the NTU value has been applied with some success to a variety of economic and economic-political models (Aumann, 1985).

Modeling a conflict situation as a cooperative game in the form of a characteristic function is determined by two conditions: creating a coalition structure by dividing players into individual coalitions and dividing the payments of players within each coalition. In this context, the question is: what properties should have the coalition structures with corresponding distribution of payments, which can be considered as "acceptable" solutions for cooperative game? Two basic requirements are (Goga, 2017):

- "Acceptable" solutions (results) should meet certain axioms of rationality.
- Each of the "acceptable" solutions should be stable in the absence of sufficient incentives to change it.

Therefore, the coalition behavior of players must be subject to agreements on the redistribution of winnings. It is clear that such agreements must at least respect the so-called axioms of rationality. The axioms of rationality take several forms. The basic conditions are axioms von Neumann – Morgenstern. We can introduce some of them (Figurová and Čičková, 2018):

- an axiom of individual rationality, i.e. the player in the coalition must earn at least as much as he could secure by stand-alone action,
- an axiom of collective rationality (Pareto optimality), i.e. players share the maximum winnings that can be earned in a cooperative game,
- an axiom of coalition rationality (Pareto optimality), i.e. no sub-coalition of coalition can provide its members a higher payment than coalition membership.

Several authors in their works provided the respect of the conditions of stability. Brandl et al. (2015) present in their paper the common notions of stability in hedonic games: core stability, Nash stability and individual stability. The coalitional stability of costs in cooperative games was investigated by Bachrach et al. (2009). Guazzone et al. (2014) devised an algorithm, which is based on cooperative game theory that can be implemented in a distributed fashion. This allows a set of cloud providers to cooperatively set up their federations in such a way that their individual profit is increased with in comparison to the case in which they work in isolation. Chun (2004) investigated the implications of the axiom of coalitional concavity for non-transferable utility coalitional form games. Inoue (2013) proved that every compactly generated non-transferable utility game can be generated by a coalition production economy.

This paper compares the situation of the cooperative game with non-transferable utility in the cooperative vehicle routing problem with homogeneous fleet. We compare the results of the two situations: an acceptance the assumption about the return of the vehicles to their initial warehouses after serving their customers or a case of reloading the vehicles in a non-initial warehouses in coalition after serving. Obviously, reloading of the vehicles can lead into the higher savings from cooperation. In case of non-transferable prizes, the condition of individual stability must be incorporated directly into the mathematical models. The proposed model includes a condition of individual stability, which means that a player cannot gain less than he would have achieved by a stand-alone situation (non-cooperative behaviour).

2 COOPERATIVE VEHICLE ROUTING PROBLEM WITH NON-TRANSFERABLE UTILITY

The basic model is model presented in Figurová and Čičková (2019). Suppose there are several depots, each one has a set of customers. The depots (players) are associated in a coalition *S*. Let: $N_S^{(1)} = \{d_1, d_2, \dots d_k\}$ is the set of nodes representing the depots. Depots offer the homogeneous goods in unlimited quantities. Let $N_S^{(i)}$ is a set of customers $of \ \ i-$th depot, $i \in N_S^{(1)}$ then $N_S^{(2)} = \bigcup_{i \in N^{(1)}} N_S^{(i)}$ is a set of nodes representing all the customers. All the nodes are represented by set $N_S = N_S^{(1)} \cup N_S^{(2)}$. Let d_{ij} be a lowest cost (distance) moving from i-th to j-th node $i, j \in N_S$. Assuming that there is unlimited number of vehicles in each i-th,

$i \in N_S^{(1)}$depot (with the same capacity g). Suppose known customer requirements q_i, $q_i \leq g$ for all $i \in N_S^{(2)}$. Customer requirements will only be realized in the whole and the size of each customer's requirement will not exceed the capacity of the vehicle.

Firstly, we will assume that all vehicles have to return back to its starting point (depot). The goal is to determine the vehicle's routes when all the customers' requirements are executed with the minimum travelled distance. In the model we will use 2-index binary variables x_{ij} $(i,j \in N_S, i \neq j)$ which are not declared as binary, but they take the values $\{0,1\}$ based on model constraints. The non-negative variables $v_{ijh} \geq 0, i,j \in N_S, i \neq j, h \in N_S^{(1)}$which determine whether edge (i,j) will be served by a vehicle from the h-th depot $h \in N_S^{(1)}$ or not are also considered. Let us summarize the model parameters and model variables:

Sets and parameters

- $N_S^{(1)} = \{d_1, d_2, \dots d_k\}$ – set of depots,
- $N_S^{(2)} = \{c_1, c_2, \dots c_m\}$ – set of customers,
- $N_S = N_S^{(1)} \cup N_S^{(2)}$ – set of all nodes (customers and depots),
- $d_{ij}, i,j \in N, i \neq j$ – shortest distance moving from node *i* to node *j*,
- q_i, $i \in N_S^{(2)}$– demand of i-th customer,
- g – capacity of all vehicles,

Variables

- $x_{ij} \in \{0,1\}$, $i,j \in N_S, i \neq j$ – determines if the node *i* immediately precedes *j* in a route of some vehicle ($x_{ij} = 1$) or not ($x_{ij} = 0$),
- $u_i \geq 0, i \in N_S^{(2)}$ – based on Miller - Tucker - Zemlin's formulation but represents vehicle load on its route to the i-th node (including),
- $v_{ijh} \geq 0, i,j \in N_S, i \neq j, h \in N_S^{(1)}$– represent the passing the vehicle of h-th depot through the edge (*i*,*j*),
- $f_k, k \in N_S^{(1)}$ – represent the individual transport costs of each player when he serves only his own set of customers.

The mathematical model can be written as (Figurová and Čičková, 2019):

$$cost = min f(X,u) = \sum_{i \in N_S} \sum_{j \in N_S} \sum_{h \in N_S^{(1)}} d_{ij} x_{ij} \tag{1}$$

$$\sum_{i \in N_S} x_{ij} = 1, j \in N_S^{(2)}, i \neq j \tag{2}$$

$$\sum_{j \in N_S} x_{ij} = 1, i \in N_S^{(2)}, i \neq j \tag{3}$$

$$\sum_{j \in N_S^{(2)}} x_{ij} \geq 1, \quad i \in N_S^{(1)} \tag{4}$$

$$\sum_{j \in N_S^{(2)}} x_{ij} = \sum_{j \in N_S^{(2)}} x_{ji}, \quad i \in N_S^{(1)} \tag{5}$$

$$u_i + q_j - g(1 - x_{ij}) \leq u_j, \; i \in N_S, j \in N_S^{(2)}, \; i \neq j \tag{6}$$

$$q_i \leq u_i \leq g, i \in N_S^{(2)} \tag{7}$$

$$\sum_{h \in N_S^{(1)}} v_{ijh} = x_{ij}, \quad i,j \in N_S, \ i \neq j \tag{8}$$

$$v_{ijh} \leq 1, \quad i,j,h \in N_S \tag{9}$$

$$v_{iji} \geq x_{ij}, \ i \in N_S^{(1)}, j \in N_S^{(2)} \tag{10}$$

$$v_{jii} \geq x_{ji}, \ i \in N_S^{(1)}, j \in N_S^{(2)} \tag{11}$$

$$\sum_{i \in N_S} v_{ijh} = \sum_{l \in N_S} v_{jlh}, \ j \in N_S^{(2)}, \ h \in N_S^{(1)}, \ i \neq j, l \neq j \tag{12}$$

$$\sum_{i \in N_S} \sum_{j \in N_S} d_{ij} v_{ijk} \leq f_k, \quad i,j \in N_S, k \in N_S^{(1)} \tag{13}$$

The scalar *cost* (1) returns the total travelled distance. Equations (2) and (3) guarantee that each customer will be visited exactly once and exactly by one vehicle. Equations (4) balance the use of vehicle of *i*-th depot, $i \in N_S^{(1)}$. Conditions (6) a (7) are the sub-tour elimination conditions and will also ensure that capacity of the vehicle is not exceeded. Equations (8), (9), (10) and (11) model the service of edge from i-th depot $i \in N_S^{(1)}$. Continuity of the routes with vehicle of *i*-th depot is secured by the equations (12). Variables $v_{ij} \geq 0, i,j \in N_S, i \neq j, h \in N_S^{(1)}$ were introduced into the model mainly because they allow to identify the vehicle's belonging to a warehouse (and there is no need to explicitly introduce a third index for binary variables $x_{ij} \in \{0,1\}, \ i,j \in N_S, i \neq j$). Equations (13) represent the condition of individual stability, where $f_k, k \in N_S^{(1)}$ represent the individual transport costs of each player (i.e. the cost of an individual player may be not higher than in the case of non-cooperative behaviour). Value f_k can be quantified as a capacitated vehicle routing problem (see in Figurová and Čičková, 2018) or as the model mentioned above where we consider a single k-th element of set $N_S^{(1)}$.

As already mentioned, further cost savings can be achieved by abandonment of the assumption of a strict round trip (i.e. returning the vehicle to their initial warehouse). In this way, the vehicle can be refilled in another player's depot in coalition. The assumption of "refilling" can be easily achieved by omitting the inequality (11). In the next section, we will consider the minimum transport cost of both cases using an illustrative example.

3 NUMERICAL EXPERIMENTS

Firstly, we will solve the cooperative model of Vehicle Routing Problem (1) – (13) with the assumption about the return of the vehicles after serving the customers to their initial warehouse. The data were obtained from work paper (see in Figurová and Čičková, 2018), where we consider the symmetric distance matrix, which represents the distance of addresses between fifteen customers and three depots in Bratislava. We obtained the shortest distance matrix from origin data by Floyd algorithm. Consider the net of 18 nodes. We will assume that there are 3 depots from which the vehicle can start its route to serve the certain customers. Suppliers or players (owners of individual depots) are expressed as $N_S^{(1)} = \{d_1, d_2, d_3\}$. Each player owns one depot with one type of vehicle. The same capacity of each vehicle is given by g = 200. Customers, who are strictly assigned to the individual depots (players) will be marked as: $D_1 = \{c_1, c_2, c_3, c_4, c_5\}$ for player d_1, $D_2 = \{c_6, c_7, c_8, c_9, c_{10}\}$ for player d_2 and $D_3 = \{c_{11}, c_{12}, c_{13}, c_{14}, c_{15}\}$ for player d_3. In the case of the creation the coalition S we know that there are exactly 7 possible coalitions between the players (including stand-alone coalitions). We solve the cooperative vehicle routing problem by using the model (1)-(13) and the model with the abandonment of assumption (11) respecting the condition of individual stability for the created coalitions S: $\{d_1\}$, $\{d_2\}$, $\{d_3\}$, $\{d_1, d_2\}$, $\{d_1, d_3\}$, $\{d_2, d_3\}$ and $\{d_1, d_2, d_3\}$ by GAMS software. To obtain the optimal solution, we used the solver Cplex 12.2.0.0 on the personal computer INTEL® Core ™ 2 CPU, E8500 @ 3.16 GB RAM for Windows 10. Our interest is to find optimal solutions based on the model of cooperative vehicle routing problem (1)-(13) and compare their results with the assumption about a case of reloading the vehicles in a non-initial warehouse. Model (1)-(13) will allow to find the optimal solution of distribution for a given coalition where condition of individual stability (13) is respected in every case (i.e. the cost of individual player is lower than that stand-alone). Our main interest is to prove that there is a reduction in individual transport costs through mutual cooperation between suppliers using the model (1)-(13) without the assumption (11) compared to the model (1)-(13) (with respect the condition of individual stability). In the Table 1, we can see the optimal routes of vehicles in each coalition and sum of the total transport costs. In the following table 1 we summarize and compare the results with the respect of the condition of individual stability in the case of returning and reloading the vehicle.

Table 1: Comparison of costs with respect of condition of individual stability

Coalition S	Costs of model (1)-(13) „case of return the vehicle“	Costs of model (1)-(13)[1] „case of reloading the vehicle“	Individual coalition player 's route costs					
			Player d_1		Player d_2		Player d_3	
			„return“	„reload“	„return“	„reload“	„return“	„reload“
$S=\{d_1\}$	23,88	23,88	23,88	23,88				
$S=\{d_2\}$	15,32	15,32			15,32	15,32		
$S=\{d_3\}$	20,50	20,50					20,50	20,50
$S=\{d_1, d_2\}$	33,82	33,82	**12,30**	**12,30**	**14,64**	**14,64**		
$S=\{d_1, d_3\}$	31,98	30,98	**13,80**	**17,20**			**18,18**	**13,78**
$S=\{d_2, d_3\}$	26,78	26,78			**12,50**	**12,50**	**14,28**	**14,28**
$S=\{d_1, d_2, d_3\}$	39,06	37,37	**13,80**	**13,80**	**11,18**	**12,09**	**14,08**	**11,48**

Source: author's calculations

In first columns of the table 1, we can see the minimum transport costs of coalitions S which represent the objective value from the presented models. Model (1) - (13) "case of return the vehicle" will find the optimal solution in the case of cooperative distribution and the return of the vehicles to their initial warehouses. Release the assumption (11) leads to reducing the transport costs ("case of reloading the vehicle"). However, coalition behavior of players must be subjected to agreements on redistribution of prices. It is clear that such agreements must respect the so-called axioms of rationality. In this case, we are interested in the axiom of individual rationality (in other words, the condition of individual stability) which requires that a player in a coalition gains at least as much as he would secure by the individual game. In both models, this condition is respected by the accepting of the assumption (13). We can also see

[1] This case is modelled with release of assumption (11) where we achieve the situation about reloading the vehicle at another depot (not the initial one).

this situation in Table 1 where we can agree that the individual costs of each player in a coalition are less than in the case of non-cooperative behavior.

4 CONCLUSION

The presented models are an extension of a model introduced by Figurová and Čičková (2018). The goal is to determine the minimum transport costs of player´s vehicles, which will satisfy the requirements of all customers. In the model (1)-(13) we assume implicitly that the size of each customer requirements will not exceed the capacity of each depot´s vehicle. The aim of this paper was to model the situation of simultaneous customer service when we compare two situations. Firstly, we calculated the minimal costs with assumption about return the vehicles to their initial warehouses. Secondly, we also respect the initial allocation of customers to the individual depot, but vehicles can be reloaded at another warehouse after serving their customers in coalition. In the both cases are the condition of individual stability respected (i.e. the cost of an individual player in coalition are lower than the stand-alone costs). Both situations in which the optimal solution of distribution within the coalition can be modelled as a model with non-transferable utility (respecting the condition of individual stability). By comparing our results, we have taken the decision that our model where we released the assumption about the return the vehicles showed better results. In model (1)-(13) with assumption about reloading the vehicle at another depot (not the initial one), we can also see the reduction when we compared the individual costs of individual players (non-cooperative behaviour) and individual costs of players in coalition. In conclusion, we can say that after the comparison our models with and without the assumption, the results of individual costs of each players have changed. Our main goal was to implement the condition of individual stability into both cases of cooperation. We can confirm that in every case is the condition of individual stability respected (they will pay less than or at least as much as they would pay if they were in a one-member coalition).

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NEIGHBORHOOD DEFINITIONS IN ECONOMETRIC MODELS: THE SEARCH FOR AN ACCURATE SPATIAL STRUCTURE WITH APPLICATION TO LABOR MARKET

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Abstract
Specification of a spatially augmented regression model is always based on a conveniently chosen neighborhood definition. Such definition depends on spatial structure information describing which regions (spatial units) are “close” and interacting and which are “distant” and thus independent. Unfortunately, exact definitions/specifications of spatial structures are not known in most economic, socio-demographic and similar application. However, reasonably well specified general spatial structures (allowing for minor deviations from optimal/true setups) lead to interpretable marginal effects and statistical inference. Still, the overall validity of a given spatial definition should always be evaluated. This article summarizes basic approaches to spatial pattern validation available from literature and provides an enhanced algorithm for spatial structure estimation, based on sectoral macroeconomic data. An empirical example is provided, based on labor market data for selected EU countries.

Keywords: *spatial structure, regional interactions, labor market, spillover effects*

JEL Classification: C23, C52, E66
AMS Classification: 91B72

1 INTRODUCTION

Modelling spatial dependencies is becoming an increasingly mainstream approach in many fields of economic and non-economic research activities. In most macroeconomic regional analyses based on empirical panel data, the underlying null hypothesis of cross-sectional independence would be violated. If not addressed properly by spatially augmented methods, this situation has important negative effects on model estimation and statistical inference (Baltagi, 2005). Neglected cross sectional dependency typically leads to inconsistent estimators (inefficient at best) and flawed test results (Elhorst, 2014).

Despite many important benefits of spatially augmented econometric analysis (Lee and Yu, 2009), this approach has some potential disadvantages. In spatial econometrics, geographic structures serve as prior information used for estimation of model parameters and marginal effects. Generally, neighborhood patterns cannot be estimated along with model coefficients. Unfortunately, a precise neighborhood specification (for a given “map” and observed economic variables) is not usually available/known with precision. Given the large number of neighborhood-definition algorithms and possible options (distance-based neighbors, *k*-nearest neighbors, contiguity patterns, etc.), researches often have to assess multiple spatial settings to evaluate model stability and robustness of their results (Formánek, 2018).

Spatial econometric models are sometimes described as a generalized form of autoregressive processes in time series analysis, with 2D dependencies (sometimes 3D interactions are modelled in environmental studies and biostatistics) being used instead of the one-dimensional time space. However, there is one remarkable difference: unlike time series analysis, where the causal direction of autoregressive processes is clear, the case of spatial dependency is much more fluid and complex. Spatial units would typically interact and influence each other (they exhibit mutual dependency), irrespective of the "direction" of spatial interactions. More importantly, the maximum distances that support spatial interactions are notoriously difficult to assess empirically (Formánek, 2018).

With respect to the ambiguity of spatial structures used for model estimation, LeSage and Pace (2014) argue that model estimates and inferences may be accurately performed for theoretically well specified models, even if slightly inaccurate spatial structures are used for model estimation. Their argument may be described using a cross sectional spatial model

$$\boldsymbol{y} = \lambda \boldsymbol{W}\boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{W}\boldsymbol{X}\boldsymbol{\theta} + \alpha\boldsymbol{\iota} + \boldsymbol{u}, \tag{1}$$

where $\boldsymbol{y}$ is the vector of all observations of the dependent variable, $\boldsymbol{X}$ is a matrix of regressors (excluding the intercept elements $\alpha\boldsymbol{\iota}$), $\boldsymbol{u}$ is the usual random element and $\boldsymbol{W}$ is a row-standardized spatial weights matrix reflecting the spatial structure (see e.g. Formánek, 2018, Furková, 2019). Maximum likelihood (ML) approach is used to estimate all model parameters: α, λ, $\boldsymbol{\beta}$ and $\boldsymbol{\theta}$; α and λ are scalars, $\boldsymbol{\beta}$ and $\boldsymbol{\theta}$ are vectors. Based on model (1), we can reasonably argue that if we compare two model estimates, each based on distinct yet reasonably similar (strongly correlated) specifications of a weight matrix $\boldsymbol{W}$, we would be highly unlikely to obtain materially different marginal effects and statistical inferences.

While the conclusions presented by LeSage and Pace (2014) may be useful in many empirical applications, they do not solve the underlying problem of unknown spatial structures. Even though two very similar $\boldsymbol{W}$-matrices may lead to mostly similar estimates, we don't know the "general quality" of spatial structure used for estimation. To address this issue, Formánek (2018) provides a relatively simple yet efficient algorithm involving an estimation of a sequence of spatial models such as (1), with varying spatial structures being used for $\boldsymbol{W}$-matrix construction. For example, a sequence of conveniently chosen maximum distances between neighbours can be used to generate different $\boldsymbol{W}$ specifications and model estimates. Subsequently, the estimated output can be summarized, and optimum spatial structures may be identified through ML-criteria. Groups with a "stable" spatial structure can be identified, with models leading to estimates robust to small changes in $\boldsymbol{W}$.

This contribution describes an empirically motivated and data-driven improvement to the distance-based spatial structures used for estimation of spatial models. Observed spatial data are used to improve $\boldsymbol{W}$-matrix definition. The approach described provides an incremental, yet reasonably useful improvement to spatial model estimation and interpretation, if one can assume temporal and sectoral stability of the spatial structure under scrutiny.

The remainder of this paper is structured as follows: Section two describes the methodology used to improve spatial specification ($\boldsymbol{W}$) used for model estimation. Section three provides an empirical illustration based on EU's labor market. Section four and the list of references conclude this paper.

2 METHODOLOGY

In this section, the discussion of potential improvements to the $\boldsymbol{W}$-matrix starts by outlining a general spatial connectivity matrix $\boldsymbol{C}$ $(N \times N)$, constructed along the following rules:

$$\boldsymbol{C} = [c_{ij}] = \begin{cases} 0 & \text{if} \quad i = j, \\ 0 & \text{if} \quad d_{ij} > \tau, \\ 1 & \text{if} \quad d_{ij} \leq \tau, \end{cases} \tag{2}$$

where $c_{ij} = 1$ means that two spatial units (say, regions) i and j are neighbors (interacting mutually) and vice versa. Matrix $\boldsymbol{C}$ is symmetric and its dimensions follow from the number of spatial units scrutinized (N). The parameter τ is a convenient threshold with empirically determined scale and value – it represents a maximum distance between neighboring units. The construction of a $\boldsymbol{W}$ matrix from the connectivity matrix $\boldsymbol{C}$ is straight forward: rows in $\boldsymbol{C}$ are standardized to sum up to unity: $\sum_{j=1}^{N} w_{ij} = 1$ for $\forall i$. It should be noted that multiple alternative methods exist for both $\boldsymbol{C}$-matrix construction (common border rule, k-nearest neighbors, etc.) and $\boldsymbol{W}$-standardization (e.g. "global" standardization) – for detailed technical discussion, see Tiefelsdorf, Griffith and Boots (2019). However, the following improvement to the method of determining a suitable spatial structure can be used in combination with most types of $\boldsymbol{C}$ and $\boldsymbol{W}$ matrices.

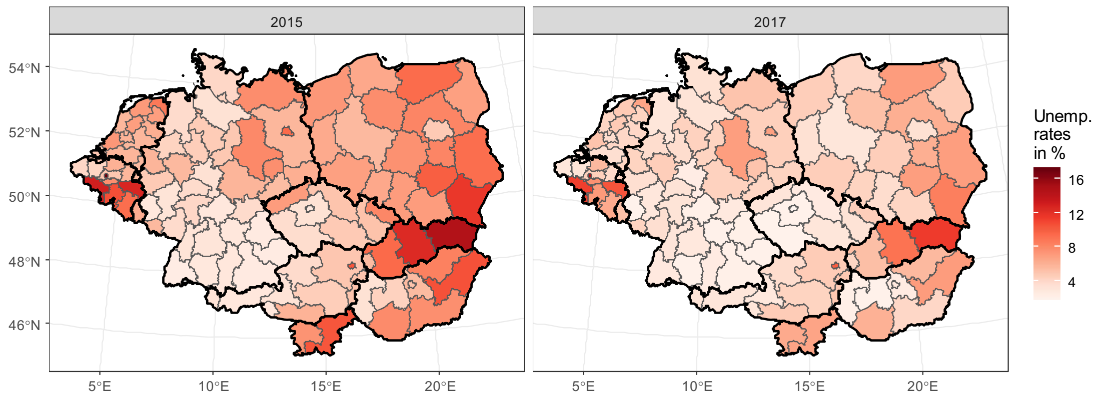

Figure 1: NUTS2-level unemployment rates (%) for years 2015 and 2017

The proposed improvement to $\boldsymbol{C}$ and/or $\boldsymbol{W}$ is designed to circumvent the inconveniences originating from the "hard" maximum neighbor distance cutoff imposed through parameter τ in expression (2). While spatially very close units would always be neighbors (mutually interacting) and distant units would be independent, there can be a prominent "grey" area where spatial units might or might not be interacting. There are multiple historical and geographic reasons for such behavior: absence or presence of communications infrastructure (highways, railways, etc.), administrative, cultural and language similarities or differences, spatial clustering affecting various industrial sectors, etc. Hence, the method proposed combines purely spatial information (distances) with observed macroeconomic (or other relevant) data to improve neighbor classification and statistical properties of spatial models.

The algorithm of improved $\boldsymbol{C}$ and $\boldsymbol{W}$ matrix construction can be motivated through Figure 1. By comparing unemployment rates for selected NUTS2 regions across 10 EU member states, we may see how spatial patterns are preserved on the map, even as individual (regional) data change over time. There may be several reasons for such empirical observations: persistent spatial heterogeneity, regional clustering, similarities in underlying socio-demographic structures, etc. If only enough data observations were available, one could perform pairwise correlation analysis or use a convenient statistical learning method (Hastie, Tibshirani and Friedman, 2001) to determine neighbors (spatially correlated or otherwise significantly bound regions) for each spatial unit.

Fortunately, the data necessary for improving $\boldsymbol{C}$-matrix specification may be available in many empirical macroeconomic applications, if we are able to make two simple assumptions:

- Static spatial structure: as spatial panel data are used for neighborhood estimation, temporal stability of spatial structures should hold over the time period under scrutiny – generally, this would be a reasonable assumption for short panels, covering e.g. 3 to 5 years or more.
- Spatial structure applies to the whole sector/field analyzed. In Figure 1, it might be reasonably assumed that an underlying spatial pattern would demonstrate itself over a range of labor market variables: unemployment and employment rates, variables describing different levels and types of socio-demographic disaggregation: labor market variables observed by male/female/total, different age groups, education levels achieved and other classification criteria upon availability. Similar assumptions may be imposed to other sectors as well: agriculture, industrial production, etc.

The above-stated conditions are inherently limiting to some extent. Yet, this is always the case with simplifying assumptions used in econometric models. However, the two assumptions can be very reasonable and non-restrictive in many empirical scenarios. At the same time, their validity should always be assessed before the next step of neighborhood analysis is performed. Clearly, the following approach depends on availability, quality, and completeness of empirical (sectoral) data.

Once the above conditions and limitations are considered, an alternative/improved $\boldsymbol{C}$ matrix may be constructed using the following formula:

$$\boldsymbol{C} = \boldsymbol{H} \circ \boldsymbol{C}^{+}, \tag{3}$$

where $\boldsymbol{C}^{+}$ follows from expression (2) with τ expanded to τ^{+} by some adequate distance range (say, 10% increase) to accommodate the "grey" zone of neighborhood definition ambiguity. $\boldsymbol{H}$ represents an empirically estimated neighborhood matrix, with binary $[h_{ij}]$ elements reflecting interacting/independent regions. The symbol $\circ$ is the Hadamard (element-wise) product and thus $[c_{ij}]$ elements in (3) equal 1 if and only if $[h_{ij}] = [c_{ij}^{+}] = 1$. Equation (3) is very general and can be used for different schemes of connectivity matrix $\boldsymbol{C}^{+}$ construction as well as with different approaches to $\boldsymbol{H}$-matrix estimation. From equation (3), $\boldsymbol{W}$ can be easily constructed (e.g. by row-standardization).

In equation (3), using an adequate $\boldsymbol{H}$-matrix specification is the key for producing improved $\boldsymbol{W}$ matrix for use in econometric models such as (1) or (5). Fortunately, this may be relatively straight-forward. Once we have a set of geo-coded data describing a given economic sector (say, labor market), we start by rescaling observed variables by demeaning and variance standardization. Next, we use the corresponding observed variables for all pairs of spatial units i and j to calculate correlation coefficients r_{ij}. Finally, we set $[h_{ij}] = 1$ if the corresponding $r_{ij} > \rho$. The parameter, ρ is a non-negative and conveniently chosen threshold: it can be zero, or it can be set more restrictively. Optimum value of ρ may be efficiently established using ML estimates of models (1) or (5), using information criteria or through cross validation (Hastie, Tibshirani and Friedman, 2001). Given the context of connectivity matrices, diagonal $[h_{ii}]$ elements may be set to zero by definition. However, as $\boldsymbol{C}^{+}$ in expression (3) already has zeros on its main diagonal, the choice of diagonal elements of the $\boldsymbol{H}$ matrix is inconsequential.

The above described approach to $\boldsymbol{H}$ matrix construction is relatively basic, and it would be adequate mainly for distance based $\boldsymbol{C}^{+}$ matrices. For a more general $\boldsymbol{C}^{+}$ construction approach (maximum distances, k nearest neighbors, contiguity, etc.), we could base $[h_{ij}]$ estimation on some form of penalized regression – with either lasso or elastic net penalty and cross-validation (Hastie, Tibshirani and Friedman, 2001) used to determine a parsimonious yet efficient set of neighbors for each spatial unit. For a quick example of such $\boldsymbol{H}$ matrix construction, consider some k-th spatial unit, selected from a map of N regions and an auxiliary regression model based on a set of L standardized sectoral variables:

$$z_{lk} = \delta_1 z_{l1} + \delta_2 z_{l2} + \cdots + \delta_{N-1} z_{l,N-1} + \omega_l, \tag{4}$$

where the first subscript in z_{lk} refers to the l-th standardized sectoral variable used for estimation (please note this is the l-th row in the dataset used for neighborhood structure estimation) and the second subscript element identifies spatial units (there are N-1 columns in a matrix of regressors $\boldsymbol{Z}$). For any given k-th spatial unit used as a dependent variable, the corresponding k-th column is excluded from the matrix of standardized sectoral observations $\boldsymbol{Z}\,[L \times (N-1)]$ on the RHS of (4). Penalized (lasso) or stepwise regression methods may be used to determine which spatial units significantly influence observed sectoral variables in the k-th spatial unit. If equation (4) is estimated by lasso, all non-zero δ_φ coefficients would implicate $[h_{k\varphi}] = 1$ and vice versa. Cross-validation can be used to determine the optimum value of lasso-penalty parameter used for estimation of (4). Please note that the above described lasso regression is possible even if sectoral variables are scarce, i.e. if $N > L$. This approach (lasso and cross validation) is used in the labor market-based empirical application provided next.

3 EMPIRICAL APPLICATION - LABOR MARKET

To illustrate the above described method of data-driven $\boldsymbol{W}$-matrix improvement, a relatively simple yet theoretically established model is estimated using annual 2015-2017 observations for NUTS2-level regions from 10 EU countries (Austria, Belgium, Czechia, Denmark, Hungary, Luxembourg, the Netherlands, Poland, Slovenia, Slovakia). Spatial panel data model is used to describe unemployment dynamics as a linear function of GDP growth, state-level effects and spatial dependencies.

A general spatial panel model used for estimation can be outlined as follows:

$$\boldsymbol{y} = \lambda(\boldsymbol{I}_T \otimes \boldsymbol{W})\boldsymbol{y} + \boldsymbol{X\beta} + \boldsymbol{u}, \tag{5a}$$
$$\boldsymbol{u} = (\boldsymbol{\iota}_T \otimes \boldsymbol{I}_N)\boldsymbol{\mu} + \boldsymbol{\varepsilon}, \tag{5b}$$
$$\boldsymbol{x}_{it}\boldsymbol{\beta} = \alpha + \beta log(GDP_{i,t-1}) + \boldsymbol{\gamma NUTS0}_i, \tag{5c}$$

where $\boldsymbol{y}$ is a NT×1 column vector of dependent variable observations (i = 1, 2, … , N identifies each of the 110 NUTS2 regions and $T = 3$ corresponds to annual observations for years 2015 to 2017). Spatial weights matrix $\boldsymbol{W}$ follows from Section 2, $\boldsymbol{I}_T$ and $\boldsymbol{I}_N$ are identity matrices (dimensions indicated by subscripts), $\boldsymbol{\iota}_T$ is a T×1 vector of ones and $\otimes$ is the Kronecker product operator. As outlined in equation (5b), the random element $\boldsymbol{u}$ contains both the unobserved individual effects $\boldsymbol{\mu}$ (spatial heterogeneity elements) and the error term $\boldsymbol{\varepsilon}$ (potentially spatially correlated). From equation (5c), the structure of $\boldsymbol{X\beta}$ can be observed - $\boldsymbol{x}_{it}$ represents a single row of the $\boldsymbol{X}$-matrix. Elements of vector $\boldsymbol{\beta}$ as well as the scalar element λ are parameters of the model, estimated by ML (Millo and Piras, 2012). Coefficients from equation (5c) are not marginal effects. Instead, direct impacts and indirect impacts (spillovers) must be calculated based on model (5a) – (5c) estimation (Elhorst, 2014).

Unemployment rate is the dependent variable in model (5): each y_{it} observation is the percentage rate of unemployment in a given NUTS2 region at time t (unemployment data come from the dataset "lfst_r_lfu3rt", Eurostat, 2020). The variable $log(GDP_{i,t-1})$ represents a log-transformed GDP per capita for a given NUTS2 unit, observations are in EUR & 1 year lags are used to account for labor market dynamic effects (dataset "nama_10r_2gdp", Eurostat, 2020). $\boldsymbol{NUTS0}_i$ is a row vector of country-level (NUTS0) binary variables. Each of its elements equals 1 if the i-th NUTS2 region is part of a corresponding state (NUTS0) and zero otherwise. $\boldsymbol{NUTS0}_i$ controls for state-level heterogeneity of the labor market (as opposed to NUTS2-level heterogeneity described by elements of vector $\boldsymbol{\mu}$). Given the hierarchical structure of NUTS regions, there are 110 NUTS2 units in the panel, belonging to 10 states. For identification purposes, $\boldsymbol{NUTS0}_i$ contains dummy variables for 9 countries and Austria is the reference "level". $\boldsymbol{NUTS0}_i$ is time-invariant, which is reflected in the subscript.

Model (5) estimation can be described as follows: First, the off-diagonal elements of $\boldsymbol{H}$ matrix from expression (3) are estimated using lasso regression such as model (4). Cross-validation is used to find optimal set of neighbors for each region. Estimation is based on a combined set of 2015-2017 labor-market sectoral data (unemployment and employment data from "lfst_r_lfu3rt", "tgs00007", "tgs00054" and "tgs00102" tables, Eurostat, 2020). Diagonal $[h_{ii}]$ elements can be set arbitrarily to either 0 or 1, as $\boldsymbol{C}^+$ from expression (3) has zeros on its main diagonal.

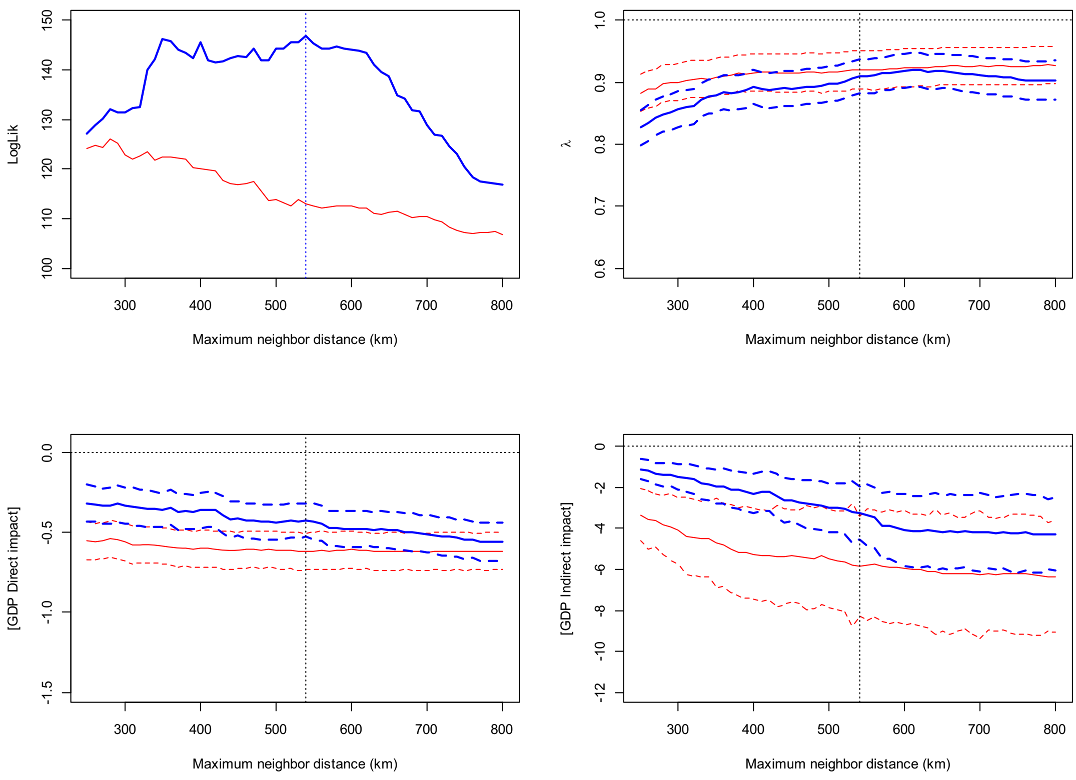

Figure 2: Model evaluation & impact estimates under varying spatial structure specifications

Next, model (5) is estimated using the enhanced $\boldsymbol{W}$ matrix based on row-standardizing $\boldsymbol{C}$ from expression (3). To determine an optimal and robust threshold τ^+ for the matrix $\boldsymbol{C}^+$ from (3), model (5) has been repeatedly estimated over a 250-800 km range of τ^+ (10-km increments used). Such repeated estimations generate many estimated parameters and statistics, which may complicate adequate model comparison across different $\boldsymbol{C}^+$ specifications. To address this problem, Figure 2 is provided, showing maximized log-likelihoods, λ-parammeters, direct and indirect impacts of GDP over the whole range of τ^+ thresholds.

From Figure 2, readers can observe how outputs from two different types of model (5) are combined for visual comparison: thicker blue lines correspond to models estimated using enhanced spatial structures, with $\boldsymbol{W}$-matrices generated from expression (3). In contrast, thinner red lines are generated by estimating model (5) using purely distance-based spatial structures, drawn from expression (2). For both model types, dashed lines mark the ± 1 standard error bands. The log-likelihood values in upper-left plot of Figure 2 show that the enhanced (data-driven) $\boldsymbol{W}$ specifications lead to superior properties of the spatial models estimated. This conclusion holds over the entire 250-800 km maximum neighbor range.

While the estimated impacts and λ-parammeters based on the two spatial structures used in Figure 2 may not seem remarkably different, the difference in maximized log-likelihood values indicate a significant improvement in predictive capabilities, if expression (3) is used to improve the definition of spatial structure used for estimation. Please note that the $\boldsymbol{\gamma}$-parameters are left out from Figure 2 (and from Table 1) due to space limitations and due to their lower informative value: while it is important to control for country-specific differences in labor market dynamics, the interpretation of multiple dummy variables is somewhat tedious. Nevertheless, all estimation output left out from this article is available from the author, along with raw datasets and R-codes used for estimation.

To provide a more detailed insight into model (5) estimation output, Table 1 shows impacts and λ-parammeters corresponding to the 540 km maximum neighbor distance threshold, which is the optimum value in terms of maximized log-likelihood values. Results for both enhanced spatial structure (3) and basic distance-based spatial structure (2) are shown.

Table 1: Estimated model (5) for maximum neighbor distance threshold of 540 km

Impact / λ	Estimate	Std. Error	z-value	Pr(>|z|)
Model with improved $\boldsymbol{W}$ structure				
GDP_Direct_Imp	-0.423	0.106	-3.992	0.000
GDP_Indirect_Imp	-3.258	1.347	-2.552	0.011
λ	0.909	0.027	33.221	0.000
LogLik = 146.740				
Model with purely distance-based $\boldsymbol{W}$ structure				
GDP_Direct_Imp	-0.620	0.114	-5.469	0.000
GDP_Indirect_Imp	-5.829	2.386	-2.626	0.008
λ	0.920	0.031	29.944	0.000
LogLik = 113.033				

In Table 1, all estimated impacts are statistically significant at the 5% significance level. Estimated indirect (spillover) effects are very prominent as compared to direct effects. Negative signs of the marginal effects concur to economic theory, which implies inverse relationship between GDP an unemployment dynamics. The estimated λ-parameters are rather high – this is mostly due to the relative simplicity and illustrative nature of model (5).

4 CONCLUSIONS

This article provides an improved method to establish/estimate spatial structures (neighborhood definitions) that may be used to enhance predictive properties and increase overall quality of different types of spatial econometric models. A relatively simple yet illustrative empirical example is provided, based on labor market data for selected EU countries, at the NUTS2-level for years 2015-2017.

The enhanced neighborhood-estimation method described in this article shows significant empirical improvement over purely distance-based approaches. However, certain limitations are present. Sectoral data availability is crucial for this type of analysis and various additional stability assumptions apply. Some ambiguity in spatial structure definition is always present in spatial econometrics and spatial analysis in general, so this feature is not "introduced" by the described methods of improving spatial models through empirical sectoral data.

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SPATIAL CLUSTER ANALYSIS OF THE REGIONAL INNOVATION IN THE EUROPEAN UNION

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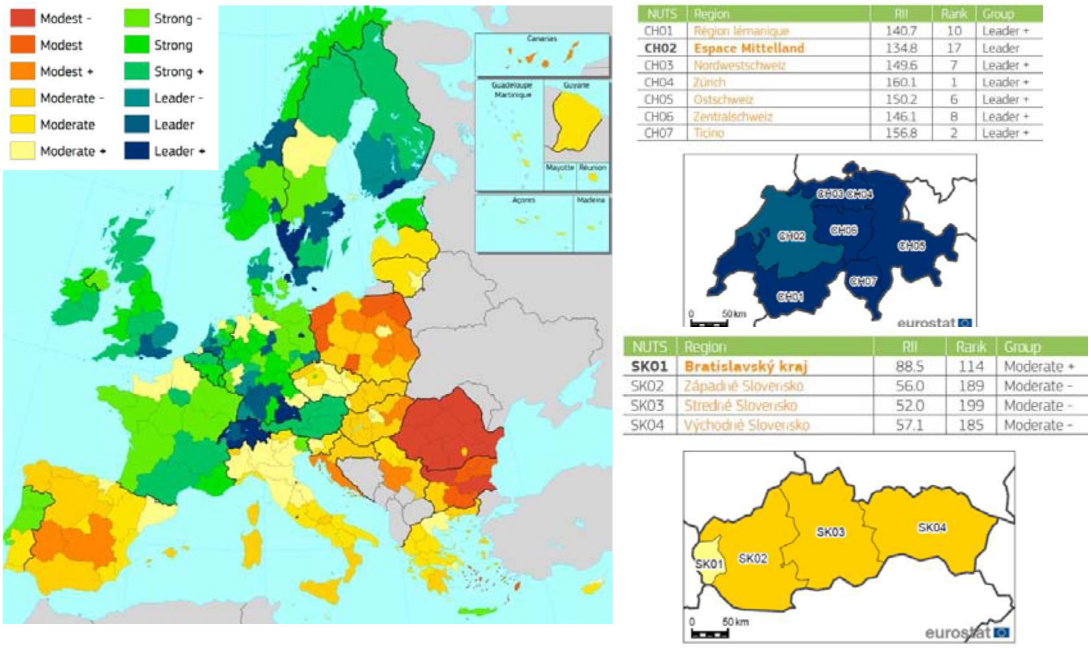
Abstract
The paper examines the role of spatial interactions in the context of regional innovation across 220 European regions based on the Regional Innovation Index (RII) 2019. The hypothesis that the level of regional innovation measured by RII at particular region affects the regional innovation processes in neighbouring regions, i.e. there exists spatial spillover effects and spatial clusters were verified based on the selected ESDA (Exploratory Spatial Data Analysis) instruments. Local univariate and multivariate Geary statistics identified statistically significant clusters corresponding mostly to regions of Switzerland, Germany, United Kingdom, Finland and Netherlands and they almost correspond to the most innovative regions – leader and strong innovative regions detected by RIS (Regional Innovation Scoreboard) 2019.

Keywords: *Local univariate and multivariate Geary statistics, Spatial autocorrelation, Regional Innovation Index*

JEL Classification: O31, R12
AMS Classification: 91B72

1 INTRODUCTION

Nowadays, regions are considered to be driving engines of economic development and measuring innovation performance at the regional level has become ever more important. The measurement and evaluation of the regional innovation have been the focus of many studies, at both, academic and policy making levels. So far, these research activities have led to three main conclusions: innovation is not uniformly distributed across regions, innovation tends to be spatially concentrated over time, and even regions with similar innovation capacity have different economic growth patterns (Hollanders, Es-Sadki and Mekelbach, 2019). Studies that support these conclusions include e.g., Moreno, Paci and Usai (2005a), Moreno, Paci and Usai (2005b) Kumar (2008), Khan (2012), Charlot, Crescenzi and Musolesi (2014) or Furková (2019a). However, a lack of the regional innovation data makes it significantly more difficult to carry out analyses on regional innovation activities. As one of the few options, the Regional Innovation Scoreboard (RIS) provides statistical facts on regions' innovation performance. The RIS is a regional extension of the European Innovation Scoreboard (EIS) which deals with comparative assessment of the performance of innovation systems at the country levels. At regional level, a composite indicator – the Regional Innovation Index (RII) is used. However, the RII is limited to using regional data for 17 of the 27 indicators used in the EIS. RIS 2019 provides a comparative assessment of performance of innovation systems across 238 regions of 23 European Union[1] (EU) member states, Norway, Serbia, and Switzerland. According to RII, Europe's regions have been classified into four innovation performance groups, i.e. groups of regional Innovation Leaders (38 regions), regional Strong Innovators (73 regions), regional Moderate Innovators (97 regions), and regional Modest Innovators (30 regions). These regional performance groups are shown in Figure 1 (left side). In addition, each performance group is splitted into a top one-third (assigned with a '+'), middle one-third, and bottom one third (assigned with a '-') regions (Hollanders, Es-Sadki and Mekelbach, 2019). Figure 1 (right side) also provides a regional performance within countries, for Slovakia and Switzerland as the most innovative country.



NUTS	Region	RII	Rank	Group
CH01	Région lémanique	140.7	10	Leader +
CH02	**Espace Mittelland**	134.8	17	Leader
CH03	Nordwestschweiz	149.6	7	Leader +
CH04	Zürich	160.1	1	Leader +
CH05	Ostschweiz	150.2	6	Leader +
CH06	Zentralschweiz	146.1	8	Leader +
CH07	Ticino	156.8	2	Leader +

NUTS	Region	RII	Rank	Group
SK01	**Bratislavský kraj**	88.5	114	Moderate +
SK02	Západné Slovensko	56.0	189	Moderate -
SK03	Stredné Slovensko	52.0	199	Moderate -
SK04	Východné Slovensko	57.1	185	Moderate -

Figure 1: Regional performance groups (left) and regional performance within countries – Switzerland and Slovakia (right)
Source: author's elaboration based on the RIS 2019 (Hollanders, Es-Sadki and Mekelbach, 2019)

The Figure 1 indicates that innovation is not uniformly distributed across the regions and probably innovation tends to be spatially concentrated. The innovation processes are often heavily localized into clusters of innovative companies, sometimes in close cooperation with

[1] Cyprus, Estonia, Latvia, Luxembourg and Malta are included at the country level, as in these countries NUTS 1 and NUTS 2 levels are identical to the country territory.

public institutions such as research centres and universities. Geographic concentration of companies allows to exploit technological development, share experiences with similar technologies, knowledge, etc.

In this paper, we focus on the role of spatial dependences among the regions in terms of regional innovation problem. Regional Innovation Index 2019 was chosen as an observed variable. Our hypothesis is that the level of regional innovation measured by RII at particular region affects the regional innovation processes in neighbouring regions, i.e. there exists spatial spillover effects and spatial clusters. Selected ESDA (Exploratory Spatial Data Analysis) tools will be used to verify this hypothesis. The paper is structured as follows: section 2 provides brief theoretical backgrounds of the study. Data and empirical results are presented and interpreted in section 3. Main concluding remarks contain section 4 and the paper closes with references.

2 METHODOLOGY

In order to test defined hypothesis, a local version of Geary´s C statistic was employed as a measure of spatial association. This decision has been greatly influenced by the fact that this statistic was recently extended to a multivariate context (see Anselin, 2018). These statistics are part of ESDA tools for evaluation spatial association of observations – spatial units (regions, countries, etc.). In general, spatial association called spatial autocorrelation corresponds to the situation where spatial units are non-independent over the space, indicating that nearby spatial units are associated in some way (for more details, see e.g. Getis, 2010). Next, a brief research method overview is provided.

The local Geary statistic as its global counterpart (see Anselin, 2019), the focus is on squared differences, rather dissimilarity than similarity. In other words, small values of the statistic suggest positive spatial autocorrelation (see Getis, 2010), whereas large values suggest negative spatial autocorrelation. Formally, the local Geary statistic can be written as follows (Furková, 2019):

$$G_i = \sum_j w_{ij} \left(x_i - x_j \right)^2 \tag{1}$$

where x_i denotes the underlying variable for region *i*, N is the number of regions and w_{ij} are the elements of $N \times N$ spatial weight matrix **W** (topics related to the spatial weight matrices can be found, e.g., Getis, 2010 or Chocholatá, 2018). A conditional permutation procedure (implemented in GeoDa [2] software) can be used to check statistical inference. The interpretation of significant locations in terms of the type of association is not straightforward

[2] In this paper, all calculations were done in GeoDa software and are presented in the form of cluster maps (Figures 2 – 5).

because the attribute similarity is not a cross-product (as it is in case of Moran´s statistics) and thus has no direct correspondence with the slope in a Moran scatter plot (Anselin, 2018; Anselin, 2019).

The local Geary statistic defined in (1) is a univariate statistic. This concept was recently extended to a multivariate context. This multivariate version measures the extent to which the average distance in attribute space between the values at a location and the values at its neighbouring locations are smaller (positive spatial autocorrelation) or larger (negative spatial autocorrelation) than what they would be under spatial randomness. Let us denote two standardized variables, z_1 and z_2. The squared distance d_{ij}^2 in two-dimensional attribute space between the values at observation (region) *i* and its geographic neighbour *j* is (Anselin, 2019, Furková, 2019b):

$$d_{ij}^2 = \left(z_{1,i} - z_{1,j}\right)^2 + \left(z_{2,i} - z_{2,j}\right)^2 \tag{2}$$

A spatial weighted average of the expression (2) is then as follows:

$$\begin{aligned} \sum_j w_{ij} d_{ij}^2 &= \sum_j w_{ij} \left[\left(z_{1,i} - z_{1,j}\right)^2 + \left(z_{2,i} - z_{2,j}\right)^2 \right] \\ &= \sum_j w_{ij} \left(z_{1,i} - z_{1,j}\right)^2 + \sum_j w_{ij} \left(z_{2,i} - z_{2,j}\right)^2 \\ &= c_{1,i} + c_{2,i} \end{aligned} \tag{3}$$

In general, then, for *k* attributes, a multivariate local Geary statistic for variable *v* can be defined as follows:

$$c_{k,i} = \sum_{v=1}^{k} c_{v,i} \tag{4}$$

3 EMPIRICAL RESULTS

The data set used in this empirical analysis comes from The Regional Innovation Scoreboard 2019 which covers 238 regions of 23 EU member states, Norway, Serbia[3] and Switzerland at different NUTS (Nomenclature of territorial units for statistics) levels. As we have already mentioned, RII was used as a main innovative indicator. However, due to missing data and isolated observations, the data set reduction must have been done. Consequently, we dealt with 220 observations at NUTS 1 or NUTS 2 levels. The RII 2019 is an aggregated indicator and consists of 17 indicators which are grouped into four main types – Framework conditions, Investments, Innovation activities, and Impacts (for more details see Hollanders, Es-Sadki and

3 For Serbia, official NUTS codes are not yet available and therefor unofficial codes will be used (see Hollanders, Es-Sadki and Mekelbach, 2019).

Mekelbach, 2019). It is necessary mention that as a spatial structure criterion among regions, a spatial weight matrix of queen case contiguity form (for more details see, e.g. LeSage & Pace, 2009) was chosen and used in all parts of our spatial analysis.

Following the univariate local Geary statistic defined by formula (1), we identified significant locations – regions with positive and negative spatial autocorrelation. Local Geary statistics for RII 2019 is presented in the form of local Geary cluster map (Figure 2 - left). A significant local Geary statistic that is less than its expected value under the null hypothesis of spatial randomness suggests a clustering of similar values (small differences imply similarity). This associations are classified as high-high or low–low. The 40 high–high and 61 low–low local clusters were identified. High–high locations are mostly regions of Switzerland, Germany, United Kingdom, Finland and Netherlands. In these regions high RII values are clustered. We can notice that these regions - clusters are more scattered across the map than the low–low clusters. In addition, high–high clusters almost correspond to the most innovative regions – leader and strong+ innovative regions detected by RIS 2019. As the calculation of local Getis statistic is based on the squared difference (see formula (1)) there may be observations for which a classification to high-high or low-low clusters is not possible. This is because the squared difference can cross the mean (expected value). These locations are referred as other positive spatial autocorrelation. One other positive location was identified in our data set. As for negative spatial autocorrelation, there are no regions with this type of association (high–low or low–high outliers).

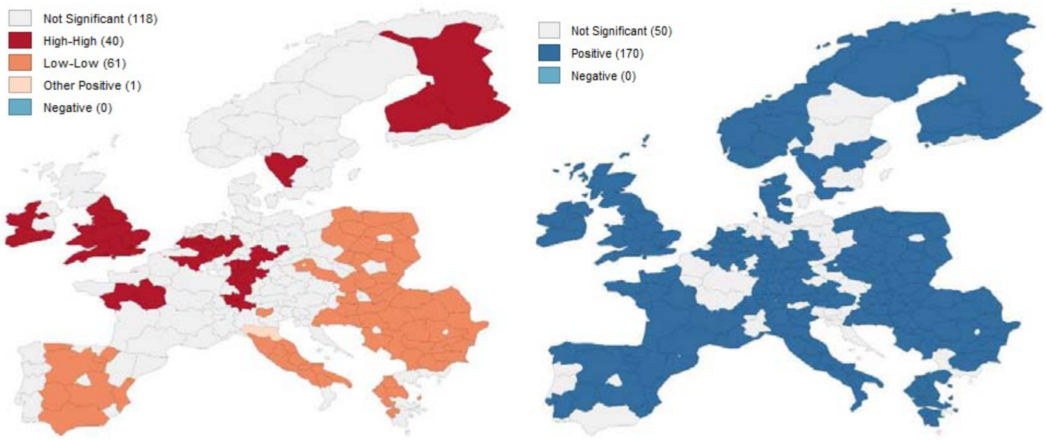


Figure 2: Univariate local Geary *clusters (left) - False Discovery Rate – 0.02318 and multivariate local Geary clusters (right) - False Discovery Rate – 0.03864 for RII 2019*
Source: author's elaboration

Figure 2 on the right side also shows a multivariate analysis. Multivariate version of Geary statistic (4) provides an additional perspective to measuring the tension between attribute similarity and locational similarity. The univariate statistic deals with distances in attribute space projected into a single dimension, whereas the multivariate statistics are based on distances in a higher dimensional space (see formulas (3) and (4)). So, a location may be identified as a cluster based on the univariate local Geary but this does not mean that it is also a multivariate cluster. The RII variable consists of 17 indicators (for more details related to these indicators see Hollanders, Es-Sadki and Mekelbach, 2019) and they were used to perform multivariate analysis of spatial association. Multivariate statistic corresponds to a weighted average of the squared distances in multidimensional attribute space (in our case 17 variables) between the values observed at a given geographic location i and those at its geographic neighbours j. Based on the formula (4) we have identified up to 170 positive multivariate local Geary clusters. Such a large number of significant locations have been obtained even though a False Discovery Rate (FDR) was used as a significance level instead of standard default p-value of 0.05. The FDR (recommended by Anselin, 2019) was also used to check a statistical inference regarding univariate version of Geary statistic.

The last part of the analysis is focused on individual indicators generating RII, namely indicators belonging to the category of innovation activities. We aimed at PCT (Patent Cooperation Treaty) patent applications, Trademark Applications and Design Applications (for more details see Hollanders, Es-Sadki and Mekelbach, 2019). Figures 3 – 5 present univariate local Geary cluster maps for these innovation outputs. On the left sides of figures standard default p-value of 0.05 was used. It is apparent that FDR yields more conservative results. Even, as for Design Applications (Figure 5), using FDR has led to only 6 statistically significant locations. But again, all high–high locations almost correspond to the most innovative regions – leader and strong+ innovative regions detected by Regional Innovation Scoreboard 2019.

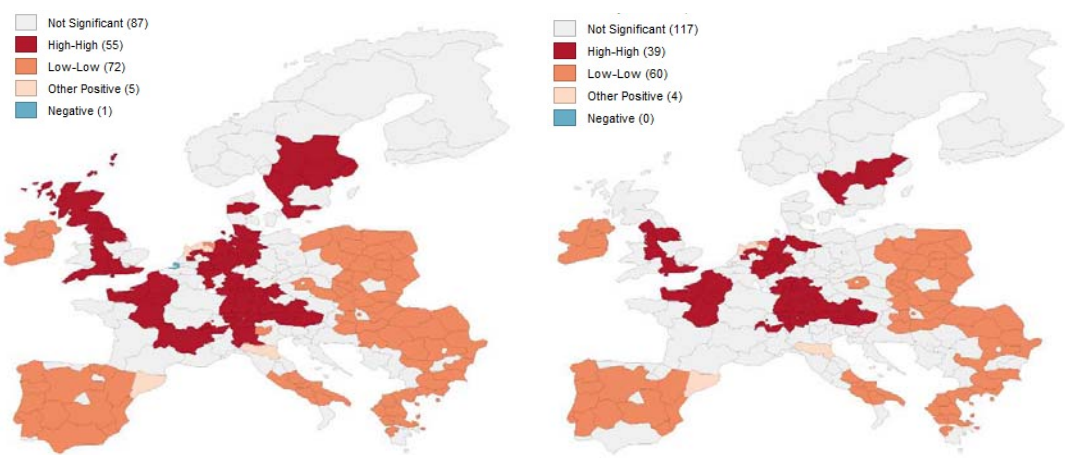


Figure 3: Univariate local Geary cluster maps for PCT patent applications: p-value 0.05 (left) and False Discovery Rate - 0.02341 (right)

Source: author's elaboration

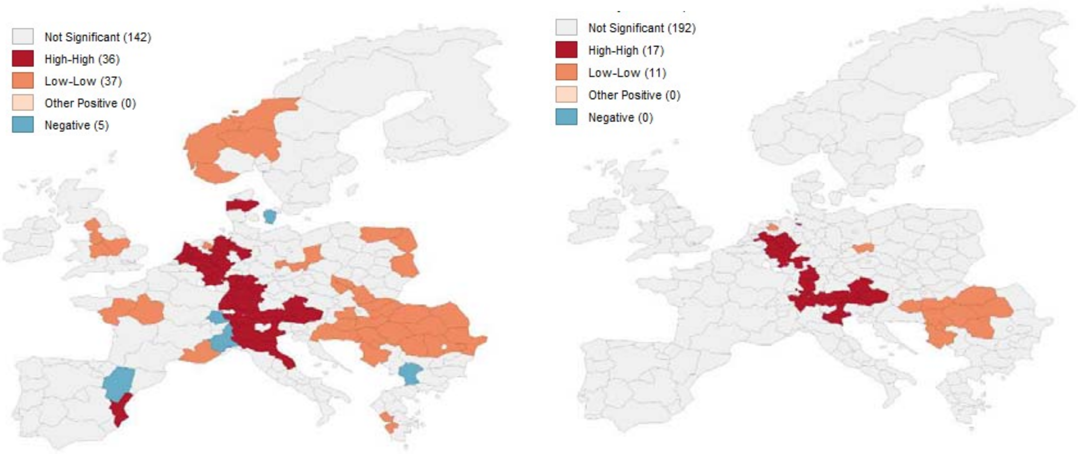


Figure 4: Univariate local Geary cluster maps for Trademark Applications: p-value 0.05 (left) and False Discovery Rate - 0.006364 (right)

Source: author's elaboration

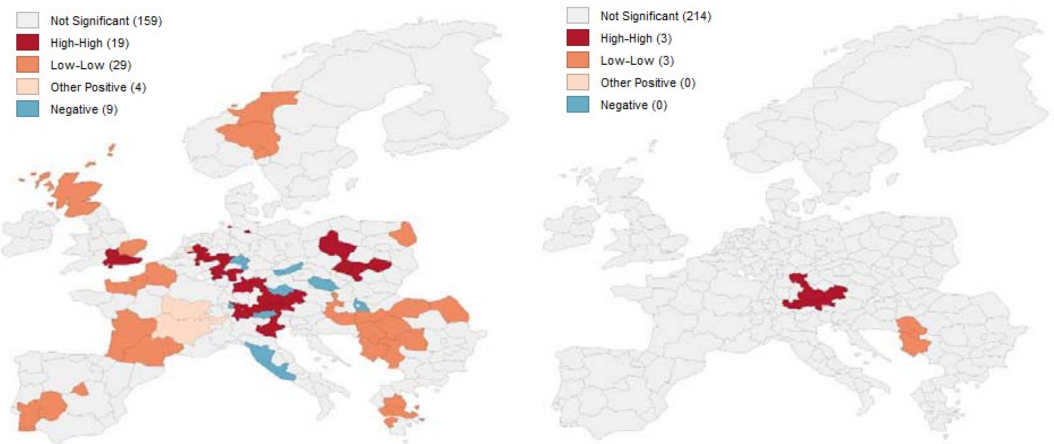


Figure 5: Univariate local Geary cluster maps for Design Applications: p-value 0.05 (left) and False Discovery Rate - 0.001364(right)

Source: author's elaboration

4 CONCLUSION

This paper dealt with spatial cluster analysis of 220 European regions based on the RII 2019. This aggregated indicator was basis for local univariate cluster analysis and individual indicators of this aggregated indicator served for multivariate cluster analysis. We concluded that our analysis based on the local univariate and also multivariate Geary statistics has confirmed the significant impact of the region's position on regional innovation process, and similar levels of RII are clustering. Our univariate spatial analysis suggested that there are significant spatial clusters as for RII in the European context. So-called high–high locations are mostly regions of Switzerland, Germany, United Kingdom, Finland and Netherlands and they almost correspond to the most innovative regions – leader and strong+ innovative regions detected by RIS 2019. According to RIS 2019 the most innovative region in Europe is region Zürich in Switzerland, followed by Ticino (Switzerland). Helsinki-Uusimaa (Finland) is the most innovative region in the EU. Taking into account multivariate context, the number of significant location increased from 102 to 170 locations. However, the number of variables included in the multivariate analysis was up to 17 and obviously, a more detailed analysis of the number as well as the relevant variables involved in multivariate analysis must be carried out.

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WAITING LINE PROBLEM IN RETAIL STORE

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Abstract
The paper deals with underlying assumptions and principles that can have an impact on the planning of needs of open cashiers in FMCG-type stores. There are basic equations of waiting line management and possible predictions related to the elimination of queues at cash desks. The paper utilizes a simulation approach to model the number of incoming customers to the cash desks and its utilization based on historical data. Paper provides an analysis of the required amount of cash desks based on the number of incoming customers.

Keywords: *Queuing theory, Customer Visit Prediction, Simulation Modeling.*

JEL Classification: C44, C63
AMS Classification: 00A72, 37M05

1 INTRODUCTION

The paper provides basic ideas, assumptions, and principles that can have a positive influence on the effective planning of open cashier needs in Fast-Moving Consumer Goods (FMCG) type stores. There are many FMCG retail outlets on the market, and most of them are part of one of the multinational chains. Therefore high competition on the market influences the number of aspects within the operation, assortment, pricing, marketing, etc. Concerning these aspects, it is difficult to correctly define the competitive advantages or factors that affect customer preferences for store selection. Existing literature provides an analysis of the impact of customers' wait time on their satisfaction in services. In general, as the customers' average wait time increases, the degree of satisfaction in services decreases (Davis and Vollmann, 1990) and (Davis and Heineke, 1998). Even when the wait time is not very long, the waiting itself has a negative impact on service evaluation (Taylor, 1994).

Self-service purchase with subsequent payment at the cash desks is the most common type of shopping, typical for most retailers, and in principle for all chains. For this reason, optimization in this area is one of the most important tasks associated with operations management, whose solution can provide a competitive advantage.

Chains often have specific rules in place that are related to customer service. Rules describe, among other things, the behavior of the cashier, but also the behavior of the management associated with the customer's waiting at the cashier. For this analysis, the waiting line is therefore defined as the situation where more customers are waiting to pay at the checkout cash desks.

One of these rules refers to the number of waiting customers. This rule is often associated with a high priority and may sound, e.g. "In case of more than three waiting customers, open another cash register."

The paper deals with the just mentioned principle of management and the possibility to meet this rule without creating a long waiting line, i.e. more than three waiting customers or without opening an unnecessarily large number of cash registers, as both situations are undesirable for retail.

2 METHODOLOGY

The primary tool for avoiding long waiting lines of customers is the prediction of waiting lines formation and predicting an adequate number of open cash registers. Specifically, this prediction aims to determine the number of customers coming to the cash registers at a particular time.

In general, it is easy to define a situation where a waiting line arises. The waiting line is created when more customers than the number of cash registers arrive at the cash registers. If the number of incoming customers is recalculated for the number of open cash registers at any given time, it is possible to obtain the value of the average number of waiting customers *AQ*, see the expression (1). *AQ* tells about waiting line creation. Respectively, its value can be compared to the given threshold value *e* (edge). If the threshold is not set, then generally, the waiting line is formed at $AQ > 1$, i.e., $e = 1$.

$$C/CD = AQ \tag{1}$$

C - number of incoming Customers

CD - the number of Cash Desks open at any given time

AQ - the average number of waiting customers, AverageQueue

The average number of waiting customers is a value that characterizes the whole store, its quality of customer service. This value does not have to be associated with previously mentioned rules; rules are applied for individual cash registers *i*, from the total number of operating cash registers *n* ($i = 1, 2, \dots n$). For individual cash registers *i*, it is possible to determine Q_i that denotes the number of waiting customers in the waiting line or Queue. For example, concerning the given threshold from the previously mentioned rule, if any $Q_i > 3$, $i = 1, 2, \dots n$, then open another cash register. However, the rule can be generalized, provided that the customers behave rationally and, when they arrive at the cash registers, always occupy the cash register that has the least waiting customers. Than expression (1) can be modified to obtain an expression that deals waiting line formation if:

$$C > eCD \tag{2}$$

or expression that deals about how many NCDs (NeedeCashDesks) are needed at a given time:

$$NCD = C/e \tag{3}$$

It is obvious from the previous expressions that the prediction of waiting line formation is, in principle, the prediction of incoming customers to the cash registers at a certain time and the subsequent determination of the necessary number of service channels. Ways how to estimate incoming customers are different in particular, given by the specifics of the chain or the type of retail, or the specifics of the store. Most often, estimates are made in the short term based on incoming customers to the retail store.

Just in connection with the arrival of customers, it is necessary to mention the procedure of counting customers. Respectively, the better name is visits counting. Counting is associated with a counter, a device that allows you to determine the number of visits in the store. Based on this information, store management can determine the number of open cash registers.

Counting procedure makes it easy to predict incoming customers to the cash registers based on the assumption that the Shopping Time (*ST*) in the store is known. The shopping time of the customer can be determined as the difference between the customer's arrival and the customer's departure times. Let specify a visit as an entity, so it is necessary to define the number of

Incoming Visits (*InV*) and the number of Outgoing Visits (*OutV*). Information on *InV*, as well as *OutV*, is provided by the counter. After abstraction from specific factors such heavy traffic, seasonality, marketing actions in particular zones or the whole store, more visitors in the passing zones or the aisles, utilization of cash registers, speed of individual employees at cash registers, etc. it is possible to determine the mean value of *ST*, respectively its estimated value.

The number of incoming visits to the cash registers would then be based on the decomposition of the *ST* into the Browsing Time (*BT*), the purchase itself and the waiting time at the cash register line, Queue Time (*QT*) and thus:

$$ST = BT + QT \quad (4)$$

After expressing the exit time of the *OutV* and then subtracting the *QT*, it would be possible to determine the time of arrival at the cash registers and, consequently, the number of visitors coming to the cash registers. If *ST* were constant throughout the day, which is a strictly hypothetical situation, it would be relatively easy to determine the number of visitors coming to cash registers. This situation is described in Figure 1.

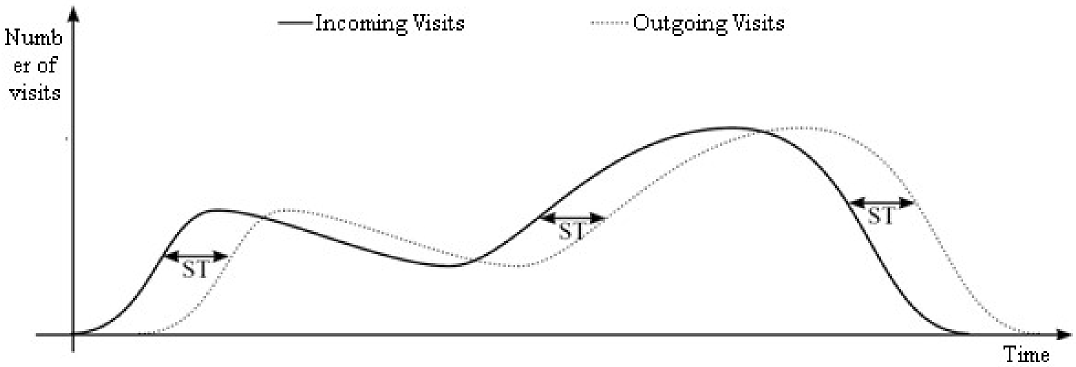

Figure 1: The arrival and departure of visitors over time with a constant shopping time

ST varies during different times of the day, not only concerning the specific factors already mentioned, but also concerning the fact that *QT* as part of the *ST*, is currently influenced by the number of open cash registers. And this is associated with the optimization of the number of cash registers. Figure 2 illustrates this.

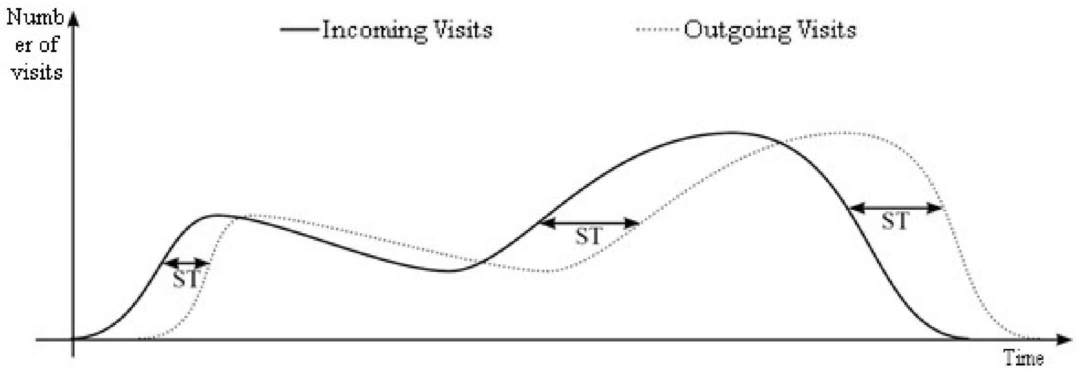

Figure 2: The arrival and departure of visitors over time with a different shopping time

Based on the simple estimation of the number of visits to the cash registers, more complex methods can be used, which will use the same principles as such a simple estimation. The choice of methods is also conditioned by the time when the forecast is made, and on this basis, the forecast can be divided into:

• Real-time predictions that continuously calculate the values of individual times (*BT, QT*), compare them with the actual number of visits (*InV* and *OutV*) and determine the expected number of incoming cash register visits based on the current values. These values should be complemented by historical values, to specify the current possible trend. These values could be estimated based on simulation models in connection with the identification of trends from historical data.

• Predictions for the next period, represent different estimates for a more extended period (e.g., next day, week, etc.) and are used in planning store operations. These predictions may also be based on simulations or econometric models, respectively trend forecasts (e.g., ARIMA).

The number of open cash registers is based in particular on the prediction of visits described above, and it is therefore assumed that the effective amount of open cash registers is one which avoids the formation of long waiting lines. There are two general methods to evaluate the queuing length, which is affecting waiting time, such as by applying a queuing theory or using simulation (Kim et al., 2013). Ray (Ray, 1988) acknowledged the queuing theory to measure the performance of the system in terms of the variables. The variables included the volume of customers in the system, an amount of customer waiting, utilization of the servers, response time, waiting time of the customers, idle time in the system. The queuing theory used a mathematical model to analyze the performance of the checkout systems (Priyangika and Cooray, 2015). Another general method utilizes simulation. Dlouhý et all. (DLOUHÝ et al., 2011) define simulation as a method of studying complex probabilistic dynamic systems using computer experimentation. Simulation means to simulate the functioning of a real system using a computer model. There are more definitions of simulation, and the interested reader can see more in literature (Mohamad and Saharin, 2019). The analyzed queuing system can be studied through the computer simulation of how the system operates. It also a tool to investigate the performance of the system virtually. There are several studies conducted for the queuing problem by using simulation. In this paper, the simulation of the queuing system is performed based on historical data and utilizing simulation as a tool for improving the efficiency of the operation at the supermarket.

3 SIMULATION CASE STUDY

The case study aims to test the rule that refers to the number of waiting customers. The analyzed rule is, "In case of more than three waiting customers, open another cash register." We have used a simulation approach to analyze the number of available employs in three scenarios. Fist scenario is four available cashiers, second scenario five available cashiers, and third scenario six cashiers. The simulation model can be used for employs capacity planning for specific days. We are comparing three scenarios based variables that measure the performance of the system, like average queue size, average queuing time, and average time in the system. For simplicity, the simulation model models only process of check-out when the customer arrives at the cash desk, and therefore the browsing time is set to be zero. The variable average time in the system represents waiting time in front of the cash desk and time needed to check-out at the cash desk.

The simulation was performed in simulation software Simul8. Simul8 is a discrete event simulation software, which can use real-world constraints and capacities. First, the sample historical data were analyzed, and based on distribution fitting tests the probability distribution was determined, specific values for inter-arrival times of customers in minutes modeled by exponential distribution are listed in table 1. Figure 3 depicts the simulation model in software Simul8 for the first scenario with four employees. Selected variables of performance measure are listed in Table 2. Based on measures, average queue size and average queuing time at the queue for cash desk four and average time in the system, it is possible to conclude that the quality of service is not sufficient and more employees are needed. For this reason, the scenario with five employs and six employees are analyzed on simulation models depicted in Figure 3 and 4. Based on the performance measure result, we can conclude that five or six employees seem to be sufficient. Measure the average time in the system does not differ too much. The only measure for an average queuing time at the queue for cash desk 5 in the second scenario with five employees is not very good, 7.33 minutes. In the third scenario, the sixth employer is awaiting the work 94.95% of the time on average. It is for the father analysis to decide if scenario with six employees is too good, and scenario with five is sufficient. For example, the sixth customer can work on other stuff while waiting for the work at the cash desk. The article aimed to focus attention on the analysis of the customer's arrival and determination of an adequate number of servers, employees.

Table1: Probability distribution for customer's arrival

Distributions

Distribution from 07:00	[Exponential: 0.8]
Distribution from 08:00	[Exponential: 0.5381]
Distribution from 09:00	[Exponential: 0.3192]
Distribution from 10:00	[Exponential: 0.2425]
Distribution from 11:00	[Exponential: 0.2239]
Distribution from 12:00	[Exponential: 0.2371]
Distribution from 13:00	[Exponential: 0.2547]
Distribution from 14:00	[Exponential: 0.2734]
Distribution from 15:00	[Exponential: 0.2941]
Distribution from 16:00	[Exponential: 0.2697]
Distribution from 17:00	[Exponential: 0.2335]
Distribution from 18:00	[Exponential: 0.2963]
Distribution from 19:00	[Exponential: 0.3192]
Distribution from 20:00	[Exponential: 0.3768]

Source: author's calculations

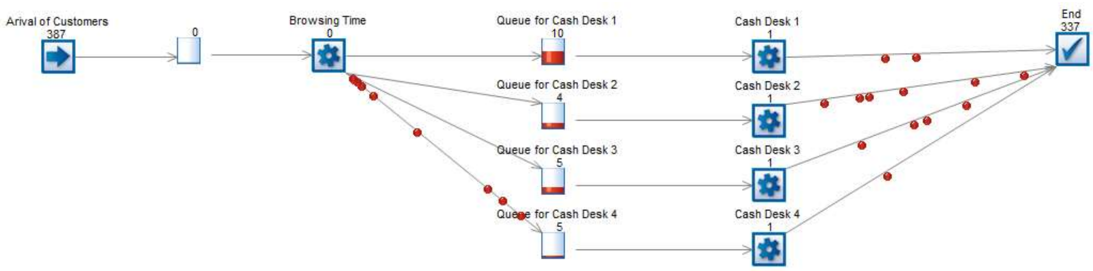

Figure 3: Simulation model in software Simul8 for the scenario with four employees.

Table 2: Performance measure for the scenario with four employees

		Low 95% range	Average Result	High 95% range
Queue for Cash Desk 1	Average Queue Size	5.5232737266574	5.54202512948483	5.56077653231226
Queue for Cash Desk 1	Average Queuing Time	5.6624901800459	5.69122937649303	5.71996857294015
Queue for Cash Desk 2	Average Queue Size	4.74318506777068	4.76354983961285	4.78391461145501
Queue for Cash Desk 2	Average Queuing Time	5.38996900826554	5.41916390191429	5.44835879556303
Queue for Cash Desk 3	Average Queue Size	3.68998950529549	3.71323427912261	3.73647905294973
Queue for Cash Desk 3	Average Queuing Time	5.07900300416853	5.11213984064654	5.14527667712455
Queue for Cash Desk 4	Average Queue Size	41.1342187800398	43.1206085904239	45.106998400808
Queue for Cash Desk 4	Average Queuing Time	52.7358889526803	54.9387371491362	57.1415853455921
End	Average Time in System	22.2144297904205	22.752280314547	23.2901308386735

Source: author's calculations

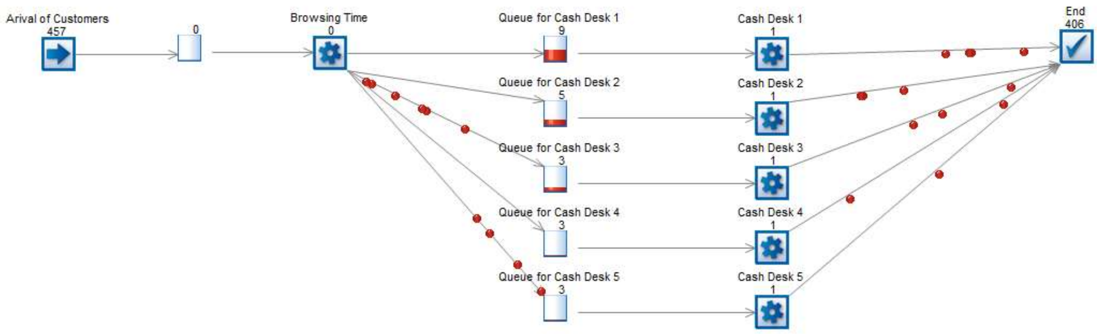

Figure 4: Simulation model in software Simul8 for the scenario with five employees.

Table 3: Performance measure for the scenario with five employees

		Low 95% range	Average Result	High 95% range
Queue for Cash Desk 1	Average Queue Size	5.5232737266574	5.54202512948483	5.56077653231226
Queue for Cash Desk 1	Average Queuing Time	5.6624901800459	5.69122937649303	5.71996857294015
Queue for Cash	Average Queue Size	4.74318506777068	4.76354983961285	4.78391461145501
Queue for Cash Desk 2	Average Queuing Time	5.38996900826554	5.41916390191429	5.44835879556303
Queue for Cash Desk 3	Average Queue Size	3.68998950529549	3.71323427912261	3.73647905294973
Queue for Cash Desk 3	Average Queuing Time	5.07900300416853	5.11213984064654	5.14527667712455
Queue for Cash Desk 4	Average Queue Size	2.22501806199524	2.25025156191204	2.27548506182883
Queue for Cash Desk 4	Average Queuing Time	4.47503663875757	4.51636923646269	4.55770183416781
Queue for Cash Desk 5	Average Queue Size	1.92982343149369	2.03875187644454	2.14768032139538
Queue for Cash Desk 5	Average Queuing Time	7.0489234952055	7.3380241874875	7.6271248797695
End	Average Time in System	11.4252510048638	11.4619843082137	11.4987176115636

Source: author's calculations

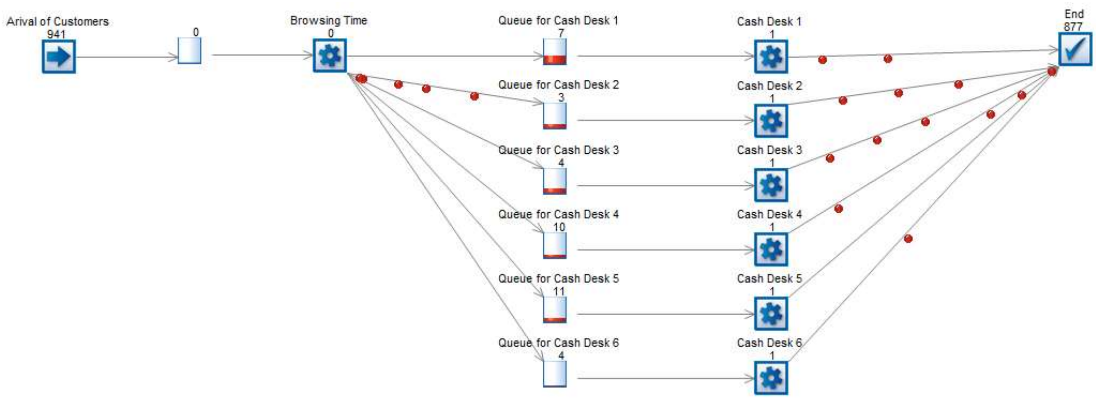

Figure 5: Simulation model in software Simul8 for the scenario with six employees.

Table 4: Performance measure for the scenario with six employees

		Low 95% range	Average Result	High 95% range
Queue for Cash Desk 1	Average Queue Size	5.5232737266574	5.54202512948483	5.56077653231226
Queue for Cash Desk 1	Average Queuing Time	5.6624901800459	5.69122937649303	5.71996857294015
Queue for Cash Desk 2	Average Queue Size	4.74318506777068	4.76354983961285	4.78391461145501
Queue for Cash Desk 2	Average Queuing Time	5.38996900826554	5.41916390191429	5.44835879556303
Queue for Cash Desk 3	Average Queue Size	3.68998950529549	3.71323427912261	3.73647905294973
Queue for Cash Desk 3	Average Queuing Time	5.07900300416853	5.11213984064654	5.14527667712455
Queue for Cash Desk 4	Average Queue Size	2.22501806199524	2.25025156191204	2.27548506182883
Queue for Cash Desk 4	Average Queuing Time	4.47503663875757	4.51636923646269	4.55770183416781
Queue for Cash Desk 5	Average Queue Size	0.841402978446675	0.862357453288559	0.883311928130443
Queue for Cash Desk 5	Average Queuing Time	3.92421629606501	3.98841819490462	4.05262009374423
Queue for Cash Desk 6	Average Queue Size	0.195752900129778	0.213341299981096	0.230929699832414
Queue for Cash Desk 6	Average Queuing Time	3.46153635433209	3.65224114083856	3.84294592734502
End	Average Time in System	11.1575074591773	11.174952123043	11.1923967869087

Source: author's calculations

4 CONCLUSIONS

The paper provides basic ideas, assumptions, and principles that can have a positive influence on the effective planning of open cashier needs in Fast-Moving Consumer Goods (FMCG) type stores. There are basic equations of waiting line management and possible predictions related to the elimination of queues at cash desks. The paper utilizes a simulation approach to model the number of incoming customers to the cash desks and its utilization based on historical data. Paper provides an analysis of the required amount of cash desks based on the number of incoming customers in software Simul8.

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GUARANTEE APPROACH IN NON-COOPERATIVE *N*- PLAYERS GAME

Marián Goga, University of Economics in Bratislava

Abstract

The article author presents some methodological problems related to the non-cooperative solution of the *n* player game, where each player progresses individually and chooses the strategy individually. This procedure in a non-cooperative game does not preclude any coordination and exchange of information between players. Players can mutually consult the choice of strategies, but the agreements in this game are not of a binding nature and its rules do not allow collusion between part of the players without the participation of others. The article shows a guarantee approach to solving such a game, explaining the important concepts and illustration of the solution in the example.

Keywords: *guarantee strategy, guarantee payment, middle value, non-cooperative game*

JEL Classification: C71
AMS Classification: 91A06, 91A10

1 INTRODUCTION

Game theory allows you to formalize (mathematically describe) conflicting phenomena, describe them in an economical way, view a complex structure of relationships and find possible results and strategies. When modelling conflicting phenomena, it is about finding the optimal way to choose strategies for each participant in a conflicting situation. This is actually how the game participant should behave to ensure the most favorable result possible.

Conflicting situations are examined in two ways:

- in the absence of cooperation between individual participants, each participant pursues an individual target – a non-operative solution in game theory [1, pp. 82],
- if the participants work together to ensure favorable conditions for the realization of the interests of all members of a particular group (coalition) – this is a cooperative solution in game theory. [5, pp. 49]

In this article we will introduce some methodological problems related to the non-cooperative solution of the *n* player, where each player progresses individually and chooses the strategy individually. Of course, this procedure in a non-cooperative game does not preclude any coordination and exchange of information between players. Players can mutually consult the choice of strategies, but the agreements in this game are not of a binding nature and its rules do not allow for collusion between part of the players without the participation of others. The article shows a guarantee approach to solving such a game, explaining the important concepts and illustration of the solution in the example.

Note that game models in normal shape display a conflicting situation as a one-time activity where strategies are chosen. However, the conflicting situation is mostly dynamic in nature, i. e. there is a progressive number of elementary activities and decisions that are carried out in a particular order. The result can also affect random factors that are usually called "nature". In analyzing the dynamic nature of the conflicting phenomenon, the degree of information of the participants (information sets of players) is also envisaged on the progress of the process.

Problems of this type are resolved in the theory of game models of strategy games in a developed (advanced) shape that are elaborated in multiple publications. [6, 8, 9]

2 NON-COOPERATIVE *N* - PLAYER GAME AND GUARANTEE STRATEGY AND GUARANTEE PAYMENT

The theory of the non-cooperative behavior of the player we analyze concerns conflicts in which binding agreements between players cannot be concluded on the choice of strategies. In such cases, each player shall, in the choice of the strategy, proceed individually so that the chosen strategy achieves as much payment as possible, without subacting its decision to any group of players. An important feature of the *n* player game is therefore the payment that the player can provide by separate procedures regardless of the behaviour of other players.

In the context of a non-cooperative game, there are two ways to work on a solution: [10]

- The guarantee principle, i. e. it is looking for a strategy for each player, which will best provide it against all the options of other players. The P_i player elects a clean strategy $x_i \in X_i$ with payment

$$g_i(x_i) = \min_{x_k \in X_k,\ k \in Q-(i)} M_i(x_1, \dots, x_{i-1}, x_i, x_{i+1}, \dots, x_n), \qquad (1)$$

which determines the smallest possible payment to which the implementation of strategy x_i may be conducted. Among the clean strategies in the set X_i can be found such $\bar{x}_i$, which maximizes the value $g_i(x_i)$ i. e.

$$g_i(\bar{x}_i) = \max_{x_i \in X_i} g_i(x_i) = g_i \qquad (2)$$

with the value of the game for gamers *i*. If the players behave according to the guarantee principle, each of them shall elect a guarantee strategy $\bar{x}_i$ with a payment that is guaranteed not to be less than the value of the game g_i. The guarantee strategies of other players $\bar{x}_j$, $j \neq i$ may not be strategies that minimize the relationship (1). However, it cannot be ruled out that the actual payment will be greater

$$M_i(\bar{x}_1, \dots, \bar{x}_{i-1}, x_i, \bar{x}_{i+1}, \dots, \bar{x}_n) \geq g_i. \qquad (3)$$

- Precautionary principle, which is linked to the equilibrium strategy of the non-cooperative game $(x_1^*, \dots, x_{i-1}^*, x_i^*, x_{i+1}^*, \dots, x_N^*)$, where the P_i player chooses a clean strategy $x_i^* \in X_i$ and expects payment

$$g_i^* = M_i(x_1^*, \dots, x_{i-1}^*, x_i^*, x_{i+1}^*, \dots, x_N^*). \qquad (4)$$

Suppose that is given the final game *n* players in normal shape $G = (P, X_i, M_i, i \in P)$. By analyzing such a game, you can find out what payment and/or the middle value of the payment can be provided by the P_i, player if the game develops at least favorably.

We note that the least favorable situation for P_i players occurs when all other players progress in the game so as to minimize payment in their favor. By choosing an appropriate strategy $x_i \in X_i$, the P_i player can maximize the payment to its advantage against such the least favorable strategy of the opposing player.

Definition 1: In the final game of n players in normal shape G we call the guarantee payment of the player P_i

$$g_i = \max_{x_i \in X_i} \min_{x_k \in X_k} M_i(x_1, x_2, \dots, x_{i-1}, x_i, x_{i+1}, \dots, x_n), \quad (5)$$

where the set $X_k = (x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_n)$ is a collection of all the files of the counter player strategies $k = 1, 2, \dots, i-1, i+1, \dots, n$.

Definition 2: The guarantee strategy of the i-th player in the final game G we call each of its strategy $\bar{x}_i \in X_i$, which satisfies the equation

$$\min_{x_k \in X_k} M_i(x_1, x_2, \dots, x_{i-1}, \bar{x}_i, x_{i+1}, \dots, x_n) = g_i, \quad (6)$$

for all $k = 1, 2, \dots, i-1, i+1, \dots, n$.

It follows from the above definition that if a P_i is chosen by a guarantee strategy, the payment will not be less than g_i in the choice of any counter-player strategies.

Definition 3: In the final game n players in normal shape $G = (P, J_i, K_i, A^i, i \in P)$ we call the guarantee medium of payment of the player P_i

$$g_i = \max_{y^i \in S_{m_i}} \min_{z^i \in S_{k_i}} E_i(y^i, z^i) = \max_{j \in J_i} \min_{k \in K_i} a^i_{jk}, \quad (7)$$

where S_{m_i} is the set of all mixed strategies y^i player P_i, z^i is the probability distribution and a^i_{jk} are the elements of the matrix A^i.

Definition 4: The net guarantee strategy of the P_i player in the final game G we call an index j_i^0 that matches the equation

$$\max_{j \in J_i} \min_{k \in K_i} a^i_{jk} = \min_{k \in K_i} a^i_{j_i^0}. \quad (8)$$

The guarantee payment or net guarantee strategy shall be calculated as the maximum line minimum of the A^i matrix, or as an index of the line of matrix A^i, for which this maximum line minimum is carried out.

Definition 5: The mixed guarantee strategy of the P_i player in the final game G is called vector $\bar{\mathbf{y}}^i \in S_{m_i}$, which meets the equation

$$g_i = \min_{\mathbf{z}^i \in S_{k_i}} E(\bar{\mathbf{y}}^i, \mathbf{z}^i), \quad (9)$$

where the vector $\mathbf{z}^i$ is some probability distribution.

The mixed guarantee strategy of the P_i player in the final game of the n player is actually a minmax optimal mixed strategy of the maximizing player in the matrix game with the A^i payment, while the guarantee middle value of the payment is the value of the game. Therefore, the guarantee joint strategy and the guarantee middle value of payments can be found, for example, by addressing the role of linear programming similar to that of matrix games. [6, 7]

We now put an illustrative example on the following definitions: suppose that the final game of three players is given in normal shape with the matrix of payments $A^1 = \begin{bmatrix} 3 & 4 & 3 & -6 \\ -2 & 5 & 1 & -3 \end{bmatrix}$,

$A^2 = \begin{bmatrix} 3 & 2 & 1 & 4 \\ 2 & -2 & 5 & 2 \end{bmatrix}$, $A^3 = \begin{bmatrix} 3 & 2 & -1 & 3 \\ 3 & -\frac{3}{2} & 4 & 5 \end{bmatrix}$.

We find out what are the net guarantee strategies of players and the corresponding guarantee payments:

matrix	line minimum	max min
A^1	–6 –3	 –3
A^2	1 –2	1
A^3	–1 $-\frac{3}{2}$	–1

The table shows that clean guarantee strategies (matrix line index) are $j_1^0 = 2$, $j_2^0 = 1$, $j_3^0 = 1$ and guarantee payments are $g_1 = a_{24}^1 = -3$, $g_2 = a_{13}^2 = 1$, $g_3 = a_{13}^3 = -1$.

The guaranteed mixed strategy players are: $\mathbf{y}^1 = (0, 1)$, $\mathbf{y}^2 = (1, 0)$, $\mathbf{y}^3 = \left(\frac{3}{5}, \frac{2}{5}\right)$, corresponding to the guarantee middle values of payments: $g_1 = -3$, $g_2 = 1$, $g_3 = \frac{3}{5}$.

We note that every player who elects a guarantee strategy is trying to maximize the payment to his advantage while assuming that the opponents are struggling together to make a payment in his favor as low as possible. Thus, opponents seek to maximize payments to your advantage.

The guarantee payment consists only of an estimate of the lower limit of payment, which the rational player in the non-cooperative n players can provide. In the result of a game that is determined by guarantee strategies, the actual payment of the players may be higher than their guarantee payments, i. e. $M_i(\bar{x}_1, \bar{x}_2, ..., \bar{x}_i, ..., \bar{x}_n) \geq g_i$.

Assuming that each player tries to maximize the payment to its advantage, the selected counter-player strategies can then be a more favorable strategy for P_i players than the guarantee strategy, i. e. it may pay the relationship

$$\max_{x_i \in X_i} M_i(\bar{x}_1, \bar{x}_2, ..., \bar{x}_{i-1}, x_i, \bar{x}_{i+1}, ..., \bar{x}_n) \geq g_i. \qquad (10)$$

We know that the result of the game is determined by the guarantee mixed strategy of players $\mathbf{y} = (\bar{\mathbf{y}}^1, \bar{\mathbf{y}}^2, \bar{\mathbf{y}}^3) = \{(0, 1), (1, 0), \left(\frac{3}{5}, \frac{2}{5}\right)\}$.

We now show the relationship between actual payments and the guarantee middle value of players' payments:

$$E_1(\mathbf{y}) = (0,\ 1).\begin{bmatrix} 3 & 4 & 3 & -6 \\ -2 & 5 & 1 & -3 \end{bmatrix}.\begin{bmatrix} \frac{3}{5} \\ 0 \\ \frac{2}{5} \\ 0 \end{bmatrix} = (-2,\ 5,\ 1,\ -3).\begin{bmatrix} \frac{3}{5} \\ 0 \\ \frac{2}{5} \\ 0 \end{bmatrix} = -\ \frac{4}{5} = -0{,}8 > g_1$$

$$E_2(\mathbf{y}) = (1,\ 0).\begin{bmatrix} 3 & 2 & 1 & 4 \\ 2 & -2 & 5 & 2 \end{bmatrix}.\begin{bmatrix} 0 \\ \frac{3}{5} \\ 0 \\ \frac{2}{5} \end{bmatrix} = (3,\ 2,\ 1,\ 4).\begin{bmatrix} 0 \\ \frac{3}{5} \\ 0 \\ \frac{2}{5} \end{bmatrix} = \frac{14}{5} = 2{,}8 > g_2$$

$$E_3(\mathbf{y}) = \left(\frac{3}{5},\frac{2}{5}\right).\begin{bmatrix} 3 & 2 & -1 & 3 \\ 3 & -\frac{3}{2} & 4 & 5 \end{bmatrix}.\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \left(3,\frac{3}{5},1,3\right).\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \frac{3}{5} = g_3,$$

while $\mathbf{z}^1 = (z_1^1, z_2^1, z_3^1, z_4^1) = \left\{\left(1.\frac{3}{5}\right),\left(0.\frac{3}{5}\right),\left(1.\frac{2}{5}\right),\left(0.\frac{2}{5}\right)\right\} = \left(\frac{3}{5},0,\frac{2}{5},0\right)$

$\mathbf{z}^2 = (z_1^2, z_2^2, z_3^2, z_4^2) = \left\{\left(0.\frac{3}{5}\right),\left(1.\frac{3}{5}\right),\left(0.\frac{2}{5}\right),\left(1.\frac{2}{5}\right)\right\} = \left(0,\frac{3}{5},0,\frac{2}{5}\right)$

$\mathbf{z}^3 = (z_1^3, z_2^3, z_3^3, z_4^3) = \{(0.1), (1.1), (0.0), (1.0)\} = (0,\ 1,\ 0,\ 0).$

The results of the game show that the actual payments of the players P_1 and P_2 are higher than their guarantee payments and the player P_3 is the same payment.

3 CONCLUSION

In this article, some methodological problems and the concept of solving non-cooperative *n* player games does not draw on existing approaches and options. There are also other concepts in the literature on game theory that solve non-operative games. [2, 4]

We note that the equilibrium point in the non-cooperative *n* player game has a similar characteristic and a characteristic of equilibrium, as in the analysis of bimatrix games. At equilibrium, we understand the state in which a particular system tends to persist under the conditions. However, not every result of a non-cooperative game with this feature can be considered an optimal solution. A result that does not have a equilibrium feature is not an optimal solution in a non-cooperative game because it is contrary to the assumption of a player's rationality. Only a set of player strategies can be considered as a satisfactory result, which in the case of a unilateral change, leads automatically to a reduction in the payment in favor of the player who caused the change. [3]

The processing of other methodological and application problems associated with the solution of non-cooperative *n* players with transferable payments is the next stage of the author's research article.

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COORDINATION OF TWO AIRCRAFT MAINTENANCE PLANS USING MATHEMATICAL MODELING

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Abstract
An integral part of modern transport aircraft are jet engines, which, like other heavily loaded technical equipment, need regular maintenance to ensure their reliable and flawless operation. Regular technical inspections of individual jet engine components are described in the Engine Maintenance Manual. However, intervals between individual inspections of the engine do not always coincide with maintenance intervals of the aircraft itself. This is, however, an undesirable phenomenon, since any maintenance will cause the aircraft is temporarily out of order, leading to a reduction in profits from its operations. The paper presents a linear mathematical model that allows the aircraft operator to optimise the maintenance plan of the aircraft as a whole, that means it enables to reconcile maintenance intervals of the jet engine and the aircraft itself.

Keywords: *Maintenance, aircraft, linear programming, mathematical modelling*

JEL Classification: C61, L9, R4
AMS Classification: 90C08

1 INTRODUCTION

Production of aircraft is one of the most complex technical activities. The latest technologies and materials are used in their production. However, the sole aim of aircraft designers is not only to ensure that the produced aircraft is capable of flight, but also to ensure that the aircraft can be maintained and, if necessary, repaired. This implies that maintenance plays an important role of each aircraft's life cycle.

Each aircraft producer defines maintenance requirements based on the MSG-3 (Maintenance Steering Group - 3) methodology. This is the third generation of methodologies developed in the 1980s to optimise the maintenance requirements for a particular aircraft type. This methodology takes into account not only maintenance and reliability requirements, but also the economic factor which is important as well. It tries to eliminate aircraft downtimes during the maintenance process, as it has been shown that more frequent maintenance does not necessarily mean increased reliability. The output of the analysis is represented by the maintenance requirements of individual aircraft components and aircraft components, which are listed in the following documents:

- Maintenance Review Board Report – it lists all maintenance works and their requested intervals.
- Maintenance Planning Document – it describes in detail all the maintenance works, which are listed in the Maintenance Review Board Report, and it is thus the main document used by the aircraft operator to plan a maintenance program.
- Aircraft Maintenance Manual – it presents the principle and operation of each aircraft system, including their service and maintenance requirements.

- Task Cards – they contain individual maintenance tasks, including detailed maintenance procedures, required material and time needed to perform the task.
- Service Bulletins – it is a continually updated document in which the aircraft producer updates all aircraft changes or new maintenance procedures. The information in this document may be obligatory or only recommended.

Based on the above-mentioned documents, the aircraft operator can create a suitable maintenance program for a specific aircraft. The program is compiled based on the division of the individual maintenance works (which are listed in the Maintenance Planning Document) into individual regular maintenance inspections. The inspections generally fall into the following 7 categories:

- Pre-flight inspection is carried out by the aircraft crew before each flight. It is only a visual inspection of the aircraft, performed during the aircraft patrol.
- Transit inspection is performed by the aircraft crew in cooperation with maintenance technicians after each landing. This inspection is also carried out before the first flight of each day.
- Daily inspection must be performed every 24 hours, usually after the last flight of the day. If it is not performed at least once within 48 hours, then the aircraft becomes unfit to fly.
- Weekly inspection is done to check the functionality of selected aircraft systems and replenishes operating fluids. The common duration of the inspection is about 5 hours.
- A – check is a more detailed inspection, which depends on the calendar time, the number of flight hours (for example after each 400 flight hours) or cycles (take-offs and landings, or the launching of aircraft power units). The duration of this inspection type is usually several days (the aircraft is out of operation).
- C – check is a very detailed check of the aircraft, which involves the dismantling of a substantial part of the aircraft. It is performed once every 4,000 flight hours (depending on the aircraft type), which is once every 1.5 to 2 years. During this check, the aircraft is out of service for several weeks.
- D – check corresponds to an overhaul, when the aircraft is almost completely dismantled, many parts are replaced and re-assembled. This check-up is performed at 25,000 flight hours (depending on the aircraft type). This usually occurs every 5 - 6 years.

The B – check inspection is no longer performed today. This is because new maintenance and operation technologies have made it possible to divide the inspections planned for the B – checks into A – check and C – check inspections. Elimination of the B – check thus made it possible to reduce the number of aircraft downtimes during its operation.

After the maintenance program has been compiled, consisting of the assignment of all the maintenance works into individual maintenance inspections, the prepared maintenance program must be submitted to the Civil Aviation Authority for its approval. Subsequently, the operator can create a maintenance plan that determines expected aircraft downtimes depending on its operation.

2 USE OF LINEAR PROGRAMMING IN MAINTENANCE

The article will present a linear mathematical model that would allow operators to effectively plan aircraft downtimes without the use of expensive maintenance planning software.

The model can be used either when the operator is creating a maintenance program or in situations when the existing maintenance program is disrupted for some reasons. Such situations are often encountered by an operator, especially when a block maintenance method is applied to an aircraft. In this maintenance philosophy, the aircraft is divided into blocks. In order to speed up its maintenance processes or reduce downtimes due to maintenance, whole blocks are changed on the aircraft in case of any problem. During maintenance, the malfunctioning block is replaced by a functional block and the aircraft is ready to fly again. Thus, the malfunctioning block can be repaired without having to stop the aircraft. When the block is put back into operation, it is stored in a warehouse for its future use.

As already mentioned, the indisputable advantage of block maintenance of the aircraft is its rapid return to service (assuming a substitute block is available). The disadvantage of this philosophy, on the other hand, is the disintegration of the original maintenance program, because each block can have a different operating state. It is therefore an effort to unify the maintenance plans of individual blocks to minimise the number of regular aircraft downtimes.

Aircraft power units represent a typical example of a block. Power unit maintenance plans are generally aligned with the service plan of the whole aircraft. The problem arises when there is a problem on the power unit due to, for example, sucking in a foreign item and subsequently damaging the turbofan blades or other technical defects. In such situations, the drive unit needs to be repaired. It will be removed and replaced by a functional drive unit, which is, however, in a different operating state than the original drive unit. Thus, it may for example occur a situation when the aircraft is scheduled to be inspected after the 40 flight hours, but the power unit is scheduled to be inspected after 80 flight hours at the latest. In such a case, it is advisable for the operator to consider uniting some aircraft downtimes even at the cost of the service being performed earlier than originally planned.

In the case of an earlier downtime, the aircraft operator incurs a financial loss resulting from insufficient utilization of the material. An example is the replacement of operating fluids (oil, hydraulic fluid etc.). These are expected to have a certain lifetime. If they are prematurely replaced, the aircraft operator has not fully utilized their lifetime, resulting in the financial loss. Conversely, if the aircraft operator allows the uniting of some service downtimes into one the aircraft operator can significantly save.

This is because the aircraft operator does not usually have a maintenance organization established under PART 145 at operator's base or in any of the destinations which the operator flies to. Such maintenance organizations focus on so-called heavy aircraft maintenance (A-check, C-check and D-check). In many cases, the aircraft operator must make an empty flight (without any passengers or cargo) to get the aircraft to a place of its downtime. Of course, such empty flight brings the considerable costs. If the operator manages to reduce the number of downtimes due to their suitable uniting, the operator can achieve the considerable savings. The decision to unite or not to unite some aircraft downtimes then depends on the difference between the early maintenance losses and the savings resulting from the reduction of empty overflights between the base and the place where the aircraft is maintained.

3 STATE OF THE ART

A well-established aircraft maintenance plan can save considerable money for the operator, so it is requested that the individual aircraft downtimes are precisely planned. One of the tools suitable for planning the maintenance processes is represented by various optimisation methods. The optimisation methods are often used in transport in general. They are used in bus transport – they can deal with planning routes [1], coordination of bus lines [2], or planning bus schedules [3].

By an appropriate adjustment of the models presented in the previously mentioned articles, the approaches can be used in other transport modes – see for example [4]. Air transport is no exception [5]. Also for air transport it is necessary to plan aircraft schedules [6] or allocate the individual aircraft to a scheduled set of flights [7]. In addition to planning aircraft operation, the optimisation methods can be also beneficial in human resource planning [8]. As already outlined in the introduction, the optimisation methods are applicable in aircraft maintenance planning as the presented article suggests.

The authors approach this issue in various ways. For example, in article [9], the authors discuss maintenance planning using the critical path method. The authors extend the classical critical path method to the use of fuzzy logic, thus considering randomness of the occurrence of some technical defects. On the contrary, the authors of article [10] do not address planning of maintenance itself, but its optimisation with regard to the return of the aircraft back to service (that means it deals with assigning the aircraft back to the served flights). The authors propose their own heuristic method. The optimisation criterion is to minimise the maintenance costs and minimise the costs associated with the return of the aircraft to service. Optimisation of the maintenance process is also dealt with by the authors of article [11]. The authors try to reschedule some maintenance inspections which are carried out at the time of aircraft turnarounds (before returning from the destination airport back to the home airport) to more suitable times or to maintain the aircraft at the home airport. The authors use methods of multi-criteria decision-making. The main optimisation criteria are the price, the remaining lifetime, the operational risk and the possible delay.
The authors of article [12] focus on block maintenance planning. The maintenance plan of the block itself, namely the drive unit, is then dealt with, for example, by article [13], where the authors try to predict the occurrence of jet engine failures.

4 PROBLEM FORMULATION AND MATHEMATICAL MODEL

Let us consider a situation when two preventive maintenance plans are applied on an aircraft; the plans differ in their time intervals of individual maintenance inspections. For each inspection it is necessary that the aircraft is put out of operation during the whole inspection. Let us assume that the first preventive maintenance plan relates to the aircraft power units and the second one to the aircraft itself. Our goal is to optimise the preventive maintenance plans so that the costs of operation of the aircraft are as minimal as possible. To achieve the goal, we can unite several maintenance inspections into single one. However, in order to ensure the process truly beneficial, it is necessary that the savings resulting from uniting the individual maintenance inspections must exceed the loss resulting from the premature shutdown of one of the components (we put the component out of operation before the time which is given by its expected technical lifetime).

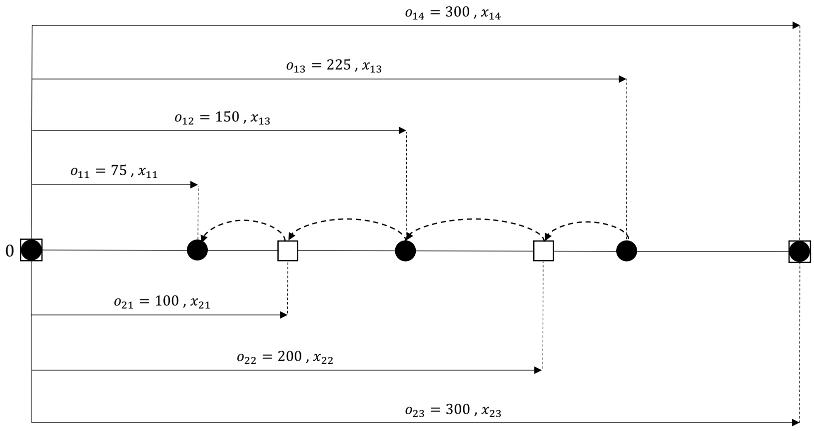

Figure 1: Example of two maintenance plans

Figure 1 graphically presents the situation when two preventive maintenance plans are optimised. The first maintenance plan is depicted as full circles and let as assume that the maintenance interval is 75 flight hours. The second maintenance plan is depicted as empty squares and the maintenance interval equals to 100 flight hours. In the figure, both maintenance plans begin at time 0 and ends at time 300 flight hours. For each maintenance plan it is depicted the latest possible time when the corresponding maintenance inspection has to be realised. That means it is only possible to shorten the preventive maintenance intervals – we can move with the individual inspections to the left (towards the beginning) in the figure. Possible uniting of the maintenance inspections in Figure 1 are depicted as dashed arcs. In our case the following uniting is possible:

- 1. inspection of maintenance plan 1 together with 1. inspection of maintenance plan 2.
- 1. inspection of maintenance plan 2 together with 2. inspection of maintenance plan 1.
- 2. inspection of maintenance plan 1 together with 2. inspection of maintenance plan 2.
- 2. inspection of maintenance plan 2 together with 3. inspection of maintenance plan 1.

To create a mathematical model a set of maintenance plans I must be given. For each maintenance plan $i \in I$ a set of inspections J_i is defined. In the presented model the first maintenance plan consists of 4 inspections. The second maintenance plan is formed by 3 inspections. In total, 6 inspections of the aircraft resulting from the set of maintenance plans I should be realised (the last inspections of both maintenance plans are realised at the same time). For each maintenance plan $i \in I$ an interval f_i of the inspections is defined. For each maintenance inspection the latest possible time when the inspection must be realised o_{ij}, where $i \in I$ and $j \in J_i$, is known.

To assess whether uniting the maintenance inspections is advantageous, the savings resulting from uniting the inspections and the losses resulting from the premature shutdown of the aircraft components must be known. Please note that only shortening the time intervals of the maintenance inspections is possible – the inspection cannot be realised after the time o_{ij}.

In the mathematical model two groups of variables must be established. The first group of variables x_{ij}, where $i \in I$ and $j \in J_i$, models the time when the inspection $j \in J_i$ of the maintenance plan $i \in I$ is realised. The value of x_{ij} is related to the beginning (time 0) which represents the moment when the both maintenance plans are initiated.

The second group is represented by variables y_k, where k is an element of a set K, which is the set of all possible uniting of the maintenance inspections. If there are four possibilities of uniting, then cardinality of the set K equals to 4 –> $|K| = 4$. The variables y_k decide whether uniting of two maintenance inspections will or not will be realised. If it holds $y_k = 1$ then the corresponding inspections will be united, otherwise ($y_k = 0$) the inspections are not united, and they are therefore realised one by one.

As discussed earlier in the article, the optimisation criterion used in the model is the difference between the losses and the savings resulting from uniting the maintenance inspections. Our goal is to minimise the difference. That means it is advantageous to unite the inspections only when the optimisation criterion takes a negative value.

The mathematical model for the described situation can be defined in the following form:

$\min f(x,y) = [1000 \cdot (75 - x_{11}) + 200 \cdot (100 - x_{21}) + 1000 \cdot (x_{11} + 75 - x_{12}) + 200 \cdot (x_{21} + 100 - x_{22}) + 1000 \cdot (x_{12} + 75 - x_{13}) + 200 \cdot (x_{22} + 100 - x_{23}) + 1000 \cdot (x_{13} + 75 - x_{14})] - 1000 \cdot \sum_{k=1}^{4} y_k$		(1)
subject to:		
$x_{ij} \leq o_{ij}$	$i \in I, j \in J_i$	(2)
$x_{ij+1} - x_{ij} \leq f_i$	$i \in I, j \in J_i \backslash \{n_i\}$	(3)
$x_{ij} \leq x_{ij+1}$	$i \in I, j \in J_i \backslash \{n_i\}$	(4)
$x_{21} - x_{11} \geq 0$		(5)
$x_{12} - x_{21} \geq 0$		(6)
$x_{22} - x_{12} \geq 0$		(7)
$x_{13} - x_{22} \geq 0$		(8)
$x_{23} - x_{13} \geq 0$		(9)
$x_{14} - x_{23} \geq 0$		(10)
$x_{14} = 300$		(11)
$x_{23} = 300$		(12)
$x_{11} - x_{21} \geq M \cdot (y_1 - 1)$		(13)
$x_{21} - x_{12} \geq M \cdot (y_2 - 1)$		(14)
$x_{12} - x_{22} \geq M \cdot (y_3 - 1)$		(15)
$x_{22} - x_{13} \geq M \cdot (y_4 - 1)$		(16)
$x_{ij} \geq 0$	$i \in I, j \in J_i$	(17)
$y_k \in \{0,1\}$	$k \in K$	(18)

Formula (1) represents the objective criterion. It consists of two parts. The first part closed in square brackets expresses the financial loss resulting from the fact the maintenance inspection is realised earlier. The financial loss is caused because of uniting two inspection into one. The second part ($1000 \cdot \sum_{k=1}^{4} y_k$) models the savings resulting for uniting two inspections. The savings can for example result from not moving the aircraft to the place when it undergoes the inspection twice.

Constraints (2) ensure that the time when the maintenance inspection $j \in J_i$ of the maintenance plan $i \in I$ is realised is not greater than the latest possible time o_{ij}. Constraints (3) ensure that the maximal time interval between the consecutive maintenance inspections of the same maintenance plan is not exceeded. That means for example that if the time interval between the inspections equals to 75 flight hours then the difference between the second and the first inspection is not greater than 75 flight hours. Constraints (4) ensure that the prescribed order of the maintenance inspections of each maintenance plan is satisfied. Constraints (5) – (10) assure that the ordered sequence of the maintenance inspections of the different maintenance plans is also satisfied. That means it cannot happen that for example the inspection of the second maintenance plan precedes the inspection of the first maintenance plan when it is not technically possible – it is more than obvious that we must observe the technical limitations and the maintenance schedule when forming the constraints. Constraints (11) and (12) ensure closing the observed time period of 300 flight hours. Constraints (13) up to (16) are auxiliary constraints ensuring that if two inspections are united then the saving resulting from their uniting is used. Constraints (17) and (18) define domain of definition of the variables used in the mathematical model.

The number of constraints (5) – (16) depends on the number of the maintenance plans and the number of the maintenance inspections. Constraints (5) – (10) and (13) – (16) must be formed individually for each maintenance schedule. To create them it is necessary to know the requested sequences of the inspections and possibilities of their uniting.

5 CALCULATIONAL EXPERIMENTS

The model presented in the article was tested in a demo version of the Xpress-IVE optimisation software. Two model experiments were performed during its testing. In order to test different intervals between the aircraft downtimes, it is necessary to partially modify the model for each experiment. The adjustments of the original model result from a change in the values of the intervals of the downtimes, which is also related to a change in their order.

5.1 Experiment I

In the first experiment, 2 maintenance plans will be coordinated. The downtime interval for the first maintenance plan is $f_1 = 60$ flight hours. For the second maintenance plan, the downtime interval is $f_2 = 75$ flight hours. The limiting time was the value of 300 flight hours, when the fifth downtime of the first maintenance plan must be performed, that means $x_{1,5} = 300$. The original downtime times of the maintenance plans are shown in Table 1. In total, according to the original plans, 8 downtimes are planned.

The model differs in constraints (3), where the values of the inspection intervals must be changed. The increase in the number of the downtimes will increase the number of the constraints for all their groups except for constraints (11) and (12). The model will be supplemented by constraints (19) – (26). The number of the terms will also increase in the objective criterion. On the other hand, constraint (12) will be removed because there are no time restrictions for the second maintenance plan downtime.

$$x_{15} - x_{14} \leq 60 \tag{19}$$
$$x_{24} - x_{23} \leq 75 \tag{20}$$
$$x_{14} \leq x_{15} \tag{21}$$
$$x_{24} - x_{14} \geq 0 \tag{22}$$
$$x_{15} - x_{24} \geq 0 \tag{23}$$
$$x_{12} - x_{23} \geq M \cdot (y_5 - 1) \tag{24}$$

$$x_{23} - x_{1,4} \geq M \cdot (y_6 - 1) \tag{25}$$
$$x_{14} - x_{24} \geq M \cdot (y_7 - 1) \tag{26}$$

The optimum solution with the value of the objective function $f(x) = -28\,000$ was found. Now there are only 5 downtimes thanks to the optimisation of the maintenance plans – see Table 2

Maintenance plan	Downtime				
	1	**2**	**3**	**4**	**5**
1	60	120	180	240	300
2	75	150	235	300	-

Table 1 Originally planned downtimes [flight hours]

Maintenance plan	Downtime				
	1	**2**	**3**	**4**	**5**
1	60	120	180	240	300
2	60	120	180	240	-

Table 2 Optimised downtimes [flight hours]

5.2 Experiment II

The second experiment also coordinates 2 maintenance plans. The downtime interval for the first maintenance plan is $f_i = 60$ flight hours. For the second maintenance plan, the downtime interval is $f_2 = 90$ flight hours. The limiting time was the value of 300 flight hours, when the fifth downtime of the first maintenance plan must be performed, that means $x_{1,5} = 300$.
The originally planned downtimes of the maintenance plans are shown in Table 3. In total, according to the original plans, 7 downtimes are planned. The original model will be modified similarly as in Experiment I.

The optimal solution with the value of the objective function $f(x) = -14\,000$ was found. Now there are only 6 downtimes thanks to optimisation of the maintenance plans – see Table 4.

Maintenance plan	Downtime				
	1	**2**	**3**	**4**	**5**
1	60	120	180	240	300
2	90	180	270	-	-

Table 3 Originally planned downtimes [flight hours]

Maintenance plan	Downtime				
	1	**2**	**3**	**4**	**5**
1	60	120	180	240	300
2	90	180	240	-	-

Table 4 Optimised downtimes [flight hours]

6 CONCLUSIONS

In air transport, maintenance processes of aircraft are very demanding – not only in terms of requested quality of maintenance works, but also in terms of minimising the amount of time the aircraft is taken out of service because of its maintenance. The aircraft operator must, based on the documents supplied by the aircraft producer, define a suitable maintenance program for each new aircraft. The operator must adhere to the inspection dates for the individual parts and blocks of the aircraft; the dates may vary considerably. For this purpose, the operator aggregates the inspections with similar time intervals. This will form a plan of the aircraft downtimes that is defined by clearly defined inspection intervals.

However, replacing a part or block of the aircraft may disrupt its original maintenance plan, and therefore it is necessary to optimise the plan after such disruptions. One of the suitable tools for optimising the service plan is linear programming, which is also used for planning in many other transport sectors. This paper presented the linear mathematical model that smaller air transport operators can use to create maintenance plans as well as to optimise the maintenance plan in case of disruption. The optimisation criterion is represented by the total savings resulting from the optimisation of the plan. This overall savings is defined as the difference between the savings resulting from uniting two aircraft downtimes into one and the financial loss that results from full non-utilization of the material due to early maintenance. The created model must always be adjusted to a specific situation. The experimental verification of model functionality was performed in the Express-IVE optimisation software.

In the future, we would like to extend the financial analysis of the savings and losses resulting from uniting aircraft downtimes to better reflect the real operating conditions. Further development of the model will consist mainly in verifying its functionality on larger optimisation tasks.

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Inequalities in the repressive state force resources: Comparison with criminality

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Abstract
This article deals with inequalities in the repressive state force resources and correlation with criminal in region of the Czech Republic. Repressive state force are the police. The objective of this study is to compare inequalities in the police resources with criminal statistics. Inequalities in the police resources are measured by three alternative methods. These methods are the separate evaluation, the common weights model, and the production frontier model based on data envelopment analysis (DEA). The data covers the year 2014 and comes from the police statistics and from the Czech Statistical office. The Czech Republic is divided into 14 regions. Inequalities in the police resources were measured against the number of inhabitants in the region. The common weights and production frontier models that take into account the possibility of resource substitution give on average higher capacity scores and show lower differences between regional capacities.

Keywords: *Inequalities, MCDA, DEA*

JEL Classification: C44
AMS Classification: 90-11

1 INTRODUCTION

The police are a repressive state authority, which take care of security in the state, protects the right of the citizen and prosecute the perpetrators of crime. The police resources are financed from the public budget. Most money in the public budget are from taxpayers. Inhabitants in the Czech Republic should have claim to equal police care across the all regions in the Czech Republic.

In this paper, I focus on regional variations in the police resources. The differences in regional publicly financed resources are observed in many publicly financed organization systems. An analysis of the regional distribution of police resources is one of the tools of evaluation of equal access. The usual analytical approach to a comparison of such regional differences is to evaluate each police resource separately. The objective of this study is to present alternative methods of regional comparisons to the method of separate evaluation. The described methods are applied to the regional distribution of police resources in Czech Republic.

2 METHODS

This study describes three alternative approaches for measurement inequalities in police resources (Dlouhý and Hanousek, 2019).

2.1 Separate evaulation

The most common method of assessment is to evaluate regional resource capacities for each police resource separately. The researchers usually compare regional resources. No substitution among resources is considered and therefore the total sum of resources does not make sense. However, the separate evaluation has some disadvantages, especially in ignoring possible substitutions among resources. For a simplicity, let us assume that more is better, although it is not always the case. Hence the population in region A has surely better access to police care than the population in region B only in the case if region A mathematically dominates region B, i.e. capacities of all police resources in region A are higher than the capacities in region B. In the case that some capacities of police resources are higher in region A and some capacities are higher in region B, the separate evaluation method is not able to decide which regional population is better-off (Dlouhý and Hanousek, 2019).

2.2 Common weights method

Unlike the method of separate evaluation, we assume that police resources are, at least to some extent, substitutes, so the total weighted sum of police resource capacities can have a police policy interpretation. It is assumed that the lower police resource capacities in one resource category can be compensated by higher capacities in another resource category. In such a case, the size of geographical inequality is expected to be lower than the inequality calculated by the separate evaluation. To cope with cases with multiple police resources, the total police resource capacity is obtained by the weighted sum of individual police resource capacities. A comparison of police resource capacities in this situation is a problem of multiple criteria decision making, specifically of the weighted sum model in which the key issue is setting of relative resource weights. Such resource weights are then used nationwide for all regions. The question is how to obtain values of these resource weights. A survey among experts that are able to give us their subjective views is one possibility. The alternative possibility is to calculate weights "objectively" by the mathematical model. The objective function of the presented model is a maximization of the total weighted sum of police resource capacities. This model is known as the common weights model. Let us have a country with regions that use m inputs to serve regional population. In this case we assume that regional population is the single output that can be omitted from the model if the inputs are expressed in the numbers per inhabitant (Dlouhý and Hanousek, 2019). The mathematical formulation of the common weights model is:

$$\text{Maximize:} \quad \sum_{j=1}^{n} \varphi_j$$

$$\text{Subject to:} \quad \varphi = \sum_{i=1}^{m} v_i x_{ij} \qquad j = 1,2,\dots,n,$$

$$\sum_{i=1}^{m} v_i x_{ij} \leq 1 \qquad j = 1,2,\dots,n, \tag{1}$$

$$v_i \geq \varepsilon \qquad i = 1,2,\dots,m,$$

2.3 Production frontier model

More flexible approaches to estimate resource weights and the rate of substitution can be based on the production frontier (Dlouhý, 2018). In this method, the weights of police resources differ for each region. In the production frontier model, the police resources can be considered as inputs and the served population is the single output. In this case, the production function is estimated by the "resource efficient" units with minimal amounts of resources. So the capacities of "resource inefficient units" are compared to the most efficient production frontier. In the second form of the production frontier, the regional population is modelled as the single input and the police resources are model outputs. In this alternative that we will use in the paper, the production frontier is estimated by the most "resource inefficient" units, i.e. the units with the maximal amount of resources. The production frontier can be estimated by the deterministic frontier analysis or stochastic frontier analysis, which are econometric methods (Kumbhakar et al., 2000), or by data envelopment analysis, which is method based on the theory of mathematical programming (see, for example, publications (Dlouhý et al., 2018, Jablonský and Dlouhý, 2004). In this paper, I will use the data envelopment analysis for estimation (calculation) of the production frontier. Data envelopment analysis (DEA) was developed by Charnes, Cooper and Rhodes in 1978 (Charnes et al., 1978) and constructs the production frontier and evaluates the technical efficiency of production units. DEA is based on the theory of mathematical programming and estimates the production frontier as the piecewise linear envelopment of the data. The production unit uses a number of inputs to produce outputs. The technical efficiency of the production unit is defined as the ratio of its total weighted output to its total weighted input or, vice versa, as the ratio of its total weighted input to its total weighted output. The DEA model permits each production unit to choose its input and output weights to maximize its technical efficiency score. A technically efficient production unit is able to find such weights that the production unit lies on the production frontier. The production frontier represents the maximum amounts of output that is produced by given amounts of input (the output maximization DEA model) or, alternatively, the minimum amounts of inputs required to produce the given amount of output (the input minimalization DEA model). Let us have n production units that use m inputs to produce r outputs. The formulation of the input-oriented version of the constant returns-to-scale DEA model for production unit q is:

$$\text{Maximize: } \varphi_{\text{q}} = \sum_{k=1}^{r} \varphi_k y_{kq}$$

$$\text{Subject to: } \sum_{k=1}^{r} u_k y_{kj} \leq \sum_{i=1}^{m} v_i x_{ij} \qquad j = 1,2,\ldots,n,$$

$$\sum_{i=1}^{m} v_i x_{iq} = 1 \tag{2}$$

$$u_k \geq \varepsilon \qquad k = 1,2,\ldots,r,$$

$$v_i \geq \varepsilon \qquad i = 1,2,\ldots,m,$$

where ϕ_q is the technical efficiency score (the normalized capacity score of the region in our case), x_{ij} is the amount of input i used by unit j, y_{kj} is the amount of output k produced by unit j, and ε represents an infinitesimal constant that assures that the weight for each police resource is greater than zero. The output weights u_k and input weights v_i are variables in the DEA model. In the input-oriented DEA model, the efficiency score ϕ_q is one if the unit q is technically efficient, and is lower than one if the unit is technically inefficient. The efficiency score calculates a size of input reduction that makes production unit q technically efficient. In the output-oriented DEA model, the efficiency score is one if the unit q is technically efficient and is greater than one if the unit is technically inefficient.

3 APPLICATION

The application of the describe methods is illustrated on the data of the police resources in the Czech Republic. The data comes from the police statistics and cover the year 2014.
The Czech Republic is divided in 14 regions and in 2014 it had 10.65 mil. inhabitants. There were 34279 (3.14 per 1000 inhabitants) police employee, 8282 (0.87 per 1000 inhabitants) police vehicles and the finance budget was 23267.5 (2.1 per 1000 inhabitants) million crowns. The regional data that were used in these calculations are presented in Table 1.

Table 1: Number of police resources

	Police employee	Number of police vehicles	Finance budget	Number of inhabitants
Praha	6 278	1 228	4 436 452 477	1 309 000
Středočeský	3 611	855	2 370 161 542	1 369 000
Jihočeský	2 192	575	1 524 316 156	642 000
Plzeňský	2 201	652	1 498 984 128	585 000
Ústecký	3 166	289	2 176 807 338	821 000
Královehradecký	1 710	753	1 183 406 231	551 000
Jihomoravský	3 430	347	2 255 263 412	1 189 000
Moravskoslezký	3 836	557	2 540 725 824	1 203 000
Karlovarský	1 265	377	851 727 064	295 000
Liberecký	1 270	289	907 278 021	442 000
Pardubický	1 212	721	822 712 090	520 000
Vysočina	1 208	304	797 021 914	509 000
Zlínský	1 217	390	790 118 009	583 000
Olomoucký	1 683	945	1 112 533 493	633 000
Czech Republic	34 279	8 282	23 267 507 699	10 651 000
Average	2448	592	1 661 964 836	760 786
Max	6 278	1 228	4 436 452 477	1 369 000
Min	1 208	289	790 118 009	295 000

Source: Police CZ, CSO

Data per 1000 inhabitants are in Table 2:

Table 2: Number of police resources per 1000 inhabitants

	Police employee	Number of police vehicles	Finance budget
Praha	4.80	0.94	3 389 192.11
Středočeský	2.64	0.62	1 731 308.65
Jihočeský	3.41	0.90	2 374 324.23
Plzeňský	3.76	1.11	2 562 366.03
Ústecký	3.86	0.35	2 651 409.67
Královehradecký	3.10	1.37	2 147 742.71
Jihomoravský	2.88	0.29	1 896 773.27
Moravskoslezký	3.19	0.46	2 111 991.54
Karlovarský	4.29	1.28	2 887 210.39
Liberecký	2.87	0.65	2 052 665.21
Pardubický	2.33	1.39	1 582 138.63
Vysočina	2.37	0.60	1 565 858.38
Zlínský	2.09	0.67	1 355 262.45
Olomoucký	2.66	1.49	1 757 556.86
Average	3.16	0.87	2 147 557.15
Max	4.80	1.49	3 389 192.11
Min	2.09	0.29	1 355 262.45

Source: Author

Results of method separate evaluation are in Table 3:

Table 3: Separate evaluation results

	Police employee	Number of police vehicles	Finance budget
Praha	1.00	0.63	1.00
Středočeský	0.55	0.42	0.51
Jihočeský	0.71	0.60	0.70
Plzeňský	0.78	0.75	0.76
Ústecký	0.80	0.24	0.78
Královehradecký	0.65	0.92	0.63
Jihomoravský	0.60	0.20	0.56
Moravskoslezký	0.66	0.31	0.62
Karlovarský	0.89	0.86	0.85
Liberecký	0.60	0.44	0.61
Pardubický	0.49	0.93	0.47
Vysočina	0.49	0.40	0.46
Zlínský	0.44	0.45	0.40
Olomoucký	0.55	1.00	0.52
Average	0.66	0.58	0.63
Min	0.44	0.20	0.40

Source: Author

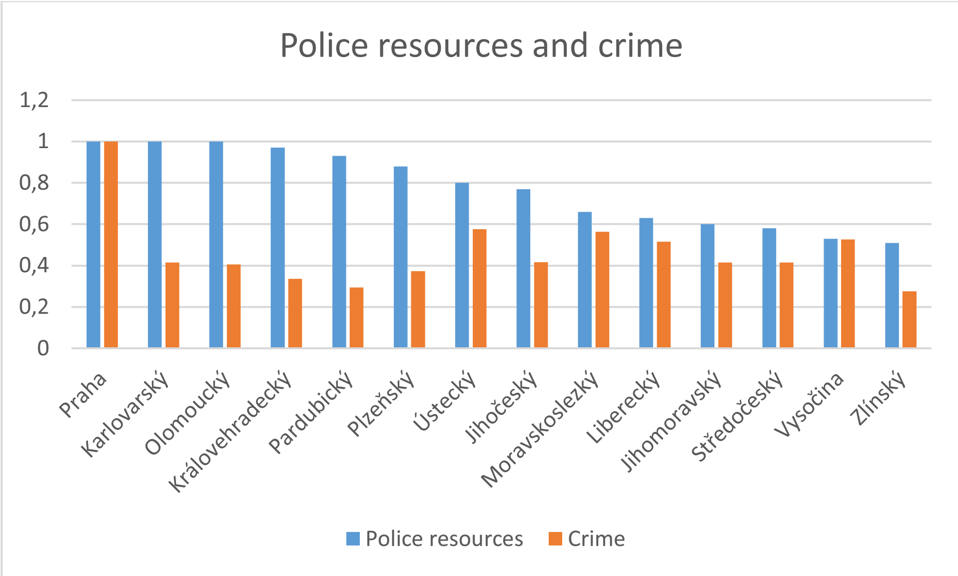
In order to better compare the results achieved by different methods, the regional capacity scores were normalized to the maximal regional capacity being 1 (see Table 3). In the case of the production frontier model we used the input-oriented constant returns-to-scale DEA model (2) with the regional population as the single input and the number of police employee, number of police vehicles and the number of finance budget as two outputs. All calculations were carried out in MS Excel, Lingo software and MaxDea software. The Czech regions with highest resource capacities are the Praha and Olomoucký regions. One region has the maximal number of police employee and finance budget 1000 inhabitants (Praha) and one region has the maximal number of police vehicles 1000 inhabitants (Olomoucký). On the other hand, the minimal police resource capacity is observed by the separate evaluation method in Zlinský region (Police employee and Finance budget) and Jihomoravský region (Number of police vehicles). Not surprisingly, the common weights and production frontier models that take into account the possibility of resource substitution show on average higher scores and thus lower differences between regional police resource capacities (see Table 4). This can be observed in majority of regions.

Table 4: Common weights method, DEA results and crime

	Common weights method	DEA model	Crime per 1000 inhabitants	Standardized crime
Praha	1.00	1.00	54.87	1.00
Středočeský	0.58	0.58	22.73	0.41
Jihočeský	0.77	0.77	22.87	0.42
Plzeňský	0.88	0.88	20.50	0.37
Ústecký	0.71	0.80	31.58	0.58
Královehradecký	0.83	0.97	18.48	0.34
Jihomoravský	0.54	0.60	22.80	0.42
Moravskoslezký	0.63	0.66	30.95	0.56
Karlovarský	1.00	1.00	22.80	0.42
Liberecký	0.62	0.63	28.29	0.52
Pardubický	0.71	0.93	16.12	0.29
Vysočina	0.53	0.53	28.85	0.53
Zlínský	0.50	0.51	15.11	0.28
Olomoucký	0.79	1.00	22.22	0.40
Average	0.72	0.78	25,58	0.47
Min	0.50	0.51	15,11	0.28
Max	1	1	54,87	1

Source: Author, CSO

In Table 4 are comparison of the police resources calculated by common weights model and DEA model and the data of crime.



Graph 1: Comparison police resources and crime. Source: Author

4 CONCLUSIONS

This paper focused on measurement of variations in regional police resource capacities in the Czech Republic. In the paper, has presented three alternative methods of assessment of variations in regional police resource capacities: the separate evaluation, the common weights model, and the production frontier model based on the data envelopment analysis. Illustrative calculations were carried out on regional data from Czech Republic that is divided to 14 regions. It is observed that both the common weights model and production frontier model that take in to account the possibility of resource substitution show on average higher capacity scores and thus lower differences between regional police resource capacities. A comparison of resource capacities could be a problem of multiple criteria decision making in which the main issue is setting of resource weights. In this paper, the calculation of resource weights were carried out objectively by the common weights and production frontier models. The dependence between amount of police resources and crime did not confirm.

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TAX ADJUSTMENTS AND INFORMAL ECONOMY IN THE CZECH REPUBLIC AND SLOVAKIA: A DSGE APPROACH

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The goal of this paper is to quantify and compare the impacts of alternative tax adjustments in the corporate tax rate on the size of the shadow economy and, consequently, on government tax revenues in Slovakia and the Czech Republic. In this contribution, we present results based on a small dynamic stochastic general equilibrium (DSGE) model consisting of both formal and informal sectors of an economy. We estimate this DSGE model in linearized form for both Slovak and Czech economies, using the quarterly data for household consumption, investments, total gross wages, probability of tax audit, and several measures for government revenues. Based on the estimated parameter values, we perform a set of simulations using the non-linear form of the underlying DSGE model to evaluate the differences in effects of changes in the corporate tax rate on the two economies of interest. These effects are evaluated with regards to the changes in steady-state values of the output in the official economy, total tax revenues, and the size of the shadow economy. The corresponding Laffer curves help us to analyze the reaction of the informal economy in response to changes in the corporate tax rates, and subsequent effects on the size of the government tax revenues.

Keywords: *DSGE model, informal economy, Laffer curve, tax evasion*

JEL Classification: C11, E26, H26
AMS Classification: 91B51, 91B64

1 INTRODUCTION

Informal economy is a part of the non-observed economy (NOE), which contains all unregistered economic activities that are not included in the official GDP estimates (Feld and Schneider, 2010). Measuring the size of the NOE is difficult, and it is only possible to estimate its components. OECD (OECD, 2012) defines four components of the NOE: unreported income arising from legal activities, unreported income arising from illegal activities, unpaid household economic activities for its own benefit, and the component reflecting shortcomings in statistical techniques and information sources. The formal definition of the shadow (underground) economy is provided by (OECD, 2017) as the *"Economic activities, whether legal or illegal, which are required by law to be fully reported to the tax administration but which are not reported and which therefore go untaxed unlike activities which are so reported."*

The first approach to estimate the size of the NOE is based on direct methods such as questionnaire surveys (Feld and Schneider, 2010). The second approach is based on indirect methods. As stated by Feld and Schneider (2010), and Schneider and Buehn (2016), these methods include (i) estimating the unobserved economy as a difference between national income and expenditure, (ii) the difference between official and actual labour, (iii) transaction approach and (iv) the demand for currency approach. The third method of evaluating the size of an unobservable economy is through a model approach. The best-known model approach is based on the Multiple Indicator Multiple Cause (MIMIC) model (Buehn and Schneider, 2016). The second type of model approach used in this paper is the dynamic stochastic general equilibrium (DSGE) model.

The goal of this paper is to quantify and compare the impacts of alternative tax adjustments in the corporate tax rate on the size of the shadow economy and, consequently, on government tax revenues in Slovakia and the Czech Republic. In this contribution, we present results based on a small dynamic stochastic general equilibrium (DSGE) model consisting of both formal and informal sectors of an economy. We estimate this DSGE model in linearized form for both Slovak and Czech economies, using the quarterly data for household consumption, investments, total gross wages, probability of tax audit, and several measures for government revenues. Based on the estimated parameter values, we perform a set of simulations using the non-linear form of the underlying DSGE model to evaluate the differences in effects of changes in the corporate tax rate on the two economies of interest. These effects are evaluated with regards to the changes in steady-state values of the output in the official economy, total tax revenues, and the size of the shadow economy. The corresponding Laffer curves help us to analyze the reaction of the informal economy in response to changes in the corporate tax rates, and subsequent effects on the size of the government tax revenues. Our paper extends the results of Němec et al. (2019) by evaluating the model for both the Slovak and the Czech economy and by treating the probability of tax auditing as a non-stationary variable.

2 MODEL

In this section, we present some of the main attributes of the DSGE model used in our analysis. The model is based on Orsi et al. (2014) and consists of the following three sectors: households, firms, and government. Firms produce goods and services in either the official economy or the NOE, employing unofficial production factors in the latter to hide some part of their production and avoid taxation. Thus, firms can employ either an official workforce, $h_{i,t}^m$, and hire an official capital $k_{i,t}^m$, or they use their unofficial counterparts, denoted $h_{i,t}^u$ and $k_{i,t}^u$, respectively. The homogeneous output of all companies is defined using the Cobb-Douglas production function (1).

$$y_{i,t}^j = A_t^j \left(\Gamma_t h_{i,t}^j\right)^{\alpha_j} \left(k_{i,t}^j\right)^{1-\alpha_j}, \; j = \{m, u\} \quad (1)$$

where $\alpha_j \in (0,1)$, A_t^j is a technological shock, Γ_t is the labour-augmenting technological progress that follows a deterministic trend, $\Gamma_t = \gamma\Gamma_{t-1}$, with $\gamma > 1$, and total production is defined as $y_{i,t} = y_{i,t}^m + y_{i,t}^u$.

Capital and labour are hired in perfectly competitive markets. Companies pay rent per unit of both, official and unofficial, capital, r_t^m or r_t^u, respectively, and similarly wage per unit of work, w_t^m or w_t^u. Besides, the official wage, w_t^m, is increased by the social and health insurance rate paid by the company, τ_t^s. Thus, the total costs of a firm are defined as in (2).

$$TC\left(h_{i,t}^m, h_{i,t}^u, k_{i,t}^m, k_{i,t}^u\right) = (1 + \tau^s) w_t^m h_{i,t}^m + w_t^u h_{i,t}^u + r_t^m k_{i,t}^m + r_t^u k_{i,t}^u \quad (2)$$

The government prevents production from shifting from the official economy to the NOE by introducing audits. Every company is at every time t facing the possibility of being subject to the audit with probability $p_t \in (0,1)$. As a result of this audit, a firm may be obliged to pay the unpaid tax based on the tax rate, $\tau_t^c < 1$, together with a penalty surcharge, $s > 1$. As a result, the net output gain of combination of official and unofficial production at time t, denoted $NR(y_{i,t})$, is a random variable defined by the following expression.

$$NR(y_{i,t}) = \begin{cases} y_{i,t}^m - \tau_t^c\left(y_{i,t}^m - w_t^m h_{i,t}^m\right) + y_{i,t}^u - s\tau_t^c\left(y_{i,t}^u - w_t^u h_{i,t}^u\right) & \text{if detected} \\ y_{i,t}^m - \tau_t^c\left(y_{i,t}^m - w_t^m h_{i,t}^m\right) + y_{i,t}^u & \text{otherwise} \end{cases} \quad (3)$$

A representative household maximizes the utility function (4) subject to capital (5) and budget constraints (6).

$$U_t^h = \sum_{t=0}^{\infty} \beta^t E_0 \left\{ \frac{(c_t/\Gamma_t)^{1-\sigma} - 1}{1-\sigma} - \xi_t^h B_0 \frac{(h_t^m + h_t^u)^{1+\xi}}{1+\xi} - B_1 \frac{(h_t^u)^{1+\psi}}{1+\psi} \right\} \quad (4)$$

$$k_{t+1} = \xi_t^x x_t + (1-\delta_k)k_t, \quad \text{where } k_t = k_t^u + k_t^m \quad (5)$$

$$c_t + x_t = \left(1-\tau_t^h\right)(w_t^m h_t^m + r_t^m k_t^m) + w_t^u h_t^u + r_t^u k_t^u \quad (6)$$

where $\beta \in (0,1)$ is the discount factor; E_0 is expectations operator; $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution; ξ_t^h denotes a transitory labour supply shock that affects the marginal rate of substitution between consumption and labour; $B_0 \geq 0$ and $B_1 \geq 0$ are preference parameters controlling for the disutility of work; $\xi > 0$ and $\psi > 0$ denote the inverse elasticities of aggregate and underground labour supplies, respectively; k_t is the capital held by households; x_t indicates the investment at time t; ξ_t^x denotes purely transitory exogenous shock that determines the efficiency with which final goods can be transformed into physical capital; $\delta_k \in [0,1]$ is the capital depreciation rate; official income is then subject to the income tax rate, $\tau_t^h < 1$.

The role of government is to finance government consumption, g_t, through taxes, and to control and penalize the informal economy as mentioned before. For simplicity, we abstract from public debt and assume that the amount of public expenditure is based on a balanced budget, which yields the government budget constraint is in (7).

$$g_t = \tau_t^h(w_t^m h_t^m + r_t^m k_t^m) + \tau_t^c \int_0^1 \left[y_{i,t}^m - w_t^m h_{i,t}^m + p_t s\left(y_{i,t}^u - w_t^u h_{i,t}^u\right)\right] di + \tau_t^s w_{i,t}^m \int_0^1 h_{i,t}^m di \quad (7)$$

where the first expression on the RHS of the equation (7) indicates the amount of fiscal revenues from personal income taxation, g_t^h, the second term is total fiscal revenues from corporate tax, g_t^c, and the last term represents total fiscal revenues from social security contributions, g_t^s. At every time t there is also total tax evasion, TE_t, defined as follows:

$$TE_t = \tau_t^s w_t^u \int_0^1 h_{i,t}^u \, di + \tau_t^h(w_t^u h_t^u + r_t^u k_t^u) + (1-p_t)\tau_t^c \int_0^1 \left(y_{i,t}^u - w_t^u h_{i,t}^u\right) di \quad (8)$$

In total, the model consists of eight stochastic processes (two technological shocks, investment efficiency shock, labour supply shock, three tax rate shocks, and shock in the probability of government control) following (in the log-linear form) AR(1) processes.

3 DATA AND METHODOLOGY

To estimate the DSGE model for the Czech Republic and Slovakia, we use seven macroeconomic variables, as depicted in Table 1. We use the variables related to household consumption, investment, government revenue from corporate tax, government revenue from the social contributions, government revenue from the taxation of individuals, total gross wages, and the likelihood of corporate tax audits to the firms. The data are obtained from the database of OECD (OECD, 2020), the Ministry of Finance of the Slovak Republic (MFSR, 2020), the Financial Administration of the Slovak Republic (FASR, 2020), the Financial

Administration of the Czech Republic (FACR, 2020), and the database ARAD of the Czech National Bank (CNB, 2020). The data cover the period from the 1st quarter of 2002 to the third quarter of 2019. The data are seasonally adjusted, expressed in the real terms, and transformed using the demeaned first logarithmic differences.

Using the described data, we set the steady states and estimate a log-linearized form of the model described in the previous section, for both countries. Steady states that are set according to the data are the probability of corporate income tax audit (0.0256 for CZ and 0.0170 for SK), corporate tax (0.2285 for CZ, and 0.2161 for SK), and personal income tax (0.15 for CZ and 0.17222 for SK). Most of the other steady-state values were computed as part of the nonlinear model. Since there is no closed-form solution for the posterior probability distribution, we apply Random Walk Chain Metropolis-Hastings (RWMH) algorithm with 2 000 000 replications and 1 000 000 burned-in samples. Selected characteristics of the prior and posterior distributions of estimated parameters are presented in Table 2. It should be noted that the log-linearized specification of the model contains the steady-states of endogenous variables that depend on the model parameters. We have set the prior information about parameters to be consistent with the steady-state values of the variables used within the estimation procedure.

Table 1: Data description

Variable	Description	Transformation	Source SVK	Source CZE
C	Private final consumption expenditure, national currency, current prices, annual levels, seasonally adjusted	Deflated by seasonally adjusted GDP deflator (OECD reference year 2015)	OECD	OECD
X	Gross capital formation, national currency, current prices, quarterly levels, seasonally adjusted	Deflated by seasonally adjusted GDP deflator (OECD reference year 2015)	OECD	OECD
G^c	Government corporate tax revenues, national currency, total revenues	Seasonally adjusted by X13-ARIMA-SEATS, deflated by seasonally adjusted GDP deflator (OECD reference year 2015)	MFSR	CNB
G^h	Government personal income tax revenues national currency, total revenues	Seasonally adjusted by X13-ARIMA-SEATS, deflated by seasonally adjusted GDP deflator (OECD reference year 2015)	MFSR	CNB
G^s	Government revenues from social security contributions, national currency, total revenues	Seasonally adjusted by X13-ARIMA-SEATS, deflated by seasonally adjusted GDP deflator (OECD reference year 2015)	MFSR	CNB
W	Hourly earnings, manufacturing, index (2105 = 100), seasonally adjusted	Deflated by seasonally adjusted GDP deflator (OECD reference year 2015)	MFSR	OECD
p	Probability of corporate income tax auditing	Number of controls related to corporate income tax divided by the number of subjects obliged by the corporate income tax	FASR	FACR

Note: SVK = Slovakia, CZE = Czechia, OECD = OECD (2020), CNB = CNB (2020), MFSR = MFSR (2020), FASR = FASR (2020), FACR = FACR (2020).

Based on the estimated mean values of the model parameters, we simulated the non-linear form of the benchmark model to get the steady-state values of all endogenous variables. This procedure was applied for various sets of the personal income tax rate and the corporate tax rate. The resulting Laffer curves for government revenues are presented in Figure 1 and Figure 2.

Table 2: Prior and posterior distributions

Parameters		Prior mean (std. dev.)	Distribution	Posterior mean (std. dev.) SVK	Posterior mean (std. dev.) CZE
α_m	share of the total number of hours worked in OE	0.66 (0.02)	beta	0.5989 (0.0006)	0.6149 (0.0078)
α_u	share of the total number of hours worked in NOE	0.66 (0.02)	beta	0.6399 (0.0022)	0.6632 (0.0030)
σ	inverse of the intertemporal elasticity of substitution	1.8 (4.20)	gamma	1.0567 (0.1009)	3.0445 (0.2236)
ξ	inverse labour supply elasticity of aggregate labour supply	1.49 (0.13)	gamma	1.7123 (0.0049)	1.6658 (0.0380)
ψ	inverse labour supply elasticity of underground aggregate labour supply	1 (0.02)	gamma	0.9682 (0.0013)	1.0015 (0.0027)
δ_k	capital depreciation rate	0.029 (0.016)	beta	0.0115 (0.0007)	0.0110 (0.0041)
B_1	disutility of working activities	6 (1.00)	gamma	5.9697 (0.0692)	6.2992 (0.4483)
Persistence of shocks					
ρ_{a^m}	technology shock in OE	0.5 (0.15)	beta	0.4857 (0.0085)	0.9611 (0.0571)
ρ_{a^u}	technology shock in NOE	0.5 (0.15)	beta	0.5497 (0.0155)	0.7019 (0.0414)
ρ_{τ^c}	tax rate shock	0.5 (0.15)	beta	0.3567 (0.0065)	0.7512 (0.0759)
ρ_{τ^s}	social security tax rate shock	0.5 (0.15)	beta	0.7191 (0.0169)	0.9139 (0.0356)
ρ_{τ^h}	income tax rate shock	0.5 (0.15)	beta	0.7959 (0.0094)	0.9134 (0.0251)
ρ_{ξ^x}	purely transitory exogenous shock	0.5 (0.15)	beta	0.8783 (0.0165)	0.8115 (0.0602)
ρ_{ξ^h}	labour supply shock	0.5 (0.15)	beta	0.4878 (0.0070)	0.5863 (0.0507)
ρ_p	probability of tax audit shock	0.5 (0.15)	beta	0.0262 (0.0014)	0.0361 (0.0032)
Standard deviation of shocks					
σ_{a^m}	technology shock in OE	0.05 (∞)	inv. gamma	0.0776 (0.0083)	0.0247 (0.0033)
σ_{a^u}	technology shock in NOE	0.05 (∞)	inv. gamma	0.7639 (0.0839)	0.1278 (0.0116)
σ_τ	tax rate shock	0.05 (∞)	inv. gamma	0.0378 (0.0034)	0.0086 (0.0010)
σ_τ	social security tax rate shock	0.05 (∞)	inv. gamma	0.0168 (0.0014)	0.0094 (0.0011)
σ_τ	income tax rate shock	0.05 (∞)	inv. gamma	0.0550 (0.0052)	0.0320 (0.0029)
σ_ξ	purely transitory exogenous shock	0.05 (∞)	inv. gamma	0.0232 (0.0024)	0.0218 (0.0028)
σ	labour supply shock	0.05 (∞)	inv. gamma	0.0499 (0.0231)	0.0236 (0.0075)
σ	probability of tax audit shock	0.05 (∞)	inv. gamma	0.4524 (0.0333)	0.2396 (0.0202)

Note: SVK = Slovakia, CZE = Czechia, OE = official economy, NOE = non-observed (informal) economy.

4 INCOME TAX AND SHADOW ECONOMY IN CZECHIA AND SLOVAKIA

In this section, we compare our results on the impact of alternative tax adjustments in personal income tax and corporate tax rates on the government tax revenues and the size of the NOE in Czechia and Slovakia. Parameter estimates of the benchmark model imply the average size of the NOE to be 3.5% of the total output for the Slovak economy and 1.9% of the total output for the Czech economy. This value for the Slovak economy is lower than the estimates of 7% provided by Němec et al. (2019). It may indicate the continued decrease in the size of the shadow economy in Slovakia and probably the higher share prior to 2002. Our model suggests that 3.8% of the Slovak labour force is working in the Slovak NOE, and 2.1% of the Czech labour force is working in the Czech NOE. This result of the labour force in both underground economies is similar when compared with the estimated value of 3% presented by Schneider (2016) from the year 2006.

Figure 1 and Figure 2 depict simulated government revenues and size of the output, relative to the benchmark model, together with the size of the NOE, measured as a share on the total production (official and unofficial) of the given economy, in our two economies of interest for alternative personal income tax and corporate tax rates. Laffer curves for both countries have the expected shape. Both personal income and corporate tax rates seem to be further away from the value maximizing the total revenues in Czechia when compared to Slovakia. From this point of view, this tax rate in Czechia would be around 40% for personal income and approximately 50% for corporate income. In contrast, in Slovakia, the model suggests the corresponding level of both tax rates at around 30%.

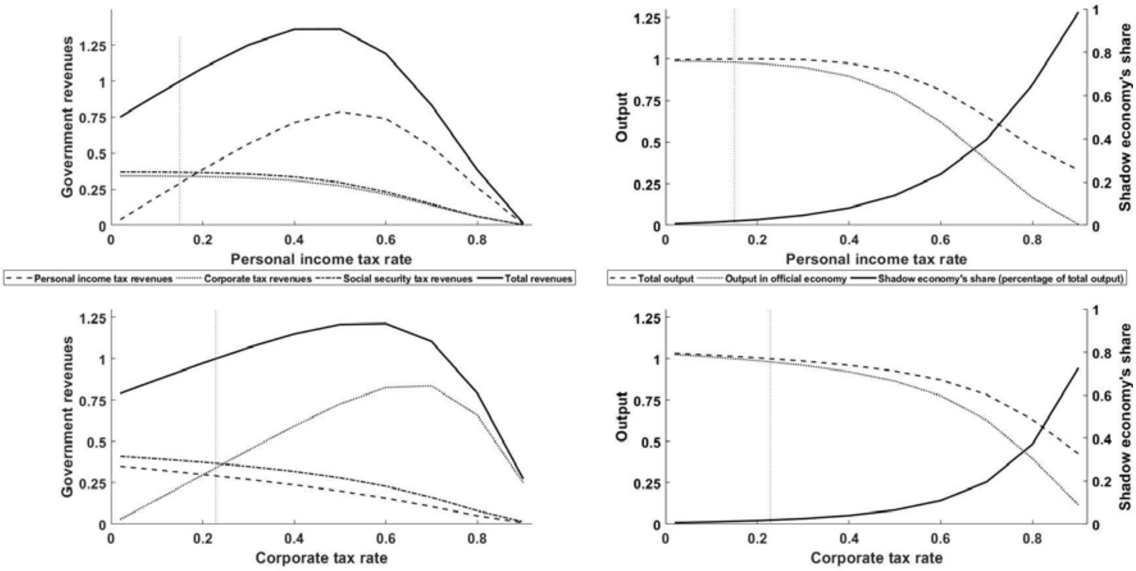

Figure 1: Simulated Laffer curves in Czechia

On the contrary, any change in either of the investigated tax rates up to the level of 40% would lead to a massive drop in the total output in Slovakia, but only a slight decline in Czechia. The slope of the curve representing the share of the NOE on total output indicates a higher sensitivity of Slovak taxpayers to any changes in the tax rates in comparison with their Czech counterparts. In both economies, adjustments of the personal income tax rate lead to a stronger response in the share of the NOE, answer to similar changes in the corporate tax rate is less pronounced.

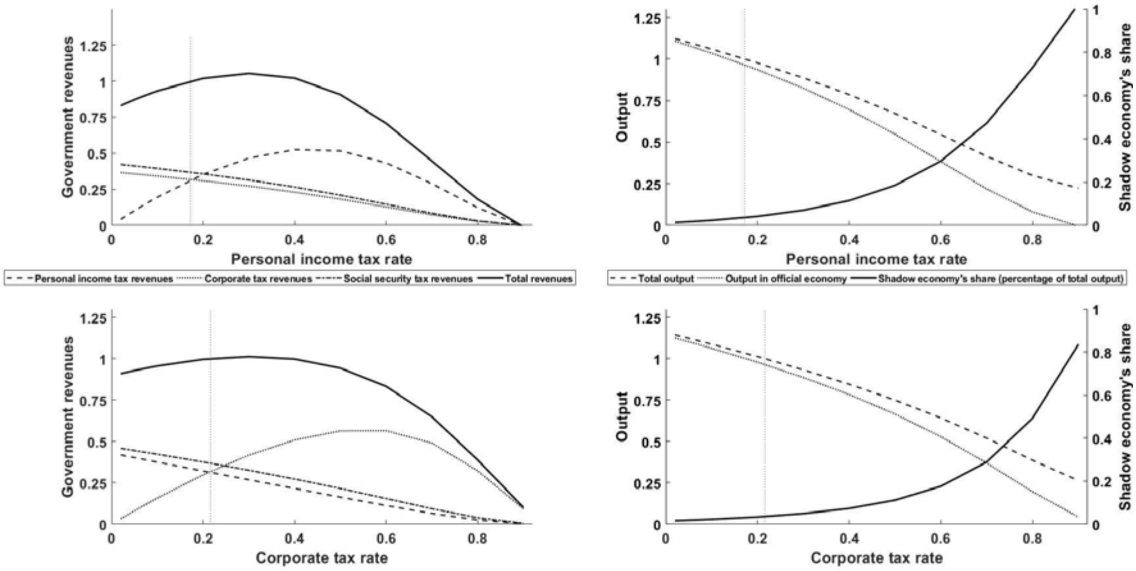

Figure 2: Simulated Laffer curves in Slovakia

5 CONCLUSIONS

In this paper, we estimated the small DSGE model consisting of both formal and informal sectors of the economy. Our estimates, based on the data of the Czech and Slovak economy, imply the size of the shadow economy to be 3.5% (Slovak) and 1.9% (Czech) of the total output where 3.8% (Slovak) and 2.1% (Czech) of the labour force is working in the underground economy. These results indicate the overall decrease in the size of the underground economy since 2010 after comparing our estimates with those from the previous studies. The Laffer curves of personal income taxation and corporate taxation have expected shape. The peak of both curves for the Slovak economy may be found at the tax rate of 30%. For the Czech economy, the peak of the personal income tax rate may be found at the tax rate of 50%, and the corporate tax rate may be found at the tax rate of 60%. All these peak estimations of taxes are higher than the actual tax rates.

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SPATIAL INSTABILITY IN THE SLOVAK LANGUAGE TEST SCORES: A GWR APPROACH

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Abstract
This paper deals with the spatial instability of the pupils' performance across the Slovak districts. To measure the pupils' performance, the Slovak language test scores of pupils of the 5th year of primary schools were used. Besides the initial spatial pattern analysis confirming the huge spatial variations in pupils' performance, the paper tries to find an explanation why pupils in some districts show better learning achievements than others, studying potential factors affecting the pupils' performance (average nominal monthly wage, unemployment rate, population density, index of economic dependence of young people and socially disadvantaged background). Global linear regression estimates showed the positive impact of wages and population density on the Slovak language test scores' achievements and negative impact of unemployment rate, amount of children (0 – 14 years old) per 100 persons aged 15 to 64 and socially disadvantaged background, respectively. To capture the apparent spatial heterogeneity, the local GWR (Geographically Weighted Regression) approach enabled to consider the geographical instability in estimation results.

Keywords: *Pupils' performance, Spatial variation, Geographically Weighted Regression*

JEL Classification: I24, C21
AMS Classification: 91B72, 62P25

1 INTRODUCTION

The issue of education plays one of the key roles in national strategies across the European countries. In line with the national priorities for the implementation of the 2030 Agenda for Sustainable Development (United Nations, 2020) in the field of education, is the improving the quality of education one of the strategic objectives in the National Program of Education Development for 2018-2027 in Slovakia (Ministerstvo školstva, vedy, výskumu a športu Slovenskej republiky, 2020). This program focuses on the modernization of the education and training system through the modernization of content and processes, in particular the modernization of educational content, methods and testing, better linking testing with the educational content and better reflecting international testing practices in national testing. Nowadays, Slovakia is involved in several international test measurements like e.g., PISA (OECD's Programme for International Student Assessment)[1] as well as national test measurements including external testing of the primary school pupils (Testing 5 and Testing 9) and the secondary school students (external part of secondary school graduation exam). Each testing serves not primary only to monitor the state of knowledge and skills of pupils/students at the international/national level, but it should serve mainly as an information for teachers to improve their education and to improve the educational process.

The analysis of education performance on different levels using various methodological approaches attracts the attention of many researchers worldwide. Some studies concentrate on identification of huge regional disparities using the ESDA (Exploratory Spatial Data Analysis) techniques enabling to have a closer look at the distribution of the educational disparities – see

[1] For more information, see Národný ústav certifikovaných meraní vzdelávania (2019).

e.g., Rodríguez-Pose and Tselios (2007) who studied the educational attainment and inequality across the European regions. Other researchers concentrate not only on the educational inequalities, but try to understand why some pupils/students show better learning achievements than others studying various factors affecting the pupils'/students' performance – for a survey of studies using various conceptual frameworks see e.g., Qiu and Wu (2011). Furthermore, during the recent two decades it has been published various studies dealing with the spatial variations in school performance using the GWR (Geographically Weighted Regression) – see e.g., Fotheringham, Charlton and Brunsdon (2001), Qiu and Wu (2011), Naidoo, van Eeden and Munch (2014), Chocholatá (2019), Chocholatá (2020).

The main objective of this paper is to investigate the spatial variation of pupils' performance and to perform both the global and local regression analysis to assess the impact of selected factors (average nominal monthly wage, unemployment rate, population density, index of economic dependence of young people and socially disadvantaged background) onto the Testing 5-2019 (T5-2019)[2] Slovak language test scores across the Slovak districts. The rest of the paper is organized as follows: section 2 explains the methodology, section 3 presents the data and empirical results and section 4 concludes.

2 METHODOLOGY

The classical approach to model the relationship between dependent variable and factors which might explain it is to use the linear regression. Even in regional data, the issue of the location of a region has not been automatically incorporated. With the advent of the new millennium, models allowing the inclusion of spatial effects – spatial autocorrelation and spatial heterogeneity has become more and more popular, various spatial econometric models and estimation techniques have been developed (Chasco, García and Vicéns, 2007). In line with the aim of this paper, the attention will be given to approaches considering the spatial heterogeneity/instability. Unlike the classical global linear regression, the GWR approach of Brunsdon, Fotheringham and Charlton (1996) enables to calculate the locally linear regression estimates for each region. The local GWR model is thus as follows (Wheeler and Páez, 2010; Furková, 2018):

$$y_i = \boldsymbol{x_i \beta_i} + \boldsymbol{\varepsilon_i} \tag{1}$$

where index $i = 1, \ldots, n,$ denotes the i-th region, y_i is the value of dependent variable at region i, $\boldsymbol{x_i}$ is a row vector of independent variables and $\boldsymbol{\beta_i}$ is a column vector of regression parameters at region i. The local regression parameters are functions of region i and can be estimated by the weighted least squares:

$$\widehat{\boldsymbol{\beta_i}} = (\boldsymbol{X^T W_i X})^{-1} \boldsymbol{X^T W_i y} \tag{2}$$

where $\boldsymbol{\beta_i}$ denotes the vector of p local regression parameters at region i, $\boldsymbol{y}$ is the n x 1 vector of dependent variables, $\boldsymbol{X}$ is the n x k matrix of independent variables (including a column of ones for the intercept) and $\boldsymbol{W_i}$ is the n x n diagonal weight matrix at region i whose off-diagonal elements are zero and diagonal elements are calculated based on a spatial kernel function giving higher weight to the nearby regions in comparison to regions farther away (Fotheringham, Brunsdon and Charlton, 2002; Wheeler and Páez, 2010).

[2] Since 2015, the nationwide Testing 5 has been based on testing of pupils' knowledge in mathematics and national language. In this paper, the Slovak language test scores of pupils of the 5th year of primary schools were used to measure the pupils' performance.

The first important step of the GWR approach is to choose the type of the kernel function and to determine its bandwidth parameter. In general, two types of kernel functions can be distinguished, fixed and adaptive. As for the bandwidth parameter determination, minimisation of a cross validation score or minimisation of the corrected Akaike Information Criterion (AICc) are the most often used methods (Fotheringham, Brunsdon and Charlton, 2002; Wheeler and Páez, 2010). To compare the fit of the global linear regression model and the local GWR model, e.g. the GWR ANOVA test can be used (Chasco, García and Vicéns, 2007). For the further testing procedures see e.g., Leung, Mei and Zhang (2000), Charlton and Fotheringham (2009).

3 DATA AND EMPIRICAL RESULTS

This section examines the spatial variation in the pupils' performance across the 79 districts of Slovakia. The pupils' performance is measured by the Slovak language test scores of pupils of the 5th year of primary schools in autumn 2019 (Testing 5 or T5-2019), the data were retrieved from Národný ústav certifikovaných meraní vzdelávania (2020). As factors affecting the pupils' performance, i.e. the Slovak language test scores, following independent variables (downloaded from Štatistický úrad Slovenskej republiky, 2020) were chosen – average nominal monthly wage in 2018 (in Euro), unemployment rate in 2019 (in %), population density in 2018 (in persons per square kilometer), index of economic dependence of young people in 2018[3]. To assess the impact of the socially disadvantaged background onto the test scores, additional independent variable, a dummy (0/1) variable, was used to denote districts with more than 5% pupils from the socially disadvantaged background (retrieved from Národný ústav certifikovaných meraní vzdelávania, 2020). The data were analysed in softwares GeoDa and GWR4[4].

Table 1 provides a view of selected descriptive statistics for individual variables – besides the mean and median values, the maximum and minimum enable to have a closer look at variables under consideration indicating huge regional differences.

	slo5	w18	un19	d18	iy18
Mean	64.0367	1047.6835	5.37702	257.2318	22.8391
Median	66.2000	1008.0000	4.4200	103.7800	21.5700
Maximum	79.5000	1696.0000	15.1400	4259.850	35.9200
Minimum	42.5000	726.0000	1.9300	27.9700	18.5100
Std. Dev.	7.1237	190.4250	3.2612	556.4348	3.8340

Table 1 Descriptive statistics for variables - Slovak language T5 test scores (slo5), average nominal monthly wages (w18), unemployment rate (un19), population density (d18) and index of economic dependence of young people (iy18)

Source: author's calculations

To assess the regional inequalities over space in more detail, the quantile maps for individual variables were constructed ranking the Slovak districts into 4 even classes (see Figure 1 and Figure 2). Figure 1 illustrates the distribution of Slovak language T5 test scores across individual districts with indication of 22 districts with more than 5% of pupils from socially disadvantaged background. Majority of these 22 districts is associated with the districts which belong to the class with the lowest Slovak language T5 test scores. The least successful were

[3] Index of economic dependence of young people = the number of persons 0 – 14 years old per 100 persons aged 15 to 64.
[4] The .shp file with Slovak districts was obtained from Freemap Slovakia (2015).

pupils in Gelnica district (42.5%), followed by pupils from Kežmarok, Sabinov and Trebišov district, respectively with an average success rate below 50 %. On the other hand, the best average success rate of 79.5 % was reached by pupils from Bratislava I district. Pupils from other 13 districts (Košice I, Bratislava II-V, Košice IV, Senec, Trnava, Trenčín, Banská Bystrica, Pezinok, Žilina and Púchov) achieved an average result above 70%. The difference between the most successful and least successful district is huge – about 37 percentage points.

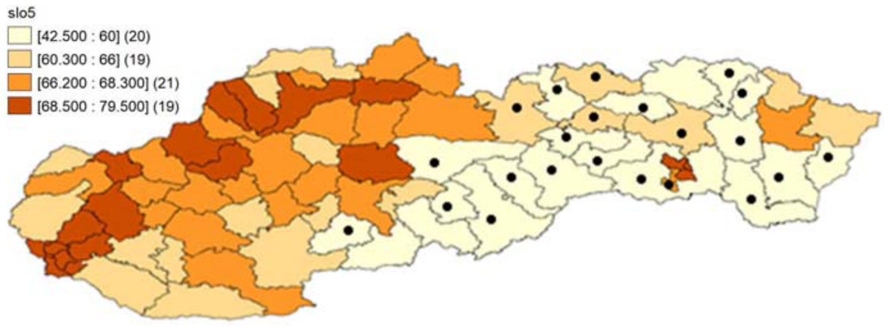

Figure 1 Quantile map of the Slovak language T5 test scores[5]

Note: Dots indicate regions with more than 5% of pupils from socially disadvantaged background.

Source: constructed by the author in GeoDa

As for the independent variables, huge territorial differences are clearly visible in Figure 2.

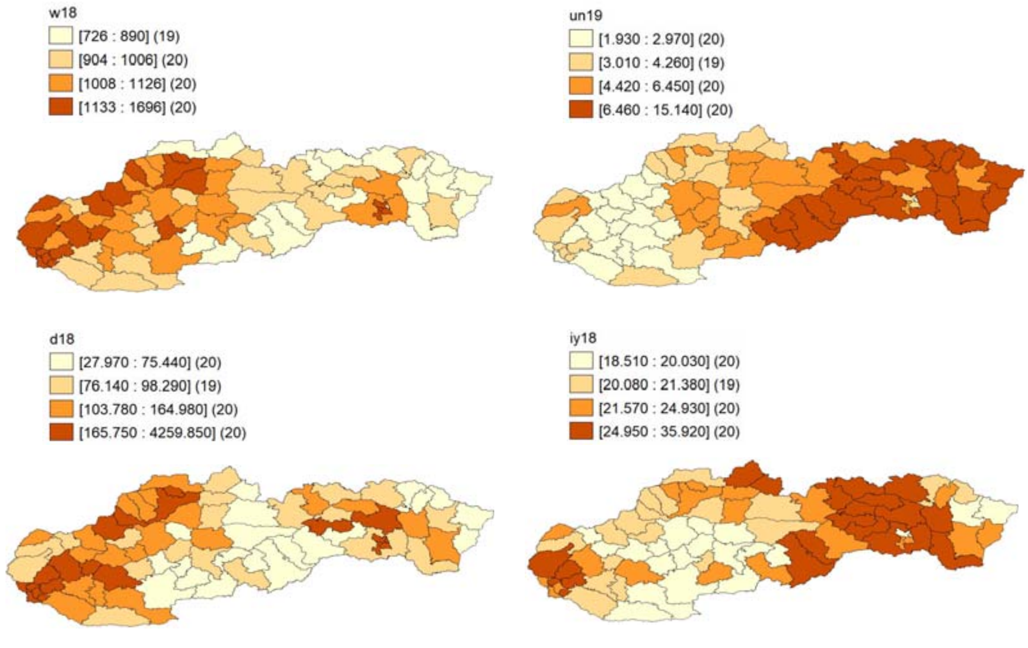

Figure 2 Quantile maps of the independent variables – average nominal monthly wages (w18), unemployment rate (un19), population density (d18) and index of economic dependence of young people (iy18)

Source: constructed by the author in GeoDa

[5] All figures presented in the paper are available in the colour version on the web-site: http://www.fhi.sk/kove/konferencie

High average nominal monthly wages (1133-1696 €) were recorded in all districts from Bratislava Region (with exception of Pezinok district), several districts from western part of Slovakia, 4 districts of Middle Slovakia as well as in the three out of four Košice districts. The lowest average nominal monthly wage of 726 € was paid in Bardejov district which indicates an enormous difference of 970 € in comparison to 1696 € of Bratislava II district. Districts with average nominal monthly wages below 900 € are located mostly in the eastern part of Slovakia and southern part of Middle Slovakia. The distribution of the unemployment rate across the Slovak districts is clearly west-east oriented, with low values in western regions and high values in eastern regions detecting a difference of 13.21 percentage points between Trenčín district (1.93 %) and Rimavská Sobota district (15.14%). Population density, a demographic variable, illustrates the differences in concentration of population in individual districts. On the one hand, a very high population density with an extreme value of 4259.85 persons per square kilometer in the district of Bratislava I and, on the other hand, a very sparsely populated district of Medzilaborce with 27.97 persons per square kilometer. Spatially considerably different is also the amount of children (0 – 14 years old) per 100 persons aged 15 to 64 in individual Slovak districts with extreme values of 18.51 (Myjava district) and of 35.92 (Kežmarok district), respectively.

In the next step, the paper tries to estimate the impact of above specified variables (average nominal monthly wage – w18, unemployment rate – un19, population density – d18, index of economic dependence of young people – iy18 and socially disadvantaged background – szp) onto the Slovak language T5 test scores – slo5. The initial specification is a global linear regression model relating the Slovak language T5 test scores' variable with the 5 independent variables:

$$slo5_i = \beta_0 + \beta_1 w18_i + \beta_2 un19_i + \beta_3 d18_i + \beta_4 iy18_i + \beta_5 szp5_i + \varepsilon_i \qquad (1)$$

where $\beta_0, \beta_1, \beta_2, \beta_3, \beta_4$ and β_5 are parameters to be estimated and ε_i represents an error term with the classical properties. OLS estimates of global model (1) are in Table 2 indicating statistical significance of all estimated parameters and acceptable goodness-of-fit ($Adjusted\ R^2 = 0.7876$). The signs of estimated parameters were as expected, i.e. indicate the positive impact of wages and population density on the test scores and negative impact of unemployment rate, amount of children (0 – 14 years old) per 100 persons aged 15 to 64 and socially disadvantaged background, respectively. Although, the OLS residuals didn't suffer from the highly statistically significant spatial autocorrelation[6], with regard to Table 1 as well as Figures 1 and 2 giving us an insight into the high heterogeneity of Slovak districts, we decided (similarly as Charlton and Fotheringham, 2009) to estimate the local GWR model of the form[7]:

$$slo5_i = \beta_{i0} + \beta_{i1} w18_i + \beta_{i2} un19_i + \beta_{i3} d18_i + \beta_{i4} iy18_i + \beta_5 szp5_i + \varepsilon_i \qquad (2)$$

where index i in parameter notation indicates the local character of estimated parameters. The geographical weighting in the estimated local GWR model was ensured by the adaptive spatial kernel. The bandwidth parameter of this model, i.e. the number of nearest neighbours to be

[6] The value of Moran's I for the OLS residuals is 0.106 with pseudo p-value 0.059 (randomization approach with 999 permutations).

[7] Variable szp was indicated to have a global character based on geographical variability tests of local parameters.

included in each spatial kernel, was calculated using AICc minimisation to be 64[8]. Selected GWR estimation results are presented in Table 2[9]. Local coefficients of determination R_i^2 calculated for individual regions were varying between 0.757 (Skalica district) and 0.842 (Tvrdošín district) indicating a very good model performance. Spatial distributions of the local GWR parameter estimates are mapped in Figure 3 indicating that the districts with similar parameter values tend to be clustered together.

Model Parameter (variable)	**Global model OLS**	**Local model GWR** Minimum	Lower Quartile	Median	Upper Quartile	Maximum
β_0 and β_{i0}	62.4226***	48.8247	54.5535	55.7828	65.9285	69.3741
β_1 and β_{i1}	0.0010***	0.0083	0.0093	0.0114	0.0130	0.0170
β_2 and β_{i2}	-0.6114***	-0.8975	-0.7761	-0.7507	-0.6914	-0.6363
β_3 and β_{i3}	0.0022***	0.0003	0.0016	0.0018	0.0023	0.0032
β_4 and β_{i4}	-0.1930*	-0.5148	-0.3849	0.1470	0.1838	0.2067
β_5	-6.0884***	-5.2285				
AICc	421.3645	411.1303				
Adjusted R^2	0.7876	0.8231				
Resid -Moran's I	0.106	0.041				

Table 2 Estimation results of global and local model

Note: Symbols ***, * indicate the rejection of H_0 hypothesis at 1% and 10 % level of significance, respectively.
Source: author's calculations

The GWR results proved the overall positive impact of wages and population density on the test scores and negative impact of unemployment rate. The impact of the variable indicating the share of children (iy18) was in all eastern districts as well as in some districts in Middle Slovakia negative, while in the rest of Slovak districts positive. Besides the spatially varying impacts of the mentioned variables, the globally negative impact of a socially disadvantaged background was confirmed. Furthermore, the GWR standardized residuals with the Moran's I of 0.041 were spatially uncorrelated. GWR model thus provides the analysis at lower level and enables to unhide some patterns hidden in the global analysis, especially in the presence of spatial heterogeneity/instability (Chasco, García and Vicéns, 2007). Better performance of the local GWR model in comparison to the global model is evident from the adjusted R^2 as well as from AICc values.

4 CONCLUSION

Spatial variations in the education performance across different regions belong clearly to important economic and social issues both at national and international level. This paper analyses the spatial patterns of the pupils' performance measured by the Slovak language T5 test scores. The spatial pattern analysis' results enabled to identify huge differences in pupils' performance across individual Slovak districts. Estimation of the global linear regression model provides an insight into the relationship between the pupils' performance and other variables

[8] Similarly, as pointed out by Charlton and Fotheringham (2009), it can be concluded that the number of 64 observations used in the estimation of each set of parameters is quite large, since each kernel contains thus 81% of the total number of 79 observations.

[9] Testing for the statistical significance of the local GWR parameters is beyond the scope of this paper – for more information see e.g., Chasco, García and Vicéns (2007), Fotheringham, Charlton and Brunsdon (2001).

(average nominal monthly wage, unemployment rate, population density, index of economic dependence of young people and socially disadvantaged background) supposing the same impact across all the Slovak districts. Due to huge spatial heterogeneity, the local GWR model was estimated showing apparent spatial variations in impact of individual independent variables under consideration (with exception of socially disadvantaged background) enabling thus to have a more detailed information for each district.

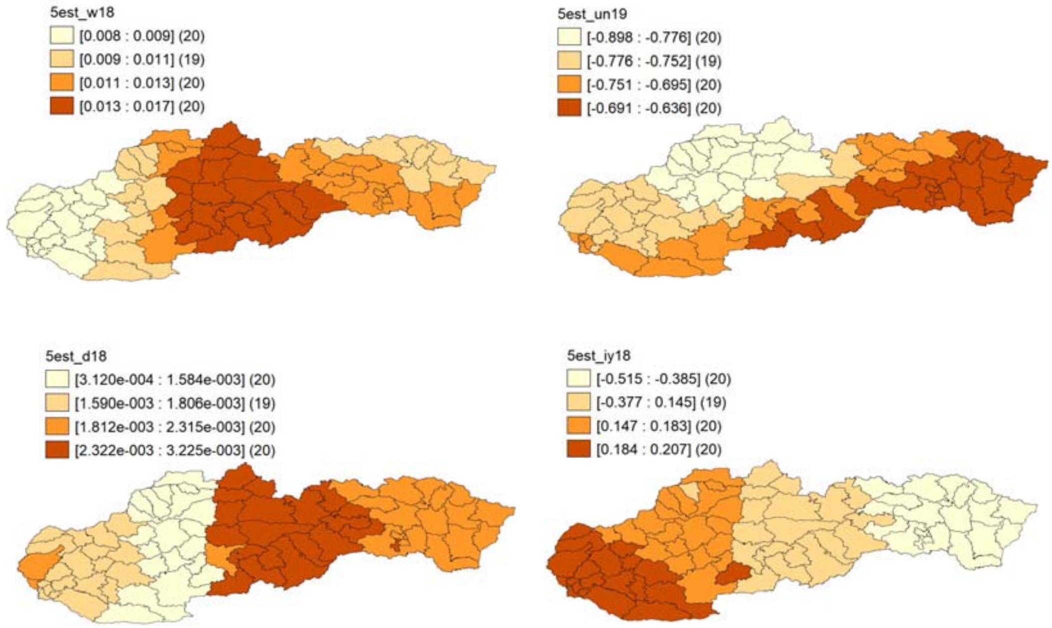

Figure 3 Local parameters of average nominal monthly wage (w18), unemployment rate (un19), population density (d18) and index of economic dependence of young people (iy18)

Source: constructed by the author in GeoDa from the GWR fit

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DATA ENVELOPMENT APPLICATIONS IN SPORTS: A CASE OF THE NATIONAL HOCKEY LEAGUE

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Abstract
Data envelopment analysis (DEA) models are general tools for relative efficiency and performance evaluation of a set of decision-making units with many real-world applications in various industries. The paper deals with applications of DEA models in sports. They are numerous and aim mainly in efficiency and performance evaluation of sports teams of various sport branches. Just a few studies are oriented on the performance evaluation of individual players. We apply traditional DEA models for performance evaluation of key ice hockey players at main positions. The results for all positions are aggregated, and final performance indicators for the teams are derived. The results of the analysis are compared with real results in the last season.

Keywords: *data envelopment analysis, performance, sports, NHL*

JEL Classification: C44
AMS Classification: 90B50

1 INTRODUCTION

Sports industry belongs to rather important industry branches in many countries. That is why efficiency and performance analysis in sports has the same importance as in other economic fields. Data envelopment analysis (DEA) is one of the traditional analytical tools for relative efficiency evaluation of a set of decision-making units (DMUs). DEA models were applied in many sports. Their application is oriented mostly in two directions: evaluation of the efficiency of sports teams on the one hand and analysis of individual players on the other hand. The first group of studies is much more numerous. Detailed information about DEA modelling in sports can be found in Bhat et al. (2019). The authors classify the studies according to the sport branches. Most of the studies are devoted to the analysis of efficiency in football (soccer), baseball and basketball, but some papers dealing with efficiency analysis of tennis or golf professional players, cricket teams, handball teams, or even in cycling and other sports. Except for studies concerned with the efficiency of teams or individual players, several papers are dealing with re-calculation of the ranking of countries in sporting events as the Olympic Games according to the number of medals. The outputs in these studies are always the number of medals (gold, silver, bronze) won by the country, and the inputs are various characteristics of the countries as population and gross national product. The first paper of this category was published by Villa et al. (2002). Jablonsky (2016) introduced an original two-stage DEA model for re-calculation of the ranking of countries at the Summer Olympic Games 2016. Many of the DEA application studies in sports have been published in prestigious operations research journals and are highly cited.

Due to the limited space for this paper, it is not possible to give a serious survey of available applications. We mention just several examples from the most popular sports. The analysis published in (Guzmán and Morrow, 2007) deals with the efficiency assessment of football clubs in the English Premier Division. Traditional DEA models, together with dynamic analysis using Malmquist index, are applied. An interesting attempt to assess scores of football matches was published by Villa and Lozano (2016). Their novel approach uses network DEA model with considering attacking and defending variables, ball possession, and economic strength of the team. The model can be run immediately after the match and finally, the season scoring efficiency for the team is derived.

Similarly to football analyses, many studies have been published about efficiency in basketball. Cooper et al. (2009) derive performance indices for individual NBA (National Basketball Association) players using an original procedure that ensures non-zero weights of variables. A dynamic network DEA model is developed for deriving scoring efficiency in each quarter of the game and for overall efficiency. In this introduction, we mentioned DEA applications in various sport branches, but we did not find any severe studies dealing with efficiency analyses in ice hockey even though detailed data sets for analyses are readily available. This paper analyses efficiency and performance of 16 NHL teams in Eastern Conference in the 2018/2019 season. The overall efficiency of the team is based on efficiency of individual players in the main positions.

The paper is organized as follows. Next Section 2 contains basic definitions and introduces the models used further in the study. Section 3 formulates in detail the problem and the data sets for the analysis. The results and their discussion are included in Section 4. Concluding section summarizes the results and discusses possibilities of future research in this field.

2 DEA MODELS

DEA is a universal tool for efficiency and performance evaluation of a set of DMUs by comparison of multiple outputs produced by the unit under evaluation and its multiple inputs. Traditional DEA model, as introduced in (Charnes et al., 1978), can be expressed as follows:

$$\text{Maximize} \qquad \theta_q = \frac{\sum_{k=1}^{r} u_k y_{qk}}{\sum_{j=1}^{m} v_j x_{qj}}$$

subject to (1)

$$\frac{\sum_{k=1}^{r} u_k y_{ik}}{\sum_{j=1}^{m} v_j x_{ij}} \le 1, \quad i = 1,\dots,n,$$

$$u_k \ge \varepsilon, v_i \ge \varepsilon,$$

where n is the number of DMUs, m is the number of inputs and r is the number of outputs, x_{ij} is the value of the j-th input for the i-th DMU, and analogously y_{ik} is the value of the k-th output for the i-th DMU, v_i and u_k are strictly positive weights of the inputs and outputs, respectively. The efficiency score of the q-th unit θ_q is derived by maximization of the above ratio under the assumption that the efficiencies of all other units do not exceed 1. Efficiency score θ_q equals 1 if the q-th unit is efficient; otherwise, the unit under evaluation is inefficient.

Another very popular DEA model is SBM (slacks-based measure) model proposed in Tone (2001). Its formulation follows:

$$\text{Minimize} \qquad \rho_q = \frac{1-\frac{1}{m}\sum_{j=1}^{m}\left(s_j^- / x_{qj}\right)}{1+\frac{1}{r}\sum_{k=1}^{r}\left(s_k^+ / y_{qk}\right)},$$

$$\text{subject to} \qquad \sum_{i=1}^{n} x_{ij}\lambda_i + s_j^- = x_{qj}, \qquad j = 1,..., m, \qquad (2)$$

$$\sum_{i=1}^{n} y_{ik}\lambda_i - s_k^+ = y_{qk}, \qquad k = 1,..., r,$$

$$s_j^- \geq 0, \qquad j = 1,...,m,$$

$$s_k^+ \geq 0, \qquad k = 1,...,r,$$

$$\lambda_i \geq 0, \qquad i = 1,...,n,$$

where λ_i is the weight of the i-th DMU, and slack (surplus) variables s_j^- (s_k^+) measure the distance of the inputs (outputs) of the q-th DMU from the target values on the efficient frontier. The unit under evaluation is efficient if the target values are the same as the real values of inputs and outputs, i.e. the efficiency score (the objective function of model (2)) equals to 1. Lower values than 1 indicate inefficiency.

Models (1) and (2) are not linear in their objective functions. Their linearization is possible using Charnes and Cooper transformation. Both models allow splitting the set of the units into two disjoint subsets - efficient and inefficient. The inefficient units can be ranked according to their efficiency scores, the efficient ones have identical efficiency scores and must be ranked by using special models. The main group of these models are super-efficiency models that assign to formerly efficient units super-efficiency scores greater than 1. A super-efficiency version of model (1) was formulated in (Andersen and Petersen, 1993), super-efficiency SBM model was introduced in (Tone, 2002).

3 PROBLEM DESCRIPTION

In our conditions, it is not necessary to introduce in detail ice hockey. An ice hockey match is played by two teams with five players plus a goalie on the ice. The fives of players on the ice are changed after a certain time and the time of the game is three times 20 minutes net, i.e. 60 minutes in total. There are players of three central positions:

- Forwards (left wings, centres and right wings),
- Defenders,
- Goalies.

There are different ice hockey leagues in the world, but the most known and prestigious is the Canadian-American National Hockey League (NHL). The regular season of the NHL is played in two conferences – each of them is divided into two divisions. In the season 2018/19, the Eastern Conference included 16 teams, the Western one 15 teams.

We aimed to evaluate the efficiency and performance of the teams based on the individual performance of players on five main positions (left wing, centre, right wing, defender, and a goalie). We decided to use available data sets for the Eastern Conference and always select the best players for each position for each team. In this way, we obtained the set of 16 players for each position (for defenders and goalies we consider two players with the highest number of matches for each team, i.e. 32 in total). It is not possible to present all the data here because of the limited space for the paper.

For all sets of players, we applied DEA models described in the previous section of the paper – models (1) and (2) including their super-efficiency modifications. The following inputs and outputs have been considered for the positions:

- Forwards: 2 inputs (the number of matches played, the total number of minutes on ice in the season), 2 traditional outputs (the number of goals scored, the number of assistances), and one undesirable output (the number of penalties in minutes).
- Defenders: The inputs are the same as in the previous case. There are 2 outputs (the number of points = goals plus assistances) and one undesirable outputs (penalties).
- Goalies: The same inputs as in the previous cases. The outputs are as follows: the number of matches without any goal, the total number of saves, the number of goals (undesirable output).

4 RESULTS

Tables 1 and 2 contain efficiency scores of selected individual players for all positions and all teams computed using CCR model (1) and SBM model (2). The last column of both tables is a simple average of all efficiency scores. The teams in both tables are ranked according to the final ranking after the regular season is finished. All calculations in this section were performed using original scripts written in LINGO modelling system that is one of the suitable tools for solving DEA models and their modifications for research purposes.

Table 1: *Efficiency scores for all positions – CCR model (1)*

Team	Left wing	Center	Right wing	Defender	Goalie	Avg. eff.
Tampa Bay	0.6821	1.0000	1.0000	0.8362	0.9808	0.8998
Washington	1.0000	0.8537	0.7502	0.8371	0.9105	0.8703
Boston	0.9515	1.0000	1.0000	0.8030	0.9385	0.9386
NY Islanders	0.6825	0.7383	0.5113	0.8732	1.0000	0.7610
Toronto	0.8077	1.0000	0.7178	0.9726	0.9868	0.8970
Pittsburgh	0.6527	1.0000	0.7944	0.5443	0.9930	0.7969
Carolina	0.9354	0.7983	0.9199	0.9347	0.8956	0.8968
Columbus	0.7702	0.9779	0.9919	0.8062	0.9226	0.8938
Montreal	0.8077	0.7842	0.7993	0.7204	0.9467	0.8117
Florida	1.0000	1.0000	0.9925	0.8187	0.8743	0.9371
Philadelphia	0.9541	0.8474	1.0000	0.8193	0.9651	0.9172
NY Rangers	0.9066	0.7490	1.0000	0.6959	0.9872	0.8677
Buffalo	0.6179	0.9409	1.0000	0.8099	0.9373	0.8612
Detroit	0.7159	0.7943	0.8505	0.7481	0.9381	0.8094
New Jersey	1.0000	0.7758	1.0000	0.8176	0.9358	0.9059
Ottawa	0.8655	1.0000	0.8687	0.9101	0.9733	0.9235

The results show a known property that the efficiency scores obtained by the SBM model are always less or equal than the efficiencies calculated by CCR model. The number of players for forwards was 16 (one for each team), i.e. the numbers in both tables are exactly the efficiency scores derived by the model. The number of players for defenders and goalies was 32 (two players for each team). In this case, the tables contain averages of two efficiency scores for two best players in these positions.

Table 2: *Efficiency scores for all positions – SBM model (2)*

Team	Left wing	Center	Right wing	Defender	Goalie	Avg. eff.
Tampa Bay	0.5200	1.0000	1.0000	0.8157	0.7857	0.8243
Washington	1.0000	0.7578	0.6989	0.6401	0.4879	0.7170
Boston	0.6650	1.0000	1.0000	0.6556	0.7792	0.8200
NY Islanders	0.3509	0.6224	0.3499	0.6160	1.0000	0.5878
Toronto	0.5835	1.0000	0.6032	0.7492	0.6519	0.7175
Pittsburgh	0.3160	1.0000	0.6699	0.2538	0.8053	0.6090
Carolina	0.6573	0.4264	0.7923	0.7773	0.7184	0.6744
Columbus	0.1319	0.9658	0.7793	0.6258	0.7866	0.6579
Montreal	0.5342	0.4349	0.4954	0.5883	0.5970	0.5300
Florida	1.0000	1.0000	0.9705	0.5637	0.4300	0.7928
Philadelphia	0.7901	0.7823	1.0000	0.5997	0.7381	0.7821
NY Rangers	0.6665	0.7006	1.0000	0.4185	0.6381	0.6847
Buffalo	0.4936	0.9153	1.0000	0.6814	0.4301	0.7041
Detroit	0.4846	0.4805	0.7137	0.4833	0.4207	0.5166
New Jersey	1.0000	0.6600	1.0000	0.6919	0.6466	0.7997
Ottawa	0.5432	1.0000	0.8031	0.7586	0.8153	0.7840

The comparison of results obtained by both models shows that the differences in efficiency scores are much higher for SBM models which is a quite questionable conclusion. For example, the efficiency scores of goalies obtained by model (1) are except for some cases higher than 0.9, i.e. all goalies are considered as very efficient. This conclusion corresponds to official statistics of goalies where the essential characteristic is the save percentage, i.e. the ratio between the number of shots saved and the total number of shots. This ratio is almost for all goalies slightly higher than 90%. On the contrary, the results derived by the SBM model show considerable differences, i.e. the efficiency scores from 1 (totally efficient) to the values lower than 0.4 that indicate a quite high level of inefficiency.

Averages of efficiency scores for all key players are presented in the last columns of both tables. Unfortunately, there is almost no correlation between real standings after the regular season and the ranking based on these average values. This approach can hardly be used for prediction of the final standings of the teams after the league is finished. The reason may consist of the limited number of players considered in our study (just three forwards only, two defenders and two goalies). Nevertheless, efficiency evaluation of individual players using DEA models returns impressive results that correspond more or less to official statistics.

5 CONCLUSIONS

Applications of operations research methods and models in sports are numerous. There are plenty of studies dealing with various aspects of sport theory. One of the popular streams of applications in sports is performance and efficiency evaluation of players or sports teams within a branch. The study presented in this paper dealt with ice hockey players and teams performance analysis based on the data from 2018/19 regular season using traditional DEA models. The proposed approach is rather applicable for evaluation of individual players than the evaluation of teams. In further experiments, we plan to extend the number of players of the teams considered in evaluation by all players with a minimum number of matches (or minutes on ice) in the season. This extension of the data set could improve the results for team efficiency.

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UNIFORMLY DEPLOYED SET KIT AND P-LOCATION PROBLEM SOLVING

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Abstract

Location problems select *p* locations from a set of *m* possible locations to optimize a given objective function. When heuristics or metaheuristics are used as a solving tool, a starting solution or a starting population of solutions play a considerable role in preventing the search from being trapped in a local minimum. Most of heuristic algorithms face to being trapped by repeated random generation of starting solutions. This contribution deals with making use of the special structure of *p*-location problem solutions trying to construct a standard uniformly deployed set of solutions depending only on parameters *p* and *m*. If a given instance of the *p*-location problem is to be solved, then the objective function is computed for each solution of the uniformly deployed set and the values form a mapping of the set into a range of the objective function values. This mapping can be used to identify the most interesting part of the set of feasible solutions for next processing. In this paper, we focus on a way of establishing a kit of the standard uniformly deployed sets and on utilization of the kit for pre-solving *p*-location problems. As the kit cannot be dense enough to cover all instances of the *p*-location problem, which parameters *p* and *m* belong to the kit range, we suggest a way of adjustment of a standard uniformly deployed set to the solved instance. We study the impact of the adjustment on the efficiency of some solving methods, which use the pre-solving phase.

Keywords: *Discrete location, p-location problems, uniformly deployed set of solutions*

JEL Classification: C61
AMS Classification: 90C27

1 INTRODUCTION

Various cases of *p*-median or *p*-center problems create a family of so-called *p*-location problems. Solution techniques for the *p*-location problems represent important tool for emergency service system designing [2, 7, 9]. Due to complexity of the integer programming problems, the exact methods [1, 3, 6] are often abandoned for their unpredictable computational time in the case, when large instances of a location problem have to be solved. That is why operational researchers focus their effort on various kinds of heuristics and metaheuristics to obtain a good solution of the emergency service system design problem. Evolutionary metaheuristics as the genetic algorithm hold an important position among the solving tools [4, 8, 10]. Also simple incrementing exchange heuristics proved to be a useful technique for *p*-location problem solving probably due to special properties of the set of feasible solutions [5]. Efficiency of heuristics and metaheuristics often depends on preliminary information about the objective function values over usually huge set of feasible solutions. Such preliminary knowledge could enable determination of a starting solution or a starting population for heuristic or metaheuristic usage. In this paper, we deal with construction and usage of a standard uniformly deployed set of *p*-location problem solutions. The objective function values of individual elements of the uniformly deployed set form a mapping of the set into a range of the objective function values. The mapping can be used to identify the most interesting part of the set of feasible solutions for next processing. We present a way of establishing a kit of the standard uniformly deployed sets and compare efficiency of the kit utilization to the approaches, which use of uniformly deployed sets „tailored“ for concrete instance of p-median problems.

2 THE P-MEDIAN PROBLEM AND UNIFORMLY DEPLOYED SET

The weighted p-median problem can be generally defined as a task of locating p centers in the set I of all possible center locations so that the sum of weighted distances from individual user locations to the nearest located center is minimal. Formulation of the problem uses denotation J for the set of user locations. The problem can be defined by (1), where decisions on locating a service center at location i is modelled by a zero-one variable y_i for each $i \in I$, where the variable y_i gets the value one if a center is located at location i and it gets the value of zero otherwise. The distance from location i to location j is denoted by d_{ij} and symbol b_j represents a weight of user j, what can be population at the location or frequency of demanded visits etc.

$$\min\left\{\sum_{j\in J} b_j \min\left\{d_{ij} : i\in I,\ y_i = 1\right\} : y_i \in \left\{0,1\right\}\ for\, i\in I,\ \sum_{i\in I} y_i = p\right\} \tag{1}$$

The problem (1) can be studied as a search in a sub-set of m-dimensional hypercube vertices, when m denotes the cardinality of the set I. A vertex corresponding to a feasible solution $\boldsymbol{y}$ has exactly p unit components.

Hamming distance between two feasible solutions $\boldsymbol{y}$ and $\boldsymbol{x}$ can be defined by (2).

$$H\left(\boldsymbol{y},\boldsymbol{x}\right) = \sum_{i=1}^{m} \left|y_i - x_i\right| \tag{2}$$

The integer p-$H(\boldsymbol{y}, \boldsymbol{x})/2$ gives the number of possible center locations occupied simultaneously by centers in both solutions. The uniformly deployed set of p-location problem solutions can be defined as a sub-set S of feasible solutions of (1) so that the inequality $H(\boldsymbol{y}, \boldsymbol{x}) \geq h$ holds for each $\boldsymbol{x}, \boldsymbol{y} \in S$ and for given h.

The previous approach to obtaining enough big uniformly deployed set was presented in [5] and consists in creation of so-called basic deployed set, which was formed by “ad-hoc” combining q – tuples of locations so that two different q-tuples have at most one common location. The basic set of p-location problem solutions satisfies the condition that no pair of solutions can contain more than r common locations and, this way, their minimal Hamming distance equals to $2(p-r)$. The basic set was enlarged step by step by solving the problem (3) – (7) and adding the optimal solution $\boldsymbol{y}$ to the current set S if the optimal value of (3) equals to zero. This enlargement process terminates either if the maximal set is obtained, i.e. the optimal value of (3) is positive, or if a demanded cardinality of the set S is reached. This model was introduced and discussed in [5]. Let the s-th solution from S be described by a series of zero-one constants e_{si} for $i = 1 \ldots m$, where e_{si} equals to one, if the solutions includes location i and it equals to zero otherwise. We introduce zero-one variables $y_i \in \{0, 1\}$ for $i = 1 \ldots m$ to described hypothetical additional solution $\boldsymbol{y}$, which could be used for extension of the set S. We also introduce auxiliary variables z_s for $s \in S$ to identify exceeding number of common locations in solution s and $\boldsymbol{y}$.

$$Minimize \quad \sum_{s\in S} z_s \tag{3}$$

$$Subject\ to \quad \sum_{i=1}^{m} y_i = p \tag{4}$$

$$\sum_{i=1}^{m} e_{si} y_i \le r + z_s \quad for\ s \in S \tag{5}$$

$$y_i \in \{0, 1\} \quad for\ i = 1, ..., m \tag{6}$$

$$z_s \ge 0 \quad for\ s \in S \tag{7}$$

The objective function (3) expresses the sum of surpluses of common locations of the suggested solution ***y*** and *s* over all solutions from *S*. If the objective function value of the optimal solution equals to zero, then the solution ***y*** has at most *d* common locations with each solution from *S* and it follows that the set *S* is not maximal and the solution ***y*** can be added to the set *S* to perform its extension. Constraint (4) is a feasibility constraint imposed upon solution ***y*** for it to be *p*-location problem solution. Constraints (5) links up suggested solution ***y*** to the variables z_s for $s \in S$. The left-hand-side of (5) for a solution *s* gives number of locations common for solution *s* and ***y***. If the number exceeds the value *d*, the associated variable z_s must obtain a value greater than or equal to the surplus.

3 THE CONCEPT OF UNIFORMLY DEPLOYED SET KIT

Even if usage of the uniformly deployed set proved to be a good idea for application of heuristic to the *p*-location problem solving, the computation of enough big uniformly deployed set of solutions for a problem specified by a number *m* of possible center locations and by a number *p* of centers to be located may be a very time demanding task.

To avoid the burden, we suggest a concept of a uniformly deployed set kit, which enables to separate the activities connected with the uniformly deployed set constitution from the *p*-location problem solving.

For bigger clarity of the next explanation, a solution of a *p*- location problem will be represented by a *p*-tuple of locations in which a center is located. The idea of a kit of uniformly deployed sets consists in determination of finite ranges *M* and P_m of parameters *m* and *p* respectively and in computing uniformly deployed standard set S_{mp} for each $m \in M$ and $p \in P_m$ so that cardinality $|S_{mp}|$ exceeds a given threshold. This system of uniformly deployed sets represents a kit and members of the kit can be published and updated independently on its further exploitation.

If a necessity to solve a $\underline{p}$-location problem with $\underline{m}$ possible center locations arises, then maximal $m \in M$ and minimal $p \in P_m$ must be chosen so that $m \le \underline{m}$ and $\underline{p} \le p$. Uniformly deployed set for the $\underline{p}$-location problem with $\underline{m}$ possible center locations can be obtained so that each *p*-tuple of S_{mp} is reduced by removing p-$\underline{p}$ locations from the *p*-tuple.

It must be noted that if the minimal Hamming distance of arbitrary couple of solutions of S_{mp} is $2(p\text{-}r)$, then the minimal Hamming distance of two reduced solutions is $2(\underline{p}\text{-}r)$.

4 CONSTRUCTION OF THE UNIFORMLY DEPLOYED SET KIT

We started with definition of the ranges *M* and P_m, which was determined so that $M=\{100, 200, \ldots, 1000\}$ and $P_m=\{10, 20, \ldots, m/3\}$. The starting instance of the kit was completed according to the following scheme.

First, an attempt for maximal uniformly deployed set S^1_q construction was performed for q-location problem with q^2 possible center locations for any integer q from the range {2, 3, …, 31} so that arbitrary two q-tuples have at most one common location.

Second, a basic set of uniformly deployed solutions for given $m \in M$ and $p \in P_m$ was composed of k q-tuples of S^1_q and of $k(p-q)$ other locations, where k and q were determined so that $k(p-q)+q^2 \leq m$ holds. Each two p-tuples of the basic set have at most one common location.

Third, each basic set was augmented by the time-restricted process making use of the model (3) – (7), where the initial value of parameter r equaled to one. If cardinality of the augmented set did not reach a given threshold, the parameter r was incremented and augmentation process continued.
The current kit for the above mentioned ranges was constructed for threshold 90 p-tuples, where the computational time limit was set at 600 seconds. The resulting values of parameter r, for which the threshold was reached, is reported in Table 1.

Table 1 Resulting values of parameter r, for which at least 90 different elements of the uniformly deployed set were obtained for different problems described by the parameters m and p.

$p \backslash m$	100	200	300	400	500	600	700	800	900	1000
10	2	1	1	1	1	1	1	1	1	1
20	5	3	2	1	1	1	1	1	1	1
30	11	6	4	3	2	2	2	2	1	1
40		9	6	5	4	3	3	2	2	2
50		14	9	7	6	5	4	4	3	3
60		19	13	10	8	7	6	5	4	4
70			18	13	11	9	8	7	6	5
80			23	17	14	11	10	9	8	7
90			28	21	17	14	12	11	10	9
100			35	26	21	17	15	13	12	10
110				31	25	21	18	16	14	13
120				37	30	25	21	19	17	15
130				44	35	29	25	22	19	17
140					40	34	29	25	22	20
150					46	39	33	29	26	23
160					52	44	38	33	29	26
170						49	42	37	33	30
180						55	47	41	37	33
190						61	53	46	41	37
200						68	58	51	45	45
210							64	56	50	49
220							70	62	55	54
230							77	67	60	59
240								73	65	64
250								79	71	69
260								86	76	74
270									82	79
280									89	85
290									95	91
300									101	97
310										104
320										110

5 NUMERICAL EXPERIMENTS

The main goal of performed computational study was to study the impact of suggested kit on efficiency of approximate solving methods. We compare results of swap algorithms getBA [8], when used together with "tailored" uniformly deployed set to the results, which were achieved using the algorithm with uniformly deployed sets obtained by reduction of the kit set. The used benchmarks were Slovak self-governing regions. The mentioned instances are denoted by the names of capitals of the individual regions and by triples (XX, $\underline{m}$, $\underline{p}$), where XX is abbreviation of the region denotation, $\underline{m}$ stands for the number of possible center locations and $\underline{p}$ is the number of service centers to be located. The list of instances is Banská Bystrica (BB, 515, 36), Košice (KE, 460, 32), Nitra (NR, 350, 27), Prešov (PO, 664, 32), Trenčín (TN, 276, 21), Trnava (TT, 249, 18) and Žilina (ZA, 315, 29).

An individual experiment was organized in such a way that the optimal solution of the min-sum location problem was obtained using the radial approach described in [6], first. The objective function value of the exact solution denoted by F^{opt} is reported in the Table 2 in the column denoted by "Optimal solution". The section "getBA tailored" contains the objective function values reported in the column denoted by F^* and solution accuracy evaluated by *gap*, which expresses a relative difference of the obtained result from the optimal solution. The value of *gap* is expressed in percentage, where the optimal objective function value of the problem is taken as the base. This section values were obtained by getBA algorithm used with "tailored" uniformly deployed set. The section "getBA kit usage" contains the objective function values reported in the column denoted by F^{**} and solution accuracy is also evaluated by *gap*. The results were achieved using the algorithm getBA with uniformly deployed sets obtained by reduction of the associated kit set.

Table 2 Results of numerical experiments to verify the usefulness of the uniformly deployed set kit usage.

Region	$\underline{m}$	$\underline{p}$	m	p	**Optimal solution**	***getBA tailored***		***getBA kit usage***	
					F^{opt}	F^*	*gap* [%]	F^{**}	*gap* [%]
BB	515	36	500	40	29873	29873	0.00	29873	0.00
KE	460	32	400	40	31200	31451	0.80	31252	0.17
NR	350	27	300	30	34041	34075	0.10	34202	0.47
PO	664	32	600	40	39073	39117	0.11	39117	0.11
TN	276	21	200	30	25099	25125	0.10	25125	0.10
TT	249	18	200	20	28206	28372	0.59	28206	0.00
ZA	315	29	300	30	28967	28967	0.00	28977	0.03

6 CONCLUSIONS

The main goal of this paper was to find whether a standard kit of uniformly deployed sets can replace the tailored uniformly deployed set in simple heuristics applications. Presented results of performed numerical experiments confirm that the both variants of heuristics give satisfactory solution accuracy. Therefore, we can conclude that construction of a kit of uniformly deployed sets can be performed once and it can be used for solving broad spectrum of p-location problems by heuristics. Future research in this field may be aimed at improvement of the kit. Another interesting topic for next research could be focused on other ways of processing elements of the uniformly deployed set different from the neighborhood search.

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COMBINED MATHEMATICAL PROGRAMMING AND COMPUTER SIMULATION APPROACH TO AMBULANCE LOCATION

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Abstract
The research compares two approaches to re-location of the current emergency medical service stations in a large-scale area. The former is a median-type model that minimizes the average response time for all patients. The latter is the maximum expected covering location model that maximizes the number of high priority patients who will probably be served within a pre-specified time limit. A computer simulation model is used to estimate the parameters of the mathematical programming model (ambulances' workload in our case) and to evaluate the proposed sets of locations. The models are verified using real-world data. The simulation study proves that the weighted *p*-median model results in a considerable improvement of the current system.

Keywords: *Emergency medical service, p-median problem, expected coverage, computer simulation*

JEL Classification: JEL C61, JEL C63
AMS Classification: AMS 65C05, AMS 90B80, AMS 90C10

1 INTRODUCTION

The output quality of an emergency system depends mainly of the number of rescue teams and their distribution in the given area. Major improvements of an existing system may be achieved by optimizing the positions of the base stations where the vehicles and their crews wait for their next call. Such a reorganization does not require substantial investments nor increased operational costs since it that preserves the current number of stations. This paper deals with the optimization of the current emergency medical system (EMS) in this sense: we try to find better locations of the ambulance stations assuming that their number is given and will not change. We do not deal with dimensioning of the stations because we use the convention applied in Slovakia, where every station houses one ambulance. To ensure the accessibility of the service in highly populated areas, large towns and city districts have multiple stations. Currently, there are 273 stations distributed in 211 towns, city districts and villages. From the reported statistics published by the Operation Centre of the EMS of the Slovak Republic follows that in 2015 the average number of patients served by one ambulance was 1811 people. We consider this value to be the capacity of an ambulance. If the expected number of patients in a municipality exceeds the capacity of all stations currently located in the municipality, we do not change their locations. We also respect previous managerial decisions about multiple stations in a town where the estimated number of patients is less than the capacity of a stations. In such a case our methods leave one of the stations in the place and seek for better locations of the other stations. When these rules are applied, we get 80 stations that cannot be relocated. These stations are denoted as fixed and are not subject to the optimization. We do not consider investment costs associated with the redeployment of the stations. Instead we use such optimization criteria that reflect the main goal of the EMS system – to save as many people as possible. Since this output cannot be measured when designing the system, surrogate optimization criteria are formulated instead resulting in different location models. The models used in healthcare facility location are surveyed in (Ahmadi-Javid et al., 2017). Since in real-world problems the facilities as well as customers are located on a transportation network, the properties of the underlying transportation network may be exploited by the solution algorithms (Czimmermann, 2016). The review of optimization problems related to emergency medical service can be found in Aringhieri et al. (2017).

The expected number of patients is derived from the age structure of population published by the Statistical Office of the Slovak Republic and a sample of ambulance interventions provided us by Falck Záchranná a.s., which was the largest EMS provider in Slovakia in the last decade. The Falck dataset contains depersonalized information on 149,474 EMS interventions performed in the year 2015. The age of the patient, his or her initial medical diagnosis, the type of intervening ambulance and time stamps of the whole EMS trip (including transport to hospital, if any) are recorded. Combining Falck data with publicly available statistics reported by the Operation Centre of the EMS of the Slovak Republic and the population statistics published by the Statistical Office of the Slovak Republic we can calculate the rates of emergency interventions for various age groups according to Eq. (1):

$$rate_k = 1000 \frac{D \cdot Falck_k}{Pop_k \cdot Falck_{total}} \quad (1)$$

where $rate_k$ is the one-year number of emergency cases per 1,000 persons in age group k, D is the total number of ambulance dispatches reported by the Operation Centre of the EMS of the Slovak Republic for the year 2015, $Falck_k$ is the number of patients in age group k in the Falck dataset, $Falck_{total}$ is the total number of patients in the dataset, and Pop_k is the number of inhabitants in age group k.

Within each age group we can further distinguish two groups of patients according to their initial clinical conditions. The most severe conditions are denoted as the First Hour Quintet (FHQ), and they include: chest pain, severe trauma, stroke, severe respiratory difficulties and cardiac arrest. Although the international definition of FHQ does not list unconsciousness, it is also a life-threatening condition. Therefore, after a consultation with emergency physicians, we decided to include it in FHQ. The FHQ conditions require immediate rescuing. If a call is recognized as a FHQ call, it gets the highest priority because every minute of delay in the response reduces patient's chance of survival. Knowing the number of highest priority calls in the Falck dataset the rates of FHQ interventions can be calculated.

The analysis of the sample data revealed that the overall rates as well as FHQ rates increase with age (Fig. 1). The Spearman correlation is $\rho = 0.95$ for overall rate and $\rho = 0.96$ for FHQ rate. The dependency curve has an exponential shape, with the acceleration from the age of 65 years.

We will distinguish three age categories: (i) children in the age of 0-14 who have the lowest emergency incident rates, (ii) teens and nonelderly adults aged 15-64, and (iii) elderly people aged 65 years and over who call EMS the most frequently. The total emergency incident rates and FHQ incident rates for these categories are shown in Table 1. Based on the age structure and the rates we can estimate the annual number of EMS patients b_j in municipality j according to Eq. (2):

$$b_j = \sum_{k=1}^{3} rate_k \, pop_{kj} \quad (2)$$

where $rate_k$ is the one-year number of emergency cases per 1,000 persons in age group k, and pop_{kj} is the number of inhabitants in age group k in municipality j (j is a member of the set of all municipalities in Slovakia). Similarly, the annual number of highest priority patients b_j^{FHQ} can be calculated using the rates of FHQ incidents.

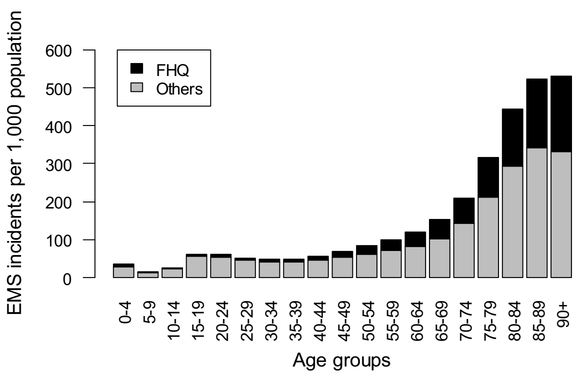

Figure 1. Emergency incident rates increase with age (Slovakia, 2015)

Table 1. Emergency incident rates

Age group	Total rate	FHQ rate
0-14	26.09	4.27
15-64	69.12	14.97
65+	267.07	89.09

2 LOCATION MODELS

We formulate the problem of optimal location of a given number of stations using mathematical programming. The output of the mathematical model defines the villages, towns and districts where the stations will be deployed (at most one station in a municipality). This output is merged with the pre-processed fixed locations, resulting in multiple stations in more populated towns and districts. However, at this moment we do not have precise addresses. The seats of the stations inside a given municipality are determined afterwards. We proceed from current locations. Addresses of fixed stations are preserved. A new station, if there is one, is placed at the municipality's centre. If one or more stations out of multiple current stations are shut down, they are selected randomly.

The models do not distinguish the types of ambulances. In Slovakia, there are two types of ambulances that differ in the equipment and the qualification of the rescue team. Most ambulances provide basic life support (BLS; Slovak abbreviation RZP) and have only a paramedic and a rescue driver on board. About one third of ambulances are well-equipped advanced life support units (ALS; Slovak abbreviation RLP). An ALS crew consists of a physician, a paramedic and a driver. It is important to specify the type of the located ambulances because the dispatching rules say that if the closest ambulance to a FHQ patient is of BLS type, then the closest available ALS ambulance is dispatched concurrently. Thus, for the sake of reducing the ambulance workload, it seems reasonable to locate ALS ambulances close to old people, who often call EMS due to life-threatening events. We distribute ambulances among the optimized station locations in the following way: first, we retain the type of fixed stations that are disregarded in the optimization, and also the type of those stations whose locations were not changed by the optimization model. As regards the relocated stations, we decide first where the remaining ALS ambulances will be sited. We proceed according to one of the following heuristic rules: (i) the distribution of ALS ambulances copies the current state as much as possible, it means they are located in the vicinity of the current ALS stations, or (ii) ALS ambulances are allocated to the stations in towns with a high number of 65+ inhabitants, unless there is already an ALS ambulance there. After allocation of ALS ambulances, the remaining stations are assigned by BLS ambulances.

The EMS infrastructure proposed by mathematical programming models is evaluated by the means of computer simulation. A detailed computer simulation model was built and verified using historical data on EMS trips by Falck (Jánošíková et al., 2019). A simulation study with the current locations of the stations is also used to estimate some parameters needed by the mathematical programming models.

We apply two classical models that are frequently used in the context of emergency systems, namely the weighted p-median model that minimizes the average response time for all patients (Garner and van den Berg, 2017, Jánošíková et al., 2019, Sasaki et al., 2010), and the maximum expected covering location model that maximizes the number of FHQ patients who will probably be served within a pre-specified time limit (Erkut et al., 2008, Ingolfsson et al., 2008). The models are formulated using the following sets, indices, parameters and decision variables.

Sets and indices

I set of candidate locations (candidate location are all municipalities and other villages that are the seats of stations today but are not self-governing units); $|I|$ = 2,934

J set of demand points (demand points are all municipalities); $|J|$ = 2,928

$i \in I$ candidate location

$j \in J$ demand point

k index corresponding to the number of stations

$N_j = \{i \in I: t_{ij} \leq S\}$ set of candidate locations in the neighbourhood of demand point j

Parameters

p number of stations to be sited

q_j probability of an ambulance in the neighbourhood of demand point j being unavailable

S the desired service standard

b_j the annual number of EMS patients in municipality j reduced by the capacity of the fixed stations

b_j^{FHQ} the annual number of FHQ patients in municipality j

t_{ij} shortest travel time from candidate location i to demand point j

st_i the number of fixed stations in candidate location i

$n_j = |N_j|$ the number of candidate locations in the neighbourhood of demand point j

Decision variables

$$x_i = \begin{cases} 1, & \text{if a station is located at site } i \\ 0, & \text{otherwise} \end{cases}$$

$$y_{jk} = \begin{cases} 1, & \text{if demand point } j \text{ is covered by at least } k \text{ stations} \\ 0, & \text{otherwise} \end{cases}$$

$$z_{ij} = \begin{cases} 1, & \text{if demand point } j \text{ is served by the station located at site } i \\ 0, & \text{otherwise} \end{cases}$$

2.1 Parameter estimation

The municipalities are represented by their central nodes, which are the closest nodes on the road network with regard to the centre of the municipality. We suppose that an ambulance always travels along the route that declares the shortest travel time. The travel times t_{ij} are calculated using the historical data on the average ambulance speed with regard to the road category and day time.

As was stated before, 80 of 273 current stations are fixed in the pre-processing phase so the models decide about new positions of the remaining p = 193 stations.

The probability q_j of an ambulance in the neighbourhood of municipality j being unavailable are set using computer simulation of the system with the current distribution of the stations. The probability q_j is calculated as the average workload of potential ambulances in the neighbourhood. If there is at least one ambulance currently operating in a candidate location, then the workload of this candidate is calculated as the average workload of these ambulances. If the candidate does not have a station today, then its workload is set to the average workload in the region.

The desired service standard S was set with regard to critical patients who are in life-threatening conditions and every minute delay in response time dramatically worsens their chance to survive. These patients should be reached within 8 minutes, which is a widely accepted standard in most European countries (Krafft et al., 2006). Thus assuming one minute pre-trip delay, we set S to the value of 7 minutes.

2.2 The weighted *p*-median model (*p*MP)

The following model is a mathematical programming formulation of the weighted *p*MP.

$$\text{minimize} \quad \sum_{i\in I}\sum_{j\in J} t_{ij}\, b_j z_{ij} \tag{3}$$

$$\text{subject to} \quad \sum_{i\in I} z_{ij} = 1 \qquad \text{for } j \in J \tag{4}$$

$$z_{ij} \leq x_i \qquad \text{for } i \in I,\ j \in J \tag{5}$$

$$\sum_{i\in I} x_i = p \tag{6}$$

$$x_i, z_{ij} \in \{0,1\} \qquad \text{for } i \in I,\ j \in J \tag{7}$$

The objective function (3) minimizes the total travel time needed by the ambulances to reach all patients. The average travel time is equal to the total travel time divided by the number of all patients. The average travel time is a lower bound of the real response time since the model assumes that there is always an ambulance available to answer to the call and therefore an ambulance from the closest station is always dispatched to the patient. Constraints (4) assign every municipality j to exactly one station i. Constraints (5) ensure that if a municipality j is assigned to a node i, then a station will be open in the node i. Constraint (6) limits the number of stations. The obligatory constraints (7) specify the definition domains of the variables.

2.3 The maximum expected covering location problem (MEXCLP)

We modify the formulation of the MEXCLP from Church and Murray (2018) that allows a single facility to be placed at a candidate location. The model is as follows.

$$\text{maximize} \quad \sum_{j\in J}\sum_{k=1}^{n_j} b_j^{FHQ}\left(1-q_j\right) q_j^{k-1} y_{jk} \tag{8}$$

$$\text{subject to} \quad \sum_{i\in N_j}(x_i + st_i) \geq \sum_{k=1}^{n_j} y_{jk} \qquad \text{for } j \in J \tag{9}$$

$$\sum_{i\in I} x_i = p \qquad (10)$$
$$x_i \in \{0,1\} \qquad for\ i \in I \qquad (11)$$
$$y_{jk} \in \{0,1\} \qquad for\ j \in J,\ k = 1, ..., n_j \qquad (12)$$

The objective function (8) maximizes the expected coverage of critical patients taking into account possible unavailability of ambulances. The term $b_j^{FHQ}(1-q_j)q_j^{k-1}$ represents the increase in expected coverage of municipality j brought about by kth station. According to Eq. (9), sitting multiple stations in the neighbourhood of municipality j enables multiple variables y_{jk} take value 1 and account for the increase in coverage in the objective function. Constraint (10) limits the number of stations to be located. Constraints (11) and (12) impose binary integer restrictions on the decision variables.

3 RESULTS AND DISCUSSION

The computational experiments were performed on a personal computer equipped with the Intel Core i7 processor with 1.60 GHz and 8 GB RAM. The solver Xpress Optimizer 8.3 (64-bit, release 2017) was used to solve the location models. The agent-based simulation model was implemented in AnyLogic simulation software.

The proposed and current sets of locations were evaluated using a computer simulation model. Computer simulation enables to estimate the impact of the proposed changes. We focus on the following performance indicators:

1. average response time for all patients, since it has been monitored by the Operation Centre of the EMS of the Slovak Republic;
2. average response time for most urgent patients;
3. percentage of all calls responded to within 15 min, because a 15-minute response time is a target for 95% calls in Slovakia;
4. percentage of highest priority calls responded to within 8 min.

The results of the simulation study are in Table 2. The simulation experiment for one set of station locations consisted of 10 replications. One replication took 91 days. The mean values from 10 replications are reported. The best values of the indicators are displayed in bold.

Table 2. Performance indicators for the current and optimized locations

Indicator	Current location (Dec 2016)	*p*MP		MEXCLP	
		ALS ambulances are close to current locations	ALS ambulances are close to old people	ALS ambulances are close to current locations	ALS ambulances are close to old people
Response time for all patients [min]	11:04	10:34	**10:23**	12:47	12:53
Response time for highest priority patients [min]	10:53	10:25	**10:13**	12:40	12:49
% of calls responded to within 15 min	77.00	79.76	**80.71**	69.83	69.39
% of highest priority calls responded to within 8 min	41.67	45.15	**46.03**	34.94	34.28

The computer simulation of the current system revealed that the system is short of the target to reach 95% of patients within 15 min. The real accessibility within this time limit is only 77%. The Slovak system also exhibits poor performance regarding the 8-minute response-time standard for critical patients. Only 41.67% of highest priority calls are responded to within 8 min, which is far less than the EU average (66.9%).

From the rest of the table it is evident that the *p*MP model reduces significantly (by more than half minute) the overall average response time, as well as response time for the most critical patients. In parallel with reducing the response time, the accessibility within a given time threshold is increasing. On the other hand, the model maximizing expected coverage is not able to improve the performance of the system. The reason is a well-known drawback of all covering models – the customers who are not covered within the pre-specified time limit may be arbitrarily distant from the service centre, since their distance does not influence the objective function. This research reveals that this feature cannot be balanced by a good estimate of ambulances' busy fractions.

As regards the two policies of allocation of ALS ambulances, we focus on the *p*MP model. Distribution of ALS ambulances to municipalities with old population is a better strategy then to allocate them to the stations that are close to their current locations. The greatest impact is observed on the average response time for highest priority calls where the difference between the two strategies is 12 seconds.

4 CONCLUSIONS

The research compares two location models for re-engineering of the current EMS infrastructure in a large-scale area. Some stations are not allowed to change their current positions due to capacity limits of the ambulances and other practical requirements on their locations. The demand volume estimation takes into account the spatial distribution of potential patients that reflects demographic changes in an ageing society. The main findings are as follow:

1. The weighted *p*MP model is better than the MEXCLP model.
2. A sufficiently accurate computer simulation model can be used to estimate missing parameters of the mathematical programming model (ambulances' workload in our case).
3. Well equipped ALS ambulances with a physician on board should be located in municipalities with old population.

Application of a hybrid bi-objective model where average response time is optimized based on minimum coverage threshold for most critical patients will be the topic of our future research.

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CLUSTER ANALYSIS OF USING BUSINESS INTELLIGENCE PRINCIPLES IN SELECTED COMPANIES

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Abstract

With the growing amount of information generated by business processes, it is no longer enough for management to rely on decision-making methods used to date. This article aims to analyse the involvement of business intelligence (BI) in companies operating in the Czech Republic. Business intelligence tools can be understood as a comprehensive and effective approach to working with business data that affects the accuracy of strategic decisions. Pressure on the speed of information processing thus ranks the principles of business intelligence among the fastest developing tools for decision support. A questionnaire survey has been conducted to meet the main objective of this paper. Obtained data were processed by cluster analysis. In total, 90 respondents were asked to answer questions about the involvement of BI principles in their organizations. Cluster analysis divides respondents into several clusters based on four parameter values. The results serve as a basis for further, more specific research in this field.

Keywords: *Business Intelligence, Cluster Analysis, Data Mining, Decision-Making Methods*

JEL Classification: JEL C380
AMS Classification: AMS 62H30

1 INTRODUCTION

Companies face technological innovations and quick development times, and management has to react promptly to this changing environment. Working well with information represents one of the most important competitive advantages on the market. This research project aims to map the current stance towards using business intelligence (BI) methods in selected companies (Bach et al., 2018a).

Business intelligence represent a complete and effective approach to working with data and information in the business environment. Outputs influence strategic decision processes. The basis of BI is to transform the source data into knowledge, with which the right decisions are made.

A survey has been conducted to meet the main goal of this paper. Data has been collected through questionnaires on a sample of 90 respondents and analysed by means of cluster analysis. Companies and workers across the industrial spectrum were addressed through the HR department or based on the author's contacts on the social network Linkedin.

In order to fulfil the objectives of this paper, its structure is as follows. After the short introduction, a brief literature review is presented, concentrating on the introduction of the term business intelligence and previous research on the usage of cluster analysis in BI research. The third section of this paper focuses on the methodology used for this study, providing an overview of the research instrument, basic characteristics and specific hierarchical clustering procedure. The fourth section presents the results of the data analysis. At the end, a short conclusion with limitations and plans for future research is given.

2 LITERATURE REVIEW

There are many definitions of the term business intelligence in literature and professional journals. It is generally known that this term was first used in 1989 by Howard J. Dresner, who defined it as a set of concepts and methods designed to improve decision-making processes in a company. He considers data analysis, reporting, and tools that help synthesize valuable and useful information as the most important aspects of the process (Buchanan and O'Connell, 2006).

Many studies and publications are devoted to the success of implementing a business intelligence project into a corporate environment where this process is often compared to the implementation of a new information system (Laberge, 2012).

However, for the purposes of this paper, the term "business intelligence" is understood as a set of processes, applications and technologies that aim to provide useful and above all effective support for decision-making processes in a company. Analytical and planning functions in the organization are supported as well. The foundations of these processes are based on the principles of multidimensional views of business data (Novotný et al., 2005).

Cluster analysis is a multivariate statistical method used to classify objects. It classifies objects into groups (clusters), so that objects that are included in one group are as similar as possible, and objects from different groups have few or no similarities. Cluster analysis is a well-known statistical method for analysing data.

Therefore, it has been previously used in many studies by numerous authors. Research studies dealing with BI are no exception to that, as can be seen in Table 1. It provides a few studies found in available journals in which cluster analysis has been applied for the analysis of data connected with business intelligence processes (Bach et al., 2018b; Flath et al., 2012; Fourati-Jamoussi and Narcisse Niamba, 2016)

Table 1: An overview of the usage of cluster analysis in BI research

Title	Author(s)	Year of publication	Objective of the paper
An evaluation of business intelligence tools: a cluster analysis of users' perceptions	Fatma Fourati-Jamoussi, Claude Narcisse Niamba	2016	The identification of the different users of BI.
Cluster Analysis of Smart Metering Data	Christoph Flath, David Nicolay, Tobias Conte, Clemens van Dinther & Lilia Filipova-Neumann	2012	The application of cluster analysis to real smart meter data to identify detailed customer clusters.
Understanding impact of business intelligence to organizational performance using cluster analysis: does culture matter?	Mirjana Pejić Bach, Jurij Jaklič, Dalia Suša Vugec	2018	To analyse the impact of the level of BI maturity on organizational performance of the company.

3 METHODOLOGY

The research was carried out in the following steps:

Step 1: Data Collection

The questionnaire survey is ranked among the methods of quantitative research which enable researchers to obtain in a short period of time a large amount of information from a larger number of respondents than for example an interview. For this study, three basic types of questions were used. The respondents usually replied to yes or no questions, chose from a finite number of alternatives, or the Likert scale representing the degree of agreement was used.

A total of 90 questionnaires were distributed among employees, of which 75 completed forms could be included in the analysis. Respondents were contacted through the HR department and through a Linkedin work network. The methods of descriptive statistics (frequencies) are applied to describe the basic set.

Tables 2 and 3 show the distribution of respondents according to the size of their organization and the classification of job positions in the organizational structure.

Table 2: Distribution of Respondents by Enterprise Size

Enterprises by Business Size	**Number of Respondents**
Micro and Small Size Enterprises (0-49 Employees)	7
Medium-sized Enterprises (50 to 249 Employees)	15
Large Enterprises (more than 250 Employees)	53

Table 3: Distribution of Respondents by Job Level

Job Level	**Number of Respondents**
Executive Management	8
Middle Management	21
Line Management	6
Specialists	40

Step 2: Cluster Analysis

Under the term cluster analysis, it is possible to find a variety (hundreds) of mathematical methods that can be used to find out which objects in a group are similar. Instead of sorting real objects, these methods sort objects described as data. Objects with similar descriptions are gathered into the same cluster (mathematically). The hierarchical methods are most widely applied in research studies (Romesburg, 2004).

Hierarchical cluster analysis is characterized as a sequence of decompositions, where on one side there is a cluster containing all objects and on the other side single element clusters. This method is better applied in finding structures rather than searching for new patterns. Hierarchical clustering can be represented using a binary tree, a dendrogram. Nodes in the dendrogram show individual clusters.

Further subdivision of hierarchical agglomerative methods is based on the criterion according to which the most similar clusters are selected. The complete linkage (furthest neighbour) method is applied in this study. This method combines objects or clusters that are furthest apart within a sorted dataset into a single burst. This means that for the distance of two clusters it takes the greatest possible distance from the distances of every two objects from two different clusters. From the calculated distances, it selects the shortest and connects the corresponding objects. It forms tight clusters of approximately the same size (Kučera, n.d.).

In most cases, when collecting data through a questionnaire survey, the data is of a qualitative nature, whether nominal or ordinal (including collecting responses using the Likert scale). Frequently, qualitative variables are represented by a text string. It is not usually possible to use these strings for mathematical-statistical evaluation. For this reason, it is necessary to convert individual variables into a group of binary numbers or, for ordinal data, to serial numbers. After this conversion, it is then possible to proceed to the actual evaluation process. Conversion increases the number of variables (Hebák, 2007).

Step 3: Data Processing

The initial set of questions presented in a questionnaire focused on obtaining basic information about each employee. The aim was to classify respondents by size of enterprise, business sector, their department and the usage of information systems in their company, not only in the decision-making process.

Another series of questions was devoted to knowledge of the concept of "business intelligence", its application in practice and the overall interconnection and usage of information systems in an organization. The respondents answered these questions using the Likert scale, which represented the degree of agreement with the statement.

The following four statements and their evaluation were selected for the cluster analysis. These statements were nominated in order to best split the sample into representative clusters.

Stat. 1: *If your department uses 2 or more information systems (e.g. production, planning, accounting, etc.): systems are interconnected regularly, data is consolidated.*

Stat. 2: *In my practice I have already encountered the term business intelligence (BI).*

Stat. 3: *Outputs from BI affect my decision-making process.*

Stat. 4: *I consider the processing and use of data sufficient in our company.*

Respondents expressed their agreement on the five-point Likert scale, where the highest value meant strong agreement and the lowest value total disagreement as shown in Table 4.

Table 4: Likert Scale

	*	**	***	****	*****
Consent Rate	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

Collected responses, presented by ordinal character of the Likert scale, were converted to alternative data as shown in Table 5. After this conversion, 16 variables in total were used for cluster analysis.

Table 5: Converting Ordinal Variables to a Group of Alternative Ones

Consent Rate/Variables	X1	X2	X3	X4
Strongly Disagree	0	0	0	0
Disagree	1	0	0	0
Neutral	1	1	0	0
Agree	1	1	1	0
Strongly Agree	1	1	1	1

Afterwards, the Statgraphics procedure of cluster analysis created three clusters from the 75 observations supplied.

4 RESEARCH RESULTS

The clusters are groups of observations with similar characteristics. To form the clusters, the procedure began with each observation in a separate group. It then combined the two observations which were closest together to form a new group. After recomputing the distance between the groups, the two groups then closest together were combined.

This process was repeated until only three groups remained. Figure 1 shows the final dendrogram.

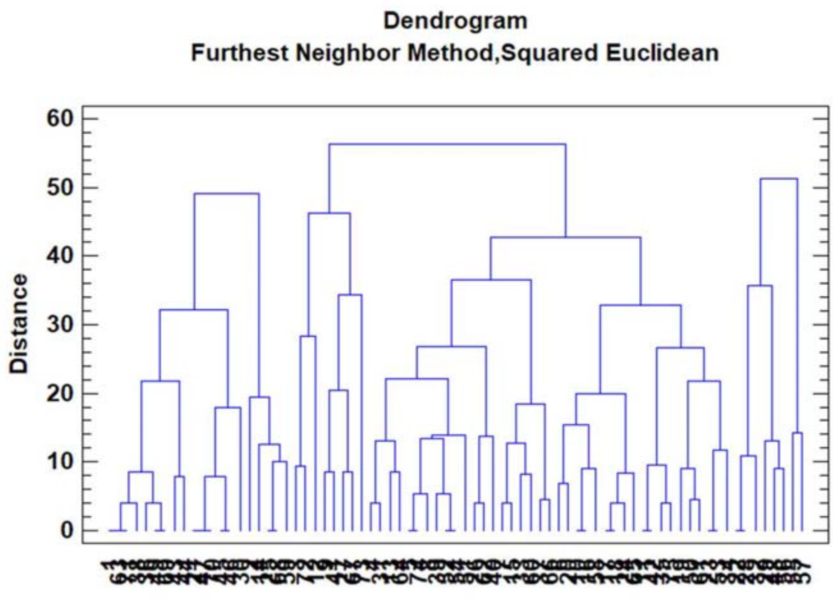

Figure 1: Dendrogram

The dendrogram represents the best way to view the output of a cluster. Working from the bottom up, the dendrogram shows the sequence of joins that were made between clusters. Lines are drawn connecting the clustered that are joined at each step, while the vertical axis displays the distance between the clusters when they were joined (Statgraphics Technologies, n.d.).

One of the ways to determine the optimal number of suitable clusters with a relatively small number of objects is according to the graphical representation of the results – dendrogram (Řezanková et al., 2007). Based on this graph, two options were considered: 3 or 4 clusters.

After reviewing both variants, an alternative with 3 clusters was finally included in the analysis. The results for this variant showed a greater logical connection within the research.

Table 6 sums up the distribution of respondents in three different clusters.

Table 6: Distribution into Clusters

Cluster	Members	Percent
1	20	26.67
2	47	62.67
3	8	10.67

Respondents in Cluster 1 rarely use principles of business intelligence in their decision-making process and they consider the utilization of data as not sufficient in their companies. The cluster is characterized by people who have never met the term business intelligence in practice. By contrast, the consolidation of data obtained from information systems is quite common in their organizations (average value of statement 1 is 2.9).

Cluster 2 contains more than half of respondents. They already encountered the term BI and quite frequently apply the outputs from BI tools to make the right decision. Consolidation of data in their enterprises can be described as average and the processing of data could be better.

In Cluster 3, respondents expressed quite strong agreement with statement 1 and probably therefore there are people who are totally satisfied with the utilization of data in their companies. The cluster is characterized by high awareness of business intelligence methods that commonly help them in decision-making processes.

Particular average values for each statement are shown in Table 7.

Table 7: Average statement's values in each cluster

Cluster	Statement 1	Statement 2	Statement 3	Statement 4
1	2.90	1.25	1.10	2.25
2	3.28	3.96	3.15	2.77
3	3.88	4.00	3.38	5.00

5 CONCLUSIONS

Cluster analysis, as a representative of multivariate statistical techniques, is an approach applied to group observations or variables into clusters based upon similarities between them. In this case study, cluster analysis has been used to divide respondents by their answers acquired through a questionnaire survey. The utilization of cluster analysis for ordinal data can be considered as a limit of the research. It was necessary to convert obtained data. Afterwards, required results were gained.

As the first step of the long-term research study, cluster analysis divides respondents into three clusters. Cluster 3 represents people who know the principles of BI methods and apply them in practice. However, this cluster contains only eight respondents.

The main objective of future research is to focus on Cluster 2, where the awareness of mentioned methods is relatively high but these procedures are not adequately involved in the corporate environment. The involvement of business intelligence principles can signify an important advantage on the market. Enterprises face enormous pressures and are continuously striving for innovative and effective material management methods. Future research can set up a model to show these organizations how to incorporate business intelligence methods to improve decision-making processes.

Respondents gathered in Cluster 1 work in companies where the acquired data is not utilized properly. People don't know how the basic principles of BI work and they are not able to exploit its advantages in making the right decisions. The designed model will focus on this group in further steps.

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THE MODELING PROCESS OF LAUNCHING A NEW SCHEDULED FLIGHT CONNECTION

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Abstract
The launch of a new flight connection is a major investment and should therefore only be launched after a thorough analysis of a number of factors and a wealth of relevant information. For carriers, the project to launch the new flight connection is only interesting when it is completed within two years. The whole process of introducing a new flight connection is a complex process, so it is necessary to plan the whole preparation well - to create a project. Network analysis methods - CPM method and PERT method are suitable tools for effective management of the created project. The paper deals with the application of the PERT method to the project of introducing a new flight connection.

Keywords: *Modeling process, Flight connection, PERT method, Network graph*

JEL Classification: C44
AMS Classification: 90C15

1 MOTIVATION TO SOLVE

In a dynamically evolving competitive environment, an airline operating regular flights must update its network of lines on an ongoing basis. The network of a flight connection is a set of all periodically recurring flights operated by a scheduled air carrier, and its updating means the cancellation of operations on existing routes whose potential has not been satisfying and the launching of new routes with promising commercial potential.

Suppose that the air carrier management has the results of a marketing survey or the initiative of a management member or a particular interest group and is considering launching a new connection. The first important moment of the whole decision-making process is the decision to start preparations for the operation of the new route. Such a decision, depending on the air carrier's internal procedures and circumstances, is made several months in advance and, for long-haul flights, up to one or two years. Before the start of preparations for the operation of the new route, the entire preparation process (preparation strategy) needs to be well planned. This actually means effectively organizing the sub-steps. The preparation strategy differs not only according to whether the company operates point to point or hub and spoke, but also if it is a low-cost air carrier or a traditional network carrier. In the event that the management of the company at the same time decides on the introduction of a new route operation to cancel an existing route, it has a fleet and staff to deploy on a new route. In other cases, when an airline is considering or is already working on the introduction of a new flight connection line without canceling another flight connection line, management must also consider whether an increase in the number of aircraft to the fleet and / or personnel will be necessary.

2 BRIEF CHARACTERISTICS OF THE MODELING PROCESS

From an operational-technical point of view, the process of launching a new scheduled flight connection can be implemented within a few weeks, a maximum of months. But the time it takes to prepare for the launch of a new route is determined primarily by the time needed by the air carrier's business units to identify potentially appropriate destinations, gathering all the necessary market information, preparing and approving the business plan, marketing, and marketing to promote and promote a new product [3]. Underestimation of the preparation may result in the project not being implemented due to, for example, its impracticability when there is the same flight connection on the market or when the market is not yet ready for a new flight connection. A number of proposals, after analyzing all the information, maybe rejected due to low potential, or the proposal may be postponed for a later period, e.g. due to operational or aeropolitic impracticability

The first stage of the preparation of the new flight connection includes the process of preparation of the business plan for the final decision of the company management to prepare the launch of the new flight connection. This stage includes a continuous activity that lasts up to several months. During this phase, information is collected (general information on the country to be flown, information on the local market, opportunities to enter the market based on the 6th Freedom of Transport), evaluation of general information on the country and territory based on PESTE analysis, on the local market, identification of a new type of clientele and proposal of flight schedule variants and calculation of expected economic result (for each flight schedule variant) divided into revenues of passengers and revenues of cargo transport and costs connected with the introduction of the new flight connection. The result of the calculation is a forecast of the given flight schedule variant revenue. This forecast and evaluation of all information serve as a basis for the preparation of management recommendations.

The second stage includes the actual implementation of the approved business plan and can be divided into two parts. The first part of the second phase begins with the approval of the business plan to start operations of the new route and ends with the issuing of a definitive decision to start or not start operation. The start of operations is subject to zero or minimal risk that the business plan will not be fulfilled. The second part of the stage begins with the final decision to commence operations and ends with the start of the first flight. During this phase, the carrier coordinates activities with the air traffic control organizations in the creation of flight schedules, contractual business cooperation, ensuring the promotion and sale of tickets, setting sales conditions (selling prices, published tariffs), technical flight support and marketing support. With sufficiently known and unchanged values of the lengths of individual activities, the second stage of the introduction of a new airline can also be solved by the CPM method. [2]

The whole process of both stages can be seen as a project because the individual activities are related to each other materially and in time. For the whole process of preparing a new airline, appropriate network analysis methods can be used, as the task is to timing the implementation of the individual activities leading to the launch of the new airline so that the start of operations takes place without undue delay. As the duration of the individual activities in both stages is difficult to estimate, the PERT method is a suitable method for modeling the course of the project. The task of the PERT method is also to identify the project completion date with a predetermined probability.

3 ANALYSIS OF THE CURRENT STATE OF KNOWLEDGE - A BRIEF OVERVIEW OF THE THEORETICAL FOUNDATIONS OF THE PERT METHOD

When performing the calculation using the PERT method, the duration of activity is considered a random variable with a β probability distribution, see Fig. 1.

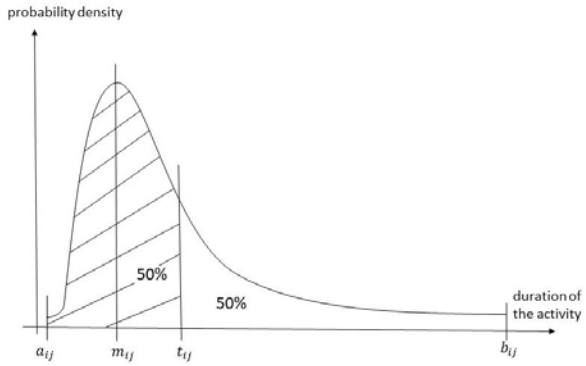

Fig. 1 - density of function β distribution

This probability distribution has a finite range and isn´t generally symmetrical.

The duration of each activity is defined using three time-related values:

a) optimistic calculation a_{ij} - the shortest possible duration of activity specified by the edge h_{ij}, duration corresponds to performed activity $[i, j]$ under ideal conditions,
b) pesimistic calculation b_{ij} - the longest possible duration of activity specified by the edge h_{ij}, duration corresponds to performed activity $[i, j]$ in situation, where the biggest problems exists,
c) most likely calculation m_{ij} – high probable duration specified by the edge h_{ij}, corresponds to performed activity $[i, j]$ under standard conditions.

Based on these three estimates, we calculate: [1]

- $E(T_{ij})$ - the expected value of a duration of activity specified by the edge h_{ij}

$$E(T_{ij}) = \frac{a_{ij}+4m_{ij}+b_{ij}}{6} \quad (1)$$

- σ_{ij} - the standard deviation of a duration of activity

$$\sigma_{ij} = \frac{b_{ij}-a_{ij}}{6} \quad (2)$$

Optimistic calculation a_{ij} and pesimistic calculation b_{ij} are expert estimates.
Most likely calculation m_{ij}, these values are given in the literature [3].

The following symbols and relationships will be introduced to calculate the critical path using the PERT method:

$t_i^{(0)}$	early start time of activity $[i, j]$
$t_i^{(0)} + E(T_{ij})$	first possible completion deadline of activity $[i, j]$,
$t_j^{(1)}$	the latest deadline, of the activity $[i, j]$,
$t_j^{(1)} - E(T_{ij})$	the latest permitted start of the activity $[i, j]$.

where: $i, j = 1,2,3, \ldots, n$, where n - is the number of network nodes, $i = 1$ corresponds to - the initial network node (project start), and $i = n$ corresponds to the end node of the network (project completion).

The solution is often performed graphically using a so-called network graph, in which the nodes represent the beginning and end of the individual project activities, the edges of the individual activity. This approach will be used in our article. The meaning of individual symbols according to their location is generally shown in Fig. 2.

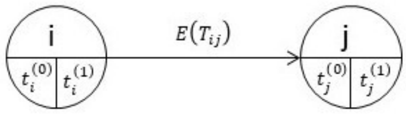

Fig. 2: General layout of symbols in the network graph

To specify the project $E(T)$ duration, we are using the expected duration values of individual activities $E(T_{ij})$. That is why, we are also talking about finding the expected value of project duration. This finding follows the rule for calculating the mean values of random variables

$$E(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} E(X_i) \qquad (3)$$

If we denote by a symbol $m(1,n)$ a critical path then for its length $E(T)$ holds:

$$E(T) = \sum_{[i,j]\in m(1,n)} E(T_{ij}) \qquad (4)$$

The PERT method also allows you to perform a probability analysis of a project. E.g. it is possible to find answers to the question, what is the probability of meeting the chosen project termination date.

For these purposes, the standard deviation from the expected value project duration $\sigma(T)$ must be known, calculated on the basis of the known variance counting rule

$$D(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} D(X_i) \qquad (5)$$

When we denote the symbol $m(1,n)$ the critical path with its length $E(T)$, then for the standard deviation from this mean value σ_T:

$$\sigma(T) = \sum_{[i,j]\in m(1,n)} \sigma_{ij} = \sum_{[i,j]\in m(1,n)} \frac{b_{ij}-a_{ij}}{6} \qquad (6)$$

To determine the above question answer, the tabulated values of the normal distribution probability distribution function $\Phi(x)$ can be used. The normalized normal probability distribution is used for situations in which a critical path consists of a sufficient number of activities. If we denote by T random variable modeling the actual project duration, $E(T)$ the expected value project duration, $\sigma(T)$ the standard deviation from the projected average, then the probability that the project time does not exceed the selected T_p value is calculated from: [1]

$$P(T \leq T_p) = \Phi\left(\frac{T_p - E(T)}{\sigma(T)}\right) \qquad (7)$$

when $\frac{T_p - E(T)}{\sigma(T)} < 0$, then we use the relationship

$$P(T \leq T_p) = 1 - \Phi\left(-\frac{T_p - E(T)}{\sigma(T)}\right) \qquad (8)$$

4 APPLICATION OF THE PERT METHOD TO THE PROJECT OF LAUNCHING A NEW FLIGHT CONNECTION

According to the theoretical description given in the chapter devoted to the brief description of the new line deployment process, the initial step providing input to the PERT application is to create a default activity table, see Tab. 1. The table shows the names of the individual activities and, if necessary, their meaning. Some of the activities listed in the table are divided into two parts (eg 4a, 4b). The division of activity into several parts is used in a situation in which some of the operations in that activity are conditional on the execution of activities other than those which make the start of the first part of the activity conditional. List of activities listed in Tab. 1 is to be understood as a maximum. In case of launching the operation of some new flight connection, it will not be necessary to carry out all activities listed in Tab. 1.

Activity number	**Activity name -** The semantic content of the activity
1	**Verifying the existence of restricted airspace** - determining whether there is a restriction on a given route, airport or airspace
2	**Licensing process for the tied market** - negotiation and preparation of contracts when airspace is bound by the agreement
3	**Identification of clientele** - determination of potential clientele
4a	**Requesting to airports** - processing and sending the request to a handling company, solving arrival and departure times, or refueling the aircraft, catering services
4b	**Response time of participating airports** - confirmation or modification of requested services by the airport
5	**Selection of aircraft in terms of proposed route, times and capacity**
6	**Adjustment of requirements according to airport dispositions -** incorporation of any comments and preparation of contracts with airports
7	**Financial analysis of the upcoming flight** - preliminary cost-benefit analysis
8	**Preparation of documents for the management's final decision on the operation of the line** - completion and sorting of all collected documents
9	**Examine the codeshare options on the link** - exploring the possibility of a commercial arrangement between several air carriers to sell tickets and promote the flight connection
10	**Conclusion of a contract on operating conditions** - Conclusion of a contract between a carrier and, for example, a region where the conditions of operation are specified in the contract
11	**Creating a flight connection network** - The activity includes a specific draft flight schedule for operations
12a	**Preparation of commercial contracts (Part 1)** - formal initiation of the process of concluding agreements between states on the opening of operations in airspace
12b	**Preparation of commercial contracts (part 2)** - time needed for bilateral negotiations between and with the countries leading to the signing of commercial contracts
13a	**Conclusion of commercial contracts (part 1)** - contractual arrangement of operations in relation to partners - necessary steps for approval of the commencement of operations by government authorities
13b	**Conclusion of business contracts (Part 2)** - contractual provision of operations in relation to a specific draft flight schedule
14	**The phase of the sale of tickets in the new market, entering the sales system**
15	**Selection and implementation of the General Sales Agent**
16	**Pricing, trade** - calculations of airline tickets for tour operators on the domestic market and abroad
17a	**Operational components, quality, and safety (part 1)** - technical feasibility of flight and checking
17b	**Operational components, quality, and safety (2nd part)** - in case of

	deficiencies of technical feasibility, the proposal of technical feasibility solution
18	**Preparation of marketing plan for line introduction**
19	**Marketing** - preparation and implementation of presentations for contractors
20	**Flight schedule** - specification of preliminary flight schedule
21	**Business contracts** - other contractual reinsurance in relation to partners
22	**Complete access to the BSP sales system**
23	**Business** - Preparation of the approaching of a possible commercial agency, ensuring the presence of the air carrier's sales representative in the territory and commencement of acquisition activities on the market
24	**Ensuring the functionality of commercial representation**
25	**Pricing, trade, distribution of prices to tour operators**
26a	**Pricing, revenue management (part 1)** - preparation of prices for individual travelers including published tariffs, revenue management settings, distribution of prices for individual travelers
26b	**Pricing, revenue management (part 2)** - pricing arrangements with other carriers
27a	**Operational components, quality, and safety (part 3)** - general phase of preparation of operational-technical provision of clearance and flight
27b	**Operational components, quality, and safety (Part 4)** - incorporation of contractual specifics, requirements, and possibilities of airports, technical handling, etc. into the preparation of operational-technical ensuring of handling and flight
28	**Marketing** - addressing agencies and business partners (eg. presentations)
29a	**Marketing** - preparation of the implementation of a marketing plan for the introduction of a line that includes marketing activities in relation to partners
29b	**Marketing** - concrete design of marketing plan including action prices
30a	**Tour operator (part 1)** - preparation of a package that includes eg. contacts with hotels
30b	**Tour operator (2nd part)** - specific preparation package according to the price
31	**Print tour operator catalog**
32	**Distribution of tour operator catalog**
33	**Preparation of the sale of the product according to the catalog**

Tab. 1. Significant content of individual activities

The network graph representing the project will be presented in more parts in Figures 3 - 6. The connection of the individual parts of the network graph is done through the numbers of the end/start nodes in the given picture. Figures 3 - 6 show the earliest possible and at the latest permissible start and end times for each activity, calculated on the basis of the average duration of each activity $E(T_{ij})$. Critical activities forming a critical path are also highlighted in the figures. There are two important milestones during the project for the launch of the new line, where it is decided whether the project will be terminated or the project will be continued. The first milestone occurs at node 12 in Fig. 4, where the business plan for launching a new flight connection is presented to the air carrier management. When the business plan is approved by the air carrier management, Stage 1 ends and Stage 2 can be started. Another milestone occurs after termination of activities 12b, 13b, 15, 17b and 19, when the air carrier management assesses the risks of continuing and issues a definitive decision to start or not to start a new airline. If it is definitely decided to start the operation of a new flight connection, the second part of Stage 2 will follow, see Fig. 6.

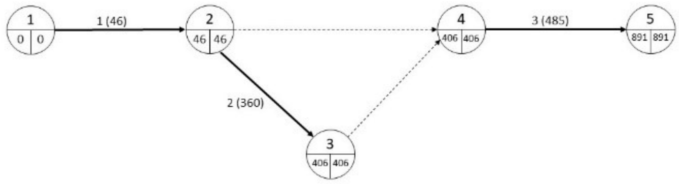

Fig. 3. Network graph Stage 1, part 1

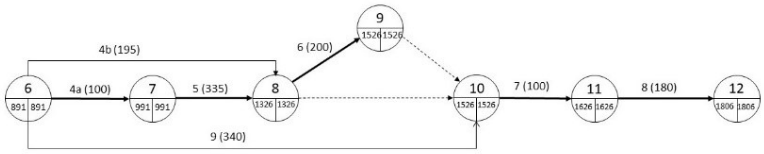

Fig. 4. Network graph Stage 1, part 2

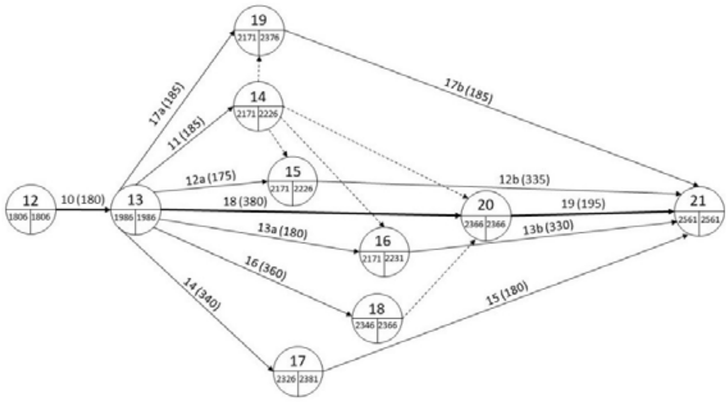

Fig. 5. Network graph Stage 2, part 1

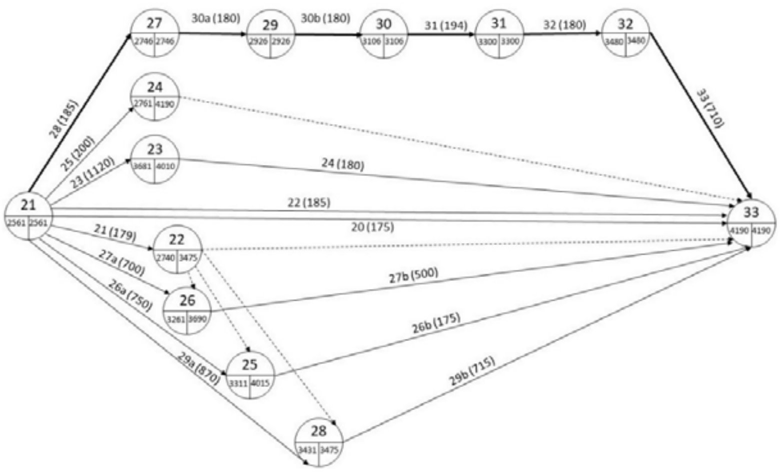

Fig. 6. Network graph Stage 2, part 2

The basis for the creation of network graphs was the data presented in Tab. 2.

Activity	Edge $[i,j]$	Duration of activity (days)			Previous activity	Following activity	$E(T_{ij})$ (days)	$\sigma(T_{ij})$ (days)
		a_{ij}	b_{ij}	m_{ij}				
1	$[1;2]$	4	14	7	x	2	46/6	10/6
2	$[2;3]$	30	90	60	1	3	360/6	60/6
3	$[4;5]$	45	120	80	2	4a, 4b, 9	485/6	80/6
4a	$[6;7]$	10	30	15	3	5	100/6	20/6
4b	$[6;8]$	15	60	30	3	6	195/6	45/6
5	$[7;8]$	5	90	60	4a	6	335/6	85/6
6	$[8;9]$	20	60	30	5	7	200/6	40/6
7	$[10;11]$	10	30	15	6, 9	8	100/6	20/6
8	$[11;12]$	15	45	30	7	x	180/6	30/6
9	$[6;10]$	10	90	60	x	7	340/6	80/6
10	$[12;13]$	15	45	30	8	11, 12a, 13a, 14, 16, 17a, 18	180/6	30/6
11	$[13;14]$	20	45	30	10	12b, 13b, 19	185/6	25/6
12a	$[13;15]$	10	45	30	10	12b	175/6	35/6
12b	$[15;21]$	25	70	60	12a	x	335/6	45/6
13a	$[13;16]$	15	45	30	10	13b	180/6	30/6
13b	$[16;21]$	20	70	60	13a	x	330/6	50/6
14	$[13;17]$	25	75	60	10	15	340/6	50/6
15	$[17;21]$	20	40	30	14	x	180/6	20/6
16	$[13;18]$	40	80	60	10	19	360/6	40/6
17a	$[13;19]$	15	50	30	10	17b	185/6	35/6
17b	$[19;21]$	15	50	30	17a	11, 12a, 13a, 14, 16, 17a, 18	185/6	35/6
18	$[13;20]$	45	95	60	10	12b, 13b, 19	380/6	50/6
19	$[20;21]$	15	60	30	18	12b	195/6	45/6
20	$[21;33]$	10	45	30	12b, 13b, 15, 17b, 19	x	175/6	35/6
21	$[21;22]$	14	45	30	12b, 13b, 15, 17b, 19	17b, 18b, 20b	179/6	31/6
22	$[21;33]$	20	45	30	12b, 13b, 15, 17b, 19	x	185/6	25/6
23	$[21;23]$	160	240	180	12b, 13b, 15, 17b, 19	24	1120/6	80/6
24	$[23;33]$	15	45	30	23	x	180/6	30/6
25	$[21;24]$	20	60	30	12b, 13b, 15, 17b, 19	x	200/6	40/6
26a	$[21;25]$	90	180	120	12b, 13b, 15, 17b, 19	26b	750/6	90/6
26b	$[25;33]$	15	40	30	26a	x	175/6	25/6
27a	$[21;26]$	60	160	120	12b, 13b, 15, 17b, 19	27b	700/6	100/6
27b	$[26;33]$	30	110	90	27a	x	500/6	80/6

28	$[21;27]$	25	40	30	12b, 13b, 15, 17b, 19	30a	185/6	15/6
29a	$[21;28]$	90	180	150	12b, 13b, 15, 17b, 19	29b	870/6	90/6
29b	$[28;33]$	100	135	120	29a	x	715/6	35/6
30a	$[27;29]$	25	35	30	28	30b	180/6	10/6
30b	$[29;30]$	25	35	30	30a	31	180/6	10/6
31	$[30;31]$	14	60	30	30b	32	194/6	46/6
32	$[31;32]$	15	45	30	31	33	180/6	30/6
33	$[32;33]$	90	140	120	32	x	710/6	50/6
∑	---	---	---	---	---	---	698,33	105,1667

Tab. 2 Initial list of project activities

Based on the network graphs, a critical path has been found, consisting of 17 activities (1, 2, 3, 4a, 5, 6, 7, 8, 10, 18, 19, 22, 28, 30a, 30b, 31, 33). Tab. 2 shows the calculated mean times of individual activities $E(T_{ij})$ and calculated standard deviations σ_{ij}. Based on the data given in the total line in Tab. 2 and relations (4) and (6), there is the expected value of the project $E(T) = 698{,}33$ days and the standard deviation is $\sigma(T) = 105{,}1667$ days. Respecting the presumption of the random variables sum normality representing the sum of the critical path activities duration, we can calculate the probability that the project will be able to complete within two years from its start (as a normal requirement of air traffic practice). We calculate this probability using one of the formulas (8) or (9). Let's put $T_p = 2$ years $= 730$ days.

$$P(T \leq T_p) = \Phi\left[\frac{T_p - T}{\sigma(T)}\right] = \Phi\left[\frac{730 - 698{,}33}{105{,}1667}\right] = \Phi[0{,}2061] \doteq \Phi[0{,}21] = 0{,}58317$$

T_p [years]	2	2,1	2,2	2,3	2,5
$P(T \leq T_p)$	0,58317	0,68793	0,83147	0,89435	0,97882

If the carrier requests, that the project should be successful with a probability of 0,8, then it must allow for a period of 2,2 years.

5 CONCLUSIONS

The establishment of a new flight connection for carriers is a very complex process, during which a number of factors must be taken into account and much different information must be provided. Because the individual activities are related to each other factually and time, it is possible to use network analysis methods. When compiling network graphs containing all the activities necessary to launch a new flight connection and identify a critical path, it was found that the average maximum time needed to start a new flight connection is 698,33 days and the project will be completed within two years (a normal operational requirement air traffic) with a probability of 0,58317. If the project implementation period is extended, the probability of its implementation will increase and vice versa.

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COMOVEMENT AND SPILLOVER EFFECT DURING CRISIS: EVIDENCE FROM PHYSICALLY DISTANCE COUNTRIES

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Abstract
In this paper we analyze comovement and spillover effect between German and Hong Kong stock indices using diagonal BEKK model during specific periods of bubble blast. The bubbles are monitored using methodology based on augmented Dickey-Fuller test. We focus on two types of bubbles, the first type in which epicenter country is not included into analyzes and the second when the epicenter is involved. Lastly, we briefly discuss possible channels of transmission.

Keywords: *financial bubbles, comovement, spillover effect*

JEL Classification: G01, G15
AMS Classification: 62P20, 91B70

1 INTRODUCTION

Shortly after World War II there was a low evidence of interconnections among stock markets which suggests a potentially significant benefit of international diversification. However, in the beginning of the 90s the correlation started to grow continuously. According to Anagnostopoulos et al. (2019) the correlation doubled based on the equity prices of 23 developed economics (from 0.40 in 1980s to 0.80 in 2010s).

The research in the field is primarily focused on measuring the degree of interactions among stock markets, also known as comovement. The study itself is relevant for assessing the risk portfolio and hedging from erroneous assumption of international diversification. Except for investors, politicians can find this information useful too. It is reasonable to assume that in case of high interaction, the shock from one market spreads to another as a disease which affects a local economy in many areas.

The first papers were presented by King, Sentana and Wadhwani (1994), Longin and Solnik (1995) and Forbes and Rigobon (2002). All of these studies proved that comovement among stock markets is not constant over time. Later on, as technical possibilities have evolved, the applied mathematical apparat has spread. Academicians moved from simple correlation coefficient to multivariate generalized autoregressive conditionally heteroskedastic models (MGARCH), wavelet analysis or copula models (Jaworski and Pittera, 2014).

In this paper we analyze comovement between German and Hong Kong stock indices. We assume it makes a difference to check possible comovement with involved and not involved epicenter country of economic/financial crisis. Hence, we first check all possible crisis within Hong Kong stock index using bubble monitoring presented by Phillips, Wu and Yu (2011) and then we calculate time evolving correlations using MGARCH model.

The paper has the following structure: in the next paragraph we present statistical bubble monitoring and diagonal MGARCH model, in the third one we make an empirical analysis based on presented methodological concept and the last paragraph concludes.

2 METHODOLOGY

In this section we focus on mathematical apparat which is fundamental for our analysis, including:

- statistical testing of explosive behavior,
- modelling stock markets interactions and spillover effects,
- analysis of key spillover factors.

2.1 Financial bubbles

This part is based on the work of Phillips, Wu and Yu (2011, abb. PWY) and Phillips, Shi and Yu (2015, abb. PSY). Asset pricing theory suggests that a bubble component manifests in dynamic of asset price and also in its stochastic process, therefore price should inherit explosive property. Test statistics presented by PWY and PSY are based on a variation of following equation:

$$y_t = \mu + \delta y_{t-1} + \sum_{i=1}^{p} \phi_i \Delta y_{t-i} + \varepsilon_t, \qquad \varepsilon_t \sim NID\left(0, \sigma_y^2\right), \tag{1}$$

where y_t is a stock price, μ intercept, δ autoregressive coefficient, p maximum number of lags, ϕ_i for $i = 1, \dots, p$ differenced lags coefficients, $\Delta y_{t-i} = y_{t-i} - y_{t-i-1}$ and ε_t is the error term. Testing for a bubble (explosive behavior) uses a right-tail variation of the standard augmented Dickey-Fuller (ADF) unit root test[1] where the null hypothesis is a unit root and the alternative is explosive autoregressive coefficient, i. e.:

$$H_0: \delta = 1,$$
$$H_1: \delta > 1.$$

Test statistics suggested by PWY uses recursive calculations of the ADF statistics with a fixed starting point and expanding window. The first observation in the sample is set as the starting point, i. e. $f_1 = 0$. Note, that f parameters (f_0, f_1, f_2) are in fraction terms. The end point (f_2) of the initial window is a minimal window size (f_0) which is set by the user. Finally, the regression (1) is recursively estimated with incrementing window size $f_2 \in \langle f_0, 1\rangle$ by one observation at a time. Test statistics is defined as the supremum value of the ADF sequence, which means:

$$SADF = \sup_{f_2 \in \langle f_0, 1\rangle} \left\{ADF_{f_1}^{f_2}\right\}. \tag{2}$$

Using SADF test statistics we can define single periodically collapsing bubble, hence PSY suggested generalized SADF (GSADF) in which multiple periodically collapsing is plausible. This test generalizes the SADF by allowing more flexible estimation windows wherein even the starting point (f_1) is allowed to vary within the range $f_1 \in \langle 0, f_2 - f_0\rangle$, formally:

$$GSADF = \sup_{\substack{f_2 \in \langle f_0, 1\rangle \\ f_1 \in \langle 0, f_2 - f_0\rangle}} \left\{ADF_{f_1}^{f_2}\right\}. \tag{3}$$

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[1] Augmented Dickey-Fuller (ADF) test uses the equation (1), however, μ is allowed to be zero, or a constant, or a deterministic function of time, such as $\mu_t = \omega_0 + \omega_1 t$. ADF test is a t-ration of $\hat{\delta} - 1$, i.e.:

$$\mathrm{ADF} = \frac{\hat{\delta} - 1}{\mathrm{std}(\hat{\delta})}.$$

PWY propose to compare each element of estimated ADF sequence to the corresponding right-tailed critical values of the standard ADF statistics to identify starting and end points of the bubble. Simply put, when the value of ADF sequence crosses the corresponding critical value from below, we have the starting point of bubble and when it crosses from above, we have the end point.

2.2 Modelling comovement and spillover effect

We use multivariate GARCH model, to be more concrete BEKK model presented by Engle and Kroner (1995). We define BEKK model for n assets:

$$\begin{aligned} \boldsymbol{r}_t &= \boldsymbol{\mu}_t + \boldsymbol{\varepsilon}_t, \\ \boldsymbol{\varepsilon}_t &= \boldsymbol{H}_t^{1/2}\boldsymbol{\eta}_t, \\ \boldsymbol{H}_t &= \boldsymbol{\Omega} + \sum_{i=1}^{p}\sum_{k=1}^{K}\boldsymbol{A}_{ik}\boldsymbol{\varepsilon}_{t-i}\boldsymbol{\varepsilon}_{t-i}^T\boldsymbol{A}_{ik}^T + \sum_{j=1}^{q}\sum_{k=1}^{K}\boldsymbol{B}_{jk}\boldsymbol{H}_{t-j}\boldsymbol{B}_{jk}^T, \end{aligned} \tag{4}$$

where $\boldsymbol{r}_t$ is $[n \times 1]$ vector of continuously compounded returns[2], $\boldsymbol{\mu}_t$ $[n \times 1]$ vector of conditional expected value of returns, $\boldsymbol{\varepsilon}_t$ $[n \times 1]$ vector of random errors, $\boldsymbol{H}_t$ $[n \times n]$ conditional variance-covariance matrix of $\boldsymbol{\varepsilon}_t$, $\boldsymbol{\eta}_t$ $[n \times 1]$ vector of IID random variable for which $\mathrm{E}(\boldsymbol{\eta}_t) = \boldsymbol{0}, \mathrm{E}(\boldsymbol{\eta}_t\boldsymbol{\eta}_t^T) = \boldsymbol{I}_{n\times n}$, $\boldsymbol{\Omega}$ $[n \times n]$ symmetric, positive definite matrix of parameters, $\boldsymbol{A}_{ik}$ and $\boldsymbol{B}_{jk}$ $[n \times n]$ are matrices of parameters, p is the ARCH order, q GARCH order and K is a whole number. This specification ensures that $\boldsymbol{H}_t$ is almost surely positive definite.

The number of estimated parameters grows exponentially therefore it is useful to assume $K = p = q = 1$. Also, another simplification without loss of generality is to use diagonal matrices $\boldsymbol{A}_{ik}$ and $\boldsymbol{B}_{jk}$. The commonly used method of estimation is quasi maximum likelihood, for more information see Bollerslev and Wooldridge (1992).

3 EMPIRICAL RESULTS

In empirical part we use daily data of Hong Kong stock index (HSI) and German stock index (DAX) retrieved from https://stooq.com/ and we show the spillover effect even for economically and physically distance countries. The reasons behind the choice of countries are fairly simple. The physical distance between Hong Kong and Germany is 8 989 km and these countries are not even in top 5 of each other list of the most important trade partners. Based on the data from World Bank (https://wits.worldbank.org/, data for 2018), Germany in terms of export has Hong Kong on 27th position, in terms of import on 38th position and it is similar in reverse. But if we start to think about Germany as part of European union and Hong Kong as part of China, the position of trade change dramatically. But the trade of physical goods itself is not a strong argument for analyzing the effects of non-weight trade.

According to Anagnostopoulos et al. (2019), foreign direct investment is the most important factor of stock market comovement. Based on OECD data (https://data.oecd.org/, data for 2017) Britain was the biggest investor in Hong Kong which comes as no surprise considering their past. In September 19, 1984 Britain and China signed an agreement on transferring Hong Kong from British to Chinese rule by 1997 and since July 1, 1997 Hong Kong has become a special administrative region under the principle of one country, two systems. It seems

[2] Continuously compounded returns are calculated as:

$$r_t = \ln(y_t/y_{t-1}),$$

where y is the price of some asset.

reasonable to assume a connection between Hong Kong and Britain. Kováč (2018) used multivariate GARCH model to estimate time evolution of correlation between German stock index (DAX) and Britain stock index (FTSE 100). The 99 % confidence interval for time evolving correlation was from 0.7 to 0.9 which stands for a high degree of interaction between these two countries. With evidence of high correlation of partner countries and their unions, the comovement and spillover effects are plausible even for the couple such as Germany and Hong Kong.

In this section we first determine financial and economical bubbles in HSI, then we continue on estimation of time evolving correlation between HSI and DAX for specific range around explosive behavior. We conclude this section by loose discussion about possible channels of transmission. The estimation is carried out in RStudio.

3.1 Explosive behavior in Hong Kong stock index

We use the SADF test to examine explosive behavior and bubble collapsing in HSI. Table 1 presents SADF test for the whole sample (24. 11. 1969 – 11. 2. 2020) and we also report the finite sample critical values obtained from 200 replication of 12 144 observations using Monte Carlo simulation. While performing the ADF regressions and calculating critical values, the smallest window contains 320 observations. The SADF value exceed their respective 1 % right-tail critical values, giving strong evidence of explosive subperiods.

Table 1: The SADF test for HIS

		Finite Sample Critical Values		
	Test Stat.	90 %	95 %	99 %
SADF	9.82	1.46	1.80	2.15

Source: author's calculations

We conduct a (pseudo) real-time bubble monitoring using PWY strategy. Within PWY dating strategy we compare the calculated ADF sequence with 95 % SADF critical values. From Figure 1 we can identify:

- in the early 1970s – global oil crisis,
- in the early 1980s – Black Saturday[3],
- around year 1987 – Black Monday[4],
- around year 1992 – Hong Kong's stock market collapse,
- around year 1997 – Asian financial crisis,
- around year 2000 – dot-com bubble,
- around year 2008 – financial crisis,
- around year 2018 – slightly explosive behavior.

[3] Black Saturday refers to September 24, 1983 and it is the name of the crisis when the Hong Kong dollar was at all-time low. From 1974 to 1983 Hong Kong was under a floating rate regime. The political talks about transfer of sovereignty handover to China contributed to a pessimistic attitude of investors. The collection of events resulted into a currency, banking and fiscal crisis.

[4] Black Monday refers to October 19, 1987 in which Dow Jones Industrial (DJI) fell exactly 508 points (22.6 %) and HSI fell 1 120 points.

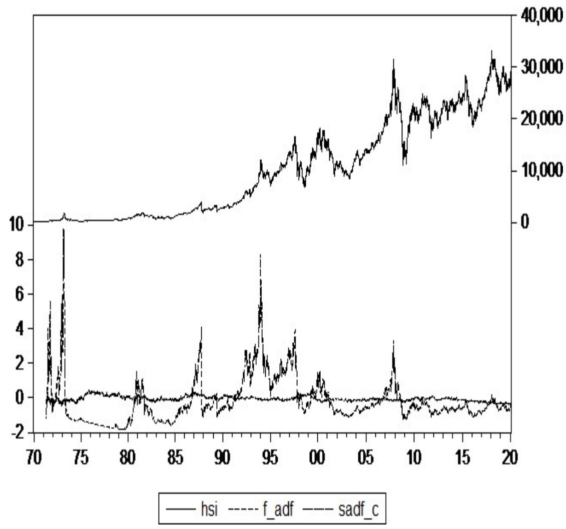

Figure 1: HSI and testing explosive behavior

Note: on the left axis there is ADF sequence and 95 % critical value sequence and on the right axis there is HSI

Source: author's calculations

Using SADF test for testing possible bubble collapse around year 2018 (03. 01. 2017 – 31. 12. 2019), we come to the conclusion, there is no evidence of collapsing (consult with the Table 2), despite Figure 1 shows there is an evidence of explosive behavior. Hence, in further analysis we focus primarily on time period around year 2008 (02. 01. 2007 – 31. 12. 2009).

Table 2: The SADF test for specified range of observations

		Finite Sample Critical Values		
	Test Stat.	90 %	95 %	99 %
SADF (2008)	2.48	1.23	1.36	1.99
SADF (2018)	0.82	1.20	1.47	1.90

Source: author's calculations

The further research in this area may suggest the most appropriate way of predicting prices or returns (e. g. using GARCH models) and use PSY or PWY strategy to determine possible future bubbles and their collapses. This might be beneficial for monetary policy makers who can adjust to the future situation and propose some solutions to moderate the consequences of bubble explosion. However, in our paper we focus on analyzing the structure, not predictions. Especially, we focus on evolution of correlations using diagonal BEKK model to capture any kind of systematic behavior around time of creating the bubble and explosion. This could be evidence for so called spillover effect even for countries which are far away from each other physically and economically.

3.2 Spillover effect

We use diagonal BEKK model to analyze comovement and spillover effect. However, we do not use time series of prices but continuously compounded returns. In Table 3 we present some descriptive statistics for prices and their returns (signed with beginning letter R) and in Table 4 we show the results of testing chosen hypothesis for a mean equation - first equation in (4). It can be seen that returns are stationary and ARCH effect is present, however, returns do not follow normal distribution hence quasi-maximum likelihood method will be applied to estimate diagonal BEKK model parameters.

Table 3: Descriptive statistics

	Mean	Median	Max	Min	Std.Dev.
DAX	3858.290	2161.100	13627.84	372.3000	3653.799
HSI	10097.16	9126.070	33154.12	147.3300	9277.516
R_DAX	0.000251	0.000561	0.107975	-0.13706	0.012786
R_HSI	0.000425	0.000610	0.180581	-0.40542	0.018349

Source: author's calculations

Table 4: Normality, stationarity and ARCH effect of mean equation

	Jarque-Bera	ADF	ARCH(1)
R_DAX	26744.82	-79.44638	567.0076
	(0.00000)	(0.000100)	(0.00000)
R_HSI	474807.1	-59.84095	398.2629
	(0.00000)	(0.000100)	(0.00000)

Note: p-value is in parenthesis
Source: author's calculations

Mean equations are not constant, we use a first order autoregressive process to be cohesive with PWY strategy. Estimated diagonal BEKK model has the form:

$$\boldsymbol{H}_t = \begin{pmatrix} 6.33\times10^{-6} & 3.23\times10^{-6} \\ 3.23\times10^{-6} & 1.34\times10^{-5} \end{pmatrix} + \begin{pmatrix} 0.07 & 0 \\ 0 & 0.32 \end{pmatrix} \boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^T + \begin{pmatrix} 0.89 & 0 \\ 0 & 0.69 \end{pmatrix} \boldsymbol{H}_{t-1}.$$

It is worth to note that the first row of estimated parameters of diagonal BEKK is related to R_DAX and second to R_HSI. The matrix which stands in front of $\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^T$ term is not a matrix $\boldsymbol{A}$, if we take the square root of diagonal values, we get matrix $\boldsymbol{A}$, nevertheless, one can immediately tell that short-term disturbances influence HSI more than DAX. On the other hand, from longer perspective, disturbances are more consistent/longer present in DAX based on the values of parameters in matrix $\boldsymbol{B}$ (the square root values of parameters standing in front of $\boldsymbol{H}_{t-1}$).

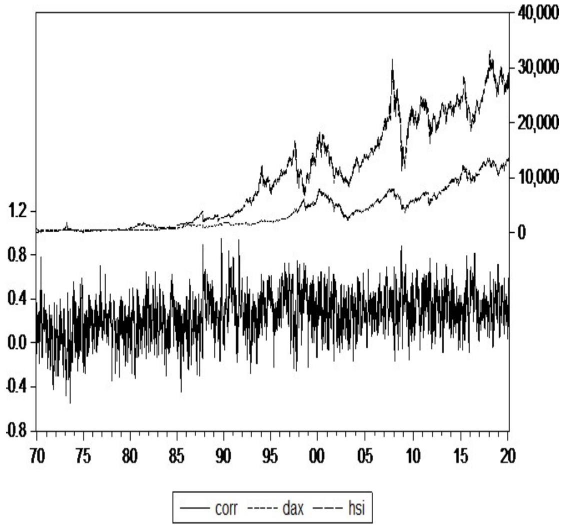

Figure 2: Time-varying correlation
Note: on the left axis there is time evolving correlation and on the right axis there is HSI and DAX
Source: author's calculations

In Figure 2 we present time evolution of correlations over a whole sample. Now we will have a closer look on evolution of correlation with respect to the epicenter of a bubble. The first one in analysis is Black Monday (on the left side in Figure 3). The epicenter country is US and we can see a sudden increase of correlation in the beginning of the 4th quarter in 1987. The highest peak during this period is around 0.8 which means that these two stock indices were strongly correlated but such conclusion would be misleading because this is not a sign of comovement. We can see a decreasing tendency over a short period after bubble blast. The shock indeed influenced both indices but this refers to the connection of HSI and DAX to DJI. So indeed, before 1990 there was a reasonable potential from international diversification. On the right side in Figure 3, we present the case of Asian financial crisis. The epicenter country is Thailand and the situation is slightly different. Before bubble blast the investors tried to distance from uncertainty (2nd and 3rd quarter in 1997) which can be reflected even through negative correlations and decreasing tendency. However, in the moment of blast, once again, the local maximum was reached in the middle of the 4th quarter in 1997. The evolution of correlation after blast has increasing tendency and this is actually a sign of comovement.

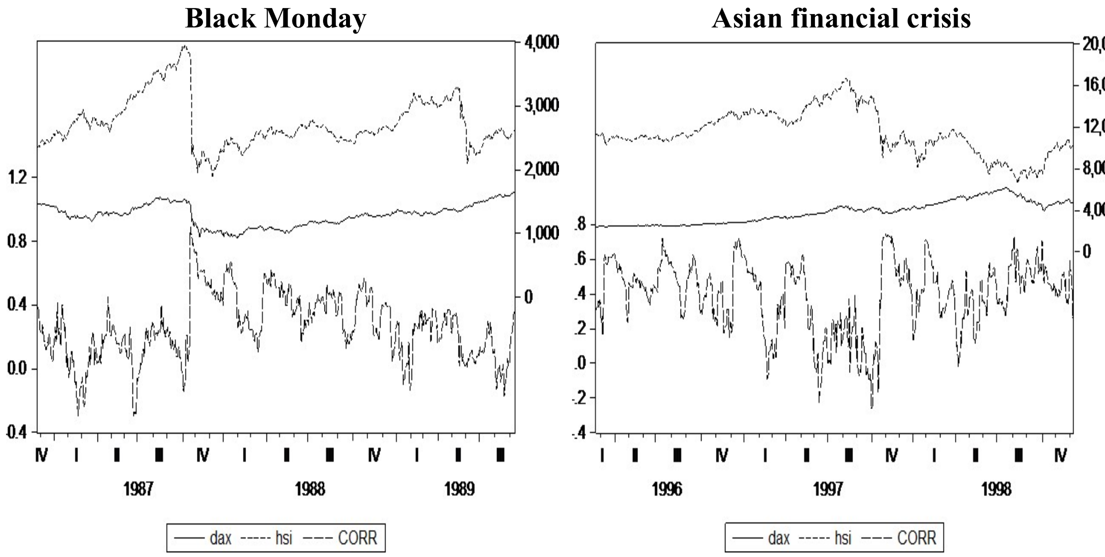

Figure 3: Comovement and spillover effect
Note: on the left axis there is time evolving correlation and on the right axis there is HSI and DAX
Source: author's calculations

We draw a simple conclusion about comovement among stock markets. The comovement of stock indices has tendency to increase if at least one country itself belongs to the epicenter of crisis, if not, the comovement has only sudden change of behavior with stabilizing tendency but still spillover effect is visible. The symmetries in periods of stabilization after different bubble blasts might be interesting for long-term investments and further research.

Lastly, we analyze briefly a period of global financial crisis around year 2008. In Figure 4 we can see a stable interaction between indices during 2007 (around 0.4), however, the sudden change happened at the beginning of 2008 when major break of crisis occurred in Europe. Again, we can see a common behavior during bubble blast, the beginning with low (sometimes even negative) correlation with increasing tendency over a whole year 2008 and stabilizing back to 0.4.

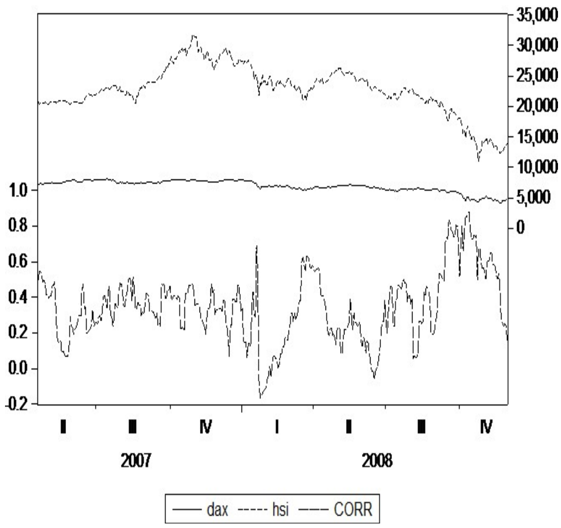

Figure 4: Comovement during global financial crisis in 2008

Note: on the left axis there is time evolving correlation and on the right axis there is HSI and DAX

Source: author's calculations

Due to a lack of data, we briefly discuss only two plausible channels – trade and foreign direct investment (data were retrieved from https://data.oecd.org/). In 2007 the inflow of foreign direct investment (FDI) from Germany to Hong Kong reached almost 1 717 mil. USD but with the blast of bubble (in 2008), the FDI turned into outflow and decreased about 140,42 %. On the other hand, trade was not so sensitive to the bubble blast. In 2008, the trade between these two countries still increased, the change came in 2009 when exports dropped about 15 % and import of Germany from Hong Kong decreased approximately 35 %. So only from comparing these two macroeconomic variables, one can say that FDI reacts to changes on stock markets much faster than trade.

4 CONCLUSIONS

In this paper we analyzed comovement and spillover effects between Hong Kong and Germany. We spotted a common behavior around time of bubble blast. The usual trend suggests that correlation before crisis dropped to a low level, sometimes even negative, with a sudden increase. The interactions have decreasing tendency when involved countries are not in the epicenter of crisis, the alternative also holds which was presented in Figure 4.

Despite the fact that involved countries are far away from each other, the shocks on stock markets have tendency to go through them. Also, we can conclude that in non-epicenter case, the stabilization will occur much faster.

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VISUAL PROMETHEE USAGE FOR THE MULTICRITERIA EVALUATION OF THE CZECH REGIONS FROM THE SELECTED ECONOMIC ACTIVITY ASPECTS POINT OF VIEW

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Abstract
Visual PROMETHEE is a software that can be used for the multi-criteria evaluation of alternatives. It is based on the PROMETHEE and GAIA multi-criteria decision-making methods. The advantage of these methods is not only the ranking of alternatives via multiple criteria, but also the possibility of defining several different preferential functions for the criteria, and graphical visualization of the results as well. In addition, Visual PROMETHEE includes the ability to categorize criteria into clusters. In this paper Visual PROMETHEE software is used to evaluate the order of 14 regions of the Czech Republic regarding such economic indices as the unemployment rate, economic activity, average age, wages, free workplaces, income, consumption and investments. Data are taken from the Czech Statistical Office web and include the years 2012-2018. The article also shows the changes in the order or the position of individual regions according to the ranking in the given years.

Keywords: *Visual PROMETHEE, PROMETHEE methods, Czech Regions, economic indicators, multi-criteria evaluation*

JEL Classification: C44, C88, R15
AMS Classification: 90B50, 91B06

1 INTRODUCTION

Multi-criteria decision-making is nowadays one of the disciplines that are still undergoing great development, especially in terms of wider usage in many areas. People, managers, companies, the government no longer consider only one criterion for a decision, but they usually need to take multiple criteria into account (Zopounidis, Pardalos, 2010). These decision problems can be divided into a choice problem to find the best alternative, the sorting problem to separate the alternatives into acceptable and unacceptable or a ranking problem to create an order of alternatives (Ishizaka, Nemery, 2013). Based on the type of the problem or the aim of the decision, several different multi-criteria decision-making (MCDM) methods can be used. The ranking problems can cover countries or regions ranking (Ulutaş, Karaköy, 2019; Kuncova, Seknickova, 2018; Latuszynska, 2014; Kuncova, Doucek, 2011; Dincer 2011), university ranking (Wu et al., 2012), companies like bank or start-up ranking (Beheshtinia, Omidi, 2017; Nikoloudis et al., 2017), renewable energy resources ranking (Lee, Chang, 2018), materials or machines ranking (Karande et al., 2016; Chatterjee, Chakraborty, 2012), people ranking (Alguliyev et al., 2019) etc. MCDM methods that were used for the calculations to obtain the final ranking cover various methods, for example, WSA and WSM (Lee, Chang, 2018; Dincer 2011), WASPAS (Karande et al., 2016), VIKOR (Lee, Chang, 2018; Wu et al., 2012), AHP (Beheshtinia, Omidi, 2017; Wu et al., 2012), TOPSIS (Lee, Chang, 2018; Latuszynska, 2014; Kuncova, Doucek, 2011), ELECTRE (Lee, Chang, 2018; Kuncova, Seknickova, 2013) or PROMETHEE (Nikoloudis et al., 2017; Koutroumanidis et al., 2002).

In this paper, we use PROMETHEE method and the software VISUAL PROMETHEE to create the order of the Czech regions based on the selected criteria: the income of region (in thousands of CZK per capita), common expenditures (mainly transferred grants and subsidies, in thousands of CZK per capita) and capital expenditures (mainly investments, in thousands of CZK per capita), as well as unemployment rate (in percentage), the ratio of economic activity (in percentage), average wage (in CZK), average age (in years) and the number of free workplaces per capita. The study is similar to Kuncova and Seknickova (2018), but now different years and different method are used. We compare all 14 Czech regions by PROMETHEE methods, and our analysis is based on statistical data from years 2012 – 2018.

For the comparison of regions, it is possible to use various kinds of techniques and methods. Dincer (2011) used TOPSIS and WSA methods for evaluation of European Union member states and candidate countries, Latuszynska (2014) and Kuncova and Doucek (2011) used TOPSIS method for the EU countries comparison from the ICT development point of view, Kuncova and Seknickova (2013, 2018) evaluated Czech regions by WSA, TOPSIS and ELECTREE III methods. PROMETHEE methods are also used for these types of evaluation: Koutroumanidis et al. (2002) compared regions of Greece using multi-criteria methods; Lopes et al. (2018) studied competitiveness of 8 tourist destinations located in the Northern Region of Portugal; Sungur and Zaranci (2018) ranked Turkey's provinces based on innovativeness, entrepreneurship, and human capital; Mlynarovič (2018) presented multiple criteria approach for identification of investment opportunities among government bonds of selected countries; Škuflić et al. (2013) analyzed 6 Southeast European countries and 27 European Union countries to assess the attractiveness of destination for the foreign direct investment.

2 DATA AND METHODOLOGY

The Czech Republic is divided into 14 regions – Prague, the capital city, Central Bohemian Region, South Bohemian Region, Plzeň Region, Karlovy Vary Region, Ústí nad Labem Region, Liberec Region, Hradec Králové Region, Pardubice Region, Vysočina Region, South Moravian Region, Olomouc Region, Zlín Region and Moravian-Silesian Region. Each region follows its own economy. The analysis compares these regions on the basis of 8 criteria via PROMETHEE method.

2.1 Data

This analysis uses official budgets of regions from years 2012 to 2018 available from official web pages of the Czech Statistical Office (www.czso.cz). For all 14 Czech regions and 7 years the data for all 8 indicators are available. The best region should have maximal income, consumption, capital expenditures, the ratio of economic activity and average wage. Vice versa it should have minimal unemployment rate, average age and number of free workplaces per capita. Each year is analysed by these 8 criteria. For our analysis, we assume (as well as in (Kuncova, Seknickova, 2018) each criterion has the same weight $v_j = 1/8 = 0.125$ (i.e. 12.5%).

For each year we obtain a ranking of regions. For the overall evaluation, we summarize the results of all years. In the first approach, we assume that the results of each year have the same weight (vector $\mathbf{w} = (0.1429, 0.1429, 0.1429, 0.1429, 0.1429, 0.1429, 0.1429)$) and therefore, we rank the regions according to the average order. The second approach respects the idea that the effect of newer data on the overall evaluation is stronger and that the effect is decreasing in the past. We assume a model of a simple arithmetic delay, which can be expressed using point-weighted weights. The year 2012 has weight 1 point, 2013 has 2 points, 2014 has 3 points, …, and year 2018 is weighted by 7 points. After normalization each point reflects weight $1/28 = 0.0357$ (i.e. 3.57%) and so years 2012-2018 are weighted by vector $\mathbf{w} = (0.0357, 0.0714, 0.1071, 0.1429, 0.1786, 0.2143, 0.2500)$.

2.2 PROMETHEE methods

The name PROMETHEE stands for "Preference Ranking Organization METhod for Enriched Evaluation". PROMETHEE is a set of methods, mainly known as PROMETHEE I and II. These methods belong to the set of MCDM methods using preference relations or so-called outranking approach (Ishizaka, Nemery, 2013). As in other MCDM methods, first, it is necessary to define a set of alternatives $(a_1, a_2, \ldots, a_p)$ and a set of criteria $(f_1, f_2, \ldots, f_n)$ and the criteria weights. PROMETHEE methods have three main steps:

a) The computation of preference degrees for every ordered pair of actions on each criterion.
b) The computation of unicriterion flows.
c) The computation of global flows.

Based on the matrix ***F*** containing the evaluation of each alternative according to each criterion, the difference between the pair of alternatives d_j is obtained:

$$d_j(a_i, a_k) = f_j(a_i) - f_j(a_k); i, k \in \{1, \ldots, m\}, j = 1, \ldots, n. \quad (1)$$

A preference degree is a score between 0 and 1 expressing how an alternative is preferred over another alternative. The value of the preference function P_j is calculated according to the type of function that must be specified by the decision-maker. Six types of preference functions exist (Alinezhad, Khalili, 2019; Ishizaka, Nemery, 2013).:

a) Usual criterion – if the difference $d_j(a_i, a_k) > 0$, alternative a_i is preferred over a_k with preference $P_j(a_i, a_k) = 1$, otherwise 0.
b) Quasi-criterion – the threshold q must be defined. If the difference $d_j(a_i, a_k) > q$, alternative a_i is preferred over a_k with preference $P_j(a_i, a_k) = 1$, otherwise 0.
c) V-shape criterion – the threshold p must be defined. If the difference $d_j(a_i, a_k) > p$, alternative a_i is preferred over a_k with preference $P_j(a_i, a_k) = 1$, otherwise $P_j(a_i, a_k) = d_j/p$.
d) Level criterion – the thresholds q and p must be defined, $q < p$. If the difference $d_j(a_i, a_k) > p$, alternative a_i is preferred over a_k with preference $P_j(a_i, a_k) = 1$. If the difference $d_j(a_i, a_k) > q$ and $d_j(a_i, a_k) < p$, alternative a_i is preferred over a_k with preference $P_j(a_i, a_k) = 0.5$, otherwise $P_j(a_i, a_k) = 0$.
e) Linear criterion – the thresholds q and p must be defined, $q < p$. If the difference $d_j(a_i, a_k) > p$, alternative a_i is preferred over a_k with preference $P_j(a_i, a_k) = 1$. If the difference $d_j(a_i, a_k) > q$ and $d_j(a_i, a_k) < p$, alternative a_i is preferred over a_k with preference $P_j(a_i, a_k) = (d_j(a_i, a_k) - q)/(p - q)$, otherwise $P_j(a_i, a_k) = 0$.
f) Gaussian criterion – the preference is calculated via Gaussian curve (more details about this type of criterion and PROMETHEE methods are in (Alinezhad, Khalili, 2019) or in (Ishizaka, Nemery, 2013).

With respect to the criteria weights the preference index matrix is calculated:

$$\pi(a_i, a_k) = \sum_{j=1}^{n} P_j(a_{i,} a_k) \cdot v_j, \ i, k \in \{1, \ldots, m\} \quad (2)$$

Finally, the leaving and entering flows are calculated as the sum of rows and columns of the preference index matrix. PROMETHEE II methods calculate for each alternative $a_1, a_2, \ldots, a_p$ the net flow (*Phi*) as a difference of the leaving (positive, *Phi*+) and entering (negative, *Phi*-) flow for each alternative and then the full rankings are performed (the higher value of the net flow is better). PROMETHEE I compares the positive and negative flows for each pair of alternatives – if the positive flow of alternative a_i is higher or equal than the positive flow of alternative a_k and if the negative flow of alternative a_i is lower or equal than the negative flow of alternative a_k, then a_i is preferred over a_k – but when both flows for both alternatives are equal, then the alternatives are indifferent. Otherwise, the alternatives are incomparable (Alinezhad, Khalili, 2019; Ishizaka, Nemery, 2013).

The extension of the previous methods is based on the weighted sum of the individual scenarios. In this paper, we assume each year as a scenario s with weight w_s that reflects the influence of the year s in the overall assessment. Overall evaluation of alternative a_i can be calculated by this formula:

$$E_i = \sum_{s=1}^{S} Phi_{is} \cdot w_s, \tag{3}$$

where S denotes the number of scenarios (years in our case), Phi_{is} is the net flow of alternative a_i in year s and w_s is the weight of year s. Similarly, as in PROMETHEE II, the higher value of the evaluation is better and alternatives (regions in our case) can be ranked.

2.3 VISUAL PROMETHEE

For the analysis of our data software Visual PROMETHEE, version 1.4 was used. This software is available at http://www.promethee-gaia.net/visual-promethee. It has implemented PROMETHEE I and PROMETHEE II, and we applied them to data (see section 2.1) from years 2012 – 2018. In GDSS part of this software, we used Scenarios Comparison, and we analysed data by both approaches described in section 2.1.

3 RESULTS

Similarly, as in (Kuncova and Seknickova, 2013) and (Kuncova and Seknickova, 2018) the aim of this contribution was to analyze the economic position of the 14 regions of the Czech Republic. In the paper (Kuncova, Seknickova, 2013), we analysed data of the year 2012 using WSA, TOPSIS and ELECTRE III methods. The same methods were used in (Kuncova, Seknickova, 2018) for analysis of data 2016 compared with DEA models results. Now we compare the results obtained by PROMETHEE methods.

Table 1: 2016 Results of MCDM methods (Kuncova and Seknickova, 2013, 2018)

Region	Phi+	Phi-	Phi	**Rank**	WSA	TOPSIS	ELECTREE	**Overall**
Prague. the Capital City	0.7759	0.0636	0.7123	**1**	1	1	1	**1**
Central Bohemian	0.4305	0.1153	0.3153	**2**	2	2	5	**2**
South Bohemian	0.2808	0.1449	0.1358	**7**	8	8	7	**7**
Plzen	0.2393	0.1932	0.0461	**4**	6	14	3	**7**
Karlovy Vary	0.1926	0.2086	-0.0160	**10**	10	13	10	**12**
Usti nad Labem	0.1779	0.2047	-0.0268	**11**	7	3	10	**6**
Liberec	0.1667	0.2090	-0.0422	**5**	5	6	8	**5**
Hradec Kralove	0.1749	0.2283	-0.0534	**9**	9	9	6	**9**
Pardubice	0.1330	0.2505	-0.1175	**8**	12	11	9	**11**
Vysocina	0.1634	0.2835	-0.1201	**6**	4	4	2	**3**
South Moravian	0.1747	0.3004	-0.1257	**3**	3	5	4	**4**
Olomouc	0.0969	0.2838	-0.1868	**12**	11	7	10	**10**
Zlin	0.1089	0.3571	-0.2482	**14**	14	12	10	**14**
Moravian-Silesian	0.0595	0.3321	-0.2726	**13**	13	10	10	**13**

Source: author's calculations

3.1 PROMETHEE methods in 2012 and 2016

Table 1 shows all the results of this research and the previous one. The grey columns show the results obtained by the PROMETHEE methods (*Phi+* and *Phi-* for PROMETHEE I and *Phi* for PROMETHEE II), the last one displays rank by PROMETHEE II. The white columns are taken from the previous analysis. The highest correlations are for WSA rank (about 91%) and ELECTRE III rank (84%). Also, the correlation coefficient between PROMETHEE rank and Overall rank is high (87%). We can see that the results of the year 2016 are less sensitive to the method used compared to the year 2012 (where the correlation coefficient between PROMETHEE II and WSA was only 23%).

3.2 PROMETHEE II method in 2012 to 2018

For PROMETHEE ranking it is preferable to use the PROMETHEE II method. The obtained net flows and the ranks for years 2012 – 2018 are in Table 2. From Table 2 it is clear that the best region in this analysis is Prague, the Capital City that is the winner over all years. The second place for Central Bohemian is also unequivocal. The other ranking is dependent on the weights of each year. Hradec Kralove and Vysocina regions improved its position from 2012 till 2018, but for example, Zlin region worsens its position.

3.3 Weighted PROMETHEE II method (2012 – 2018)

For each year, the net flow obtained by PROMETHEE II was calculated and the ranking of regions set (see Table 2). For overall evaluation, we summarized the results of all years for each region. In the first approach, we assume that the results of each year have the same weight. As we have 7 years, weight vector $\mathbf{w} = (0.143, 0.143, 0.143, 0.143, 0.143, 0.143, 0.143)$. Weighted sum of flows is easily calculated as average flow, as well as the weighted sum of ranks is average rank. Both values for each region we can see in the penultimate column of Table 2. The ranks according to average flow and average rank are very similar.

The second approach respects the idea that the effect of newer data on the overall evaluation is stronger and that the effect is decreasing in the past. As we described above the vector of weights is $\mathbf{w} = (0.0357, 0.0714, 0.107, 0.143, 0.179, 0.214, 0.250)$. The weighted sum of flows and weighted sum od ranks are in the last columns of Table 2. Note that ranking of regions according to the non-weighted and weighted flows differs only at places no. 8-10. Similarly ranking of regions according to the non-weighted and weighted ranks differs only at places no. 7-9.

We can see that Prague, the Capital City is a clear winner and the second place belongs to the Central Bohemian for the whole analyzed time. Prague, the Capital City is the heart of Central Bohemian, and so the economy of Central Bohemian is closely linked to the Prague economy. The third, fourth and fifth places are for South Moravian, Plzen and Vysocina Region. The centre of South Moravian Region is Brno, that is the second-largest city of the Czech Republic, and centre of Plzen Region is the fourth largest city of the republic.

It might seem that the assessment depends on the size of the central cities, although we recalculated the data per capita. In this respect, however, the rank of Vysocina Region is a bit surprising. Vysocina is the third smallest region in the Czech Republic and Jihlava, as the largest city in the region, ranks 18th among the largest cities. Nevertheless, in 2018 it occupied third place. The last three places are intended for economically weak regions – Usti nad Labem, Zlin and Moravian-Silesian Regions. Note that Moravian-Silesian is the third-largest region in terms of population and sixth area. Also, Usti nad Labem Region and Zlin Region are not the smallest.

Table 2: 2012-2018 Flows and rankings according to PROMETHEE II (own calculations)

	2012	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	Avg. flow	Weig.. flow
Region	Flow (rank)	Flow (rank)	Flow (rank)	Flow (rank)	Flow (rank)	Flow (rank)	Flow (rank)	Flow (rank)	Flow (rank)
Prague. the Capital City	0.589 (1)	0.635 (1)	0.694 (1)	0.498 (1)	0.712 (1)	0.587 (1)	0.592 (1)	0.615 (1)	0.613 (1)
Central Bohemian	0.423 (2)	0.387 (2)	0.371 (2)	0.432 (2)	0.315 (2)	0.380 (2)	0.332 (2)	0.377 (2)	0.365 (2)
South Bohemian	-0.019 (5)	-0.047 (6)	-0.002 (6)	-0.016 (7)	-0.042 (7)	0.019 (6)	0.011 (6)	-0.014 (6)	-0.007 (6)
Plzen	0.143 (3)	0.135 (3)	0.152 (3)	0.100 (4)	0.046 (4)	0.084 (4)	0.015 (5)	0.097 (4)	0.076 (4)
Karlovy Vary	-0.071 (8)	-0.126 (10)	-0.142 (9)	-0.049 (8)	-0.120 (10)	-0.101 (8)	-0.137 (10)	-0.106 (10)	-0.111 (10)
Usti nad Labem	-0.178 (12)	-0.128 (11)	-0.260 (13)	-0.287 (14)	-0.126 (11)	-0.189 (12)	-0.247 (14)	-0.202 (12)	-0.209 (12)
Liberec	-0.094 (9)	-0.077 (7)	-0.096 (8)	-0.086 (9)	-0.016 (5)	-0.137 (9)	-0.120 (9)	-0.090 (8)	-0.094 (9)
Hradec Kralove	-0.117 (10)	-0.167 (12)	-0.146 (10)	-0.133 (10)	-0.118 (9)	-0.010 (7)	-0.046 (7)	-0.105 (9)	-0.085 (7)
Pardubice	-0.069 (7)	-0.081 (8)	-0.031 (7)	-0.005 (6)	-0.053 (8)	-0.148 (10)	-0.143 (11)	-0.078 (7)	-0.089 (8)
Vysocina	-0.032 (6)	-0.024 (5)	0.065 (5)	0.036 (5)	-0.027 (6)	0.067 (5)	0.136 (3)	0.032 (5)	0.053 (5)
South Moravian	-0.013 (4)	0.074 (4)	0.129 (4)	0.175 (3)	0.136 (3)	0.112 (3)	0.100 (4)	0.102 (3)	0.117 (3)
Olomouc	-0.157 (11)	-0.275 (14)	-0.287 (14)	-0.228 (12)	-0.187 (12)	-0.156 (11)	-0.065 (8)	-0.193 (11)	-0.172 (11)
Zlin	-0.202 (13)	-0.119 (9)	-0.190 (11)	-0.162 (11)	-0.273 (14)	-0.266 (14)	-0.228 (13)	-0.206 (13)	-0.222 (13)
Moravian-Silesian	-0.203 (14)	-0.190 (13)	-0.258 (12)	-0.275 (13)	-0.248 (13)	-0.243 (13)	-0.200 (12)	-0.231 (14)	-0.234 (14)

Source: author's calculations

4 CONCLUSIONS

In this paper, we analysed 8 economic indices for all 14 regions of the Czech Republic during the years 2012 – 2018. We used MCDM methods, especially PROMETHEE methods applied to all years. For the calculations we used the software Visual PROMETHEE. We proved that year is not so significant for evaluation and all regions follow their trend. The best region with respect to analysed criteria is clearly Prague, the Capital City and the second place is reserved for Central Bohemian region. The third place is designed for South Moravian Region with Brno City.

Based on this analysis (see net flows in Table 2) we can however also conclude the same as in Kuncova and Seknickova (2018), that South Moravian Region (see negative net flows in 2012 and 2013 and positive later) starts to be better. The same conclusions can be drawn for South Bohemian, and Vysocina with positive net flows from 2017. For all years the ranks of regions are very similar, and so the results between non-weighted and weighted PROMETHEE methods are negligible.

Visual PROMETHEE software appears to be a handy tool for PROMETHEE analysis. This software can, among other things, perform sensitivity analysis to change the criteria weight and is able to display all results graphically (we did not use it because of the length of the text). Its great advantage is the possibility to use scenarios and change the evaluation of alternatives according to the given individual criteria during the years, as well as the possibility to change the weights of these years (scenarios). VP offers the possibility to import data from MS Excel and export the results back. Unfortunately, the installed version 1.4 failed to import data (without error message), which made it much more difficult to process a considerable amount of data. On the other hand, the software control itself is very intuitive, and data analysis is easy.

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FINANCIAL PERFORMANCE OF THE CZECH TOUR OPERATORS EVALUATION: COMPARISON OF DEA AND ELECTRE METHODS

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Abstract
Evaluating the performance, efficiency or financial health of a company is one of the central topics in business economics. For the comparison of firms, several different methods and principles can be used, including data enveloping analysis models (DEA) and multi-criteria decision-making methods (MCDM). The principle of DEA models is to estimate an efficient frontier based on the "best ratio" of the compared decision-making units' (DMUs) outputs and inputs. A DMU which is marked as efficient has better performance than other DMUs. MCDM methods evaluate alternatives via more criteria and the main aim is to choose the best alternative, to rank alternatives or to sort them into acceptable and unacceptable. However, sorting may also result in obtaining effective/efficient variants – ELECTRE methods use this principle. The main aim of the paper is to compare the results of selected DEA and ELECTRE methods used to assess the financial performance of Czech tour operators. The data were taken from database Albertina CZ Gold Edition.

Keywords: *Multi-criteria evaluation, DEA methods, ELECTRE, Czech tour operators*

JEL Classification: C67, C44, L83
AMS Classification: 90B50, 90C08, 91B06

1 INTRODUCTION

The evaluation of business performance and financial health is one of the central topics of business science and is also the subject of a lot of scientific articles and studies (Steigenberger, 2014; Richard et al., 2009; Hult et al., 2008). The methods used to measure the performance vary from one expert study to another and the principles applied in these methods also differ. The traditional approach to financial health assessment is the use of financial analysis tools. The most frequently used indicators are then selected indicators of profitability, especially return on equity (ROE), return on assets (ROA) and return on sales (ROS) (Hult et al., 2008). An alternative is offered by bankruptcy models that allow a company to assign a single value based on a set of ratios. These models make it possible to distinguish between firms that show good financial health and firms that are at risk of bankruptcy. As an example, the "Altman Z-score" models and the index IN05 can be mentioned (Altman, 1983; Neumaierová, Neumaier, 2005). Furthermore, selected methods of the multi-criteria decision-making could be used to assess the financial performance of businesses. These methods can be used to compare a larger number of alternatives based on selected criteria to divide them into groups (good and bad or efficient and inefficient) or to find the order or the winner (Figueira et al., 2005). The group of multi-criteria decision-making methods covers multi-criteria evaluation of alternatives methods, sometimes also called multiple attribute decision-making (MADM) methods (Alinezhad, Khalili, 2019), but also methods and models that measure the efficiency such as Data Envelopment Analysis (DEA) models (Cooper et al., 2006).

DEA models are widely used in a comparison of companies, countries, regions or districts from various points of view to find a set of efficient units (Cooper et al., 2006). MADM methods, especially those where the pairwise comparison and preference relations are used, try to separate the set of alternatives into effective or efficient and ineffective or inefficient. Such methods include, for example, the family of methods called ELECTRE (Elimination Et Choix Traduisant la REalite) that were first introduced by Bernard Roy in 1990. These methods, covering ELECTRE I, II, III, IV, IS or TRI, evaluate all alternatives using outranking comparisons to eliminate low-attractive alternatives (Figueira et al., 2016). Some authors (Belton and Vickers, 1993; Stewart, 1996) have highlighted the similarities of the DEA and MCDA models especially from the mathematical point of view to see that these the two approaches could be viewed as competing, but finally, both approaches can also be taken as complements (Belton and Steward, 1999).

In this paper, we apply the approaches of DEA models and ELECTRE methods on the data of the Czech tour operators not only to find out the efficient operators but also to compare the principles of the selected methods when the financial performance of the tour operators is the main aim.

A similar approach was used by Madlener et al. (2008) to determine the relative performance of 41 agricultural biogas plants located in Austria in terms of economic, environmental, and social criteria via DEA and IRIS/ELECTRE TRI methods. Damaskos and Kalfakakou (2005) presented the application of ELECTRE III and DEA as a part of a small commercial bank's ongoing effort to reengineer its branch network. Giannoulis and Ishizaka (2010) invented a customized ranking of British universities via ELECTRE III and DEA. Many researchers have emphasized in complicated decision problems more than one MCDM methods to obtain a more trustworthy and safer decision – for example, Kuncová and Sekničková (2013) evaluated the economic efficiency of the Czech regions by WSA, TOPSIS, ELECTRE III and DEA models; Mousavi-Nasab and Sotoudeh-Anvari (2017) applied TOPSIS, COPRAS and DEA for material selection problems.

There are also studies that applied DEA and selected multi-criteria decision-making methods to assess the performance of firms in tourism industry. Ramírez-Hurtado and Contreras (2017) used DEA to examine the efficiency of travel agencies in Spain. Hedija et al. (2017) applied DEA methods to assess the efficiency of Czech travel agencies and tour operators. Chen et al. (2011) used MCDM approach for evaluation of spring hotels performance. Hedija and Kuncová (2018) used selected MCDM methods (specifically TOPSIS and WSA) to evaluate the financial performance of Czech travel agents. This paper partly builds on these studies. It compares the results of the DEA and MDCM approach in evaluating the financial performance of Czech tour operators. The ELECTRE methods were selected as the aim of some of them is to separate alternatives into indifference classes and the 1.indifference class is supposed to cover the efficient alternatives. That is why we decided to compare the DEA methods and ELECTRE I and III as all has the main aim to find the efficient DMUs/alternatives. Data from 2014 was taken because of the higher number of companies for the comparison but still with the acceptable number of companies to be able to use SANNA application for calculations.

2 DATA ENVELOPMENT ANALYSIS

Data Envelopment Analysis (DEA) belongs to the operational research methods, especially to the linear programming models, that have been used many times in private or public sector to evaluate the performances of many different kinds of entities (countries, regions, enterprises, schools, hospitals, insurance companies, military units etc.) engaged in many different kinds of activities in many different contexts (Cooper, Lawrence, Zhu 2004). The basic idea of DEA models consists of the estimation of an efficient frontier which is created by the best relative ratios of the compared units (so-called decision-making units DMUs). DMU is taken as efficient when it lies on the efficient frontier. Otherwise, it is inefficient, which means it can improve its efficiency by changing parameters that were compared with other DMUs. These parameters are separated into inputs and outputs based on the main idea of the comparison. Usually, it is assumed that the inputs are used to create the outputs, and that is why the inputs should be minimized and outputs maximized (Charnes et al., 1978). Depending on which parameters are to be considered fixed and which can be better moved, the DEA models are separated into the input and output-oriented models. In the case of input-oriented models, the fixed level of all outputs (CCR-I) is assumed, the output-oriented model assumes a fixed level of all inputs (CCR-O) (Cooper, Lawrence and Zhu, 2004). DEA models are also classified by the type of returns to scale. The CCR models (Charnes, Cooper, Rhodes) are based on the constant return to scale, the BCC models (Banker, Charnes, Cooper) suppose variable returns to scale. The review and detailed information about DEA models were described by Cooper, Lawrence and Zhu (2004) and by Cooper, Seiford and Tone (2006). The basic idea for the efficiency calculation is to maximize the rate of the weighted sum of outputs divided by the weighted sum of inputs (Charnes, Cooper and Rhodes, 1978).

The mathematical model of DEA considers r DMUs $U_1, U_2, \dots, U_r$ with m inputs ($i = 1,2,\dots,m$) and n outputs ($j = 1,2,\dots,n$). The vector of input values of DMU k ($k = 1,2,\dots,r$) is denoted as $\mathbf{x}_k = (x_{1k}, x_{2k}, \dots, x_{mk})^{\mathrm{T}}$ and matrix of all input values for all DMUs is denoted as $\mathbf{X} = \{x_{ik}, i = 1,2,\dots,m, k = 1,2,\dots,r\}$. Similarly, the vector of output values of DMU k is denoted as $\mathbf{y}_k = (y_{1k}, y_{2k}, \dots, y_{nk})^{\mathrm{T}}$ and matrix of all output values for all DMUs is denoted as $\mathbf{Y} = \{y_{jk}, j = 1,2,\dots,n, k = 1,2,\dots,r\}$. The relative technical efficiency of given DMU q can be generally expressed as ratio of weighted sum of outputs and weighted sum of inputs

$$TE_q = \frac{\sum_{j=1}^{n} u_j y_{jq}}{\sum_{i=1}^{m} v_i x_{iq}}, \tag{1}$$

where v_i, $i = 1, 2, ..., m$ is a weight for i-th input and u_j, $j = 1, 2, ..., n$ is a weight for j-th output.

CCR input oriented model (CCR-I) has the following form (the other models are determined from this model and are described by Cooper, Lawrence and Zhu (2004)):

maximize

$$z = \mathbf{u}^{\mathrm{T}}\mathbf{y}_q,$$

subject to

$$\begin{aligned} \mathbf{v}^{\mathrm{T}}\mathbf{x}_q &= 1, \\ \mathbf{u}^{\mathrm{T}}\mathbf{Y} - \mathbf{v}^{\mathrm{T}}\mathbf{X} &\le 0, \\ \mathbf{u} &\ge \varepsilon, \\ \mathbf{v} &\ge \varepsilon. \end{aligned} \tag{2}$$

The efficient unit U_q lies on the efficient frontier in case that the optimal efficiency (calculated by the model) $z = 1$. In the CCR output-oriented model (CCR-O) it is similar. Still, it minimizes the objective function with the sum of the weighted inputs, where vector $\boldsymbol{v}$ and vector of inputs $\boldsymbol{x_q}$ of the DMU q are used, and the product of the vector $\boldsymbol{u}$ and vector $\boldsymbol{x_q}$ should be equal to 1 in the first constraint. The inefficient units have z lower than 1 in CCR-I model, higher than 1 in CCR-O model. The BCC models are slightly different with additional parameter μ (BCC-I) and ν (BCC-O) connected with the convex efficiency frontier and added to the objective function and to the left-hand side of the inequalities in constraints (Cooper et al., 2004).

3 MULTI-CRITERIA EVALUATION AND ELECTRE METHODS

Multi-Criteria Decision Making (MCDM) belongs to the areas that are still rapidly growing. In theory, many methods have been proposed and developed to solve various kind of problems in numerous ways (Triantaphyllou, 2000). This stream focuses on the issues with discrete decision spaces, i.e. with countable decision alternatives (a_1, a_2, …, a_p) evaluated by a set of criteria (f_1, f_2, …, f_k) defined by the decision-maker. For the solution to this type of problems, it is necessary to know the preferences of the decision-maker. These preferences can be described by aspiration levels (or requirements), criteria order or by the weights of the criteria (Figueira, Greco and Ehrgott, 2005; Zopounidis and Pardalos, 2010). The aim of the decision process can be following (Ishizaka, Nemery, 2013): to find the winner (the choice problem); to find the order/ranking (the ranking problem); to sort the alternatives into predefined groups (the sorting problem). The results of the sorting problem can be the separation of the alternatives into acceptable/good/efficient ones and unacceptable/bad/inefficient ones. The sorting problem is usually based on methods that use the outranking relations, i.e. using pairwise comparisons among alternatives under each one of the criteria separately. The outranking relation theory has been widely used in several methods, such as ELECTRE methods or PROMETHEE methods (Ishizaka, Nemery, 2013). As a result of ELECTRE methods could be a set of efficient alternatives, we decided to use these methods for the comparison not only to find the efficient tour operators but also to compare the results with DEA models.

The ELECTRE (Elimination Et Choix Traduisant la REalite) methods begin with pairwise comparisons of alternatives under each criterion to create a set of binary relations, so-called outranking relations (Triantaphyllou, 2000). The outranking relationship of the two alternatives A_i and A_j describes that the relationship is equal to 1 if A_i is better than A_j. To set the relationship to be equal to 1, several previous steps should be done and sometimes also several thresholds should be set by the decision-maker to express the preferences even when the *i-th* alternative does not dominate the *j-th* alternative. Alternatives are said to be dominated if there is another alternative which is better at least in one criterion and not worse in any other criterion. The non-dominance testing is very important as the first step before other steps of the methods are started as the dominated alternatives can never be among the best/efficient ones. So, the elimination of dominated alternatives will both reduce the size of the task and the number of pairwise comparisons and eliminate the impact of these unsuitable alternatives on the results.

The set of ELECTRE methods covers ELECTRE I, II, III, IV, IS or TRI (Ishizaka, Nemery, 2013. Although the first step of ELECTRE methods should be the change of real data into normalized decision matrix (Triantaphyllou, 2000), it is not necessary to do this step because of the pairwise comparison where the decision-maker sets when the alternative is better/preferred regardless of whether it is real or normalized data. So, the main advantage of the ELECTRE method is that they avoid any normalization process, which distorts the original data, and also avoid compensation between criteria (Ishizaka, Nemery, 2013).

The selection of the type of ELECTRE method is dependent on the data set and also on the software available. If the number of alternatives is high enough, it is necessary to use the software, as the pairwise comparison matrices are too big. As in our case, more than 100 alternatives are compared, the MS Excel add-in application SANNA (Jablonský, 2009) was used for the comparison. That is why the ELECTRE I and ELECTRE III methods were chosen.

For the ELECTRE I the decision matrix could be normalized (Alinezhad, Khalili, 2019) but it is important especially for the dominated matrix ***D*** and the threshold *d*, but as the threshold is set by decision-maker, the normalization is not necessary. First, the dominant matrix ***C*** is calculated. The c_{ij} value is the sum of weights where the alternative *i* is better or the same than alternative *j*. The dominated matrix D is calculated as the ratio of maximal differences – the maximum difference between two alternatives i and j (for differences in all criteria) is selected in the denominator, and in the numerator, there is selected the maximum difference only from those differences where the alternative *i* is worse than *j*. Detailed description can be found in (Alinezhad, Khalili, 2019).

ELECTRE III method also uses a pairwise comparison of the alternatives and summarizes the weights of the criteria where the alternative is better than the other one. Based on the Fiala (2006), according to the strength of the preference, the final indifference classes are made. Although Alinezhad and Khalili (2019) describe more steps for this method, we used the basic principle from Fiala (2006) as these steps are included in SANNA add-in application. It is necessary that all the alternatives are nondominated (so there is no alternative that is better in at least one criterion and no worse in any other) because the existence of dominated alternatives could influence the results.

4 CZECH TOUR OPERATORS DATA

Data from the Albertina CZ Gold Edition database are used to assess the financial health of selected companies. The Albertina database contains selected data on all enterprises to which an identification number has been assigned. Currently, this database contains information on more than 2.7 million companies. We chose the data of companies which as the main activity according to the Classification of Economic Activities (CZ-NACE) report the activities of travel agencies (CZ-NACE group 79.12). The sample was further narrowed to companies that had the necessary data to assess financial health. The final sample includes 236 travel agency data from the year 2014. According to data from the Czech Statistical Office, these represent approximately 13 percent of all registered travel agencies in the Czech Republic.

Based on our previous research (Hedija et al., 2017), we used 3 inputs and 1 output for DEA CCR-I and BCC-I models and the same as 4 criteria for ELECTRE methods. As output, we used sales which are calculated as a sum of revenues from sold goods and production. As the inputs, we employed personnel expenses (which represents labor), tangible and intangible fixed assets (that are the proxy for capital) and expenses on sold goods and production consumption (as another proxy for capital).

5 RESULTS

First, the calculations of DEA models were made when 3 inputs and 1 output were used, and CCR-I and BCC-I models were applied. All calculations were made in STATA software. From all 236 tour operators the models with constant returns to scale (CCR-I) found out only 4 efficient tour operators and models with variable returns to scale (BCC-I) 18 efficient tour operators (see Table 1 and Table 2). When it comes to data, there is no need to adjust the input data. It is only important that the data is not negative. Since the method itself calculates the ratios of outputs and inputs, it is ideal to enter data in absolute values. From the tour operators' point of view, the results suggest rather the worse position of most companies surveyed.

Table 1: DEA results

	Score CRS-I	Score VRS-I
number of efficient	4	19
avg.eff.score	0.339	0.461
No.of eff.score>0.8	12	56
% of eff.score>0.8	5.08	23.73

Source: author's calculations

Second, the calculations via ELECTRE I and ELECTRE III were made. In this case, the situation is different. As it is known from the previous chapter, it is necessary to set the type of each criterion (min or max) and the criteria weights. The criterion type can be derived from assumptions of the DEA models – the inputs should be minimized, the output maximized. With regard to the criteria weights, we chose two ways of calculating - setting the same weights for all 4 criteria (each criteria weight is equal to 0.25) and setting the equivalent weights for inputs (0:5 as a whole, so 1/3 for each) and outputs (0.5).

Table 2: Results of DEA and ELECTRE

DMU No.	personnel expenses	tangible and intangible fixed assets	expenses on sold goods and production consumption	sales	Score CCR-I	Score BCC-I	all tour operators: ELECTRE I equal weights	ELECTRE III equal weights	ELECTRE I half weights	ELECTRE III half weights
9	193	0	86	467	1	1				
12	47	0	363	897	1	1		efficient 1		
13	3778	601	38684	333111	1	1	efficient		efficient	
16	2	0	1265	1309	1	1		efficient 2		
2	372	0	37336	38754	0,91944	1				efficient 1
11	4374	0	164219	158122	0,73159	1				efficient 2
4	973	0	1666	3522	0,68038	1				
15	5320	109	200058	207632	0,6724	1				
8	521	146	56626	58333	0,62982	1				
10	35851	866	841542	886532	0,54702	1				
6	3340	0	32731	37261	0,52631	1				
3	2619	0	21195	26004	0,51021	1				
7	2778	0	5564	8014	0,48195	1				
14	5344	0	38491	44323	0,46354	1				
1	35431	58378	989404	959267	0,25966	1				
5	139629	117286	2414364	2479437	0,18773	1				
149	48	0	222	126	0,21893	1				
157	20	145	346	393	0,20772	1				
113	3299	237	12208	30042	0,28578	0,28815	efficient			

Source: author's calculations

Another problem may be the inclusion of dominated alternatives in the calculations, as in particular the results according to ELECTRE III may be affected. That is why the non-dominance testing was the first step, and only the non-dominated alternatives were left for further calculations. Out of the 236 tour operators, 56 were dominated and 180 non-dominated. ELECTRE I method evaluated (with preference parameter c=0.5 and dis-preference parameter d=0.9) two alternatives as efficient (see Table 2). In addition to alternative 13, which is also efficient for CCR and BCC, alternative 113, that is also efficient, did not work very well for DEA. The reason is that this alternative is via ELECTRE I evaluated to be better than 135 other alternatives and no worse than any other alternative – it is not better than most of the alternatives efficient in BCC model, but also these alternatives are not evaluated to be better than No. 113. When equal weights of inputs and outputs were set, the only efficient alternative is the tour operator No.13. This alternative was evaluated to be better than other 178 alternatives (only with 1 alternative there is no preference relation.

ELECTRE III method separated alternatives into the indifference classes, so the 1st indifference class is taken as “efficient 1”, second as “efficient 2” etc. When equal criteria weights were set, there were only 4 indifference classes found, but the 4th covered 177 tour operators. Table 2 mentions only the first and second places. A similar situation was in case of equal weights of inputs and output, when 6 indifference classes were calculated, but again the 6th covered 175 alternatives. The differences between the results of ELECTRE III with different weights lie in the higher weight of the output so the alternatives (no.2, no.11) with higher sales are better rated. The results when equal weights were used are similar to CCR results.

As it was mentioned above, ELECTRE methods are based on pairwise comparison and the sum of criteria weights. A lot of alternatives can also influence the results – that is why we decided to apply ELECTRE I and III only on the alternatives with the DEA (CCR, BCC) efficiency score higher than 0.8. There are 59 such alternatives, out of these, 3 are dominated. We have also retained no.113 (as efficient from previous calculations). The results of the ELECTRE I method were the same. The results of the ELECTRE III method were slightly different; the alternatives no.12 and no.16 switched positions. Keeping 3 dominated, the alternative no.9 will still get 2nd, but only because it is better than one of the dominated alternatives in all criteria. This result confirms that the retention of dominated alternatives may affect the outcome. On the other hand, the results of the DEA are also based on a comparison of the relative positions of the alternatives (DMUs). Perhaps we would have obtained more efficient alternatives by keeping all 236 alternatives in a file, but unfortunately, the add-in application we used does not allow to include more than 200 alternatives. So it would be necessary to find more suitable software.

6 CONCLUSIONS

The main aim of the paper was to compare the results of DEA models and ELECTRE methods used to assess the financial performance of Czech tour operators. Both principles confirmed that the situation in the sector of the tour operators was not good, and most of the companies were inefficient. In terms of the need to add additional information, ELECTRE methods have a disadvantage. In addition to determining the type of criterion, it is necessary to set criteria weights and for ELECTRE I also thresholds. If we compare the results of DEA models and ELECTRE methods, it can be stated that the results of CCR and ELECTRE III method are closer to each other. However, the reason might be the data in the given case, so it would be necessary to verify this fact on other examples and, as mentioned earlier, to use other software for the possibility of analyzing a large file while retaining the dominated alternatives.

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PATH-RELINKING METHOD FOR WEIGHTED P-MEDIAN PROBLEM SOLUTIONS

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Abstract
The path-relinking method is an excellent tool for solving zero-one programming problems, solutions of which correspond to vertices of a unit hypercube. The method is based on inspecting one of the shortest paths connecting given starting and ending solutions. In the paper we suggest and study a special version of the path-relinking method, which is adjusted for the *p*-median problem. The suggested method avoids infeasible solutions, and makes use of fastened neighborhood search, which exploits information obtained by previous objective function computation. The piece of information enables to reduce complexity of the objective function enumeration of a neighboring solution. The research reported in this contribution is focused on a way of path-relinking method application. The standard way consists in random generating a pool of starting population and in selecting pairs of the generated solutions, which are submitted to the path-relinking method. Contrary to the way, we make use of so-called uniformly deployed set of *p*-location problem solutions. The uniformly deployed set was established for *p*-median problem sizes *p* and *m*. The number *p* is the number of centers, which are to be located, and the number *m* is the total number of possible center locations. When the uniformly deployed set of solutions is used, the application of the path-relinking method proceeds some pairs of the predetermined solutions, where one of the solutions is the best found solution. The process is repeated until all solutions of the uniformly deployed are used. The above mentioned path-relinking method applications are compared and their efficiency is studied.

Keywords: *Discrete location, p-median problem, uniformly deployed set, path-relinking method*

JEL Classification: C61
AMS Classification: 90C27

1 INTRODUCTION

The family of *p*-location problems includes a broad spectrum of location problems, which consist of a task to choose *p* locations of *m* possible facility locations in a transportation network so that an objective function is minimal. The considered objective function usually expresses the disutility perceived by users located at nodes of the transportation network and serviced by the located facilities [1, 3, 10]. Models of the *p*-location problem play an important role in public service system designing, where the number of disposable facilities is limited and demands of all users represented by dwelling places of a serviced region must be taken into account [2, 8]. As the basic decisions on facility location can be described by a vector of *m* zero-one decision variables, the *p*-location problem are solved either by integer linear programming methods [1, 3, 7] or by broad spectrum of evolutionary metaheuristics or heuristics [4, 11]. The set of all feasible solutions of the *p*-location problem can be represented by a sub-set of vertices of *m*-dimensional unit hypercube, where corresponding zero-one vectors contain exactly *p* unit components. This special characteristic gave raise of methods, which enables preliminary mapping of the set of feasible solution using so-called uniformly deployed sets [6, 9]. In addition, the characteristic gave raise of smart methods for neighborhood search, where the neighborhood of a current solution is determined by an exchange operation. Within this paper, we present a special kind of the path-relinking method, which makes use of the characteristic of the *p*-location problem solutions and we study the efficiency of the method in combination with preliminary mapping of the set of feasible solutions, which is provided by a uniformly deployed set of p-location problem solutions.

2 UNIFORMLY DEPLOYED SETS

The p-median problem is often defined by (1), where decisions on locating a service center at location i is modelled by a zero-one variable y_i for each i=1, …, m, where the variable y_i gets the value one if a center is located at the location i and the variable gets the value of zero otherwise.

$$\min\left\{ f\left(\boldsymbol{y}\right): y_i \in \{0,1\} \;\; for\, i = 1,...,m, \; \sum_{i=1}^{m} y_i = p \right\} \tag{1}$$

The problem (1) can be solved by a search in a sub-set of m-dimensional hypercube vertices. A vertex corresponding to a feasible solution $\boldsymbol{y}$ has exactly p unit components. Hamming distance between two feasible solutions $\boldsymbol{y}$ and $\boldsymbol{x}$ can be defined by (2).

$$H\left(\boldsymbol{y},\boldsymbol{x}\right) = \sum_{i=1}^{m} \left| y_i - x_i \right| \tag{2}$$

The integer $r=p-H(\boldsymbol{y}, \boldsymbol{x})/2$ gives the number of possible center locations common for both solutions. The uniform deployed set of p-location solutions can be defined as a sub-set S of feasible solutions of (1) so that the inequality $H(\boldsymbol{y}, \boldsymbol{x})\geq h$ holds for each $\boldsymbol{x}, \boldsymbol{y}\in S$ and for given h. The previous approach to obtaining enough big uniformly deployed set was presented e.g. in [6, 9] and consists in creation of so-called basic deployed set, which was formed by “ad-hoc” combining q–tuples of locations so that two different q-tuples have at most one common location. The basic set of p-location problem solutions satisfies the condition that no pair of solutions can contain more than r common locations and, this way, their minimal Hamming distance equals to $2(p-r)$. The basic set was enlarged step by step by solving an integer programming problem followed by adding the optimal solution $\boldsymbol{y}$ to the current set S. This enlargement process terminates either if either the maximal set is obtained or a demanded cardinality of the set S is reached.

3 PATH-RELINKING METHOD FOR P-LOCATION PROBLEM

In general, the path-relinking method was suggested as a solving tool of zero-one programming problems, solutions of which correspond to vertices of a unit hypercube [4]. The method searches one of the shortest paths, which connects two given input vertices and returns the best feasible solution, which is met during the search. The inspection of the shortest path connecting two input vertices $\boldsymbol{x}$ and $\boldsymbol{y}$ of a unit hypercube is based on the fact that the path consists of hypercube edges, which connect neighboring vertices. Neighboring vertices differ exactly in one component of the associated vectors. Thus the shortest path connecting the two input vertices consists of exactly $|\Delta|$ edges, where Δ is set of components, in which the vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ differ. The original mixed path-relinking method [4] can be described by the following steps.

0. Initialize $\boldsymbol{x}^{best}$ by $argmin\{f(\boldsymbol{x}), f(\boldsymbol{y})\}$ and determine $\Delta=\{i : i=1, \ldots, m, x_i \neq y_i\}$.
1. Determine $i^*=argmin\{f(inv(\boldsymbol{x}, i)): i\in\Delta\}$.
2. Update $\boldsymbol{x}$, Δ, and $\boldsymbol{x}^{best}$ according to $\boldsymbol{x}=inv(\boldsymbol{x}, i^*)$, $\Delta=\Delta-\{ i^*\}$, $\boldsymbol{x}^{best} = argmin\{f(\boldsymbol{x}), f(\boldsymbol{x}^{best})\}$.
3. If $|\Delta|=1$, then terminate and return $\boldsymbol{x}^{best}$, otherwise exchange $\boldsymbol{x}$ and $\boldsymbol{y}$ and go to 1.

Comment: The operation *inv*($\boldsymbol{x}$, *i*) adjusts the vector $\boldsymbol{x}$ so that it inverts the *i*-th component according to x_i=1 - x_i. Operation *argmin*{*f*($\boldsymbol{x}$), *f*($\boldsymbol{y}$)} returns the argument $\boldsymbol{x}$ or $\boldsymbol{y}$, which has less function value.

Based on the idea of the original path-relinking method, we suggested a special version for the min-sum *p*-location problem, which evaluates a solution **y** according to (3).

$$f\left(\boldsymbol{y}\right)=\sum_{j=1}^{n} b_j \min\left\{d_{ij} : i=1, ..., m,\ y_i=1\right\} \tag{3}$$

In the expression (3), b_j stands for a weight of users located at location *j*, *n* is the total number of users' locations, and d_{ij} denotes a distance between possible facility location *i* and users' location *j*.

From the point of the *p*-location problem, the above original version of the path-relinking method visits also infeasible and inadmissible vertices of the hypercube on the shortest path inspection. This drawback has been removed by the suggested version, which skips the inadmissible or infeasible vertices. To describe the suggested version, we introduce two sets u and $\boldsymbol{v}$ of locations for the input solutions $\boldsymbol{x}$ and $\boldsymbol{y}$ instead of common set Δ. Set $\boldsymbol{u}$ contains the subscripts of unit components of $\boldsymbol{x}$, which correspond to zero components in $\boldsymbol{y}$ (4). The same way, set $\boldsymbol{v}$ is defined for input solution $\boldsymbol{y}$ (5).

$$\boldsymbol{u}=\left\{i: i=1, ..., m,\ x_i=1, y_i=0\right\} \tag{4}$$

$$\boldsymbol{v}=\left\{i: i=1, ..., m,\ x_i=0, y_i=1\right\} \tag{5}$$

The newly suggested path-relinking method performs according to steps described below.

0. Initialize $\boldsymbol{x}^{best}$ by *argmin*{*f*($\boldsymbol{x}$), *f*($\boldsymbol{y}$)} and determine $\boldsymbol{u}$ and $\boldsymbol{v}$ according to (4) and (5).
1. Determine i^* and j^* by (i^*, j^*) =*argmin*{*f*(*ex*($\boldsymbol{x}$, *i, j*)): $i\in\boldsymbol{u}, j\in\boldsymbol{v}$}.
2. Update $\boldsymbol{x}$, $\boldsymbol{u}$, $\boldsymbol{v}$, and $\boldsymbol{x}^{best}$ according to $\boldsymbol{x}$=*ex*($\boldsymbol{x}$, i^*, j^*), $\boldsymbol{u}$=$\boldsymbol{u}$-{i^*}, $\boldsymbol{v}$=$\boldsymbol{v}$-{j^*}, $\boldsymbol{x}^{best}$ = *argmin*{*f*($\boldsymbol{x}$), *f*($\boldsymbol{x}^{best}$)}.
3. If $|\boldsymbol{u}|$=1 and $|\boldsymbol{v}|$=1, then terminate and return $\boldsymbol{x}^{best}$, otherwise exchange $\boldsymbol{x}$, $\boldsymbol{u}$ and $\boldsymbol{y}$, $\boldsymbol{v}$ and go to 1.

Comment: The operation *ex*($\boldsymbol{x}$, *i, j*) adjusts the vector $\boldsymbol{x}$ so that it inverts the *i*-th component according to x_i=1- x_i and it inverts also the *j*-th component according to x_j=1- x_j.

To accelerate step 1 of the suggested path-relinking method, we used a special approach to the neighborhood search [5], which enabled to reduce complexity *O*(*pn*) of the objective function computing to complexity *O*(*2n*).

4 APPLICATION OF PATH-RELINKING METHOD

Let us define function *Path-Relinking*($\boldsymbol{x}$, $\boldsymbol{y}$) according to the algorithm described in the previous section. The function returns the resulting solutions obtained by inspection of the shortest path connecting the input solutions $\boldsymbol{x}$ and $\boldsymbol{y}$.

The below application makes use of a uniformly deployed set S constructed for the set of all feasible solutions of the p-location problem defined for m possible facility locations.

We assume that the solutions of the uniformly deployed set $S = \{\boldsymbol{x}^1, \ldots, \boldsymbol{x}^{|S|}\}$ are subscripted and ordered by permutation U so that $f(\boldsymbol{x}^{U(1)}) \leq f(\boldsymbol{x}^{U(2)}) \leq \ldots \leq f(\boldsymbol{x}^{U(|S|)})$ holds.

Then, the suggested application performs according to the following steps:

0. Initialize $\boldsymbol{x}^{best}$ by $\boldsymbol{x}^{U(1)}$.
1. For k=2, …, $|S|$ perform subsequently $\boldsymbol{x}^{best}$ = *Path-Relinking*($\boldsymbol{x}^{best}$, $\boldsymbol{x}^{U(k)}$).
2. Terminate and return $\boldsymbol{x}^{best}$.

5 NUMERICAL EXPERIMENTS

The main goal of performed computational study was to study efficiency of the suggested path-relinking application in comparison with other heuristic, which exploits the uniformly deployed sets. We compare results of swap algorithms getBA [9], when used together with "tailored" uniformly deployed set to the results, which were achieved using the suggested version of the path-relinking method embedded into the simple application. The used benchmarks were Slovak self-governing regions. The mentioned instances are denoted by the names of capitals of the individual regions, which are reported by abbreviations of the region denotations. The list of instances is Banská Bystrica (BB), Košice (KE), Nitra (NR), Prešov (PO), Trenčín (TN), Trnava (TT) and Žilina (ZA). The sizes of the individual benchmarks are m and p, where m stands for the number of possible center locations and p is the number of service centers to be located.

An individual experiment was organized in such a way that the optimal solution of the min-sum location problem was obtained using the radial approach described in [7], first. The objective function value of the exact solution denoted by F^{opt} is reported in the Table 1 in the column denoted by "OptSol". The section "getBA" contains objective function values reported in the column denoted by F^* and solution accuracy evaluated by *gap*, which expresses a relative difference of the obtained result from the optimal solution. The value of *gap* is expressed in percentage, where the optimal objective function value of the problem is taken as the base. This section values were obtained by getBA algorithm used with "tailored" uniformly deployed set. The section "Path-Relinking" contains objective function values reported in the column denoted by F^{**} and solution accuracy is also evaluated by *gap*. The results were achieved using the uniformly deployed sets used by swap algorithm getBA. The computational time of the studied heuristic approaches are reported in the columns denoted by *CT* [s]. Their values are given in seconds.

Table 1 Results of numerical experiments for the self-governing regions of Slovakia

Region	*m*	*p*	*OptSol*	**getBA**			**Path-Relinking**		
				*F**	*gap* [%]	*CT* [s]	*F***	*gap* [%]	*CT* [s]
BB	515	36	29873	29873	0.00	1.32	29873	0.00	3.48
KE	460	32	31200	31451	0.80	0.79	31280	0.26	0.75
NR	350	27	34041	34075	0.10	0.32	34041	0.00	0.51
PO	664	32	39073	39117	0.11	1.74	39073	0.00	4.39
TN	276	21	25099	25125	0.10	0.10	25099	0.00	0.36
TT	249	18	28206	28372	0.59	0.07	28206	0.00	0.36
ZA	315	29	28967	28967	0.00	0.29	28967	0.00	0.69

6 CONCLUSIONS

The paper studied the efficiency of a special version of the path-relinking method applied to a uniformly deployed set of the *p*-location problem solution. The resulting approach enables to obtain the optimal or a near-to-optimal solution of a min-sum *p*-location problem. The suggested approach proved to be a bit more computational time demanding in comparison to a classical exchange heuristic method. As far as the solution accuracy is concerned, the path-relinking approach proved to be a very useful tool. Therefore, we can conclude that suggested path-relinking approach can be recommended for practical use and further research as well. Future research in this field could be focused on more general ways of obtaining the maximal uniformly deployed set and on other schemes of path-relinking method applications.

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DATA ENVELOPMENT ANALYSIS OF PROJECT PORTFOLIO

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Abstract
Project management is the approach to managing resources in order to successful achieve specific goals of an organization. Project portfolio is set of all projects that are implemented in the organization at a time. Possible projects are characterized by sets of inputs and outputs, where inputs are resources for project realization and outputs measure multiple criteria of goals of the organization. The data envelopment analysis (DEA) is an appropriate approach to select efficient projects. The organization has its total resources in limited quantities. Designing a portfolio of efficient projects not exceeding the limited resources does not always lead to the most efficient portfolio. The paper proposes a new approach for project portfolio designing based on DEA models. Performance measures of the designed project portfolio are the efficiency of the portfolio and the effectiveness of outputs. Possible extensions of the concept are formulated and discussed.

Keywords: *Project portfolio, Multiple criteria, Resources, DEA*

JEL Classification: C44
AMS Classification: 90C15

1 INTRODUCTION

Project management is the discipline of planning, organizing, securing and managing resources to bring about the successful completion of specific project goals. In an accelerating economic world, projects become tools for promoting the goals of the organization. There is a very extensive literature on the management of individual projects and project portfolios. We start from the publication (Larson and Gray, 2013) that describes very clearly project management as a managerial process. Projects are in accelerating world rhythm the right option of solving problems of a lot of organizations. Nothing is permanent, everything is temporary, and that makes pressure on companies to finish new products or services faster, cheaper and definitely not to fail. Risk is a very important factor in project management. Most project organizations exist in a multi-project environment. This environment creates the problems of project interdependency and the need to share resources. Projects are the way for implementing the organization's strategy. Strategic alignment of projects is of major importance to effective use of organization resources. Selection criteria need to ensure each project is prioritized and contributes to strategic goals.

We propose a new approach for project portfolio designing based on a Data Envelopment Analysis (DEA) and De Novo optimization approach. Possible projects are characterized by sets of inputs and outputs. The DEA is an appropriate approach to select efficient projects. Inputs are resources for project realization. Charnes, Cooper, and Rhodes (Charnes et al., 1978) developed the first DEA. The DEA model is based on the reduction of the multiple inputs and multiple outputs to that of a single ”virtual” input and a single ”virtual” output using weights. The model searches for the set of weights which maximize the efficiency of the project. The DEA may be characterized as a method of objective weight assessment. The DEA includes a number of models and methods to evaluating performance (Cooper et al., 2006, Charnes et al., 2013).

The rest of the paper is organized as follows. In Section 2, the project portfolio problem is formulated. Using the Data Envelopment Analysis (DEA) for searching efficient projects is summarized in Section 3. Section 4 formulates DEA model for searching efficient project portfolios. An illustrative example is presented in Section 5. Discussion and conclusion are summarized in Section 6.

2 PROJECT PORTFOLIO MANAGEMENT

The portfolio management domain encompasses project management oversight at the organization level through the project level. Full insight of all components of the organization is crucial for aligning internal business resources with the requirements of the changing environment. Project portfolios are frequently managed by a project office that serves as a bridge between senior management and project managers and project teams. Project opportunities come in time and it is necessary to decide which will be accepted for creating a dynamic portfolio of projects and which will be rejected (Fig. 1).

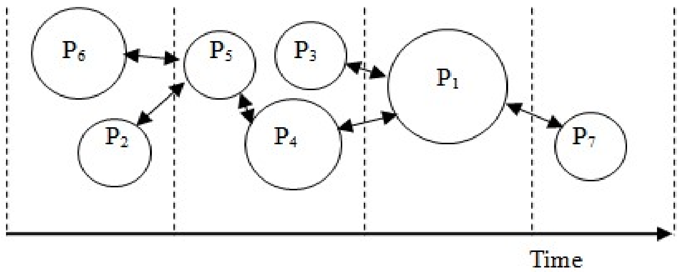

Figure 1 Dynamic flow of projects

Project portfolio is set all projects that are implemented in the organization at that time. The basic objectives of the project portfolio management include:

- Optimize the results of the entire project portfolio and not individual projects
- The selection of projects to start
- Interruption or discontinuation of projects
- Defining priorities for projects
- Coordinate internal and external sources
- Organization learning from each other project

It is generally expected that the portfolio should be designed in such a way as to maximize the possibility of achieving the strategic goals of the company. This is consistent with the notion that portfolio selection problem is a multi-criteria decision making. The main goal of each project is to increase the value of the organization, so most managers prefer financial criteria for project evaluation. The most commonly used indicators include net present value, internal rate of return, payback period, rate of return.

In addition to these financial indicators, however, in selecting a portfolio of projects should be considered other characteristics, which include for example:

- The probability of completing the project on time, within budget and within the proposed quality
- Consistency between strategic and tactical plans
- The balance between investment projects and maintenance projects
- Efficient use of resources

- Relations between projects
- The scope of each project
- Time-dependent consumption of resources on projects
- Allocation of expenditure and resources for research and development
- Allocation of marketing spending and resources

Lots of professionals tried to find sophisticated way to improve techniques for project management in different ways (Larson and Gray, 2013).

3 EFFICIENT INDIVIDUAL PROJECTS

For our problem, there is supposed a set $P = \{P_1, P_2, \dots, P_n\}$ of n projects each consuming r inputs and producing s outputs; (r, n)-matrix X and (s, n)-matrix Y are observed input and output measures. The CCR (Charnes, Cooper, and Rhodes) model with supposed constant return to scale was used for project evaluations. Constant return to scale means that changing the amounts of inputs results in similar changes in the amounts of outputs. For a particular project, the ratio of the single output to the single input provides a measure of efficiency that is a function of the weight multipliers (u,v). The relative efficiency e_k of the project P_k is maximised to the condition that the relative efficiency of each project is less than or equal to one.

A DEA-based approach allows each project to evaluate itself, relative to all the projects under consideration. The formulation leads to a linear fractional programming problem.

$$e_k = \frac{\sum_{i=1}^{s} u_i y_{ik}}{\sum_{j=1}^{r} v_j x_{jk}} \to max, \qquad k = 1, 2, \dots, n \tag{1}$$

$$\frac{\sum_{i=1}^{s} u_i y_{ih}}{\sum_{j=1}^{r} v_j x_{jh}} \le 1, h = 1, 2, \dots, n \tag{2}$$

$$u_i, v_j \ge 0, \qquad i = 1, 2, \dots, s, \qquad j = 1, 2, \dots, r \tag{3}$$

If it is possible to find a set of weights for which the efficiency ratio of the project P_k is equal to one, the project P_k will be considered as efficient otherwise it will be considered as inefficient. The set of efficient projects is designed in this way.

Solving this nonlinear nonconvex problem directly is not an efficient approach. The following linear programming problem with new variable weights (u, v) that results from the Charnes - Cooper transformation gives optimal values that will also be optimal for the fractional programming problem.

$$e_k = \sum_{i=1}^{s} u_i y_{ik} \to max, \quad k = 1, 2, \dots, n, \tag{4}$$

$$\sum_{j=1}^{r} v_j x_{jk} = 1 \tag{5}$$

$$\sum_{i=1}^{s} u_i y_{ih} - \sum_{j=1}^{r} v_j x_{jh} \le 0, h = 1, 2, \dots, n \tag{6}$$

$$u_i, v_j \ge 0, \qquad i = 1, 2, \dots, s, \qquad j = 1, 2, \dots, r \tag{7}$$

The efficiency scores e_k might be used to rank the projects. Implementing the most effective projects until resources are consumed will not always lead to the most effective portfolio. The reason is the same as for the knapsack problem.

4 EFFICIENT PORTFOLIOS

A portfolio as a subset C of the set of possible projects P ($C \subseteq P$) can be taken as a single combined project. The combined project is defined by combinations of outputs and combinations of inputs. The combination vector is $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$ where $\lambda_i = 1$(the individual project P_i is included in the portfolio) or $\lambda_i = 0$ (the individual project P_i is not included in the portfolio). Total inputs of the combined project denoted as $x_j(C) = \sum_{h=1}^{n} \lambda_h x_{jh}$, $j = 1, 2, \dots, r$, and total outputs denoted as $y_i(C) = \sum_{h=1}^{n} \lambda_h y_{ih}$, $i = 1, 2, \dots, s$, are determined by the combination vector λ. The set of all combined projects is the so-called power set of P and the set is denoted as $R(P)$ where the number of elements in $R(P)$ is $2^n - 1$.

DEA-approach can be used for evaluation of each combined project relative to the power set $R(P)$.

$$e_C = \sum_{i=1}^{s} u_i \sum_{h=1}^{n} \lambda_h y_{ih} \rightarrow max \tag{8}$$

$$\sum_{j=1}^{r} v_j \sum_{h=1}^{n} \lambda_h x_{jh} = 1 \tag{9}$$

$$\sum_{i=1}^{s} u_i \sum_{h=1}^{n} \lambda_h y_{ih} - \sum_{j=1}^{r} v_j \sum_{h=1}^{n} \lambda_h x_{jh} \leq 0,\ C \in R(P) \tag{10}$$

$$\lambda_h \in \{0, 1\}, h = 1, 2, \dots, n \tag{11}$$

$$u_i, v_j \geq 0, \quad i = 1, 2, \dots, s, \quad j = 1, 2, \dots, r \tag{12}$$

The model (8)-(12) is a non-linear one with variables λ_h, u_i ,v_j where λ_h are elements of an unknown project combination vector and u_i ,v_j are weights of outputs and inputs. Due to the large number of constraints (10) it is difficult to solve.
Introducing new variables

$$c_{ih} = u_i \lambda_h,\ d_{jh} = v_j \lambda_h, i = 1, 2, \dots, s,\ j = 1, 2, \dots, r,\ h = 1, 2, \dots, n \tag{13}$$

linearizes this problem. The portfolio total inputs and outputs are compared against the set of all portfolios $R(P)$ but it is easy to see that the general constraints (10) are additive combination of constraints for individual projects and it is sufficient to compare them with individual projects from the set P given by the constraint (16) (Cook and Green, 2000). Constraints for combined projects are redundant. The constraints (19) and (20) link new variables c_{ih}, d_{jh} and old variables u_i, v_j, λ_h, where M is a large number. The constraint (19) links the variables c_{ih}, u_i, λ_h. If the binary variable $\lambda_h = 1$, then $0 \leq c_{ih} \leq M, u_i = c_{ih}$ and if the binary variable $\lambda_h = 0$, then $0 \leq u_i \leq M, c_{ih} = 0$. The constraint (20) analogically links the variables d_{jh}, v_j, λ_h. The problem is then formulated as follows:

$$e_C = \sum_{i=1}^{s} \sum_{h=1}^{n} c_{ih} y_{ih} \rightarrow max \tag{14}$$

$$\sum_{j=1}^{r} \sum_{h=1}^{n} d_{jh} x_{jh} = 1 \tag{15}$$

$$\sum_{i=1}^{s} u_i y_{ih} - \sum_{j=1}^{r} v_j x_{jh} \leq 0, h = 1, 2, \dots, n \tag{16}$$

$$\lambda_h \in \{0, 1\}, h = 1, 2, \dots, n \tag{17}$$

$$u_i, v_j \geq 0, \quad i = 1, 2, \dots, s, \quad j = 1, 2, \dots, r, \tag{18}$$

$$c_{ih} \geq 0, c_{ih} \leq M\lambda_h,\ u_i \geq c_{ih}, u_i \leq c_{ih} + M(1 - \lambda_h), i = 1, 2, \dots, s, h = 1, 2, \dots, n \tag{19}$$

$$d_{jh} \geq 0, d_{jh} \leq M\lambda_h,\ v_j \geq d_{jh}, v_j \leq d_{jh} + M(1 - \lambda_h), j = 1, 2, \dots, r, h = 1, 2, \dots, n \tag{20}$$

5 ILLUSTRATIVE EXAMPLE

The proposed analysis will be illustrated on a simple example to make the procedure and calculations easy to understand and to make detailed analyses of all portfolio structures. An organisation considers 5 potential projects $(P_1, P_2, \ldots, P_5)$ that are characterized by two inputs $(I1, I2)$ and two outputs $(O1, O2)$.

The parameters of potential projects are given in Table 1.

Table 1 Parameters of potential projects

	P_1	P_2	P_3	P_4	P_5
$I1_i$	6	3	8	9	5
$I2_i$	8	4	2	4	6
$O1_i$	9	7	6	10	8
$O2_i$	12	10	15	8	12
e_i	0.643	1	1	1	0.761

Source: author's calculations

The efficiency ratios e_i of projects were computed using the model (4)-(7). The set of efficient projects consists of projects P_2, P_3, P_4.

There are 31 possible project portfolios from the 1-project to the 5-projects structure. The model (14)-(20) was used for efficiency evaluation of portfolios with different structures. Table 2 captures portfolio structures, the total number of possible portfolios in the specific structures, maximal efficiency ratios e_C in the structure, and the portfolios with maximal efficiency ratios.

Table 2 Portfolio structures

Structure	**Number**	**Max e_C**	**Portfolios**
1-project	5	1	$P_2,\ P_3,\ P_4$
2-projects	10	1	$(P_2,\ P_3), (P_2,\ P_4), (P_3,\ P_4)$
3-projects	10	1	$(P_2,\ P_3,\ P_4)$
4-projects	5	0.920	$(P_2,\ P_3,\ P_4,\ P_5)$
5-projects	1	0.838	$(P_1, P_2,\ P_3,\ P_4,\ P_5)$

Source: author's calculations

6 DISCUSSION AND CONCLUSION

An approach for efficient project portfolio designing is proposed in the paper. The problem can be formulated as a multi-objective linear binary programming problem. The experiments show that this approach can be an appropriate instrument for analysing project portfolio designing and can produce interesting results in comparison with other approaches. The approach can be used for various types of projects, such as ICT projects. It can also be used for other types of problems, such as selecting an efficient subset of supply chain members with subsequent profit allocation (Fiala, 2016).

The basic model allows possible extensions. The approach can be refined with weight restrictions according to preferences. Analytic Hierarchy Process (AHP) (Saaty, 1990) can be applied for a restriction of weights in the DEA by the decision maker's judgements. The judgements are captured in the comparison matrix $C = \left(c_{jk}\right)$, where elements c_{jk} are judgements of w_j / w_k. The preference region W is structured by column vectors of the comparison matrix C. Any weight vector from W is possible to get as a linear combination of column vectors

$$w = C\mu , \tag{21}$$

where μ is a nonnegative vector of coefficients $\mu = \left(\mu_1, \mu_2, \ldots, \mu_n\right)$. If the matrix C is consistent, the consistency index $C.I. = 0$, the preference region is a line through the origin. If the matrix C is inconsistent, the consistency index $C.I. > 0$, the preference region is a convex cone, the greater consistency index, the greater preference cone.

Another area to approximate reality is to capture the uncertainty in the data used. It is possible to apply imprecise DEA with interval data for inputs and outputs (Smirlis et al., 2004). The basic DEA model can be combined with other approaches, as De Novo optimization (Fiala, 2018). Duality theory (Fiala, 1981) can be applied on the multi-criteria model for searching of efficient project portfolios. The analysis of the dual problem can bring interesting information for managers. The portfolio designing is a multi-agent problem. A consensus can be found by negotiation process with multiple evaluation criteria (Fiala, 1999).

Combinations of the methods for searching an efficient project portfolio and modelling of specific requirements give a powerful approach to capture managerial problems in project portfolio management.

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COMPARISON OF SELECTED MACROECONOMIC INDICATORS IN THE CZECH AND SLOVAK REPUBLICS AT THE NUTS 2 LEVEL

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Abstract
We compare Czech and Slovak macroregions in our article. The comparison will be made at the level of the NUTS 2 regions. As a comparison tool we use 3 basic macroeconomic indicators: Gross domestic product (GDP) at current market prices (purchasing power standard per inhabitant), Disposable income of households (purchasing power standard per inhabitant) and Unemployment rates (%). These indicators express the economic strength of the regions. There are 8 macroregions in Czech Republic an 4 macroregions in Slovak Republic. We work with annual data, each indicator was monitored over different periods. The data source is the Eurostat database. We will compare each indicator separately, at first. Then we make a comparison for all indicators at once using cluster analysis. The result will be a grouping of regions with several homogeneous groups. Each group will include regions with a similar level of macroeconomic indicators.

Keywords: *NUTS 2 Regions, GDP, Unemployment rate, Disposable income of households*

JEL Classification: C500, O520

AMS Classification: 62-07

1 INTRODUCTION

The article compares Czech and Slovak macro-regions at the NUTS2 level in terms of selected macroeconomic indicators. Three basic macroeconomic indicators serve as a comparison tool. These are Gross domestic product (GDP) at current market prices (purchasing power standard per inhabitant), Disposable income of households (purchasing power standard per inhabitant) and Unemployment rates (%). The data source is the Eurostat database. We will use cluster analysis as a tool for measure of similarity. The aim is to find macro-regions of similar economic level. Because the data analyzed are time series, we are also interested in the development of the indicators over time. We will apply cluster analysis to data in different years to assess how stable the classification of individual regions into common clusters is over time.

2 METHODOLOGY

The source of data for our analyzes is the Eurostat database (Eurostat, 2020). The data are presented here in the form of time series in EUR and are comparable. They are recalculated according to purchasing power parity per capita (GDP and Disposable income of households), or are given in percent (%, Unemployment rate). Unfortunately, the time series are not the same length. The GDP series is presented in the Eurostat database for the period 2000-2017, Disposable income of households for 2000-2016 and Unemployment rate for 1999-2018. In case of individual comparison this does not matter. Only in the application of cluster analysis were we limited by the shortest time series, ie 2000-2016. We performed calculations in MS Excel and Statgraphiscs Centurion 18.

The breakdown of regions at NUTS2 level means that we work with 8 regions in the Czech Republic and 4 regions in Slovakia. Their territorial distribution is well visible in Figure 1 (Czech Statistical Office, 2018).
Comparison of the V4 regions was also dealt with in other works (Kuttor, 2009) and (Widuto,2019). A comparison of the Czech regions can be found in (Marek, 2016).

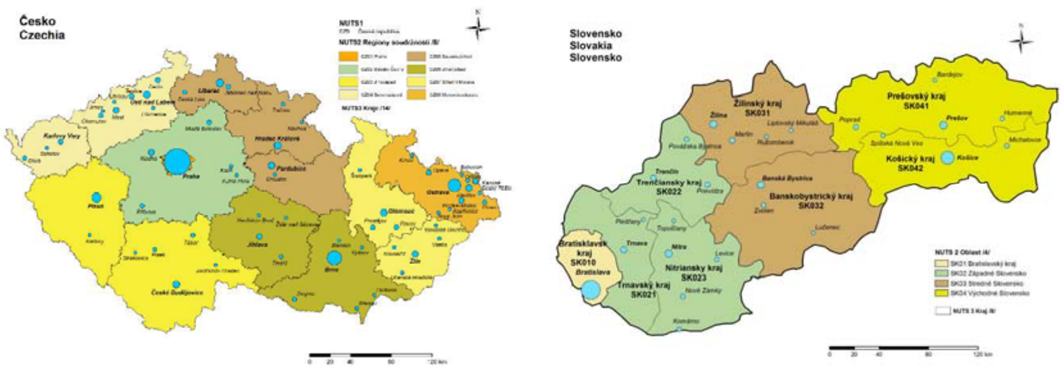

Figure 1: Regions at NUTS 2

Since it is not possible to use the full names of regions in the tables and graphs, the following Table 1 lists the names and their codes, which we will continue to use. It is worth mentioning the regions CZ01 and SK01, which contain the capitals of both countries. These regions will differ significantly from all others, as we will see below.

Czechia		Slovakia	
CZ01	Praha	SK01	Bratislavský kraj
CZ02	Stredni Cechy	SK02	Západné Slovensko
CZ03	Jihozápad	SK03	Stredné Slovensko
CZ04	Severozápad	SK04	Východné Slovensko
CZ05	Severovýchod		
CZ06	Jihovýchod		
CZ07	Stredni Morava		
CZ08	Moravskoslezsko		

Table 1: Regions at NUTS 2

3 ANALYSIS

3.1 GDP

We work with Gross domestic product (GDP) at current market prices by NUTS 2 regions (Purchasing power standard per inhabitant). GDP is probably the most important macro-economic indicator for comparing the economic power of regions. We analyze data for the period 2000-2017. As already mentioned, the capital cities - CZ01 Praha and SK 01 Bratislava region - are very different from other regions. The distance from other regions is enormous at all times. The other regions are not very different, especially in recent years. The long-term lowest value of GDP is in the Slovak regions SK03 Central Slovakia and SK04 Eastern Slovakia.

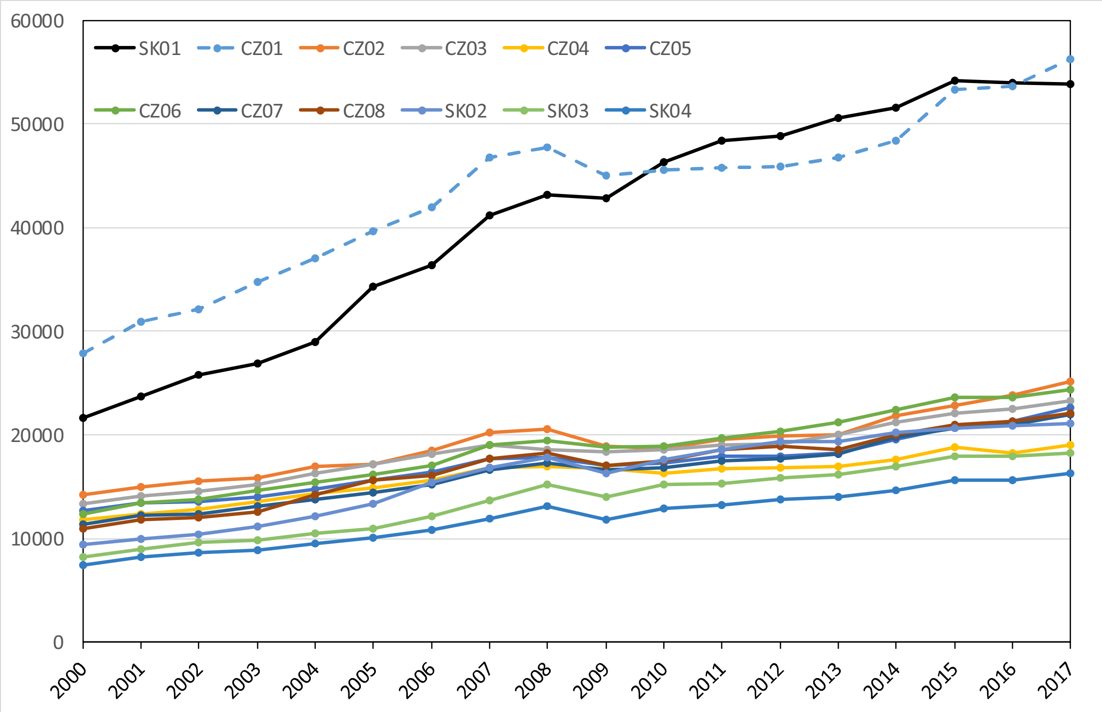

Figure 2: GDP

3.2 Disposable income of private households

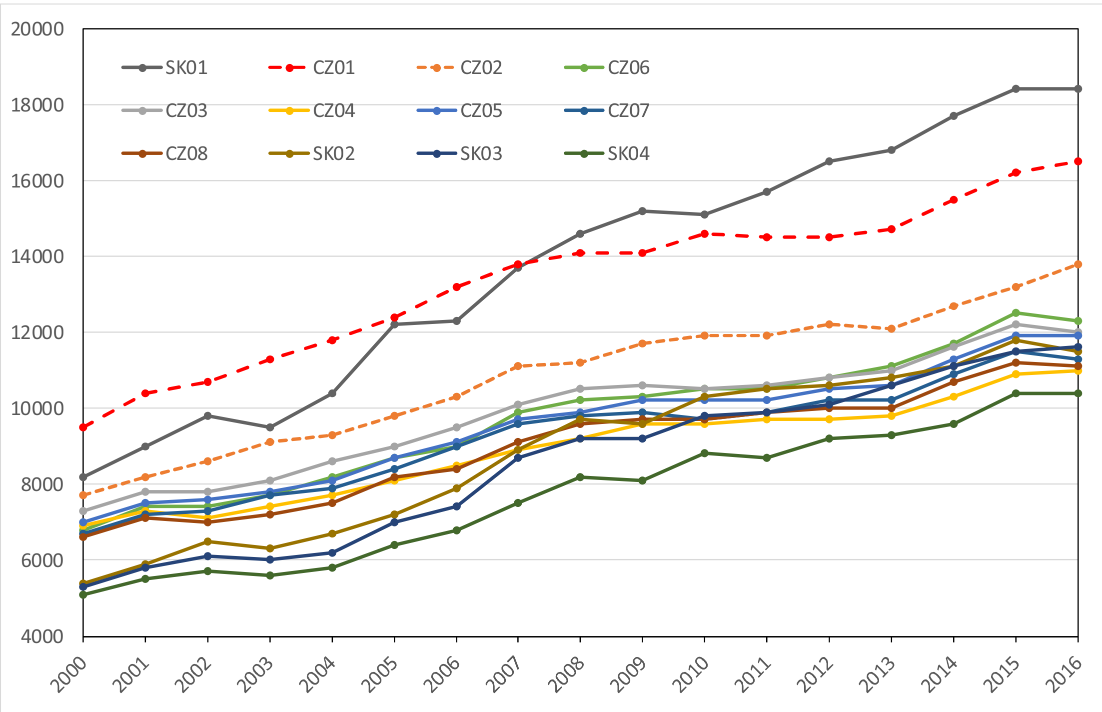

Figure 3: Disposable income

Disposable income is another important macroeconomic indicator that is often used for comparing territorial units. Now we work with data for the period 2000-2016. Again, it is clear that the regions CZ01 Praha and SK 01 Bratislava are doing best. The values in the Slovak region have been significantly higher than in the Czech region in recent years. The Czech region CZ02 Stredni Cechy holds third place from other regions. This region has its economic power based on the existence of two car factories (Škoda Mladá Boleslav and TPCA in Ovčáry u Kolína). As in GDP, the last position is taken by the Slovak region SK04 Eastern Slovakia.

3.3 Unemployment rates

Unemployment rates is economic indicator that is very often mentioned by economists and journalists and is regularly published in the media. Let's look at its trend over time. This time we have the longest time series available for the period 1999-2018.

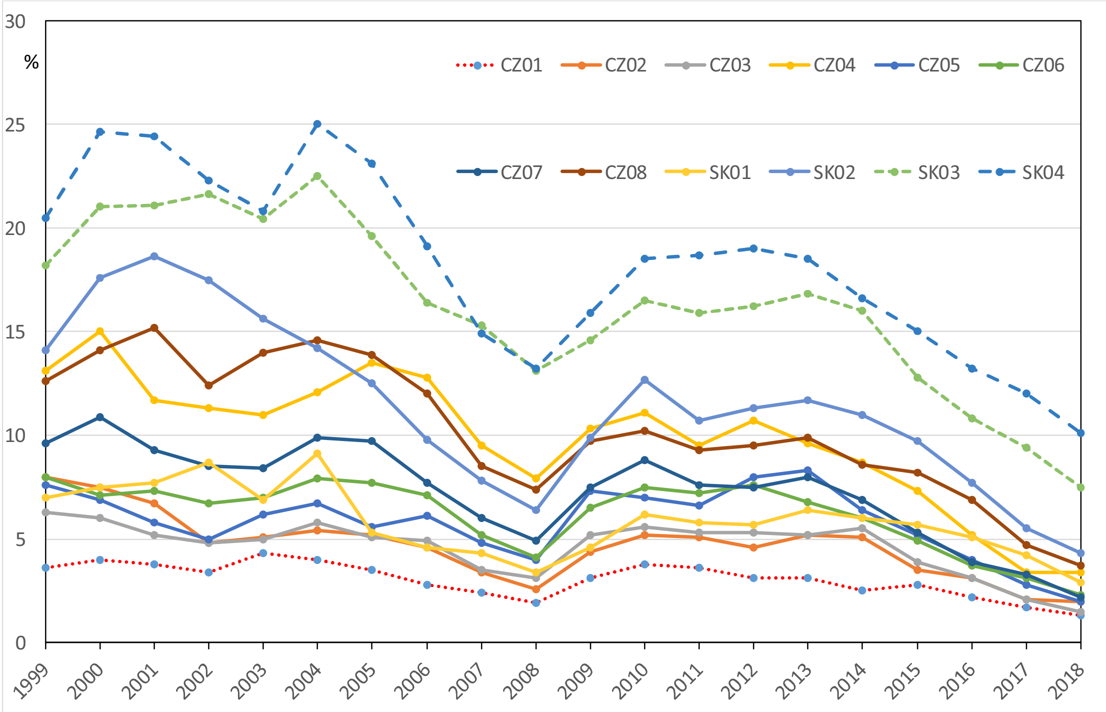

Figure 4: Unemployment rate

The CZ01 Prague region has the lowest unemployment rate for 20 years. Two other Czech regions CZ02 Stredni Cechy and CZ03 Jihozápad are very close. The highest values are clearly in the two Slovak regions SK03 Central Slovakia and SK04 Eastern Slovakia.

3.4 Cluster analysis

For further analysis we will use cluster analysis methods. This analysis is applied to all three indicators in a given year at once. We try to divide the regions into 4 homogeneous clusters. The resulting clusters should confirm the results already achieved. We work with time series and so it is logical that we are interested in development over time, too. Therefore, we chose three different years in which to perform cluster analysis. Based on the results, we will assess how stable the clusters are over time. For comparison we have chosen years 2000, 2008 and 2017. Results are displayed in dendrograms. Of course, we have received a large number of other outputs, but these are not given for capacity reasons.

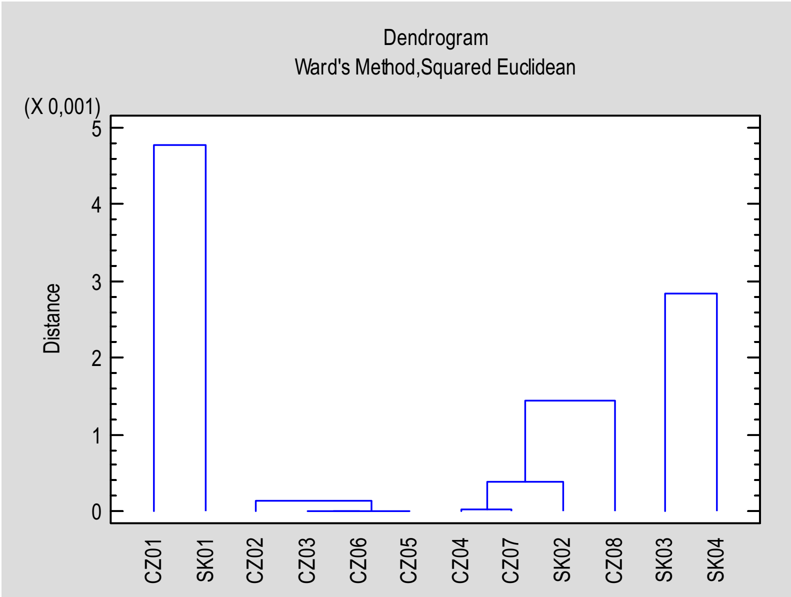

Figure 5: Year 2000

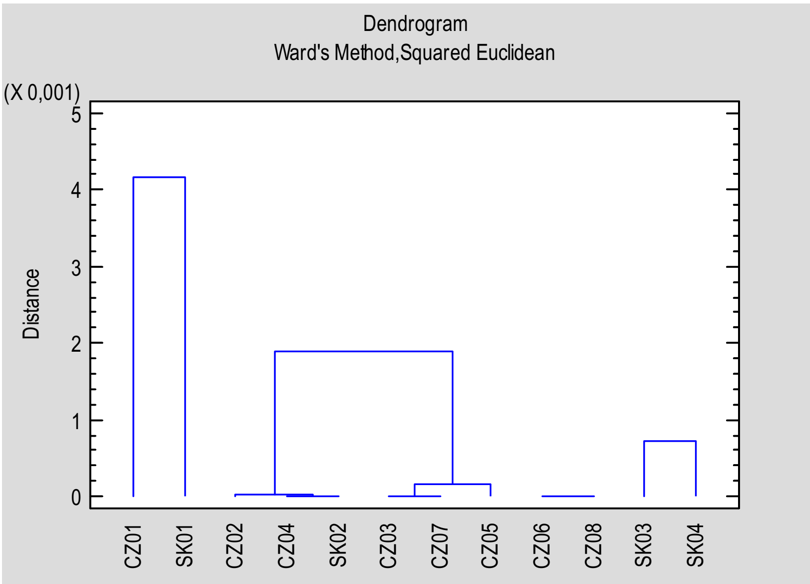

Figure 6: Year 2008

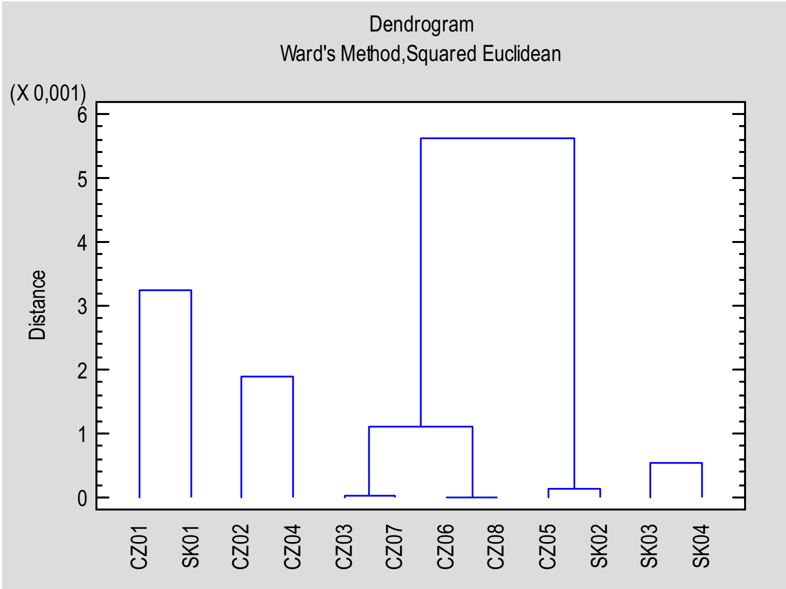

Figure 7: Year 2016

The results of the cluster analysis confirmed our previous considerations. In all years, 4 clusters were formed which have the same or very similar composition. The CZ01 Praha and SK 01 Bratislavský kraj created a single cluster. The next cluster is made up of the regions SK03 Stredné Slovensko and SK04 Východné Slovensko. The situation has not changed in 17 years. In the other two clusters the regions exchange places occasionally, but in all three years the regions CZ07 Strední Morava and SK02 Západné Slovensko stay together in one cluster. If we were limited to 2008 and 2016, the situation is much more stable - in one cluster in both years there are regions CZ03 Jihozápad, CZ05 Severovýchod, CZ07 Strední Morava and SK02 Západné Slovensko. This means that the situation has been very stable over the past 9 years. Out of the total of 12 compared regions, 8 regions hold their position in one cluster, only in 4 regions the economic indicators changed so that their position changed.

4 CONCLUSIONS

The aim of the article was to compare the regions of the Czech Republic and Slovakia at the NUTS 2 level. We first performed this comparison graphically, and then confirmed our conclusions using cluster analysis. We grouped the regions into four clusters and examined the representation of individual regions in these clusters. It has been confirmed that regions with capital cities have a privileged position. The affiliation to individual clusters was largely determined by the economic situation in each region. Mostly the same regions appeared in individual clusters. Two clusters remained stable for 18 years. However, if the comparison was limited to 10 years, the situation was much more stable. This suggests that, despite extensive subsidies to these regions, the position of the regions does not change much.

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TRANSPORT INFRASTRUCTURE IN THE MODELS OF ECONOMIC GROWTH

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Abstract
Transport infrastructure plays an important role in the process of economic growth, especially in developing countries. In addition to being considered as a direct production factor, improved transport infrastructure can reduce transport costs and raise productivity. Moreover, transport infrastructure investment can stimulate construction sector and manufacturing industry, as well as create positive expectations and favourable business environment for private investment. This paper presents two different approaches for incorporating infrastructure into the analysis of economic growth: the production function approach and the cost function approach. Based on the production function approach, the model of economic growth, which includes various types of transport infrastructure, is defined. Due to the possible non-linear impact of transport infrastructure on economic growth, two different types of models are proposed: a model which includes the interactions of transport infrastructure and other important variables and a fixed effects panel threshold model. The advantages of the econometric analysis of panel data are highlighted, as well as the need for testing all econometric assumptions, considering that ignoring of econometric problems can cause biased results and wrong conclusions.

Keywords: *transport infrastructure, economic growth, production function, econometric analysis, panel data.*

JEL Classification: H54, O41, C23
AMS Classification: 91B62

1 INTRODUCTION

Transport infrastructure plays a very important role in the process of economic growth and development, as confirmed by numerous empirical researches during the last three decades. Fedderke and Garlick (2008) distinguish three main channels of impact of transport infrastructure on economic growth.

Firstly, infrastructure can be considered as a direct input in the production process. Secondly, infrastructure is a complementary factor to other factors of production. On the one hand, improved infrastructure can reduce transportation costs, while inadequate infrastructure creates many additional costs related to more expensive transportation of raw materials and finished products. On the other hand, appropriate infrastructure increases the productivity of other factors of production, i.e. well-developed transport infrastructure can increase the total factor productivity. Thirdly, development of infrastructure represents a stimulus for the accumulation of factors of production. All these channels are related to the supply side.

Within the mechanisms on the supply side, the role of telecommunications infrastructure in reducing transaction costs, and thereby creating positive impact on the production and economic growth, must be emphasized. Moreover, development of telecommunications infrastructure, primarily development of broadband technologies, increases the productivity of labour force (Koutroumpis, 2009).

Investment in transport infrastructure also provides positive signals to the key sectors of the economy (Miljković and Petrović-Vujačić, 2016). Infrastructure can also be used as a tool for industrial policy. States can create positive expectations, especially during recessions, by specific infrastructure projects, which can attract additional private sector investment. This mechanism is recognized by the European Commission (2014).

There are also effects of transport infrastructure on economic growth, which are related to the demand side. Infrastructure investment can stimulate aggregate demand, given that large infrastructural projects, which require considerable expenditures during the construction phase, can create a stimulus for construction sector, as well as for a series of related branches of manufacturing industry (Miljković and Petrović-Vujačić, 2016).

Considering the important role of transport infrastructure in the process of economic growth, the main aim of this paper is to present some quantitative approaches which could be applied in the empirical analyses of economic growth.

2 INFRASTRUCTURE IN QUANTITATIVE ANALYSIS OF ECONOMIC GROWTH

There are two main methods for incorporating infrastructure in the analysis of economic growth: the production function approach, and the cost function approach, which are presented in this chapter of the paper.

2.1 Production function approach

Most empirical researches related to the impact of infrastructure on economic growth done to date were based on the production function. Munnell (1990) defined the total production (Y) as a function of private capital (K), labour (L), total factor productivity as an indicator of technological development (T) and public capital as an indicator of public infrastructure (G):

$$Y = T \times f(K, L, G) \tag{1}$$

This function can be presented in the form of the Cobb-Douglas production function:

$$Y = T \times K^{\alpha} L^{\beta} G^{\gamma} \tag{2}$$

which after taking logarithms of both sides gets the following form:

$$\ln Y = \ln T + \alpha \ln K + \beta \ln L + \gamma \ln G \tag{3}$$

where α, β and γ represent coefficients of output elasticity with respect to the included factors of production.
Munnell (1990) suggested that constant returns to scale should apply only on labour and private capital (α+β=1), while the entire function should have increasing returns to scale, i.e.

$$\ln Y = \ln T + \alpha (\ln K - \ln L) + \ln L + \gamma \ln G \tag{4}$$

Increasing returns to scale for all factors, while at the same time constant for labour and private capital, mean that just public capital, i.e. infrastructure, contributes to the increasing returns to scale.
On the other side, if constant returns to scale would apply to all production factors (α+β+γ=1), the production function would be as follows:

$$\ln Y = \ln T + \alpha (\ln K - \ln L) + \ln L + \gamma (\ln G - \ln L) \tag{5}$$

Moreno et al. (1997) suggested the same framework, but they distinguished two forms of public capital: basic (economic) infrastructure and social infrastructure. Basic infrastructure (G_b) includes transportation, water supply and sewerage, and electric power infrastructure, while social infrastructure (G_s) includes infrastructure of the health and education sector. The equation of Munnell (3) can now be expanded:

$$\ln Y = \ln T + \alpha \ln K + \beta \ln L + \gamma_1 \ln G_b + \gamma_2 \ln G_s \tag{6}$$

Madden & Savage (2000) proposed a Cobb-Douglas production function with directly included telecommunications infrastructure (Tel):

$$Y = K^{\alpha} H^{\beta} Tel^{\gamma} (T \times L)^{1-\alpha-\beta-\gamma} \tag{7}$$

where H stands for human capital, and technology (T) creates impact on labour efficiency.

2.2 Cost function approach

In addition to the production function approach which treats infrastructure as a direct factor in the production process, there is also a cost function approach. This approach was developed by Morrison and Schwartz (1996), given the advantages of the wide framework for the microeconomic analysis of firms.

Cost function provides the opportunity for a detailed analysis of the impact of infrastructure on production performance. A starting point is the function of total costs (C):

$$C = VC + p_k K + p_g G \tag{8}$$

where p_k stands for the price of private capital (K), p_g for the price of public capital (G), and VC for variable costs, which are the function of total level of capital (X=K+G), prices of variable inputs (p, which includes price of productive labour, price of non-productive labour and price of energy), time as an indicator of technological progress (t) and output (Y):

$$VC = f(X, p, t, Y) \tag{9}$$

Given that firms do not bear the cost of public capital, the function of total cost can be presented as follows:

$$C = VC + p_k K \tag{10}$$

Valuation of the contribution of infrastructure investment to the firm's efficiency starts from defining the level of marginal benefit of public capital (Z_g), which represents the reduction of variable costs as a result of incremented level of public capital by one unit:

$$Z_g = \frac{-\partial VC}{\partial G} \tag{11}$$

Given that the level of public capital is not a choise of firms, the equality between marginal benefit of public capital and price of public capital (p_g), cannot be an optimization condition which firms can achieve. Moreover, firms do not pay for the amount of public capital (p_g=0), because taxes are not explicitly related to the public infrastructure investment. Therefore, if Z_g is larger than zero, a firm would benefit from each increase of the level of public capital.

However, given that infrastructure is not free of charge from the social perspective, the benefits have to be compared with the social user cost of public capital. But the evaluation of these costs is very demanding, which is the main obstacle for application of this approach to some empirical researches.

3 MODEL OF ECONOMIC GROWTH WITH VARIOUS TYPES OF TRANSPORT INFRASTRUCTURE INVESTMENT

Starting from the production function approach developed by Munnell (1990), which included public capital as an indicator of the level of infrastructure in the production function, we propose the following economic growth model with transport infrastructure investment as a direct production factor:

$$y_{it} = \beta_1 + \beta_2 TII_{it} + \beta_3 I_{it} + \beta_4 Emp_{it} + \beta_5 Edu_{it} + \beta_6 Open_{it} + \beta_7 FDI_{it} + u_{it} \quad (12)$$

Dependent variable (y_{it}) represents the growth rate of gross domestic product (GDP) in real terms, expressed in percentage. Independent variables include, first of all, investment in transport infrastructure (TII_{it}), which include investment in road infrastructure ($Road_{it}$), investment in rail infrastructure ($Rail_{it}$) and investment in telecommunications infrastructure (Tel_{it}). All these variables are expressed in percentage of GDP. Therefore, the economic growth model could also be defined as follows:

$$y_{it} = \beta_1 + \beta_2 Road_{it} + \beta_3 Rail_{it} + \beta_4 Tel_{it} + \beta_5 I_{it} + \beta_6 Emp_{it} + \beta_6 Edu_{it} + \beta_7 Open_{it} + \beta_8 FDI_{it} + u_{it} \quad (13)$$

On the left side of the equation (12) and (13) is the growth rate of GDP, because on the right side of the equation is investment (as an indicator of flow). If on the right side of the equation would be some indicator of infrastructure stock, then on the left side should not be the growth rate, but an indicator of level of gross domestic product.

Investment in fixed assets (I_{it}) represent an independent variable which is related to private sector investment. It is calculated as total investment in fixed assets of the entire economy minus investment in transport (road, rail and telecommunications) infrastructure, and it is also expressed as a percentage of GDP.

Independent variables related to labour force should include employment rate (Emp_{it}), and some indicator of the quality of labour force, such as level of education, i.e. gross enrolment rate in tertiary education (Edu_{it}). The model includes also other important macroeconomic variables, such as trade openness (value of total exports and imports as percentage of GDP), as well as foreign direct investment (as percentage of GDP, too). These variables are included, given that many empirical researches of economic growth identified these variables as highly significant, especially in developing countries.

The sign "i" stands for different countries, so that i = 1, 2, …., N, while the sign "t" stands for different time period, so that t = 1, 2, ……., T. Therefore, we propose the analysis of panel date, because there are many benefits from using panel data, as listed by Baltagi (2008):

- Controlling for individual heterogeneity.
- More informative data, more variability, less collinearity among the variables, more degrees of freedom and more efficiency.
- Panel data are better able to study the dynamics of adjustment.
- Panel data are able to identify and measure effects that are simply not detectable in pure cross-section or pure time-series data, etc.

The model should be applied on countries, but not on regions. The problem with using a production function at the regional level is that the impact of the infrastructure is often spilled over from one to the other regions, and that it has a positive effect on the wider area (Rietveld, 1989).

The equation (12) is related to the pooled panel regression, without individual and time effects. In the case of fixed individual and time effects, the coefficient of β_1 should be replaced with β_{1it}:

$$\beta_{1it} = \beta_1 + \mu_i + \lambda_t \quad (14)$$

where μ_i denotes the time-invariant unobservable individual specific effect and λ_t the individual-invariant unobservable time specific effect which are not included directly into regression. On the other hand, in the case of stochastic individual and time effects, the individual and time effects are components of the disturbance u_{it} from the equation (12), i.e.:

$$u_{it} = \mu_i + \lambda_t + v_{it} \quad (15)$$

4 MODELLING OF NON-LINEAR IMPACT OF TRANSPORT INFRASTRUCTURE INVESTMENT ON ECONOMIC GROWTH

Impact of transport infrastructure investment on economic growth can be quite different depending on the achieved level of transport infrastructure, given that the law on diminishing returns states that adding more of one factor of production will ceteris paribus yield lower incremental per unit returns. That means than transport infrastructure investment could have greater and more significant impact on economic growth in the countries with less developed infrastructure, i.e. in the countries with lower per capita income. On the other hand, transport infrastructure investment would create small, maybe even insignificant impact on economic growth in the countries with well-developed transport infrastructure, i.e. in the countries with highest level of per capita income.

Moreover, based on the opinion of Banister and Berechman (2001), that transport infrastructure investment could have a significant impact on economic growth only if some preconditions related to economic, investment and institutional environment would be fulfilled. Therefore, we start from the assumption that transport infrastructure investment has a greater impact on economic growth in the countries with higher trade openness and with better educated workforce.

In order to consider the possibility of such non-linear impacts, we propose two groups of models: models with interaction of variables and fixed effects panel threshold model.

4.1 Models with interaction of variables

Starting from the equation (12), we add the interaction between transport infrastructure investment (TII_{it}) and a dummy variable (M_{it}), which takes the value of 0 if a country has high level of income and the value of 1 if a country has middle or low income, according to the classification of World Bank:

$$y_{it} = \beta_1 + \beta_2 TII_{it} + \beta_3(TII_{it} \times M_{it}) + \beta_4 Iit + \beta_5 Emp_{it} + \beta_6 Edu_{it} + \beta_7 Open_{it} + \beta_8 FDI_{it} + u_{it} \quad (16)$$

Moreover, we add the interaction between transport infrastructure investment (TII_{it}) and trade openness ($Open_{it}$) in the equation (12):

$$y_{it} = \beta_1 + \beta_2 TII_{it} + \beta_3(TII_{it} \times Open_{it}) + \beta_4 Iit + \beta_5 Emp_{it} + \beta_6 Edu_{it} + \beta_7 Open_{it} + \beta_8 FDI_{it} + u_{it} \quad (17)$$

And finally, we also add the interaction between transport infrastructure investment (TII_{it}) and level of education (Edu_{it}) in the equation (12):

$$y_{it} = \beta_1 + \beta_2 TII_{it} + \beta_3(TII_{it} \times Edu_{it}) + \beta_4 Iit + \beta_5 Emp_{it} + \beta_6 Edu_{it} + \beta_7 Open_{it} + \beta_8 FDI_{it} + u_{it} \quad (18)$$

The key challenge is to detect whether there is a joint significance between transport infrastructure (TII_{it}) and its interaction with other variables ($TII_{it} \times M_{it}$, $TII_{it} \times Open_{it}$, $TII_{it} \times Edu_{it}$). If there is a joint significance, then it should be looked at the sign of the parameters β_2 and β_3 and define adequate conclusions.

4.2 Fixed effects panel threshold models

If we identify the significance of the fixed individual effects, we can also apply the fixed effects threshold model, based on the model defined by Hansen (1999). In order to detect possible non-linear impact of transport infrastructure investment on economic growth with respect to achieved income level, the following threshold model is defined:

$$y_{it} = \beta_{1i} + \alpha_1 TII_{it} + \beta_2 I_{it} + \beta_3 Emp_{it} + \beta_4 Edu_{it} + \beta_5 Open_{it} + \beta_6 FDI_{it} + u_{it}, \quad \text{for } Inc_{it} \leq \gamma$$
$$y_{it} = \beta_{1i} + \alpha_2 TII_{it} + \beta_2 I_{it} + \beta_3 Emp_{it} + \beta_4 Edu_{it} + \beta_5 Open_{it} + \beta_6 FDI_{it} + u_{it}, \quad \text{for } Inc_{it} > \gamma \qquad (19)$$

It means that below some threshold level (γ) of income per capita (Inc_{it}), the coefficient which describes the nature of impact of transport infrastructure investment on the growth rate of GDP amounts to α_1, while for the level above the threshold level, the mentioned coefficient amounts to α_2.

The same logic is applied for identifying the threshold level of trade openness:

$$y_{it} = \beta_{1i} + \alpha_1 TII_{it} + \beta_2 I_{it} + \beta_3 Emp_{it} + \beta_4 Edu_{it} + \beta_5 Open_{it} + \beta_6 FDI_{it} + u_{it}, \quad \text{for } Open_{it} \leq \gamma$$
$$y_{it} = \beta_{1i} + \alpha_2 TII_{it} + \beta_2 I_{it} + \beta_3 Emp_{it} + \beta_4 Edu_{it} + \beta_5 Open_{it} + \beta_6 FDI_{it} + u_{it}, \quad \text{for } Open_{it} > \gamma \qquad (20)$$

As well as for identifying the threshold level of education:

$$y_{it} = \beta_{1i} + \alpha_1 TII_{it} + \beta_2 I_{it} + \beta_3 Emp_{it} + \beta_4 Edu_{it} + \beta_5 Open_{it} + \beta_6 FDI_{it} + u_{it}, \quad \text{for } Edu_{it} \leq \gamma$$
$$y_{it} = \beta_{1i} + \alpha_2 TII_{it} + \beta_2 I_{it} + \beta_3 Emp_{it} + \beta_4 Edu_{it} + \beta_5 Open_{it} + \beta_6 FDI_{it} + u_{it}, \quad \text{for } Edu_{it} > \gamma \qquad (21)$$

The significance of the threshold should be tested by the bootstrap method. The insignificant threshold would mean that there is a linear impact of transport infrastructure investment on economic growth.

5 CONCLUSIONS

Development of transport infrastructure is very important for economic growth. There are different ways for incorporating transport infrastructure in the analysis of economic growth, including the production function approach and the cost function approach. Based on the production function approach, we defined and proposed an economic growth model which includes various types of transport infrastructure investment, i.e. investment in road, rail and telecommunications infrastructure. Moreover, we defined different models with interactions of transport infrastructure investment and other variables, as well as fixed effects panel threshold models.

Finally, we have to emphasize that all econometric assumptions have to be tested at the very beginning of the empirical analysis, including tests for individual and time effects, autocorrelation, heteroscedasticity, specification, cross-sectional dependence, endogeneity, and stationarity of all variables in the model. All identified econometric problems must be solved, given that ignoring of econometric problems can cause biased results and wrong conclusions.

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SLOVAK FIRMS ROBUST MULTIPLE CRITERIA RISK AND PERFORMANCE ANALYSIS

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Abstract
A firm position on the selected market is very important for trading and investment decisions. The paper presents a decision support system methodology for the selection of a firms set that satisfy person interested (investor or business partner requirements. It is assumed that the requirements are stated in the form of relative or absolute financial indicators for a specified classification of the goal firms set. The experimental applications of combined PROMETHEE methods use data form the Slovak Republic register of balances of accounts for the year 2018 for over 100 000 firms. The data are adapted from a view point of mutual comparability. The applied approach shows how one can use multiple criteria approaches in confrontation with selected benchmark for risk minimization decisions concerning investment opportunities or business relation looking for.

Keywords: *financial indicators, outranking methods, benchmark, investment opportunities*

JEL Classification: G32
AMS Classification: 90C29

1 INTRODUCTION

In business environment a distinguished attention is devoted to business subject analyses that are available from public information. The financial analyses are needed mainly for qualified decisions about financial flows direction among businessmen, banks, investor, state, and so on. The paper offers the methodological tool for using financial data from the Slovak register of balance accounts for such aims. This tool on the base of evaluation of statistical relevant financial data and selected criteria for examination of importance compared set gives possibility to help in decisions concerning business partner risk evaluation, investment opportunities looking for, and so on.

For mutual comparisons of firms in the whole Slovak firms system (over 450 000 units) is important to start from comparable quality of financial data. The company CRIF (Slovak Credit Bureau) suggest on the base of analyses to create for statistical analysis for year 2018 the firms set that have equal properties from the view point of continues business period (at least three years), marketing season, and approved balanced of accounts. For the year 2018 this basic set consists of more than 105 000 firms. The selected firms then serve for definitions of statistical benchmarks for sector characteristics of relative financial indicators. This set of firms with their financial data we assume as a representative one for all type of tasks that will satisfy described modeling methodology.

2 MODEL STRUCTURE CREATION

For a purpose to define a feasible (requested) firms field we will use

a) database of absolute financial data from the register of balance accounts from book of profit and loss and form balance sheet;
b) database of relative financial indicators;

c) database of administrative firms classification according economic activities SK NACE and database of regional classification.

These databases were made available by the non-profit organization PROFINI and CRIF – Slovak Credit Bureau as part of the project "Sectoral Standards for Double-Entry Entrepreneurs" project code 314011L717, Call code: OP EVS DOP-PO1-SC1.1-2017-1.

Let us note that a specification of the set of firms can be also stated by the specific selection of individual firms according to own decisions. From the practical point of view the specific databases can be linked to one in such forms, where each firm will be characterized by data from all blocks. The firms set creation wit specified properties can by restricted by simple filters. For example one can be interested from specified reasons in firms from car industry with property over 4 million EUR in Košice region. For selected field of firms we will formulate analytical tasks concerning our decisions.

To selected firms set we assign absolute and relative value if financial indicators that really influence source of complex information for decision process. These data are used for a complex evaluation of financial quality of the selected set. One of the known approaches to complex ranking of selected alternatives (firms) represents the family of PROMETHEE methods. The methodology is well known a one can finds details e.g. in [1], [2], or [4].This approach application requires defining a group of evaluation criteria. For this purposes on can select some or all from relative indicators, absolute indicators or otherwise defined criteria as well. It depends on the purpose for which the model is constructed. For example, one can select criteria for:

1. liquidity – an ability to pay liabilities in one year horizon,
2. activity – assets liability or exploitation,
3. indebtedness, which describe a level of external sources using, their structure and repayment ability,
4. rent ability and performance – a firm efficiency or firm capital appreciation.

The paper uses model structure for the PROMETHEE II method where in the form of criteria we use a selected group from relative indicators described in the Table 1.

The importance role in the PROMETHEE method pays the selection of preference functions for selected criteria that reflect the preference power of the criterion values difference between each couple of variants. In our case the preference function value for criterion $C_j, j = 1, \dots, k,$ will depend on the difference of criteria C_j values for firms x_p and x_q, it means from $C_j(x_p) - C_j(x_q)$. Let us denote shortly this difference as d_j. Owing to great number of variants we use for all criteria *Gaussian* preference function that for $d_j \geq 0$ can be written in the form

$$P(d_j) = 1 - e^{\frac{d_j^2}{2\sigma_j^2}} \tag{1}$$

where σ_j is the standard deviation of data set for selected criterion. In this case the power of preference is ascending function of criterion values differences that for high values of d approaches to 1 and in the value of difference equal σ_j the convexity of the function changes into concavity and growth of the preference slows down. The advantages of such function consists in the fact that the function take into account statistical properties of criterion data and for higher value of standard deviation the preference *P(d)* is less sensitive on the difference *d.* These properties of preference function are illustrated on the Figure 1. It is well known that besides preference function selection in PROMETHEE approach one can define the nonnegative weight of importance for each criterion. Usually one can start with same weights for all criteria and then on the result of sensitivity analysis can they change in desired selection. Corresponding multiple criteria approach results are then confronted with specified benchmark as it was suggested e.g. in [3].

Table 1: Relative indicators

Criterion	Name	Type
C_1	liabilities/assets * 100	Min
C_2	Inventories/turnover * 360	Min
C_3	short term trade receivables/turnover * 360	Min
C_4	short term trade payables/ turnover * 360	Min
C_5	receivables/turnover * 360	Min
C_6	payables/turnover * 360	Min
C_7	long term payables/turnover * 100	Min
C_8	EBIDTA/revenues * 100	Max
C_9	gross profit/ assets * 100	Max
C_10	short term property without inventories/short term external sources	Max
C_11	short term property/ short term external sources	Max
C_12	turnover/assets	Max
C_13	PO_EBITDA/revenues * 100	Max
C_14	newly-formed value/ revenues * 100	Max
C_15	value-added/revenues * 100	Max
C_16	profit or loss from ordinary activities/revenues * 100	Max
C_17	net profit/equity * 100	Max
C_18	liabilities/ balance cash flow	Min
C_19	gross profit and interest expense/interest expense	Max
C_20	bank credits/ assets * 100	Min
C_21	equity/liabilities	Max

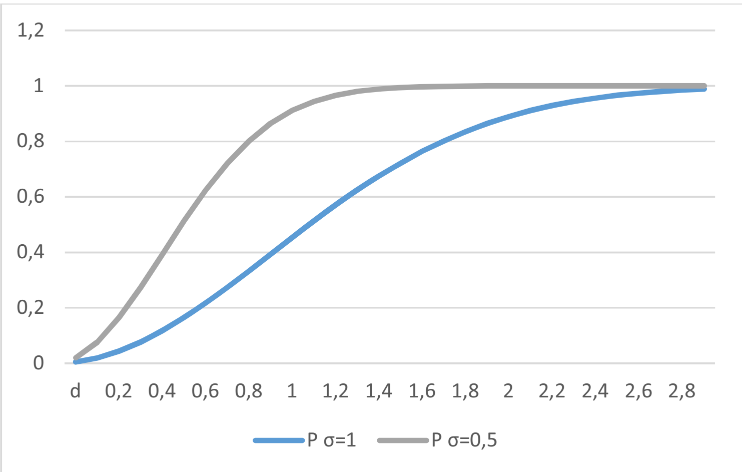

Figure 1: Gaussian preference function

3 DECISION MAKING APPLICATIONS OF MODEL STRUCTURE

At the first step we select such firms from the whole set of firms that satisfy decision maker requirements. From the set of criteria we select decision maker relevant ones assign weights of their importance. Application of PROMETHEE II method then provide ordering of the firm on the base so called net flows. The net flows can be interpreted as the difference between average value of firm preference index owing to the other firms and average preference index of the other firms owing to the firm. As a result we have firms with positive net flow (*good firms*) and firms with negative net flow (*bad firms*)

For each couple of firms x_p and x_q we compute the index of multiple criteria preference (the preference of the firm x_p owing to the firm x_q) $\pi(x_p, x_q)$ according to the relation

$$\pi(x_p, x_q) = \sum_{j=1}^{k} P\big(d_j(x_p, x_q)\big)\, w_j, \qquad (2)$$

where w_j are the criteria importance weights. Then so called outgoing and ingoing flows are computed for each firm x_p, $p = 1, 2, \ldots, n$, in the form

$$F^+(x_p) = \frac{1}{n-1}\sum_{x_q} \pi(x_p, x_q), \qquad F^-(x_p) = \frac{1}{n-1}\sum_{x_q} \pi(x_q, x_p), \qquad (3)$$

and the net flow in the form

$$F(x_p) = F^+(x_p) - F^-(x_p) \qquad (4)$$

Ordering of the selected set of the firms gives possibility to choice for decisions firms among so called *good firms* with positive net flows. Such firms can be from the viewpoint of decision maker assumed as best candidates that have the least risk for selected criteria. In the following stage the investor, producer or trader looks for its goals the most advisable partner or investment opportunity, where the advisability can be described trough values of absolute and relative financial indicators.

Table 2: Selected criteria

Criteria	Name	Type	Weight	sigma
C_1	liabilities/assets * 100	min	7,692%	22,351
C_7	long term payables/turnover * 100	min	7,692%	23,500
C_8	EBIDTA/revenues * 100	max	7,692%	4,847
C_9	gross profit/ assets * 100	max	7,692%	12,945
C_12	turnover/assets	max	7,692%	1,754
C_13	PO_EBITDA/revenues * 100	max	7,692%	4,787
C_14	newly-formed value/ revenues * 100	max	7,692%	5,392
C_15	value-added/revenues * 100	max	7,692%	16,021
C_16	profit or loss from ordinary activities/revenues * 100	max	7,692%	4,639
C_17	net profit/equity * 100	max	7,692%	23,500
C_19	gross profit and interest expense/interest expense	max	7,692%	6190,4
C_20	bank credits/ assets * 100	min	7,692%	2,081
C_21	equity/liabilities	max	7,692%	0,709

4 INVESTMENT OPPORTUNITY SELECTION

As an illustration of the decision process for investment opportunity selection assume the intention to buy a share of the firm in the Slovak car industry at about for 5 million EUR. We create the group of the firms from car industries and as a result we have 148 potential firms. Then we select firms with property at least 4 million EUR, turnover at least 8 million EUR and work in Bratislava region. Finally we have 11 firms. We would like to stress that such reduction of the size is only owing to illustration purposes. Developed decision support system can really process problems with high dimensions. Selected criteria are present in the Table 2. PROMETHEE II results are presented in the Table 3 and graphical illustration on the Figure 2. As a benchmark form PROMETHEE II results the difference between equity and liabilities was selected. One can see that this benchmark is positive for firms on the first, third and eight places. It says that final decision will between firms on the first and third places.

Table 3: Results

Sorted results			
ID Firms	Ranking flows (%)	Equity less liabilities (mil EUR)	ranking of companies
44996365	44,61	8,139021	1
35881704	23,04	-1,887987	2
31364217	19,17	12,726491	3
35825251	18,70	-30,230066	4
35799218	4,36	-100,882635	5
36859893	-2,85	-34,458793	6
35785136	-4,66	-4,629802	7
35798513	-9,45	7,431317	8
35779594	-11,08	-2,743019	9
31392482	-35,83	-4,408073	10
35811650	-46,00	-27,450555	11

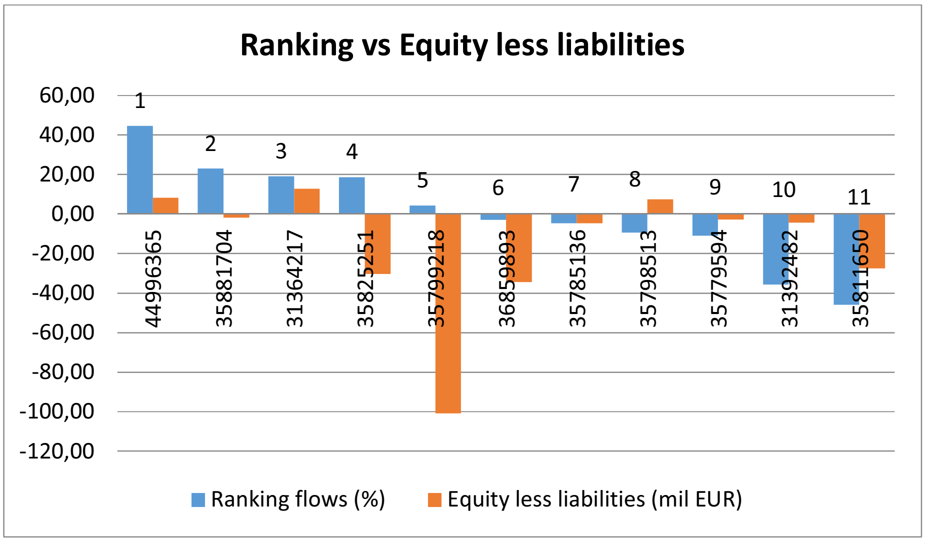

Figure 2: Ranking flows vs Equity less liabilities

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EXCHANGE RATE UNCERTAINTY AND ECONOMIC GROWTH: CASE STUDY OF CZECH REPUBLIC

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Abstrakt

Způsob, jakým je řízen měnový kurz, je důležitý pro ekonomický růst. V letech 2014 – 2017 Česká národní banka regulovala měnový kurz s cílem ekonomický růst podpořit. V článku je zpracován časový úsek 1996 – 2018, jehož převážná část tedy regulaci nepodléhala, a zkoumán vliv monetárních nejistot na vývoj HDP. Na základě charakteru dat je pracováno jednak s modelem ARDL, součástí jehož výstupů jsou i relevantní dlouhodobé informace. Veličina REER je zde použita jako vysvětlující, což může být teoreticky diskutabilní. Oprávněnost je doložena Hausmanovým testem. Oboustranný vliv mezi HDP a REER není potlačen v alternativním přístupu pomocí VAR modelu. Oba přístupy potvrzují, že HDP České republiky ve svých přírůstcích není ovlivněno monetárními nejistotami souvisejícími s měnovým kurzem.

Klíčová slova: *měnový kurz, monetární nejistoty, ARDL a VAR modely*

JEL Classification: C26, C30, E58
AMS Classification: 62H12, 60H30

Abstract

The way how exchange rate is managed is important for economic growth. The Czech National Bank, in 2014 – 2017 period regulated the exchange rate of the Czech crown with the goal to maintain an economic growth. The role of exchange rate uncertainties and the exchange rate rule application is studied as for their effect on GDP concerning 1996 – 2018 time span the most part of what was not under exchange rate regulation. First, the ARDL model is used and justified giving the relevant long-run information. The REER variable is used as an exogenous one what theoretically may be controversial. The procedure is justified by the Hausman test. Second, the VAR model is applied allowing for a mutual influence between GDP and REER changes. According to both approaches, no influence of exchange rate uncertainties on the GDP changes of Czech Republic is apparent.

Keywords: *exchange rate, monetary uncertainties, ARDL and VAR models*

JEL Classification: C26, C30, E58
AMS Classification: 62H12, 60H30

1 INTRODUCTION

The economists have long known that the way how exchange rate is managed is important for economic growth. A more strictly formulation is presented by Rodrik [10]: poorly managed exchange rates can be disastrous for economic growth. Some sort of macroeconomic targeting is applied in many economies, with the goal of generating macroeconomic benefits and the choice of a target variable is important because there is no direct relationship between monetary instruments and the ultimate goal of monetary policy. The 2007–09 financial crisis was a milestone in relation to the macroeconomic targeting concept and gave rise to some important conclusions (e.g. [3] or [9]). So, the exchange rate rule became a subject of new interest of central bankers. In 2014 the CNB started to weaken the exchange rate of the Czech crown because of a disproportionately low inflation rate. The goal of the exchange rate regulation was to maintain an economic growth. This policy was dropped in April 2017 and there is no unified opinion about its suitability among the Czech economists.

Not regulated, the exchange rate is one of the sources of monetary uncertainties. Uncertainty is a characteristic of the real world that affects the decision-making process; it contributes negatively to economic activity. Primarily, the higher uncertainty the lower willingness to invest. In Czech Republic, the demand for investment coming from abroad is rather high that is why exchange rate (as well as inflation) uncertainty can play an important role.

Amisano and Tristani in [1] summarize: an increase in uncertainty induces firms to temporarily reduce investment and hiring. Higher uncertainty over future shocks induces households to increase their precautionary saving. Consumption demand will tend to fall, this will bring down output and inflation. Uncertainty shocks therefore act like demand shocks.

Though the impact of monetary uncertainties is undeniable this entity is not measurable; moreover, there is no consensus on an adequate measure of volatility. Problematic is the choice of the exchange rate itself. There are no consensual criteria regarding the use of nominal or real exchange rate. Nevertheless, some studies suggest that the use of nominal or real exchange rate does not significantly affect obtained (see [2]).

To incorporate uncertainties in a mathematical model some additional assumptions usually are necessary. E.g. Dixit and Pindyck ([6]) present uncertainties as a discount rate decreasing future prices. The equations of a model are then the first order optimum conditions. Byrne and Davis in [4] propose to replace unmeasurable uncertainties with the concept of permanent and transitory parts of the variable.
Monetary uncertainties are rising from a volatility of relevant variable. Stochastic volatility leads to uncertainty shocks that contribute to affect macroeconomic fluctuations. Creal and Wu in [5] use the first, respective second, moments of macroeconomic variables to model uncertainty as extracted from their volatility.

2 DATA AND METHOD

Quarterly data (source: Eurostat and ČSÚ/Czech Statistical Office) referring to 1996Q4 – 2018Q4 interval are seasonally adjusted and describe *Y* as real Gross domestic product, *EX* is real effective exchange rate (deflator: consumer price index - 19 trading partners - euro area, Index, 2010=100), *LY* and *LEX* meaning the logarithms.

To study the empirical importance of exchange rate uncertainty, *VOL* is constructed as second root of variance *Var* of a sequence of nine monthly data (real effective exchange rate, deflator: consumer price index - 19 trading partners - euro area, Index, 2010=100) involving actual quarter as the middle part.

Using Phillips – Perron test it was found that $LY \sim I(1), LEX \sim I(1), VOL \sim I(0)$.

The mix of $I(0)$ and $I(1)$ variables could be treated by an Autoregresive Distributed Lag (ARDL) model as a standard least square regression containing lags of both the dependent variable and independent variables as regressors (see e.g. [7] or [8]). ARDL models have been shown to provide a very valuable vehicle for testing for the presence of long-run relationships between economic time-series.

The variables *DY*, *DEX* and *DVOL* are differences of *LY*, *LEX* and *VOL* respectively.

$$DY_t = \beta_0 + \sum_{i=1}^{q} \beta_{1i} DY_{t-i} + \sum_{i=0}^{q} \beta_{2i} DEX_{t-i} + \sum_{i=0}^{q} \beta_{3i} DVOL_{t-i} +$$

$$+\delta_1 LY_{t-1} + \delta_2 LEX_{t-1} + \delta_3 VOL_{t-1} + u_t \qquad (1)$$

In the equation (1)

- u is the white-noise error term
- β are short-run coefficients
- δ are long-run coefficients.

The long run effects are obtained by setting the non-first-differenced lagged component of eq. (1) to zero. Than it is

$$\delta_1 LY_{t-1} + \delta_2 LEX_{t-1} + \delta_3 VOL_{t-1} = 0$$

$$\delta_2 LEX_{t-1} + \delta_3 VOL_{t-1} = -\delta_1 LY_{t-1}$$

and the effect of exogenous variables on the endogenous one is given by normalizing δ_2 , resp. δ_3, on $-\delta_1$. Hence, the long-run effect of exchange rate on GDP is $-\delta_2/\delta_1$. The long-run effect of volatility on GDP is $-\delta_3/\delta_1$. As for an interpretation, parameters δ are elasticities, their ratios are dimensionless.

3 RESULTS

The convenient number of lags is 6 according to the AIC. Using OLS, the output of Eviews11 is in Table 1.

Table 1 Estimation of ARDL model (*source: own computation*)

Dependent Variable: DY
Method: Least Squares

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DY(-1)	-0.094248	0.126821	-0.743159	0.4602
DY(-2)	-0.129168	0.114246	-1.130609	0.2626
DY(-3)	-0.497190	0.098155	-5.065334	0.0000
DY(-4)	0.531001	0.096261	5.516248	0.0000
DY(-5)	-0.352134	0.116176	-3.031048	0.0036
DY(-6)	-0.323980	0.120976	-2.678056	0.0095
DEX	-0.018970	0.085458	-0.221979	0.8251
DEX(-1)	0.144223	0.087101	1.655812	0.1029
DEX(-2)	0.113617	0.084188	1.349560	0.1821
DEX(-3)	0.033553	0.081036	0.414055	0.6803

DEX(-4)	-0.004646	0.076795	-0.060501	0.9520
DEX(-5)	-0.031062	0.074337	-0.417858	0.6775
DEX(-6)	-0.019013	0.072891	-0.260844	0.7951
DVOL	-0.000366	0.002726	-0.134261	0.8936
DVOL(-1)	-0.002399	0.003695	-0.649245	0.5186
DVOL(-2)	0.004317	0.003455	1.249514	0.2163
DVOL(-3)	0.001408	0.003280	0.429362	0.6692
DVOL(-4)	-0.000283	0.003131	-0.090416	0.9283
DVOL(-5)	-0.001661	0.002930	-0.567050	0.5728
DVOL(-6)	0.005242	0.002906	1.803881	0.0762
LY(-1)	0.046525	0.024085	1.931675	0.0580
LEX(-1)	-0.156644	0.055125	-2.841618	0.0061
VOL(-1)	-0.000840	0.003729	-0.225221	0.8226
C	0.087320	0.115996	0.752784	0.4545

R-squared	0.950060	Mean dependent var	0.012662
Adjusted R-squared	0.931231	S.D. dependent var	0.051228
S.E. of regression	0.013434	Akaike info criterion	-5.549113
Sum squared resid	0.011009	Schwarz criterion	-4.859424
Log likelihood	259.8373	Hannan-Quinn criter.	-5.271701
F-statistic	50.45550	Durbin-Watson stat	1.993701
Prob(F-statistic)	0.000000		

$-\delta_2/\delta_1 = 3.367$ effect of REER on GDP

$-\delta_3/\delta_1 = 0$ effect of volatility on GDP

As changes in GDP can trigger changes in REER, the Hausman test of eventual endogeneity of variable *DEX* in equation (1) was performed (see e.g. [11]).

Using *Var* as an additional instrumental variable, the reduced form of *DEX* was computed by OLS. Obtained residuals *r* are inserted as a further explanatory variable into equation (1). The result of OLS estimate is

parameter-r = -0.769

s.e. = 0,489

t-statistics = -1.574.

We do not reject H_0: *parameter-r* = 0. Hence, we cannot conclude the endogeneity of *DEX*.

The choice of a methodology can always be a subject of a discussion. The question if the results stay similar when using an alternative method is justified. If we cannot *a priori* exclude an impact of GDP on REER, e.g. a VAR model technique could match the relations especially after the founding that granger causality (level of lags 2, 3, 4) in both directions exists (the Granger causality is not contradictory to the Hausman test founding). Applying it to *DY*, *DEX* as well as to *DGDP*, *DREER* the outputs are in Table 2.

Table 2 Estimation of VAR models (*source: own computation*)

Vector Autoregression Estimates
Included observations: 89 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	DY	DEX	DGDP	DREER

DY(-1)	-0.086909	0.143370	DGDP(-1)	-0.124248	1.73E-05
	(0.06033)	(0.08772)		(0.06713)	(9.1E-06)
	[-1.44052]	[1.63432]		[-1.85087]	[1.89615]
DY(-2)	-0.094683	0.046469	DGDP(-2)	-0.128790	8.88E-06
	(0.06021)	(0.08754)		(0.06842)	(9.3E-06)
	[-1.57266]	[0.53082]		[-1.88229]	[0.95509]
DY(-3)	-0.111450	0.224221	DGDP(-3)	-0.129013	2.84E-05
	(0.06065)	(0.08819)		(0.06971)	(9.5E-06)
	[-1.83757]	[2.54250]		[-1.85068]	[2.99971]
DY(-4)	0.860202	0.094145	DGDP(-4)	0.852126	1.27E-05
	(0.06169)	(0.08971)		(0.07127)	(9.7E-06)
	[13.9430]	[1.04948]		[11.9561]	[1.31065]
DEX(-1)	0.096769	0.184805	DREER(-1)	977.6220	0.222183
	(0.07709)	(0.11209)		(828.126)	(0.11250)
	[1.25527]	[1.64868]		[1.18052]	[1.97487]
DEX(-2)	-0.017443	-0.117104	DREER(-2)	-116.7942	-0.175765
	(0.07739)	(0.11252)		(842.402)	(0.11444)
	[-0.22541]	[-1.04072]		[-0.13864]	[-1.53581]
DEX(-3)	-0.025322	-0.210295	DREER(-3)	-276.2218	-0.178536
	(0.07828)	(0.11382)		(860.932)	(0.11696)
	[-0.32349]	[-1.84764]		[-0.32084]	[-1.52645]
DEX(-4)	-0.114227	-0.184346	DREER(-4)	-2109.923	-0.199674
	(0.07764)	(0.11290)		(834.289)	(0.11334)
	[-1.47115]	[-1.63283]		[-2.52901]	[-1.76169]
C	0.006643	-0.008143	C	7672.857	-0.905771
	(0.00419)	(0.00610)		(3920.30)	(0.53259)
	[1.58425]	[-1.33555]		[1.95721]	[-1.70069]
VOL	-0.000857	0.006200	VOL	-1025.560	0.583550
	(0.00209)	(0.00305)		(1923.09)	(0.26126)
	[-0.40931]	[2.03575]		[-0.53329]	[2.23359]
R-squared	0.916430	0.251241	R-squared	0.919873	0.283698
Adj. R-squared	0.906662	0.163723	Adj. R-squared	0.910508	0.199975
Sum sq. resids	0.018661	0.039455	Sum sq. resids	1.52E+10	280.9985
S.E. equation	0.015568	0.022636	S.E. equation	14061.50	1.910322
F-statistic	93.82051	2.870752	F-statistic	98.21953	3.388510
Log likelihood	244.0057	211.4371	Log likelihood	-949.0902	-174.4488
Akaike AIC	-5.379441	-4.630738	Akaike AIC	22.04805	4.240203
Schwarz SC	-5.096003	-4.347300	Schwarz SC	22.33149	4.523641
Mean dependent	0.013260	0.004722	Mean dependent	10642.77	0.382184
S.D. dependent	0.050956	0.024753	S.D. dependent	47004.41	2.135771

Effect of volatility on GDP equals zero in both cases while a small positive effect on REER is apparent. All the computations were repeated after introducing a dummy variable for the exchange rate regulation period. No significance of such a special treatment occurred.

4 CONCLUSIONS

Two alternative methodological approach were applied to 1996 – 2018 data with the goal to study exchange rate uncertainties and their impact on economic growth in Czech Republic. Though in the most part of data sample (periods 1996 – 2013 including and 2017Q2 – 2018) no exchange rate regulation was performed, the role of uncertainties is insignificant independently on a method chosen.

The VAR formulation does not excludes a mutual influence between REER and GDP. An impact of volatilities on REER is admitted but GDP changes seem intact.

The long-run effect of exchange rate itself on GDP is quantified by the help of the ARDL model as a positive one. It relates to a common expectation, the higher real effective exchange rate what means the weaker Czech Crown, the higher GDP. The transmission mechanism is evidently the export for which a weak home currency is a supporting factor.

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PROBABILITY DISTRIBUTION OF DJIA STOCK MARKET INDEX IN THE POST-CRISIS PERIOD

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Abstract
The probability distribution of returns of financial assets is an important part of financial modeling itself, as well as in estimating financial risk measures. Normal distribution of returns is an assumption that is often considered, but the large number of papers associated with empirical observations suggests that reality is different. Plenty of authors have been looking for the probability distribution which fits the financial data the best, but to date there is no consensus whether there is a probability distribution or a family of distributions that would describe the probability distribution of the returns of all financial assets in different markets and different market conditions. The aim of this thesis is to examine the different types of probability distributions in order to find the distribution that best describes the characteristics of stock returns in the post-crisis period.

Keywords: *Fitting distributions, skewed generalized t distribution, normal distribution, power exponential distribution, general lambda distribution, hyperbolic distribution*

JEL Classification:C13, G19
AMS Classification:62-07, 62E17, 91B84

1 Introduction

One important part of financial modeling, as well as in estimating financial risks, is an analysis of empirical probability distribution. An important assumption in the early days of financial modeling was the assumption of a normal distribution of stock returns. However, such an assumption is not consistent with empirical observations for daily returns that clearly indicate that the empirical distribution of financial asset returns is more spiked in the center and wider in the tails compared to the normal distribution. A large number of authors, especially in the 60s and 80s of the last century, were looking for alternative probability distributions that would suitably describe the behavior of changes in stock prices. A number of different alternative distributions have been proposed to provide a better description of the financial asset returns behavior than the normal distribution provided. Such alternative distributions include, for example: Student-t distribution (Blattberg and Gonedes, 1974), stable Pareto distribution, scaled-t distribution (Praetz, 1972), mixture of several normal distributions, generalized hyperbolic distribution (Behr and Pötter, 2007), skewed generalized-t distribution (Harris and Küçüközmen, 2001), generalized lambda distribution (Chalabi et. al, 2009), and others. Despite the large number of contributions, the question of what probability distribution is best for modeling stock returns remains open. In the first part the authors describe descriptive statistics of analyzed data, in the next part they briefly describe selected probability distributions, in the third part they present the results of the analysis and at the end they summarize the achieved results and present ideas of possible future research.

2 Data

The stock price data we use in our analysis is data on the individual components of the Dow Jones Industrial Average index as well as data on the index itself, with the exception of Dow, Inc., since the stock price data of this company is only available from April 2019. Individual components of DJIA include: *Dow, Inc.*(DOW), *Walgreens Boots Alliance, Inc.*(WBA), *Chevron Corporation*(CVX), *Walmart, Inc.* (WMT), *Verizon Communications, Inc.* (VZ), *International Business Machines Corporation* (IBM), *American Express Company* (AXP), *Apple, Inc.* (AAPL), *The Boeing Company* (BA), *Exxon Mobil Corporation* (XOM), *McDonald's Corporation* (MCD), *Johnson & Johnson* (JNJ), *Cisco Systems, Inc.* (CSCO), *The Coca-Cola Company* (KO), *NIKE, Inc.* (NKE), *The Travelers Companies, Inc.* (TRV), *JPMorgan Chase & Co.* (JPM), *Merck & Co., Inc.* (MRK), *The Home Depot, Inc.* (HD), *The Procter & Gamble Company* (PG), *The Walt Disney Company*(DIS), *3M Company* (MMM), *Pfizer, Inc.* (PFE), *United Technologies Corporation* (UTX), *Microsoft Corporation* (MSFT), *Caterpillar, Inc.*(CAT), *UnitedHealth Group Incorporated*(UNH), *Intel Corporation*(INTC), *The Goldman Sachs Group, Inc.*(GS), *Visa Inc.*(V). Daily values of the adjusted closing positions of the individual stocks and the index itself are collected from finance.yahoo.com. The daily values we use are from the period after the global financial and economic crisis from 01/01/2013 to 20/02/2020. The behavior of the index in this period is mainly characterized by stable upward trend, as can be seen in Fig. 1.

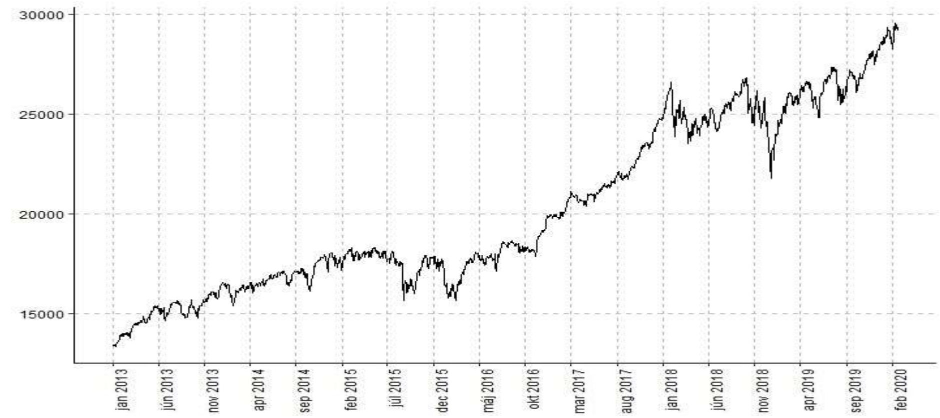

Fig. 1. Development of DJIA levels in the analyzed period
Source: author´s calculations

Daily returns are calculated as:

$$R(t) = \log\left(\frac{S_t}{S_{t-1}}\right) \quad (1)$$

where S_t is the adjusted closure level of price in time t. Therefore, these are logarithmic returns that are consistent with continuous interest.

Descriptive statistics of daily returns for individual stocks as well as for the index itself are in the tab. 1. In addition to the above statistics, the table also include the p-values of two statistical normality tests. Specifically, it is the Shapiro-Wilk test and the Jarque-Bera test. As can be seen in the tab. 1. for all cases, the p-values are close to 0, which means that at the significance level of 0.01 we reject the hypothesis of a normal distribution of returns on individual shares. The fact that the daily returns do not have a normal distribution is also indicated by the Skewness and Kurtosis values themselves, which in all cases strongly deviate from the values characteristic for normal distribution, concretely Skewness = 0 and Kurtosis = 3. The comparison is shown in fig. 2.

Tab. 1. Descriptive statistics of each daily stock returns, index itself and p-values of normality tests.

	WBA	CVX	WMT	VZ	IBM	AXP	AAPL	BA	XOM	MCD	DJI
Mean	0,000268	0,00016	0,000393	0,000334	0,000003	0,000528	0,000864	0,000927	-0,000069	0,000605	0,000436
Std	0,015808	0,012861	0,01122	0,010534	0,012621	0,012531	0,015787	0,015108	0,011373	0,009833	0,008023
Var	0,00025	0,000165	0,000126	0,000111	0,000159	0,000157	0,000249	0,000228	0,000129	0,000097	0,000064
Skewness	-1,238393	-0,085721	-0,07892	-0,049786	-0,648041	-0,682735	-0,632586	-0,265522	-0,123021	0,292418	-0,523453
Kurtosis	14,685041	5,546635	21,520712	5,8725	10,740299	13,145499	9,236155	6,722004	5,68201	9,594417	6,74782
Max	0,068885	0,061446	0,103444	0,073995	0,084933	0,08644	0,078795	0,094214	0,053692	0,078105	0,048643
Min	-0,154754	-0,057276	-0,107399	-0,047941	-0,086419	-0,128981	-0,131885	-0,093531	-0,058586	-0,051732	-0,047143
Shapiro_test_p.value	0	0	0	0	0	0	0	0	0	0	0
JarqueBera_test_p.value	0	0	0	0	0	0	0	0	0	0	0
	JNJ	CSCO	KO	NKE	TRV	JPM	MRK	HD	PG	DIS	
Mean	0,000523	0,00058	0,000381	0,000833	0,000432	0,000733	0,000504	0,000836	0,000456	0,00062	
Std	0,009804	0,014019	0,009111	0,014223	0,010273	0,012751	0,011962	0,011489	0,009515	0,012166	
Var	0,000096	0,000197	0,000083	0,000202	0,000106	0,000163	0,000143	0,000132	0,000091	0,000148	
Skewness	-1,160152	-0,275452	-0,753254	0,852665	-0,929187	-0,082646	0,257036	-0,167572	0,070092	0,046366	
Kurtosis	13,748912	13,554225	11,982188	12,454688	9,057843	5,76927	8,622946	5,919986	9,425989	11,474867	
Max	0,048395	0,118987	0,058947	0,115342	0,048397	0,079999	0,099013	0,062149	0,084328	0,109247	
Min	-0,105781	-0,116066	-0,088126	-0,073114	-0,086506	-0,072009	-0,062526	-0,057617	-0,060607	-0,09619	
Shapiro_test_p.value	0	0	0	0	0	0	0	0	0	0	
JarqueBera_test_p.value	0	0	0	0	0	0	0	0	0	0	
	MMM	PFE	UTX	MSFT	CAT	UNH	INTC	GS	V	Average	
Mean	0,000396	0,000331	0,000418	0,00116	0,000341	0,001024	0,000757	0,000385	0,001008	0,000542	
Std	0,01167	0,011052	0,011314	0,014449	0,015603	0,013469	0,015519	0,014239	0,012997	0,012528	
Var	0,000136	0,000122	0,000128	0,000209	0,000243	0,000181	0,000241	0,000203	0,000169	0,000161	
Skewness	-1,430858	-0,133202	-0,548996	-0,179871	-0,316117	0,110472	-0,096829	-0,274209	-0,006018	-0,294139	
Kurtosis	17,5879	6,362251	6,633088	11,216853	6,547152	5,741434	8,686868	5,47656	7,763864	9,681798	
Max	0,057447	0,068282	0,052445	0,099413	0,075671	0,078441	0,100315	0,091153	0,097527	0,080326	
Min	-0,138631	-0,066325	-0,072925	-0,121033	-0,095697	-0,058117	-0,095432	-0,077482	-0,078368	-0,087623	
Shapiro_test_p.value	0	0	0	0	0	0	0	0	0	0	
JarqueBera_test_p.value	0	0	0	0	0	0	0	0	0	0	

Source: author´s calculations

The individual stocks are relatively heterogeneous in the view that some stocks have spikier center and thicker tails of the distribution than others. Higher kurtosis in most of the data is caused by outliers. Walmart (WMT) has the highest kurtosis, while Goldman Sachs (GS) has the lowest one. Despite the fact that GS has the lowest kurtosis, it is still significantly higher than that of normal distribution.

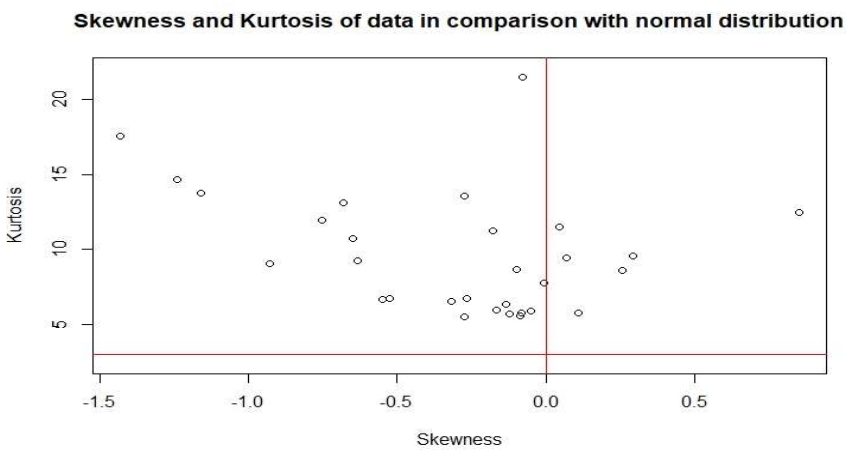

Fig. 2. Comparison of skewness and kurtosis of individual stocks and index itself with statistics characteristic for normal distribution.
Source: author´s calculations

3 Probability distributions

The goal of the paper is to analyze the returns of the DJIA index and its components. In this section, we present the probability distributions that were used in the analysis, with the exception of the normal distribution (norm).

Student´s t-distribution

The Student´s t-distribution is again widely used distribution. Compared to the normal distribution, it has thicker tails of the distribution, which is one of the reasons why it is often used. The t-distribution has only one parameter, concretely degrees of freedom df. Probability density function of Student-t distribution:

$$f(z|df) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\sqrt{\pi v}}\left(1 + \frac{z^2}{v}\right)^{-\frac{v+1}{2}} \quad (4)$$

where v are degrees of freedom and Γ is the gamma function. The problem of utilization is that the properties of the Student´s t-distribution itself are relatively restrictive for needs of financial data. Since the Student´s t-distribution has mean equal to zero and the variance is defined through the degrees of freedom parameter. Therefore, it is preferable to use a scalable version of the t-distribution (t.scaled) where the mean value, the standard deviation and the tails of the distribution may be customizable for financial data modeling purposes.

Skewed generalized Student´s t-distribution

It is a highly flexible probability distribution that covers a large number of shapes and other distributions as special cases. This distribution has five parameters. Probability density function (PDF) of the skewed generalized Student´s t-distribution (sgt_2) with parameters $\mu, \sigma, \lambda, p, q$:

$$f((x|\mu, \sigma, \lambda, p, q) = \frac{p}{2v\sigma q^{\frac{1}{p}} B\left(\frac{1}{p}, q\right)\left(\frac{|x-\mu+m|^p}{q(v\sigma)^p(\lambda sign(x-\mu+m)+1)^p}+1\right)^{\frac{1}{p}+q}} \quad (5)$$

where B is the beta function, μ is the location parameter, $\sigma > 0$ is the scaling parameter, $-1 < \lambda < 1$ is the skewness parameter, $p > 0$ and $q > 0$ are the kurtosis control parameters. The remaining parameters, m and v are functions of the other parameters. The special cases of this distribution are shown in fig. 3, taken from (Hansen et al., 2010).

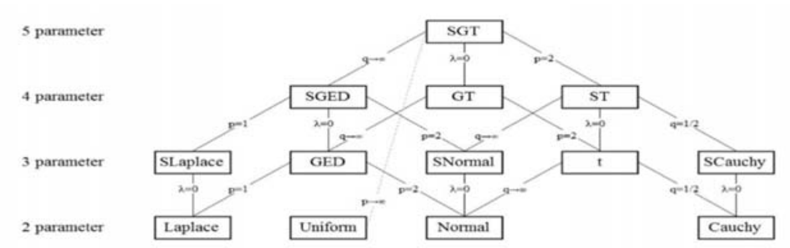

Fig. 3.: Special cases of skewed generalized Student´s t-distribution

Source: Hansen, et al. (2010).

One special case is the Hansen skewed t-distribution (sgt_1), (Hansen, 1994). It is a frequently used probabilistic distribution in the financial literature.

Generalized normal distribution

This is a generalized version of the normal distribution, often referred to as the exponential power distribution or general error distribution (GED). The probability density function (PDF) of a generalized normal distribution with two parameters λ, κ:

$$f(x|\lambda, \kappa) = \left(e^{1-e^{\lambda x^{\kappa}}}\right) e^{\lambda x^{\kappa}} \lambda \kappa x^{\kappa-1} \quad (6)$$

where λ is the positive scaling parameter and κ is the positive parameter affecting the shape.

Generalized lambda distribution

In our paper, we consider modified parameterization of the generalized lambda distribution (gld) by Freimer et al. (1988). It is a flexible probability distribution that provides a wide range of different shapes. Distribution function (CDF) of generalized lambda distribution with parameters $\lambda_1, \lambda_2, \lambda_3, \lambda_4$:

$$F^{-1}(u|\lambda_1, \lambda_2, \lambda_3, \lambda_4) = \lambda_1 + \frac{1}{\lambda_2}\left(\frac{u^{\lambda_3}-1}{\lambda_3} - \frac{(1-u)^{\lambda_4}-1}{\lambda_4}\right) \quad (7)$$

There is a single boundary for the parameter $\lambda_2 > 0$.The types of shapes that this generalized lambda distribution can take are divided into five categories, and as stated in Chalabi et al. (2009), the appropriate version for modeling the distribution of stock returns is the first category which requires additional boundaries to $\lambda_3 < 1$ and $\lambda_4 < 1$.

Generalized hyperbolic distribution

The generalized hyperbolic distribution (hyp) is also one of the flexible distributions where the probability density function can take a large number of shapes. For this reason, attempts have been made to apply them to the modeling of the distribution of stock returns such as in Behr and Pötter (2007). The probability density function (PDF) of a generalized hyperbolic distribution with five parameters $\alpha, \beta, \lambda, \delta, \mu$:

$$f(x|\alpha, \beta, \lambda, \delta, \mu) = \kappa\{\delta^2 + (x-\mu)^2\}^{\frac{1}{2}\left(\lambda-\frac{1}{2}\right)} K_{\lambda-\frac{1}{2}}\left(\alpha\sqrt{\delta^2 + (x-\mu)^2}\right) e^{\beta(x-\mu)} \quad (8)$$

it holds that$0 \leq |\beta| < \alpha, \delta > 0$, a κis a function of the other parameters.. $K_\lambda$is a modified Bessel function of the third kind with indexλ.

4 Analysis

Before analysis of the goodness of fit of estimated probability distributions to the empirical distribution, we need to estimate the parameters of the individual probability distributions, such that they fit the probability distribution to the empirical distribution the best. We used the maximum likelihood estimation method, in package fitdistrplus (Delignette-Muller and Dutang, 2015), to estimate the parameters of individual distributions. Each calculations contained in this article are from R-Studio. The procedure for estimating parameters as well as the packages used are described in more detail in Pekár and Pčolár (2019).

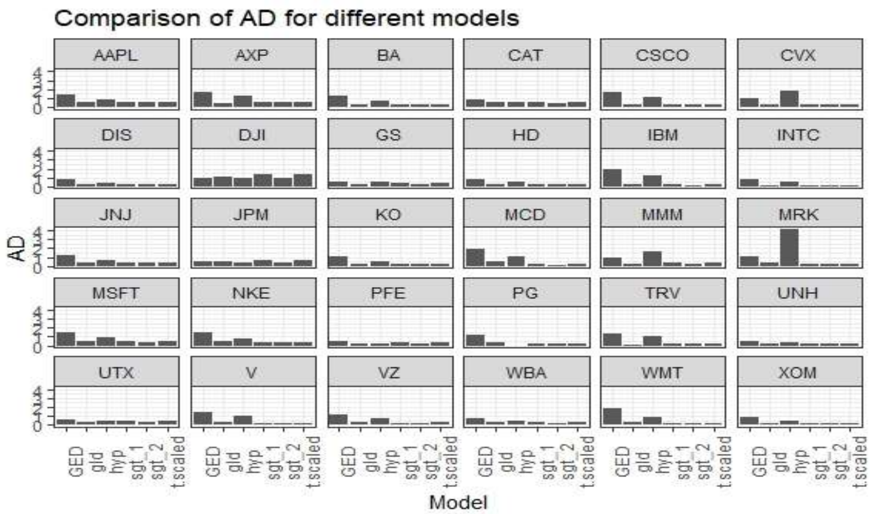

Fig. 4.: Comparison of the AD statistics of each fitted distribution model for each stock
Source: author´s calculation

The goal of this section will be to compare the goodness of fit of each tested probability distribution. We will use Anderson-Darling statistics (AD) to evaluate the goodness of fit of probability distribution to the empirical distribution. This statistic measures the deviation between the fitted cumulative distribution function (CDF) $\hat{F}$and the empirical distribution function F_n. AD statistics are calculated as the weighted average of square deviations $[F_n(x) - F(x)]^2$. The weights are selected so that they place more emphasis on deviations at the tails of the distribution. Such a measure may be more appropriate for assessing the strength of adjustment, if we pay particular attention to goodness of fit at the tails of the distribution, which may be particular interest for risk management. The lower the value of statistic is, the better the goodness of fit to the empirical distribution

In order to reflect the difference between numbers of parameters of each estimated model, we also provide statistics based on information criteria, which also considers the complexity of the model itself. In the analysis we present the Akaike Information Criterion (AIC). The lower the AIC value, the better.

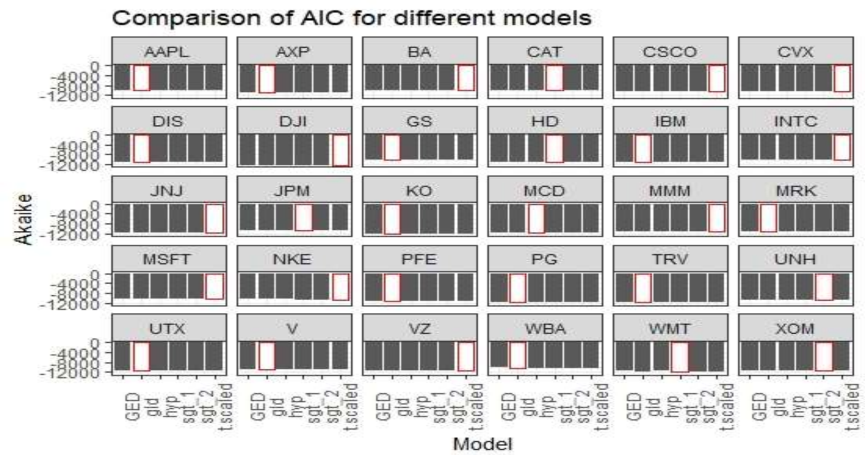

Fig. 5.: Comparison of the AIC values of each fitted distribution model for each stock
Source: author´s calculation

A comparison of these statistics for each model and stock is shown in fig. 4. And fig. 5. In both cases, we have omitted the normal distribution from the comparison as it is significantly worse in all statistics compared to other models. Such values would distort the scale suitable for comparing other models. For greater clarity when comparing AIC statistics, the highest values are highlighted in each graph. Normal distribution has not been confirmed in all cases, and the calculated statistics of goodness of fit alone provide incomparable worse values for normal distribution than the rest of the fitted models. It is evident from the analysis that several of the distributions tested seem promising for the modeling of stock returns in the post-crisis period. The t.scaled, sgt_2, sgt_1, and gld distributions provide the highest goodness of fit of the estimated distribution to the empirical distribution. Hyp, GED, and norm distribution look like as inappropriate models for daily returns of individual stocks. This finding is not true for the index itself and the JPM stocks, where we may consider the norm, t.scaled, and sgt_1 distribution to be inappropriate. Other models provide equal good power of fit to empirical distribution.

5 Conclusions

The results of our analysis point to several findings. The normal distribution of the daily stock returns was rejected in all of analyzed stocks, nor in the index itself. Goodness of fit statistics point to multiple adepts for the model that best describes distribution of the daily returns over the analyzed period. Equally good power of fit for most stocks provide the Student's *t*-

distribution, Hansen skewed *t*-distribution, skewed generalized *t*-distribution, and generalized lambda distribution. These finding are not relevant in part to JPM stock returns. and stock returns of the DJIA itself. In order to identify the best distribution for the modeling of daily returns of DJIA components, it is necessary to carry out a more detailed analysis, at sub-intervals of the analyzed period and periods in different market conditions. The direction for further research may be to examine the relevant of the above models in modeling daily returns distribution in different market conditions, i.e. during bearish or bullish market behavior. Another way of research may also be to examine the suitability of such models for different markets (both developed and developing).

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PURCHASER'S VEHICLE ROUTING PROBLEM

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Abstract
Standard problem in operations research is vehicle routing problem, but there are many modifications of the problem that were caused by the specifics of case studies. Vehicle routing problem studied in the paper consists of nodes, which represent producers, supplies offering goods and depot, customer with a list of demanded quantity of goods. The price list of goods of producers is given and the distance matrix of nodes too. The goal is to acquire the requested goods and deliver them to depot with minimal costs. The mathematical model is proposed which is demonstrated by numerical example. In addition to the model a heuristic method is shown.

Keywords: *vehicle routing problem, integer programming, heuristic method*

JEL Classification: JEL C44

AMS Classification: 90C15

1 INTRODUCTION

The article proposes a new modification to vehicle routing problem (VRP hereafter). The basic form of VRP consists of *n* nodes, where the first node represents the depot and the remaining nodes the customers. The goods are transported using the routes both starting and ending in depot. The transport itself is realized by the vehicles with certain capacity and the customer-node demands are given by the volume of the demanded goods in a vehicle. The route length depends on the order of the nodes of a given route and can be calculated using the distance matrix between each pair of nodes. Instead of the node distance one can also calculate the transport cost of a given vehicle from one node to another. The aim is to minimize the total sum of route distances or eventually to minimize the transport cost of these routes using given vehicles. But first we must assure that a) each one of the nodes is included in at least one of the routes, b) the sum of node demands for each route is not exceeding the capacity of the vehicle.

This problem can be solved using a mathematical model, which is in its nature linear integer programming problem. VRP and linear integer programming problem are both NP hard, i.e. if the number of nodes is higher, in reality more than ~ 30 nodes, it is impossible to get optimal solution in reasonable time using standard computer and branch and bound method. Except for the mathematical model one can use heuristic methods such as nearest neighborhood method, insert method or savings method, which can provide us with good solution in reasonable time.

There are a lot of modifications of the basic form of VRP, which arise as a results of merchandise transport in praxis. We list the following: VRP involving vehicles with different capacities and transport cost; VRP with more than one depot; split delivery VRP; VRP involving stochastic conditions for the merchandise once the vehicle reaches the node; VRP with time windows, where the time of vehicle arrival in the node must be equal to a certain time interval denoted as time window.

The VRP problem is described in the literature, heuristic methods are proposed, and a summary of these approaches is described in [1], [2], [3].

VRP described in this article concerns the goods transport from nodes that represent the suppliers. The goods are being transported to the depot, in our case noted as node 1. The capacity and transport cost of each vehicle used for the transport is identical. The goods being transported to the depot involves demanded products in demanded volume. The product cost vary from node to node and the choice of supplier influences the product as well as the transport cost. The objective function is then comprised of both transport and product cost.

The mathematical model assumes the demanded product to be available in every node. In case there is a product that is not available in a given node, we set the product cost to be prohibitively high.

Except for the numerical examples and simultaneous case study VRP can be used in case study concerning the delivery and assembly services. The demanded goods are being transported from the depot, which consists of several sections (palettes), in our case nodes. Each type of goods can be stored in different section. The product cost in each of the nodes is zero in case the product is located in the node or prohibitive if it is the other way around.

2 MATEMATICAL MODEL

Parameters of the model:

n number of nodes, node 1 is depot,

d_{ij} vehicle transport cost from node i to node j,

m number of products: $P_1, P_2, \ldots, P_m$,

q_k customer demand for k-th product,

c_{ki} price per unit of the k-th product in the i-th node,

W vehicle capacity,

$M > 0$ prohibitive value.

Variables of the model:

x_{ij} binary variable, equals 1 if the vehicle travels from node i to node j, where $i \neq j$,

y_{ik} binary variable, equals 1 if product k is loaded in node i,

z_i binary variable, equals 1 if the vehicle visits node i,

u_i anticyclic conditions variable in (6).

$$f(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}x_{ij} + \sum_{i=2}^{n}\sum_{k=1}^{m} c_{ki}q_k y_{ik} \rightarrow min \tag{1}$$

$$\sum_{j=1}^{n} x_{ij} = z_i \quad i = 2,3,\dots,n \tag{2}$$

$$\sum_{i=1}^{n} x_{ij} = \sum_{i=1}^{n} x_{ji} \quad i = 1,2,\dots,n \tag{3}$$

$$\sum_{i=2}^{n} y_{ik} = 1 \quad k = 1,2,\dots,m \tag{4}$$

$$\frac{1}{M}\sum_{k=1}^{m} y_{ik} \le z_i \quad i = 2,3,\dots,n \tag{5}$$

$$u_i + \sum_{k=1}^{m} q_k y_{jk} - W(1 - x_{ij}) \le u_j \quad i = 1,2,\dots,n, j = 2,3,\dots,n, i \ne j \tag{6}$$

$$u_j \le W \quad j = 2,3,\dots,n \tag{7}$$

$$x_{ij}, i,j = 1,2,\dots,n, i \ne j, y_{ik}\ i = 2,3,\dots,n, k = 1,2,\dots,m \ binary, \tag{8}$$

$$u_i \ge 0, \quad x_{ii} = 0, \ i = 1,2,\dots,n$$

Objective function (1) is defined as a sum of the transport cost from one node to another and the product cost. Function (1) is being minimized. Equation (2) ensures that once the node i is visited then z_i=1 and the vehicle has to arrive to this particular node, et vice versa. Condition (3) dictates that once the vehicle arrives to a certain node it will have to depart from that particular node as well. Equation (4) prescribes that the k-th product has to be picked up in one of the nodes. If the product is picked up in the i-th node the vehicle has to pass through (5). Conditions (6) and (7) state that the vehicle capacity will not be exceeded. At the same time (6) prevents partial cycles.

The mathematical model (1) – (8) can be modified for the case of divided demand, i.e. if the demand after the k-th product q_k is divided into several fractions and these fractions are being transported by different vehicles from different nodes.

3. EXAMPLE

The number of nodes is 11, whereas node 1 is the depot. The vehicle capacity is W=100. The aim is to pick up and transport five products $P_1, P_2, \dots, P_5$ in amount $\boldsymbol{q}$ = (24, 35, 42, 20, 45) to the node 1. The individual product cost in each node is given in below table:

c_{kj}	j=2	3	4	5	6	7	8	9	10	11
k=1	18	10	13	8	12	5	4	13	2	3
2	19	15	14	6	12	5	4	9	5	2
3	17	12	15	7	18	4	2	11	5	6
4	19	15	18	6	9	3	5	4	14	5
5	8	19	2	4	6	3	5	4	6	4

We will decide in which node each product is purchased so that total costs, i.e. sum of the transport and the product cost, is minimized.

Transport cost matrix between the nodes is C:

0	13	6	55	93	164	166	168	169	241	212
13	0	11	66	261	175	177	179	180	239	208
6	11	0	60	97	168	171	173	174	239	209
55	66	60	0	82	113	115	117	117	295	265
93	261	97	82	0	113	115	117	118	333	302
164	175	168	113	113	0	6	4	2	403	374
166	177	171	115	115	6	0	8	7	406	376
168	179	173	117	117	4	8	0	2	408	378
169	180	174	117	118	2	7	2	0	409	379
241	239	239	295	333	403	406	408	409	0	46
212	208	209	265	302	374	376	378	379	46	0

The mathematical model solution to (1)-(8) are two routes 1-4-3-1 and 1-7-8-1 with travel cost 463 units. Product P_1 is picked up in node 3, product P_5 in node 4, product P_4 in node 7 and both products P_2 and P_3 in node 8. The product cost is equal to 614 units and taking into account the transport cost, the total cost amounts to 463+614=1077 units. This final cost provides the minimal value.

3 HEURISTIC METHOD

Given the fact that the VRP is NP hard, it is advisable to use heuristic method rather than mathematical model, especially once the complexity of the problem rises. The designed heuristics consists of the first phase, i.e. proposing the first draft of the feasible solution whereas in the second phase the solution is being corrected.

Phase1:

As a starting solution one can use either the solution with minimal product cost, i.e. we will first look for the node with minimal product cost of the given product and then find the optimal or heuristic solution to the VRP. Another approach would be to choose the node with minimal transport cost and pick all demanded products in this particular node. In this case the transport cost of the route depot – chosen node is minimized.

Phase 2:

The total cost of the feasible solution can be decreased using three methods:

a) by changing the node where a certain product is picked up,

b) by adding a node to one of the routes,

c) by excluding a node from one of the routes.

Add a) if node i offers product P_k and the product cost of the same product is lower in node j, we can change the supplier of the given product, i.e. switch to node j. For this to hold the node j has to be present in one of the routes and the vehicle capacity cannot be exceeded. This can be also achieved by switching product P_k and P_l so that P_k and P_l are available in node i and j respectively.

Add b) inserting an extra node s between the nodes u and w will increase the transport cost by $d_{us} + d_{sw} - d_{uw}$, The inserting node can however lead to the change of previous place of product purchase which can lead to product cost decrease. Then the total cost can decrease as well.

Add c) excluding a node from a certain route move all products supplied by this node to other nodes of routes, it can lead to decrease in total cost. For this to hold the vehicle capacity cannot be exceeded.

Example.

Phase 1: Node 3 is the closest node to the depot, the transport cost of the two routes 1-3-1 and 1-3-1 equals to 24 units. If we can purchase all the demanded products in the node 3 the product cost equals to 2424 units. The total cost is then given as sum of the latter 2424+24=2448 units.

Different approach is to choose for every product the node with the lowest product cost. Therefore the product P_1 is purchased in node 8, P_2 in node 11, P_3 in node 8, P_4 in node 9 and P_5 in node 4. The product cost is then equal to 376 units. We also have to take into account the transport cost for the chosen nodes 8, 11, 8, 9 and 4 bearing in mind the vehicle capacity W=100. As a result we are presented with two routes: 1-9-8-1 and 1-11-4-1 with transport cost 872 units, making the total cost equal to 1248 units.

Phase 2: a) let us take the route 1-3-1 and change the node 3, supplying us with product P_3 and P_5, to node 8. The transport cost will increase by 324 units but the product cost of the products P_3 and P_5, decreases by 1050 units. This action will lead to the total cost decrease 1050-324=726 units to final value 1722 units.

b) Let us take the route 1-3-1 with the node 3 supplying us with products P_1, P_2 and P_4. Now let us add the node 11 to the inspected route. The resulting route is 1-3-11-1 and its transport cost will increase by 415 units. Once we choose the node 11 to be the product P_2 supplier the product cost will decrease by 455 which will amount for the total cost decrease 455-415=40 units.

c) The transport cost of the route 1-11-4-1 is 532 units with the product P_2 being purchased in node 11 and product P_5 being purchased in node 4. By excluding the node 4 the transport cost will decrease by 108 units. However, we have to switch from node 4 to node 11 as the product P_5 supplier which will result in product cost increase by 90 units. In total the exclusion of the node 4 from the initial route will result in the total cost decrease by 108-90=18 units.

4 CONCLUSIONS

The article is proposing and solving a new modification to the VRP, where the total cost consists of the transport cost from one node to another and purchase cost based on the product cost in the nodes. We proposed both mathematical model and heuristic method.

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TWO HEURISTICS FOR VEHICLE ROUTING PROBLEM WITH UNIFORM PRIVATE FLEET AND COMMON CARRIER

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Abstract
Optimizing the distribution of goods to customers is discussed on non-split delivery modification of Vehicle routing problem (VRP) with uniform private fleet and common carrier. Private fleet has all vehicles with the same capacity and it costs are proportional to the sum of distances traveled by its vehicles. Common carrier has no capacity limit and costs are proportional to the quantity of transported goods only. The goods and all vehicles are located in the same depot. A set of customers with given demand is known. The goal is to minimize the overall costs. The model is formulated, modified Insert heuristic is described, and a new heuristic based on division nodes into two subsets is proposed. This heuristic which solves both subsets separately gives good results on tested examples. The performance of the mathematical model and both heuristics are compared on some numerical experiments.

Keywords: *vehicle routing, integer programming, insert heuristic, non-split delivery model*

JEL Classification: C61
AMS Classification: 90C59

1 INTRODUCTION

Optimizing the costs of transporting goods to customers has recently become an increasingly important factor in ensuring the competitiveness of companies on the market. Growing transport costs force managers to look for new ways of transporting goods using not only their private fleet of vehicles, but also the services of specialized carriers. Vehicle routing problem (VRP) deals with optimizing the distribution of goods to customers and is a rapidly developing area of operations research during last years. Its general purpose is to design a set of routes at minimal costs to serve the needs of customers. This means deciding which vehicles to serve customers and in what order all routes are completed with minimal financial costs or execution time. Many variants of this problem have been studied over the years.

These include:

- VRP with Pickup and Delivery (VRPPD) - there are more depots where the loading is in progress,
- VRP with Time Windows (VRPTW) - the goods can be delivered only within a certain time interval,
- Capacitated VRP (CVRP or CVRPTW) - vehicles have a limited load space capacity,
- VRP with Multiple Trips (VRPMT) - vehicles may have more than one route,
- Open VRP (OVRP) - vehicles do not have to return to the starting point,
- VRP with private fleet and common carrier (VRPPC) - the services of an external carrier are also used for delivery.

Detailed classification of variants of the VRP can be found in the articles (Eksioglu, Vural and Reisman, 2009; Braekers, Ramaekers and Van Nieuwenhuyse, 2016). The VRPPC variant according (Chu, 2005) consists in serving a set of customers using the next rules. Each customer is served only once, all routes associated with the private fleet have a start and end in the depot, each vehicle of the private fleet rides only one route, the serviced route contains a demand not exceeding overall vehicle capacity and total costs are minimized. Due to numerous applications, a number of new mathematical models and original numerical methods were created. A number of heuristic algorithms are currently being developed that are effective in terms of time and implementation complexity. One of the first proposed heuristics is the savings heuristics (Clarke and Wright, 1964). A heuristic proposed in (Plevný, 2013) selects customers according the distance from depot. In (Pelikán, 2016) is described a modified insert heuristic for solving split and non-split demand with a uniform and heterogeneous fleet of vehicles.

We consider a modification of VRP known as non-split delivery VRP with uniform private fleet and common carrier (VRPPC). In this case, it is necessary to decide whether it is more advantageous to reduce costs by optimizing the routes of the fleet of own vehicles or to look for cheaper common carriers, or to use a combination of both variant. Matters such as customer distances, fleet capacity, lead time or warehouse location play an important role in deciding which variant to choose. The advantage of using a private fleet is that it can reduce costs compared to normal carrier prices or transport shipments directly from origin to destination through multiple stops without having to collect them in central warehouses of a common carrier. However, the use of your private car fleet may mean higher costs in some specific cases. For example, the situation where customers are in hard-to-reach areas where, thanks to their savings, a common carrier can be able to offer a lower price especially for small shipments.

To formalize the problem more precisely, assume that we are given customers' demands with known quantity. The goods can be transported as a complex from the depot to the customers by private fleet with limited capacity or by common carrier. The customers are represented as nodes on a given graph with weighted edges, where weights correspond to distances. The goods are placed in the depot where also the private fleets and vehicle of common carrier are located. The private fleet has all vehicles with the same capacity. This fleet can be used in the TSP-style and the price of delivery is proportional to the sum of distances traveled by the vehicles. The common carrier has no capacity limit and costs are proportional to the quantity of transported goods only. The goal is to minimize the overall costs.

2 MATHEMATICAL MODEL OF VRPPC

VRPPC contains a binary variable z_i which is equal 1 if common carrier assures whole demand q_i of node i. This model can be formulated as integer programing problem. Let $G=\{V,E\}$ be an undirected complete graph, $V=\{1,2,\ldots,n\}$, node 1 is the depot and nodes $2,3,\ldots,n$ are customers.

Parameters of the model are:

- d_{ij} is costs of transport from node i to node j,
- q_i is demand in node i,
- W is capacity of private fleet vehicles,
- c_c is costs of transport of a unit of goods by the common carrier.

Variables of the model are:

- $x_{ij} \in \{0,1\}, x_{ij} = 1$ if a vehicle of the private fleet travels from node i to node j, $x_{ij} =$ 0 otherwise,

- $z_i \in \{0,1\}, z_i = 1$ if the common carrier serves whole demand q_i of node i, $z_i = 0$ otherwise,
- u_i are auxiliary variables in anti-cyclic constraints.

ILP model of VRPPC is following

$$f(x,z) = \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij} + c_c \sum_{i=1}^{n} q_i z_i \rightarrow \min \tag{1}$$

subject to

$$\sum_{i=1}^{n} x_{ij} = \sum_{i=1}^{n} x_{ji}, \qquad j = 1, \dots, n, \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} = 1 - z_i, \qquad i = 2, \dots, n, \tag{3}$$

$$u_i \leq W, \qquad i = 1, \dots, n, \tag{4}$$

$$u_i + q_j - W(1 - x_{ij}) \leq u_j, \qquad i = 1, \dots, n, j = 2, \dots, n, i \neq j, \tag{5}$$

$$x_{ii} = 0, \qquad i = 1, \dots, n. \tag{6}$$

The objective function (1) minimizes the sum of costs of vehicles of the private fleet and the common carrier. Constraint (2) states that a vehicle entering node j has to leave it. Equation (3) means that a node not served by the common carrier has to be served by a vehicle of the private fleet. Inequality (4) assures that capacity of the vehicle is not exceeded. Anti-cyclic conditions and defining of load u_i of the vehicle entering node i are in (5).

3 HEURISTICS USED FOR VRPPC SOLUTIONS

We consider two new heuristics for VRPPC. The first is a modification of the standard insert heuristic. The latter called **node subset heuristic**, is based on dividing of the set of nodes V into two subsets, subset served by private fleet and subset served by common carrier.

The following notation is used for the heuristics:

- the route of s-th vehicle is $R^s = \{v_1^s, v_2^s, \dots, v_{h^s}^s\}$, where $v_1^s = v_{h^s}^s = 1$ and (v_j^s, v_{j+1}^s) $j = 1,2, \dots, h^s - 1$ are arcs of route R^s,
- h^s is number of nodes in the route of s-th vehicle,
- n is number of nodes,
- S is number of vehicles of private fleet,
- c_c is the transport costs of unit of goods by the common carrier,
- q_i is demand at the node i,
- w^s is load of the private fleet vehicle s,
- W is capacity of the private fleet vehicles,
- d_{ij} is costs of transport from node i to node j.

3.1 Insert heuristic

The following algorithm is used for modified insert heuristic:

Step 0: $\mathrm{s} := 0$,

Step 1: {initial route s} $\mathrm{s} := \mathrm{s} + 1, \mathrm{k} := 0, \delta := 0, w^s := 0$, If s $>$ S then go to Step 3.

Repeat for i $= 2, 3, \dots, n$:

If $c_c q_i - (d_{1i} + d_{i1}) > \delta$ and $q_i > 0$ then k := i, $\delta := c_c q_i - (d_{1i} + d_{i1})$.

If k = 0 then go to Step 3,

else $v_1^s := v_3^s := 1, v_2^s := k, h^s := 3, w^s := q_k, q_k := 0$.

Step 2: {insertion} $\delta := 0, k := 0$,

Repeat for i $= 1, 2, \dots, n$:

If $q_i > 0$ then

Repeat for j $= 1, 2, \dots, h^s - 1$:

If $c_c q_i - \left(d_{v_j^s i} + d_{i v_{j+1}^s} - d_{v_j^s v_{j+1}^s}\right) > \delta$ and $w^s + q_i \le W$

then k := i, $\bar{j}$:= j, $\delta := c_c q_i - \left(d_{v_j^s i} + d_{i v_{j+1}^s} - d_{v_j^s v_{j+1}^s}\right)$.

If k = 0 then $h^s := h^s + 1, v_{h^s}^s := 1$, go to Step 1,

else remove arc $\left(v_{\bar{j}}^s, v_{\bar{j}+1}^s\right)$, add new arcs $\left(v_{\bar{j}}^s, v_k^s\right), \left(v_k^s, v_{\bar{j}+1}^s\right)$ to the route R^s.

Set $w^s := w^s + q_k, h^s := h^s + 1, q_k := 0$, go to Step 2.

Step 3: {common carrier transport}
The remaining demand $q_i > 0$, i $= 1, 2, \dots, n$ is assigned to transport by common carrier.

3.2 One feature of the VRPPC problem and Insert heuristic

The following notation is used for the modified insert heuristic: the route of *s*-th vehicle $R^s = \{v_1^s, v_2^s, \dots, v_{h^s}^s\}$, where $v_1^s = v_{h^s}^s = 1$.
The symbol $\Delta(i, j, k)$ means $\Delta(i, j, k) = (d_{ij} + d_{jk} - d_{ik})$.

Proposition 1 If $\{R^s; s = 1, 2, \dots, m\}$ is optimal, then inequality

$$\Delta(v_j^s, v_{j+1}^s, v_{j+2}^s) = \left(d_{v_j^s v_{j+1}^s} + d_{v_{j+1}^s v_{j+2}^s} - d_{v_j^s v_{j+2}^s}\right) \le c_c q_{v_{j+1}^s} \tag{6}$$

for all s $= 1, 2, \dots, m$ and j $= 1, 2, \dots, h^s - 2$ is valid.

Proof Proof by contradiction.

If the inequality (6) did not apply to some *s* and *j*, then

$$\left(d_{v_j^s v_{j+1}^s} + d_{v_{j+1}^s v_{j+2}^s} - d_{v_j^s v_{j+2}^s}\right) > c_c q_{v_{j+1}^s} \tag{7}$$

and by omitting the node v_{j+1}^s on the route *s* would reduce the value of the function, which is contrary to the assumption that the solution $\{R^s; s = 1, 2, \dots, m\}$ is optimal.

3.3 Node subset heuristic

VRPPC with non-split demand means that the set of nodes $V = \{1, 2, \dots, n\}$ is divided into two subsets, subset *V'* and subset *V-V'*. The first subset *V'* contains nodes which are served by the private fleet. Common carrier transports goods from the depot to the nodes contained in the second subset *V-V'*. In the optimal solution of VRPPC, there is an optimal subset of nodes *V'* and optimal routes of vehicles of the private fleet on the subset *V'*. If a subset of nodes *V'* is created, we have to optimize transport costs of the private fleet. Costs of the common carrier for nodes from subset *V-V'* are given by the sum of q_i for *i* in *V-V'* multiplied by costs c_c.

VRPPC consists from two sub-problems:

a) finding the optimal subset of nodes *V'*,
b) solving the vehicle routing problem on this subset of nodes *V'*.

Proposed heuristic consists of two parts: a heuristic for point a) and a heuristic for point b). We can utilize standard heuristics (for example insert heuristic, nearest neighborhood heuristic, savings heuristic, …) for the problem b). For the problem a) we have to choose a subset *V'* from all subsets of the set *V*. Because the number of all *V* subsets is roughly 2^n, it is not possible in polynomial time to go through all subsets and choose the subset with the lowest costs (sum of costs of the private fleet and common carrier). The proposed heuristic will test only a portion of these subsets, according to the chosen strategy, and determine the routes of the private fleet for each of them and calculate the total costs. From these subsets then selects subset with the lowest costs. This paper describes the following four sorting strategies for selecting a subset of nodes *V'*.

3.3.1 *dist-1* sorting rule

The first strategy creates a subset of V' from those nodes closest to the depot because it is assumed that the costs of delivering goods will be low for these nodes.

$$D_i^1 = d_{1i} \qquad \mathrm{i} = 2, 3, \dots, \mathrm{n}.$$

Since the distance from the depot to which the nodes should belong to the V' subset cannot be precisely determined, these subsets are gradually created according to the distance of the nodes from the lowest to the highest.

3.3.2 *dist-2(m)* sorting rule

This rule adds to the node's distance from the depot also distances to the next few adjacent nodes because the private fleet vehicle is expected to continue to its nearest *m* neighbors after visiting this node. The number of neighbors included in this consideration is parameter *m*.

$$D_{im}^2 = d_{1i} + \sum_{k=1}^{m} d_{ij_k} \qquad \mathrm{i} = 2, 3, \dots, \mathrm{n}, \mathrm{m} = 1, 2, \dots, \mathrm{n} - 1,$$

where j_k is k-th nearest neighbor of *i* and $j_k \neq 1$.

3.3.3 *km-tons* sorting rule

Another method of selecting nodes to subset V' assumes that it will be advantageous for the private fleet to visit those nodes that are close to the depot and the requirements of q_i are large. If a common carrier transported the goods to this node, the transport costs would be high due to the large volume of transport. Therefore, the selection of nodes to V' is valued both by the distance from the depot and by the volume of goods.

$$D_i^3 = d_{1i}\left(\max_{2 \le j \le n} q_j - q_i + \overline{q}\right) \qquad \mathrm{i} = 2, 3, \dots, \mathrm{n},$$

where $\overline{q} = \sum_{j=2}^{n} q_j / (n-1)$.

Since the volume of goods is maximized, we must put in this product instead of the demand volume q_i the difference of this requirement from the maximum requirement of q_j still increased by a constant (here the average requirement was chosen), for the reason that this product can be minimized and positive.

3.3.4 *costs-dist* sorting rule

The last strategy selecting nodes to the subset *V'* is based on the difference between the private fleet's costs if it uses direct route for delivery from the depot to node *i*, and the common carrier's costs.

$$D_i^4 = (d_{1i} + d_{i1}) - c_c q_i \qquad \text{i} = 2, 3, \dots, \text{n}.$$

3.3.5 Heuristic

The heuristic first sorts the list of nodes (in the descending order) according a particular rule. Then it calculates in the cycle for $\ell = 2, 3, \dots, \text{n}$ the total travel costs as the sum of the private carrier's travel costs *TP* for the first ℓ nodes calculated by some heuristic or mathematical model and the common carrier travel costs *TC* for the last $\text{n} - \ell + 1$ nodes of sorted list.

The following algorithm is used for all variants.

Step 0: $\ell := 0, T^* := \infty$.

Create sorted list $(v_1 = 1,\ v_2, \dots, v_n)$ of nodes $\{1, 2, \dots, \text{n}\}$ according to the selected rule.

Step 2: {private and common transportation costs}

Repeat for $\ell = 2, 3, \dots, \text{n}$

2a) solve VRP for the node subset $(v_1,\ v_2, \dots, v_\ell)$ using ILP (for small number of ℓ) or some heuristic method (i.e. nearest neighbor method, savings Clarke Wright heuristic, insert heuristic). Results are a set of routes *R* and private carrier's costs *TP*.

2b) The remaining demand $q_i > 0, i = v_{\ell+1}, v_{\ell+2}, v_n$ is assigned to the common carrier with costs $\text{TC} = \sum_{i=\ell+1}^{n} q_{v_i}$.

2c) If $T^* < TP + TC$ then $T^* := TP + TC, \ell^* := \ell, R^* := R$.

Results of the heuristic are the set of routes R^* and the total costs T^*. The heuristic can be applied with different values of parameter *m*, so with different choices of *m* we can get different values of T^* and take the lowest.

4 NUMERICAL EXAMPLES

The model and both heuristics were testing on examples VAL3, VAL6 and E3 published on http://www.uv.es/belengue.carp.html. The ILP model is solved by CPLEX 12.0. The heuristics are written in VBA language. PC (IntelCore2Quad, 2.83GHz) is used for computation. Computation of the model ran for three times and interrupted gradually after 1 minute, 10 minutes and 1 hour. Parameters and computational results are summarized in *Table 1*.

Table 1: Summary data for test instances.

sample	VAL3	VAL6	E3

n	24	31	77
c_c	10	15	300
W	50	50	50
model (1 min)	1200 (31%)	2690 (29%)	151260 (72%)
model (10 mins)	1200 (31%)	2690 (18%)	138020 (71%)
model (1 hour)	1200 (31%)	2540 (14%)	-
insert	1560	3175	155340
dist-1	1340	2950	**136520**
dist-2 (m)	1270 (7)	2630 (7)	**125740 (12)**
km-tons	1230	2735	**128180**
consts-dist	1310	2780	**127360**

The columns stand for the tested examples VAL3, VAL6 and E3. The first three rows represent parameters of examples, *n* is number of nodes, c_c is costs of the transport of unit of goods by the common carrier, *W* is capacity of private fleet vehicles. The next three rows contain results for the model (with different time of interruption in brackets in the first column). Result values are followed in parentheses by gap (difference between best feasible solution found so far and the corresponding best lower bound found so far). In one case (E3/1 hour) the result was not reached because the calculation was terminated due to insufficient memory. The remaining rows contain results for Insert heuristic and Node subset heuristic (4 rows for different type of nodes sorting). In case of the *dist-2* variant, the number of adjacent nodes whose sum of distances is applied to sorting is written in parentheses.

5 CONCLUSIONS

We consider non-split delivery modification of VRP with uniform private fleet and common carrier. ILP of the problem has been presented. Using of Insert heuristic for VRPPC, the feature of this problem and algorithm of Insert heuristic was presented. The new type of a heuristic approach has been designed. The model, the modified insert heuristic and a new type of heuristic were tested on instances from a benchmark dataset. The results show that the ILP formulation need not be suitable for solving the real problems. The proposed heuristic method gives us good solutions, better than results from the modified insert heuristic.

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AN ASSESSMENT OF THE EFFICIENCY OF SELECTED CZECH AND SLOVAK INTERNATIONAL AIRPORTS: DEA APPROACH

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Abstract
The paper deals with the evaluation of technical efficiency of international airports in the Czech Republic and Slovakia using the DEA method. Data about individual airports were collected for 2015–2018. The data envelopment analysis with two inputs and one output was used to assess the technical efficiency. The number of employees and costs were selected as inputs, the number of passengers was used as output. The first part of the paper is devoted to a literature review on the use of the DEA method in the evaluation of airport and air transport performance. The second part describes the methodology of own research. Finally, the research results are presented and discussed.

Keywords: *data envelopment analysis (DEA), international airports, technical efficiency*

JEL Classification: C14, C44, R40
AMS Classification: 90-06

1 INTRODUCTION

Air transport is one of the most important and fastest-growing forms of transport. In many areas, air transport is a major driver of regional development and has become indispensable for both the transport of persons and for many types of goods. Following the accession of the Czech Republic and Slovakia to the European Union in 2004, aviation infrastructure has grown significantly. An indispensable role in air transport is played by airports. The main objective of this paper is to evaluate the performance of the main international airports in the Czech Republic and Slovakia.

In 2019, 24 international airports were registered in the Czech Republic. However, there are only eight public international airports, five of which have regular air traffic. The largest airport in the Czech Republic and also the first airport to be evaluated is Vaclav Havel Airport Prague (hereinafter Prague-Ruzyne Airport). The second evaluated airport is Brno-Turany Airport. It is the second largest airport handling international public traffic in the Czech Republic. The third evaluated and the third largest international public airport is Leos Janacek Ostrava Airport. Another evaluated airport is the fourth largest international public civil airport in the Czech Republic, Karlovy Vary Airport. Other airports subject to the evaluation were the Pardubice and Hradec Kralove International Airports. Kunovice Airport was the last Czech airport to evaluated; it currently belongs to a group of international non-public airports with an external Schengen border.

14 public civilian airports, 14 non-public civilian airports, and 2 military airports were registered in Slovakia in 2019. Slovakia currently has 6 international airports. The largest Slovak airport and also another airport to be evaluated is M. R. Stefanik – Airport Bratislava (hereinafter Bratislava Airport). It is the main and largest international airport in Slovakia. The second evaluated airport is the second largest Slovak international airport located in Kosice. The most common routes from here are to Istanbul, Warsaw, Vienna, and London. The third evaluated airport is an international airport of regional importance – Piestany Airport. The catchment areas of this airport are the towns of Trnava, Nove Mesto nad Vahom, Trencin, and Topolcany. The fourth evaluated airport, Poprad – Tatry International Airport, is the highest

located airport for medium- and short-haul flights in Central Europe. Another airport that was evaluated was Silac International Airport, used mainly by charter flights, among other things. Zilina Airport is the last evaluated international airport. In addition to handling international and domestic flights, it is also used for private, sports and other special flights.

Most airports in the Czech Republic are owned by the various regions. The only airport owned by the Czech Republic is Prague-Ruzyne Airport. While the number of passengers at Prague-Ruzyne Airport has been increasing every year, the regional international airports have typically run at a loss and have been experiencing growth problems. Despite the fact that regional airports are not tremendously profitable, they bring a certain amount of secondary profit to each region in the form of tourism for the entire area (The Ministry of Transport of the Czech Republic, 2019). However, this raises the question whether such a large number of domestic international airports is too large. In Slovakia, the most important airports are M. R. Stefanik Airport and Kosice Airport, which together serve 97% of all passengers, while Sliac, Piestany, Zilina, and Poprad – Tatry Airport share the rest. Which one of them has the greatest perspective?

2 LITERATURE REVIEW

Research into the measurement and comparison of airport performance uses a wide range of quantitative techniques. Almost all commonly used approaches to measure airport productivity and effectiveness are based on the ratio between the airports' outputs (results) and inputs (resources). This principle is also used for conducting data envelopment analysis (hereinafter DEA), which is flexible and relatively easy to use.

The DEA method searches for the weights of input and output parameters to maximize the degree of technical efficiency, which is assigned a closed interval value between 0 and 1. Units with a degree of technical efficiency equal to 1 are marked as effective and form the technical efficiency threshold. Units with a technical efficiency value of less than 1 are marked as inefficient and lie below the efficiency threshold (Cooper et al., 2006). Depending on the production options and characteristics of input/output variables, we can consider several different types of DEA models: the Charnes, Cooper and Rhodes model (CCR), and the Banker, Charnes and Cooper model (BCC). The CCR model differs from the BCC model by considering a constant return to scale, while the BCC considers a variable return to scale.

Some authors apply the DEA method in their work when evaluating the performance and efficiency of airports and airlines. In their paper, Danesi and Lupi (2008) applied input-oriented DEA models to evaluate the performance of Italian airports. They saw inputs as: the number of gates in the passenger terminal and the total length of runways. They chose three variables as outputs: the number of aircraft movements, the number of passengers, and the quantity of carried cargo. A paper by Koçak (2010) used the DEA model to explore the efficiency of 40 airports in Turkey (2008). Fageda and Voltes-Dorta (2012) evaluated 42 Spanish airports. Research has found that small airports are unable to benefit from economies of scale. Lai et al. (2015) used a combination of AHP methods and the DEA method to assess the efficiency of 24 international airports around the world. A set of six input parameters (the number of employees, the number of gates, the number of runways, terminal area, runway length, and operating expenses) and four output parameters (the number of passengers, the amount of cargo and mail, aircraft movements, and total revenue).

3 DATA AND METHODOLOGY

The first step was to select an airport sample for analysis. Six Czech airports and six Slovak airports were selected. When defining the sample, it was decided to focus primarily on international civilian airports. The main source of data were the annual reports and financial statements of the evaluated airports. The annual reports and financial statements for 2015 to 2018 revealed data on the number of transported persons, the number of employees and the total costs).

Tab. 1: Assessed Czech and Slovak airports and a sample of variables for 2018

Airport name	IATA	Employees	Costs (CZK/EUR)	Number of passengers
Praha-Ruzyne	PRG	2 654	157 103 000	16 797 006
Brno-Turany	BRQ	143	99 532 000	500 727
Leos Janacek Ostrava	OSR	176	278 735 000	377 936
Karlovy Vary	KLV	43	44 902 400	45 003
Pardubice	PED	32	78 288 000	147 572
Kunovice	UHE	15	27 347 000	195
Bratislava	BTS	608	32 010 022	2 292 712
Piestany	PZY	25	1 229 560	768
Poprad – Tatry	TAT	60	2 987 274	88 542
Kosice	KSC	134	10 068 424	542026
Sliac	SLD	38	2 317 236	41 866
Zilina	ILZ	19	28 131	523

Source: author's calculations

Based on the available data from the above sources, it was also necessary to select appropriate sets of inputs (I) and outputs (O). The following inputs were considered: I1 – Number of full-time employees. I2 – Costs (in CZK). The costs of Slovak airports were converted at the exchange rate of CZK 26/EUR to CZK. The following outputs were considered: O1 – Number of transported persons. Another important step is to choose the appropriate number of inputs and outputs, taking into account the number of evaluated units (a total of 12 airports). Bowlin (1998) stated that the number of DMUs should be three times the number of inputs and outputs being considered. The total number of variables had not to be reduced.

The next step was to construct DEA models and determine the technical efficiency (TE) score. Input-oriented DEA models were used to measure efficiency through the DEA method, namely the BCC-I and CCR-I models. The classic CCR-I model works with the assumption of a constant return to scale and can be recorded as a relation (1); the BCC-I model works with the assumption of a variable return to scale, and the relationship is supplemented by the condition of convexity (2).

$$\mathrm{E}_0 = \min.\theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)$$

$$\text{s.t.} \sum_{j=1}^{n} \lambda_j X_{ij} + s_i^- = \theta X_{i0}, i = 1, \dots, m \qquad (1)$$

$$\sum_{j=1}^{n} \lambda_j Y_{rj} - s_i^+ = Y_{r0}, r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j = 1 \qquad (2)$$

$$\lambda_j, s_i^-, s_i^+ \geq 0, j = 1, \dots, n, i = 1, \dots, m, r = 1, \dots, s.$$

Where λ_j, $j = 1, 2, \ldots, n$ are weights of all DMUs, s^-_i, $i = 1, 2, \ldots, m$ and s^+_r, $r = 1, 2, \ldots, s$ are slack/surplus variables, $\varepsilon > 0$ is an infinitesimal constant (so-called non-Archimedean element) defined to be smaller than any positive real number, and θ is the efficiency score that expresses the reduction rate of inputs in order this unit reaches the efficient frontier.

A score of 1 was awarded to any airport that did not show signs of inefficiency when compared with the other related airports. A score of less than 1 indicated an inefficient airport. The BCC model measures pure technical efficiency (PTE). The CCR-I measures the overall technical efficiency (OTE) by aggregating pure technical efficiency and scale efficiency (SE) into one value (see relationship 3).

$$OTE = PTE \cdot SE \tag{3}$$

The CCR model assumes that a business operates at the optimal scale, i.e. under conditions of constant returns to scale (CRS). Scale efficiency (SE) measures the extent to which a business can improve its efficiency by changing its size.

4 RESEARCH RESULTS

First, the BCC-I model was applied to the data. The average rate of pure technical efficiency (see PTE score in Table 2) of airports during the whole period of 2015-2018 was relatively high. To achieve efficiency, the airports would have to reduce inputs by 21% to 36%. Of the 12 airports in total, 3 to 6 airports were near the efficiency threshold. Three airports were reached the efficiency threshold during the entire reference period (PRG, UHE and ILZ). These three airports are able to effectively transform given inputs into an output. The remaining airports, which were classified as inefficient according to the BCC-I model, required an amount of inputs that was much greater than optimal in order to generate the given level of outputs. In an input-oriented model, the general recommendations for similarly inefficient airports include changes to practices and processes in controlled operations so as to reduce the level of inputs at the current level of outputs.

In the second step, the CCR-I model, which provides overall technical efficiency scores (see OTE score in Table 2), was applied to the data. Since this model assumes constant returns to scale, it can be used to identify airports that operate at the optimal scale, i.e. the size of their operations is optimal and most productive. The CCR-I model identified 1 to 2 airports as effective every year. No airport was on the efficiency threshold during the entire reference period. Detailed results are shown in Table 2.

Tab. 2: Pure and overall technical efficiency scores

	2015		**2016**		**2017**		**2018**		**AVG OTE**
Airport	**PTE**	**OTE**	**PTE**	**OTE**	**PTE**	**OTE**	**PTE**	**OTE**	
PRG	**1.000**	0.563	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**0.891**
BRQ	**1.000**	**1.000**	0.415	0.346	0.484	0.385	0.628	0.553	**0.571**
OSR	0.571	0.550	0.226	0.172	0.315	0.230	0.388	0.339	**0.323**
KLV	0.577	0.360	0.393	0.072	0.404	0.059	0.527	0.165	**0.164**
PED	**1.000**	0.663	0.587	0.126	0.873	0.395	**1.000**	0.729	**0.478**
UHE	**1.000**	0.004	**1.000**	0.002	**1.000**	0.003	**1.000**	0.002	**0.003**
BTS	**1.000**	0.874	0.371	0.370	0.438	0.418	0.608	0.596	**0.565**
KSC	0.742	0.735	**1.000**	**1.000**	0.519	0.429	0.702	0.639	**0.701**
TAT	0.575	0.431	0.395	0.178	0.426	0.174	0.465	0.233	**0.254**

PZY	0.493	0.025	0.774	0.009	0.652	0.004	0.644	0.005	**0.011**
SLD	0.456	0.173	0.460	0.075	0.498	0.119	0.549	0.174	**0.135**
ILZ	**1.000**	0.069	**1.000**	0.038	**1.000**	0.041	**1.000**	0.007	**0.039**
AVG	0.785	0.454	0.635	0.282	0.634	0.271	0.709	0.370	X

Source: author's calculations

Table 2 also shows that airports that were classified as efficient according to the CCR-I model are also classified as efficient according to the BCC-I model. This means that the size of these airports' operations is optimal and, at the same time, that these airports are able to efficiently transform the given inputs into outputs thanks to appropriate management methods, practices and processes. Prague-Ruzyne Airport can be designated as the most efficient airport for the entire period of review. Bratislava Airport was on the efficiency threshold in 2016-2018 and can be described as efficient in terms of traffic size; at the same time, this airport is able to effectively transform the inputs into outputs thanks to appropriate procedures, processes and methods of management. Other airports identified as effective according to the CCR-I model included Brno and Kosice. For these airports, however, while the size of their operations was optimal, the airports were also able to effectively transform the given inputs into outputs only one year at a time. The tables clearly show that the PTE score is higher than the OTE score, as the CCR-I model takes into account scale inefficiency (SE), which reduces the OTE value. Scale efficiency scores are listed in Table 3 and calculated according to the relationship (3).

For airports that are classified as inefficient according to the CCR-I model but are classified as efficient according to the BCC-I model, we can say that their technical inefficiency is only due to scale inefficiency. It is to be expected that, while these airports use best practices in operations management, their size is not optimal. The management recommendations for these selected operators should focus on changing the scale of operations (i.e. size) depending on the type of returns to scale.

Tab. 3: Scale efficiency scores and returns to scale of each airport (own processing)

Year	**2015**			**2016**			**2017**			**2018**		
Airport	**SE**	**∑λ**	**RTS**	**SE**	**∑λ**	**RTS**	**SE**	**∑λ**	**RTS**	**SE**	**∑λ**	**RTS**
PRG	0.563	4.358	DRS	1.000	**1.000**	CRS	1.000	**1.000**	CRS	1.000	**1.000**	CRS
BRQ	1.000	**1.000**	CRS	0.834	0.214	IRS	0.795	0.031	IRS	0.881	0.030	IRS
OSR	0.963	0.663	IRS	0.761	0.205	IRS	0.730	0.021	IRS	0.874	0.023	IRS
KLV	0.624	0.111	IRS	0.183	0.009	IRS	0.146	0.001	IRS	0.313	0.003	IRS
PED	0.663	0.127	IRS	0.215	0.033	IRS	0.452	0.006	IRS	0.729	0.009	IRS
UHE	0.004	0.000	IRS	0.002	0.000	IRS	0.003	0.000	IRS	0.002	0.000	IRS
BTS	0.874	3.357	DRS	0.997	0.934	IRS	0.954	0.126	IRS	0.980	0.136	IRS
KSC	0.991	0.881	IRS	1.000	**1.000**	CRS	0.827	0.032	IRS	0.910	0.032	IRS
TAT	0.750	0.183	IRS	0.451	0.036	IRS	0.408	0.005	IRS	0.501	0.005	IRS
PZY	0.051	0.004	IRS	0.012	0.001	IRS	0.006	0.000	IRS	0.008	0.000	IRS
SLD	0.379	0.044	IRS	0.163	0.016	IRS	0.239	0.002	IRS	0.317	0.002	IRS
ILZ	0.069	0.001	IRS	0.038	0.000	IRS	0.041	0.000	IRS	0.007	0.000	IRS
AVG	**0.578**	**x**	**x**	**0.471**	**x**	**x**	**0.467**	**x**	**x**	**0.543**	**x**	**x**

Source: author's calculations

Table 3 shows that the solution to technical inefficiency could be to expand traffic, because all airports, except Prague-Ruzyne Airport and Bratislava Airport in 2015, worked with an increasing return to scale during the time evaluated. Only Kosice in 2016 and Prague-Ruzyne Airport in 2016-2018 worked to the optimal extent. With the exception of above named airports, all other airport operated in a situation of increasing returns to scale, and so the same recommendation (expanding operations) applies to them.

5 CONCLUSION

Based on the analyses conducted, it can be concluded that the overall technical efficiency of airports is relatively low. On average, airports would have to reduce inputs by 54% to 73% to become technically efficient, while maintaining the current volume of transported passengers. However, the average pure technical efficiency value was higher. The difference is due to low scale efficiency. An average airport is smaller than its optimal size and, at the same time, it operates under conditions of increasing returns to scale. Only two airports reached an optimal scale size – Prague-Ruzyne Airport in 2016-2018 and Kosice in 2016. Therefore, these airports can be described as the most powerful among the evaluated international airports. Other relatively well-rated airports were the Bratislava and Brno airports.

Poprad Airport - Tatry is doing quite well, because it has had a clear strategy for several years and is supported by the city in active tourism. The Karlovy Vary Airport is mainly focused on Russia and Ukraine. Due to the short and narrow runway, Karlovy Vary Airport cannot yet consider routes to more distant destinations, as larger types of aircraft cannot land on it. Due to the unfavorable development, the Karlovy Vary region must also save its airport from financial difficulties (Sůra, 2019). Sliac Airport is having problems. Its only advantage is that the airport's infrastructure is also operated by the army and the civilian portion of the airport is not concerned about the cost of operating the runway. Therefore, this airport needs relatively low-performance from charter flights to avoid a loss, but on the other hand, it does not generate any profit. Piestany Airport is dealing with problems. These are related, among other things, to the proximity of competing airports in Bratislava and Vienna. One positive aspect is the change of leadership in the region, which is interested in developing the airport and has begun to revive it, especially through charter flights. Zilina Airport finds itself in the worst situation. It is the only international airport in Slovakia that does not have a long enough runway, and this limits its operations to turboprop aircraft. Of all the airports, Zilina has the greatest potential in the field of business aviation and business travellers. The composition of the region's industry and economy provide a strong background for this. Most airports have been found to be operating under conditions of an increasing return to scale, where efficiency can be increased by increasing performance. Zilina Airport, for example, is effective in terms of transforming inputs into outputs and its inefficiency only caused by scale inefficiency (Vojtaššáková, 2019). The results of the research may serve as a methodological proposal for assessing airport performance, such as for domestic airports.

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APPLICATION AND COMPARISON OF NETWORK DEA MODELS IN BANKING SECTOR

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Abstract

Unlike traditional models, Network DEA (NDEA) captures internal structure of the unit as a set of interacting sub-processes, connected by links, or intermediaries. However, a particular real system can be modelled in multiple ways with differing structures, and possibly, results. In a previous paper, the author proposed ways to measure model complexity and differentiating power. In this paper these are applied to a real problem, consisting of 20 banks on the Czech market, defined by 11 factors. A traditional model is examined, as are 3 versions of a network model, with 3 different structures – 2- and 3-stage series model, and a 4-process hybrid model with a shared factor. For these, the proposed measures are calculated – 4 for complexity and 3 for differentiating power, and based on them, a suitable model is chosen.

Keywords: *data envelopment analysis, Network DEA, NDEA, comparison, complexity, banking*

JEL Classification: C44, C61, C67

AMS Classification: 90-08, 90C05, 90C60

1 INTRODUCTION

Data envelopment analysis (DEA) was first proposed by Charnes and Cooper [2] as the CCR model (with Rhodes). It is regarded as a versatile theoretical and mathematical tool, but has also achieved lasting popularity in practical applications. It is used for evaluating the performance of an individual production unit (decision-making unit, DMU) among a set or grouping of like units. Over the years there have been published many modifications of the original CCR model, most notably the BCC model [1] which introduced variable returns to scale and SBM model which used slacks to measure the efficiency. All of these, and the ones that followed, however, did not take into account the possibility that DMUs could have some internal structure. They are therefore referred to as black-box models, as illustrated by figure 1. And while such abstraction of the real system may often be justified, in some cases the internal processes are too significant a factor that their omission produces skewed results and they need to be taken into account. In 2000, Färe and Grosskopf [3] proposed a model in which this internal structure is captured as a series of interconnected processes (divisions, sub-processes), each of which acts as a DMU itself, with its own set of inputs and outputs. The processes can be connected to each other via links, factors that are output from one process and then consumed as an input of another, as seen in figure 2. This forms the basis of Network DEA (NDEA).

Figure 1: Example black-box model structure

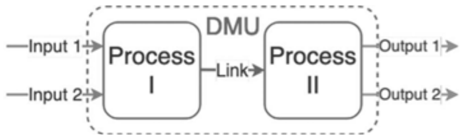

Figure 2: NDEA model, with internal structure

One of the issues with Network DEA is that the structure and indeed number of processes can be adjusted based on a multitude of factors, such as availability of data, perceived flows or desired precision. In particular, adding further processes and more intricate links, in an effort to bring the model closer to reality, translates to disproportionately more variables and constraints in the final model. This added complexity requires more computational resources, longer time and is more difficult to both construct and interpret. In a previous paper [5], the author proposes several ways to measure both the complexity of the model and the quality of the results it gives. This paper serves as a follow-up and applies some of the methods to a practical example, based on data from the banking sector. It starts by introducing the models that will be compared – a single traditional, black-box model and three NDEA models of varying complexity, to be precise. What follows is a presentation of actual data and three network structures under consideration. The models are then applied on the data and results are calculated and briefly discussed. Several of the methods proposed in the previous paper are then used to measure the complexity and the quality of the results, and these measures are then used to compare all models in an attempt to choose one of them to serve as a compromise. Finally, a discussion of obtained results and some other remarks, as well as proposals for further research in the field, conclude the paper.

2 USED MODELS

The models under consideration are a single traditional, black-box model and three network models of varying complexity – one as simple as possible (for NDEA), an intermediate one and one relatively complex. This scheme was chosen to better illustrate differences between the models, especially with regards to computational requirements and their ability to differentiate the units based on calculated efficiency scores, as outlined in the previous paper. All chosen models utilize the slacks-based measure, as introduced by Tone.

2.1 Black-box model

The traditional model is Tone's landmark SBM model [6], in which he introduced slack variables for each factor, as a way to measure the absolute difference of a given DMU with respect to its efficient reference. One way the various SBM models can differ is in the precise form of the objective function. Based on whether it is the input or output slacks that are optimized, they can be input-oriented, output-oriented or non-oriented. Chosen model is non-oriented, so that both kinds of slacks appear in the objective, as found in [6, p. 5]. Both the constant returns-to-scale (CRS) and variable ones (VRS) are considered. While it is common to use a linear form (via Charnes-Cooper transformation), it is not necessary in this case. The solver program used for calculations is perfectly capable of handling the non-linear form as is.

2.2 Network model

Chosen network model also uses slacks-based measure and is in fact a generalization of the SMB model. Proposed by Tone and Tsutsui, [7] NSBM incorporates links between individual processes and comes in multiple varieties, based on whether and how the various slacks, especially link slacks, appear in the objective and affect the resulting efficiency. The one chosen here is the most comprehensive one, where all factors, that is inputs, outputs and links, are optimized – the non-oriented, link flow adjusted model. Also, variable returns to scale are assumed. As with other NDEA models, it allows one to calculate not only the overall efficiency of the unit, but also to decompose it into the efficiencies of each individual process, dubbed divisional efficiencies. They can be calculated by optimizing the following objective:

$$\rho_o = \frac{\sum_{k=1}^{K} w^k \rho_o^{k-}}{\sum_{k=1}^{K} w^k \rho_o^{k+}} \to \text{MIN}, \qquad \begin{aligned} \rho_o^{k-} &= 1 - \frac{1}{m^k + \sum_{f\in F^k} t^{(f,k)}} \left(\sum_{i=1}^{m^k} \frac{s_i^{k-}}{x_{io}^k} + \sum_{\substack{i=1 \\ f\in F^k}}^{t^{(f,k)}} \frac{s_i^{(f,k)-}}{z_{io}^{(f,k)}} \right), \\ \rho_o^{k+} &= 1 + \frac{1}{r^k + \sum_{g\in G} t^{(k,g)}} \left(\sum_{i=1}^{r^k} \frac{s_i^{k+}}{y_{io}^k} + \sum_{\substack{i=1 \\ g\in G^k}}^{t^{(k,g)}} \frac{s_i^{(k,g)+}}{z_{io}^{(k,g)}} \right), \end{aligned} \qquad \rho_o^k = \frac{\rho_o^{k-}}{\rho_o^{k+}},$$

where the overall efficiency score of unit o is given as ρ_o, while its k-th divisional efficiency is $\rho_o{}^k$. Constraints can be found in [7, pp. 246, 251], with all terms defined therein. In the objective function, one may see weights w^k for each division – these are set exogenously and reflect the relative importance of each division to the decision-maker. They are set so that $\sum^K{}_{k=1} w^k = 1$.

3 APPLICATION IN THE BANKING SECTOR

To compare the models presented in the previous chapter, a real world scenario needed to be found. Banking and finance routinely make use of DEA, both traditional and network. There is a natural need for evaluation of various entities, like banks, companies, branches, stocks and so on. The availability of large quantities of precise data is also important. That is why this particular area was chosen. To be precise, it is the evaluation of the performance of various banks on the Czech market that was studied. The dataset comprises a total of 20 banks operating at one point on the Czech market and can be found in its entirety in [4, p. 63 - addendum B].

3.1 Structures considered

To use the SBM model, the 11 factors that are available in the data needed to be sorted into inputs and outputs. Of course, they could have been filtered, but for the sake of meaningful comparison it seems more appropriate that the number of factors stays constant and only the structure changes. A single bank was therefore modelled as transforming its assets – workforce, branches, equity, deposits and renown (as measured by number of clients), as well as operating costs – into loans, income (from fees and interest) and health indicators of ROE (return on investment) and CAR (capital adequacy ratio). This simple, black-box structure is illustrated by figure 3. By separating it into two processes - marketability and profitability – the same bank can be modelled using a network structure, as illustrated by figure 4.

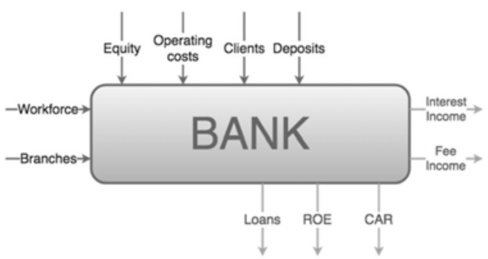

Figure 3: Black-box model of a bank

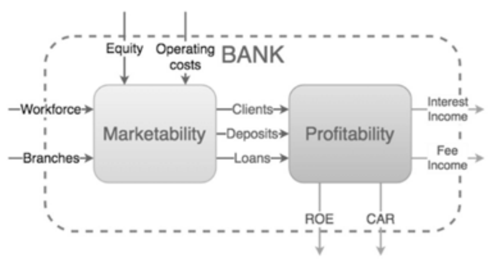

Figure 4: 2-stage series network model of a bank

The links in the 2-stage model can be further separated by another process, which represents conversion of clients (as potential sources of income) into loans and deposits (as real sources of income), as seen in figure 5. Finally, the most complex structure considered incorporates several changes, as evident in figure 6. Profitability is considered solely as an income generating process (despite the slightly misleading name) and the health indicators of return on investment and capital adequacy ratio are considered to be the only two final outputs. They are modelled as a result of bank's ability to turn income and costs (and thus profit) and equity into future growth and financial stability. This process is dubbed (stakeholder) desirability, because it is by these measures that the bank ultimately will be judged by investors and governments. Because there is also a parallel flow and equity as an input is shared, this is a hybrid model.

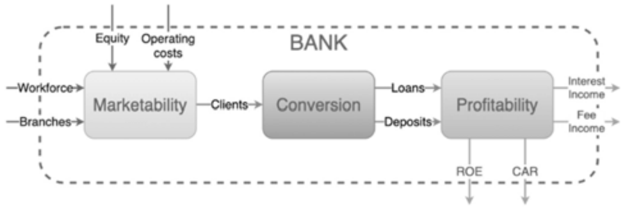

Figure 5: 3-stage series model of a bank

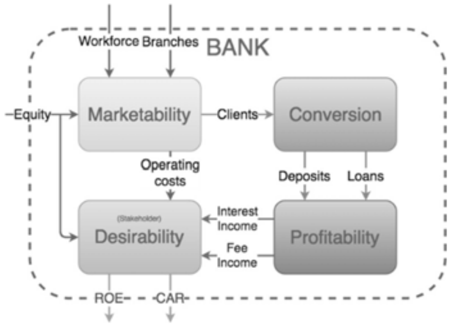

Figure 6: Hybrid series-parallel model with 4 processes

3.2 Calculation and results

All mathematical models were transcribed into the LSP language and solved by LocalSolver. The number of units that were calculated to be efficient, with both overall and for each division (in brackets) for network models, can be seen in table 1. Also included are the computational resources needed to calculate solution for each model. Shown are aggregate resources (time and number of solver iterations) needed to solve for all 20 units combined, as output by LocalSolver. Included are also the number of decision variables and constraints for each individual model.

Table 1: Number of efficient units, computational resources and model size

Model	Efficient units	Time	Iterations	Variables	Constraints
black-box (CRS)	14	00:00:07	169 728	31	11
black-box (VRS)	20	00:00:09	211 747	31	12
2-stage	17 (m.: 18; p.: 19)	00:00:24	732 173	54	19
3-stage	10 (m.: 12; c.: 20; p.: 13)	00:00:55	1 623 693	74	20
4-stage hybrid	9 (m.: 10; c.: 20; p.: 16; d.: 13)	00:00:59	1 571 262	97	26

Source: author's calculations

4 COMPARISON OF MODELS

At first glance, it is obvious that as the number of stages increases, fewer units are identified as efficient, keeping in mind that results for network models must be compared primarily to VRS black-box model.This supports a hypothesis that more realistic (complex) internal structure leads to higher differentiating power. It is important to note that differentiating power is not the only, nor main reason to prefer NDEA – there are after all non-network models that allow full ranking [4]. Nevertheless, it is a useful attribute, especially when comparing several models. As mentioned, the comparison utilizes methods outlined in the previous paper, where they are presented in detail. All except the entropy-based measure of differentiating power have been calculated, and their values for each model can be seen in table 2.

Table 2: Measures of complexity and differentiating power

Γ, Δ	Γ_{VC}	Γ_F	Γ_P	Γ_D	Γ_T	Γ_{T*}	Δ_N	Δ_S	Δ_C	$\Delta_{C'}$=1-Δ_C
black-box (CRS)	42	11	1	1	7	169 728	0,300	0,287	0,169	0,831
black-box (VRS)	43	11	1	1	9	211 747	0,000	0,000	1,000	0,000
2-stage	73	11	2	3	24	732 173	0,150	0,106	0,077	0,923
3-stage	94	11	3	4	55	1 623 693	0,500	0,181	0,100	0,900
4-stage hybrid	123	12	4	7	59	1 571 262	0,550	0,149	0,092	0,908

Source: author's calculations

All terms are as defined in the paper, with an addition of $\Gamma T*$, which represent solver iterations, rather than time in seconds, as in ΓT. Also, in calculating clustering measure ΔC, average distance to cluster centroid as determined by k-means algorithm, weighted by cluster size, has been used. Lower values then represent better differentiating power, and so $\Delta C' = 1 - \Delta C$ was used in further calculations. On the complexity side, all measures reflect the intuition with regards to increasing complexity with additional stages. In differentiating power measures, that situation seems much the same, with the notable exception of a slight edge for 2-stage model over 3- and 4-stage models. This can either indicate more distinct efficiency scores or possible issues in the measure itself. In any case, to properly compare all models, measures of both complexity and differentiating power must be brought together into one compromising measure $\Pi'^{(r)}_{\alpha}$, as given in (4, p. 6). This final compromising measure for each pair of differentiating power and complexity measure can then be seen in table 3, with no special preference for either.

Table 3: Compromising measure for pairs of Δ and Γ, α=0,5

Δ	Δ_N						Δ_S						$\Delta_{C'}$					
Γ	Γ_{VC}	Γ_F	Γ_P	Γ_D	Γ_T	Γ_{T*}	Γ_{VC}	Γ_F	Γ_P	Γ_D	Γ_T	Γ_{T*}	Γ_{VC}	Γ_F	Γ_P	Γ_D	Γ_T	Γ_{T*}
black-box (CRS)	**0,38**	0,21	**0,44**	**0,54**	**0,63**	**0,67**	**0,60**	**0,40**	**0,66**	**0,74**	**0,80**	**0,83**	**0,40**	0,24	**0,46**	**0,56**	**0,63**	**0,67**
black-box (VRS)	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
2-stage	0,11	0,10	0,11	0,09	0,09	0,08	0,13	0,15	0,12	0,09	0,09	0,07	0,26	**0,26**	0,25	0,21	0,20	0,17
3-stage	0,28	0,34	0,25	0,23	0,13	0,12	0,17	0,25	0,14	0,12	0,06	0,05	0,19	**0,26**	0,16	0,15	0,09	0,08
4-stage hybrid	0,24	**0,35**	0,20	0,14	0,14	0,13	0,11	0,19	0,09	0,05	0,05	0,05	0,15	0,24	0,12	0,09	0,08	0,08

Source: author's calculations

It is apparent by looking at the results (best compromising measure is highlighted) that when no preference is given, the CRS black-box model is almost unanimously declared as best. That is because it is easiest to model and calculate, yet provides results of reasonable quality. However, when one decides to replace the practically questionable assumption of constant returns to scale, the resulting VRS model becomes useless, as evidenced by 0 score – all units efficient. Network models are preferred only when comparing against Γ_F (number of factors) measure. 4-stage model is preferred when comparing Δ_N (number of inefficient units), 2- and 3-stage models when using cluster analysis, but only narrowly so. It is also worth noting that the 3-stage model is in many cases better performing than the 4-stage model. This might suggest that, all other requirements being equal, there is little advantage to further modelling the intricacies of the system by adding another process, and so 3-stage model might be best suited.

5 CONCLUDING REMARKS

This paper follows from the previous publication and set out to apply the methods proposed therein on real-world data. First, the models that were going to be used for such comparison were presented. After presenting the data in question, consisting of 20 banks on the Czech market, what followed were the actual structures to be modelled and compared, along with the rationale on why they were chosen. After calculating the results, the complexity and differentiating power measures proposed previously were computed and used to compare the models in all their combinations. Finally, this comparison was interpreted and the appropriate model chosen for each scenario. What remains to be done in future research is to also include the entropy-based measure or find out if other measures could be useful. Also worth revisiting would be the measure based on cluster analysis, as there may yet be issues previously not considered. Another possibility is to compare various wildly different models, not just multiple variants of (N) SBM model and see what the results would be like. Of course, using the proposed measures for comparison in other real-world applications is always an interesting prospect.

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COUNTERFACTUAL IMPACT EVALUATION OF RURAL RENEWAL AND DEVELOPMENT SUPPORT PROGRAM

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Abstract

The main objective of this study is to show how various direct effects of grant programs can be estimated using econometric evaluation methodologies. The empirical analysis is focused on evaluating the effects of the Czech national grant program for rural renewal and development. A propensity score methodology is applied to identify differences in specific indicators on a municipality level comparing the group of program beneficiaries with an appropriately selected control group.

Keywords: *Counterfactual Analysis, Grant Programs Evaluation, Matching, Propensity Scores*

JEL Classification: C51, H54
AMS Classification: 91B68, 91B32

1 INTRODUCTION

Development programs and policies are typically designed to change outcomes; for example, to raise incomes, to improve learning, or to reduce illness. Whether or not these changes are actually achieved is a crucial public policy question but one that is not often examined. More commonly, program managers and policymakers focus on controlling and measuring the inputs and immediate outputs of a program rather than on assessing whether programs have achieved their intended goals of improving well-being (White and Raitzer, 2017).

This problem is present in regional, national, and European grant programs as well. Until recently, the major criticism of the existing EU standard evaluation system and common indicators concerned the relevance and appropriateness of particular indicators suggested by the European Commission; the lack of a coherent evaluation framework linking inputs, outputs, and outcomes; gaps in data in the programs' monitoring systems, and the lack of prioritization between many indicators (Michalek, 2012). While the EC has addressed some of these problems in the evaluation guidelines prepared for the programming period 2007-2013, the major issues remained open. As mentioned by Michalek (2012), the vast majority of studies concerned with the quantitative assessment of socio-economic impacts of programs for rural development in EU countries (programming period 2000-2006) was based on "naïve" evaluation techniques, which compare the observed changes in selected performance indicators in a sample of program areas with arbitrarily selected comparison groups. This kind of evaluation is subject to many distortions.

Fortunately, advanced evaluation methodologies based on causality theory are able to eliminate the distortions. Some of them were successfully applied in studies that focused on the measurement of effects of various structural, social and rural programs in several countries worldwide, e.g., Argentina (Ravallion, 2001), USA (Dehejia and Wahba, 2002), and Bolivia (Newman et al., 2002). Studies performing program evaluations based on counterfactual analysis are also emerging in Europe: Switzerland (Jalan and Lechner, 2002), Finland (Venetokis, 2004), Germany (Pufahl and Weiss, 2009), and Slovakia (Bartova and Hurnakova, 2016; Michalek et al., 2018). This paper adds to this overview with the counterfactual evaluation of the effects of the Czech national rural renewal and development support program.

2 METHODOLOGY

2.1 Propensity score matching

Principles of counterfactual impact evaluation can be used for determining the effects of various interventions in the area of social sciences, economy, public health, and others. The term counterfactual describes hypothetic situation expressing what has not happened but could, would, or might have occurred under differing conditions (Švarc, 2016). In the case of random assignment of individuals into treatment and control group, the treatment effect can be calculated as an average difference between the change in observed variable for treatment and control group. However, in field of development program evaluations, the assumption of random assignment to treatment and control groups is not valid. Since municipalities self-select themselves to apply or not for the support in the program, a selection bias may emerge. In addition, the selection procedure of beneficiaries may favor particular types of projects. Thus another important issue is a simultaneity bias since the support is not assigned randomly.
Due to these facts, new methods for estimation of the treatment effect were developed to address the problem of missing counterfactual. The methods differ in the basic assumptions concerning the way in which they treat the selection bias in the estimated treatment effect. The best known and most used methods include randomized control trials, differences in differences, instrumental variables, regression discontinuity, and propensity score matching (PSM), which was used in this study. In the PSM framework, we consider binary variable Tr representing two possible treatments (Tr = 1 for active treatment, and Tr = 0 for control treatment), covariates X, and an outcome variable Y. A causal effect on an individual i is a difference between $Y_i\ (1)$ (potential response under an active treatment) and $Y_i\ (0)$ (potential response under control treatment). The intervention as a whole can be then evaluated using population average treatment effect:

$$ATE = E[Y(1) - Y(0)] \tag{1}$$

Another useful characteristics is the average treatment effect on the treated:

$$ATT = E[Y(1) - Y(0)|\mathrm{T} = 1] \tag{2}$$

Both characteristics have to be estimated because one of the potential responses $Y_i\ (0),\ Y_i\ (1)$ is always unobservable.

Pairing methods consist of four key steps (Rosenbaum and Rubin, 1983):
1. Definition of the distance used to determine whether an entity is suitable for matching with another entity considering the observed covariates. We used the distance given by the formula $D_{ij} = |e_i - e_j|$, where e_i is the propensity score of the subject i with covariates X_i defined as the probability of treatment assignment conditional on observed covariates:

$$e_i = e(X_i) = P(Tr = 1|X_i). \tag{3}$$

The propensity scores are usually estimated by logistic regression.

2. Introduction of the method of matching with respect to the degree of similarity: We selected one of the most common methods; 1:1 nearest neighbor matching (*NNM*), which assigns each subject i of the treated group to the element of the control group having the smallest distance from i.
3. Assessing the quality of the resulting pairs, and repeating the first two steps until sufficiently similar pairs are achieved: A standardized difference of means (SMD) was used for numerical evaluation of the covariates balance in the matched groups. Graphical diagnostics was also used

to assess the assumption of overlapping ranges of propensity scores in the treatment and control group.

4. Response analysis and estimation of the effect of the treatment with respect to the pairing performed in the previous steps: The unknown counterfactual response is for every subject i in the active group estimated by Y_{i*}, where i^* denotes the element of the control group matched with i. If we compute the differences between these responses and average them over all N_T matched pairs, then we get the estimate of the overall effect ATT defined as

$$\widehat{ATT} = \frac{1}{N_T} \sum_{i:Tr_i=1} [Y_i - Y_{i*}], \tag{4}$$

2.2 Data for the analysis

The matching process described in the previous section was applied to the dataset containing 1,025 municipalities whose representatives applied for a national subsidy under the sub-program Support for Rural Renewal and Development in 2015 (Ministerstvo pro místní rozvoj: národní dotace, 2019). Specifically, it was a subsidy title to support the involvement of children and youth in community life in the municipality. Our goal was to test whether this subsidy has an impact on the total annual increase in population in the municipality by estimating the Average Treatment Effect on the Treated (ATT) based on propensity score matching.
In this case, assignment to treatment was represented by the binary variable $Tr \in \{0,1\}$, where $Tr = 1$ denotes municipalities that has been granted a subsidy and $Tr = 0$ denotes i.e. municipalities with rejected applications for subsidy. The number of beneficiaries of the subsidy was 413, and the number of non-supported municipalities was 612. As the aim of a contrafactual impact evaluation is to measure the long-term effects, the outcome of interest was variable CP_17, which indicates the total increase in the population of the municipality for 2017. The role of pre-treatment covariates, i.e. confounding variables X, represented the population in the municipality 2014 (PO_14), the number of registered enterprises in the municipality in 2014 (RP_14), the share of unemployed persons in the municipality 2014 (PNO_14), the municipality's property at the end of 2014 (Majetok_14), the municipality's liabilities at the end of 2014 (Zavazky_14), and the value of the requested subsidy (DT).
The averages and standard deviations of these covariates on the original treated and control subpopulations, as well as the standardized mean differences (SMD) between the group averages of the individual covariates are given in Table 1.

	0	**1**	**SMD**
n	612	413	
PO_14 (mean (SD))	786.64 (678.72)	840.46 (612.83)	0.083
RP_14 (mean (SD))	168.44 (147.55)	177.92 (135.01)	0.067
PNO_14 (mean (SD))	7.60 (3.60)	7.30 (3.08)	0.091
Majetok_14 (mean (SD))	95606.12 (102798.63)	103142.72 (90085.53)	0.078
Zavazky_14 (mean (SD))	7745.00 (20209.89)	8858.27 (14939.55)	0.063
DT (mean (SD))	316310.46 (99375.55)	327750.39 (90573.05)	0.120

***Table 1*:** Before Matching

The treatment effect on the original unmatched set before matching obtained as the difference between the mean outcome value of the treatment group and the mean outcome value of the control group was 0.44. That means, supported municipalities have on the average higher annual increase in population by 0.44.

3 RESULTS

The first step in the matching process is to define distance between subjects given by pre-treatment covariates. Hence we conducted logistic regression of the treatment (Tr) upon confounding variables X mentioned above to estimate propensity scores, that is the probability of assignment the subsidy. The results are shown in *Table 2.*

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	-6.025e-01	2.742e-01	-2.197	0.028
PO_14	3.695e-04	4.091e-04	0.903	0.366
RP_14	-1.572e-03	1.800e-03	-0.874	0.382
PNO_14	-2.954e-02	1.973e-02	-1.497	0.134
Majetok_14	2.546e-07	1.296e-06	0.196	0.844
Zavazky_14	1.550e-06	4.479e-06	0.346	0.729
DT	1.126e-06	6.876e-07	1.637	0.102

Table 2: Estimation of propensity scores based on the logistic regression model

Estimates of the logistic model parameters are not informative and not interpretatively interesting, only estimates of the propensity scores are interesting, and the quality of the model is assessed by the balance of the matched groups. However, it is necessary to assess the overlap assumption before matching based on the estimated propensity scores. A perfect overlap will ensure that for each subject in the observed treatment group, a subject from the observed control group can be found that has the same probability of being assigned to the treatment group as the subject treated. This assumption can be evaluated visually based on the histogram of the estimated propensity scores for the observed treated and observed control group. (*Figure 1*)

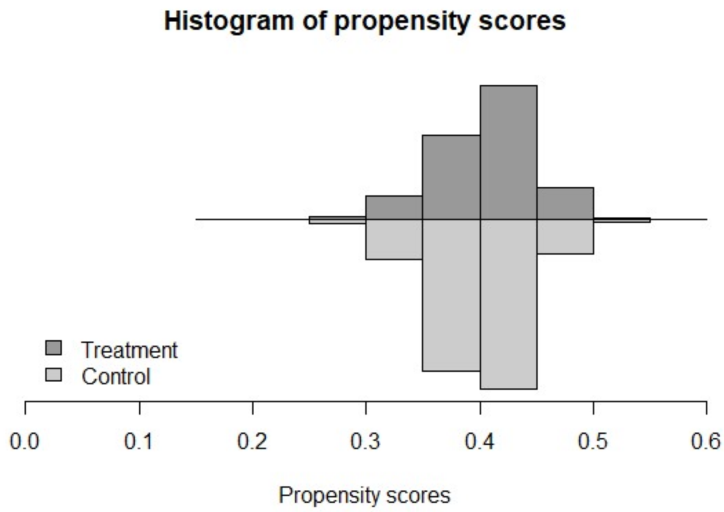

Figure 1: Histogram to assess the assumption of overlap of estimated propensity scores.

The graph above shows that the propensity score values estimated for the treatment group reflect the estimated score values in the control group and the overlap assumption was therefore satisfied.

The next step of the matching process was to choose a matching algorithm and to perform matching. We selected one of the most common methods; the 1:1 nearest neighbor matching. The resulting descriptive characteristics of individual covariates after 1:1 NNM matching are shown in *Table 3.*

	0	1	SMD
n	413	413	
PO_14 (mean (SD))	805.42 (695.65)	840.46 (612.83)	0.053
RP_14 (mean (SD))	170.56 (147.91)	177.92 (135.01)	0.052
PNO_14 (mean (SD))	7.19 (3.11)	7.30 (3.08)	0.035
Majetok_14 (mean (SD))	96186.88 (100276.79)	103142.72 (90085.53)	0.073
Zavazky_14 (mean (SD))	7630.72 (14083.49)	8858.27 (14939.55)	0.085
DT (mean (SD))	327503.88 (95806.00)	27750.39 (90573.05)	0.003

***Table 3*:** After 1:1 Nearest Neighbor Matching

As we can see, there are 413 control subjects matched with all 413 treated subjects, and 199 control subjects were discarded. These matched groups should be balanced with respect to confounding variables because the propensity scores have a balancing property. However, they were estimated, so the balance needs to be verified before we can proceed with outcome analysis. To this check were used standardized differences in means. From Table 3 we can see very small values of SMD in the pre-treatment covariates; hence we can conclude that the balancing property was achieved. The results of the matching we also assessed by using a graph that compares how many subjects and which subjects from the control and treatment groups were used for matching. Figure 2 is a jitter chart showing how many subjects from the treatment group with the matching algorithm found the "pair" from the control group, and how many subjects from the control group control remained unused.

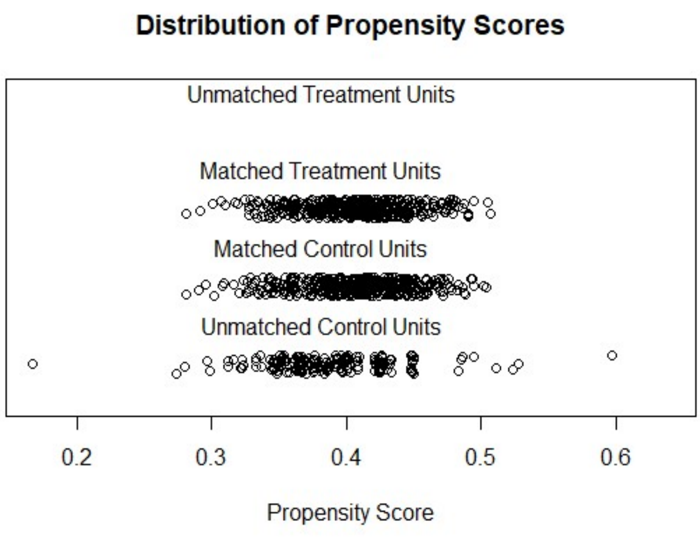

Figure 2: Jitter chart showing propensity scores of matched and unmatched subjects

Finally, the last step was estimation of the Average Treatment Effect on the Treated, i.e. effect of subsidies on total population growth for 1:1 nearest neighbor matching. *Table 4* shows that $\widehat{ATT}$ is equal to 1.19, which means that municipalities with subsidies have on average 1.19

higher total annual increase in population as municipalities without the subsidy. The computations were performed using the R package MatchIt (R-project, 2019).

Matching method	$\widehat{ATT}$ (Confindence intervals)	t- test (p-value)
1:1 *NNM* without replacement	1.19 (-0.73; 3.11)	0.2223

Table 4: Resulting estimates by conducting 1:1 nearest neighbor matching without replacement with a logistic regression-based propensity score

Subsequently, to test whether this effect is statistically significant we performed a paired t-test on the difference in the outcome of the matched subjects. From Table 4 above we can see that p-value of this t-test is not significant on the 5% level, which indicates that direct impact of subsidies for supporting the involvement of children and youth in community life in the municipality on the total annual increase in population was not proven.

4 CONCLUSIONS

Based on the results, it can be concluded that municipalities with subsidies show on average a higher total population growth than municipalities without subsidies. The result after matching is even almost three times higher than the difference between the original observed groups of municipalities with and without subsidies. However, this result is still not statistically significant.

There are several limitations to the methodology we used. Some of them are related to the fact that we performed ex-post evaluations because impact evaluations are best designed before a program begins to be implemented. Matching methods are in general less robust than the other evaluation methods such as randomized selection, regression discontinuity design, or difference-in-differences, and in practice, they are typically used when the other options are not possible. Another problem of matching approaches is that they require large samples and relevant characteristics with sufficient common support for the treated and untreated group. Since we had enough data, this issue was of minor relevance for us. But one significant issue cannot be eliminated: Although matching helps to control for observed background characteristics, we can never control for bias coming from unobserved characteristics. Despite these limitations, our method, including at least some available characteristics relevant for identifying a counterfactual, provides evidence for more informed decision-making and is much more useful than naive approaches. Application of inadequate methodologies for evaluation of program impacts would lead to bad decisions with a number of negative consequences, including inappropriate policy measures and inefficient allocation of public resources, and in the final consequence may call into question not only the credibility of program evaluations but also that of all institutions involved.

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THE IMPORTANCE OF INTERNET OF THINGS DEVICES IN LOGISTICS

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Abstract
Internet of Things – IoT concept is based on the application of information and communication technologies that enable marking, identification, communication and intelligent management of things. IoT consists of a large number of devices located on physical objects, capturing the characteristics of objects and the environment, and sending and receiving communication messages. IoT devices may be RFID tags (Radio Frequency Identification), RF readers (Radio Frequency), card readers, smart sensors, actuators, cameras, GPS devices (Global Positioning System), etc. Realization of IoT in a company enables the creation of a virtual reality model where real-time process management is possible based on timely information about the current state of physical objects. The aim of this paper is to develop a model for evaluating the importance of different IoT devices in planning and managing logistics processes in a company. The model is based on the application of AHP and FAHP methods. The model allows defining importance and priorities of different IoT devices that meet the requirements and needs of the company.

Keywords: *Internet of Things, Logistics, AHP, FAHP.*

JEL Classification: C, L
AMS Classification: 90-08

1 INTRODUCTION

The development of information and communication technologies has created new possibilities and challenges in the field of logistics. Internet of Things – IoT includes the application of contemporary information and communication technologies which enable identification and location of objects, generating data on their state, sending and receiving information. Various IoT devices are placed on physical objects and enable their integration, connectivity in real time and detection of the surroundings. The application of the IoT concept in logistics and supply chains enables data processing on objects, automatization, monitoring, supervision and control of logistic activities and processes. The effects of applying IoT devices are multiple for all participants in logistics. The aim of this paper is to develop a model for evaluating the importance of different IoT devices in planning and managing logistic processes in a company. The model is based on applying AHP and FAHP methods. The model enables evaluation of the significance and defining priorities of different IoT devices which meet the requirements and needs of a company.

The paper consists of five parts. The second part describes different IoT devices in logistics and the significance of their usage. The third part describes the description of the model for the choice of IoT devices in logistic processes. A numerical example and analysis of output results are presented in the fourth part. The conclusion is presented in the fifth part.

2 IOT DEVICES IN LOGISTICS

Supply chain management involves planning and managing all processes and activities of procurement, production, transport, demand, order processing and payments. This implies the coordination and cooperation of all participants in the supply chain – suppliers, manufacturers, distributors, intermediaries, logistic service providers and users (He et al., 2009). According to the 7R concept, a successful supply chain means that the right product, in the right condition, in the right quantity, in the right quality is at the right place at the right time and at the right price. Supply chain success requires the support of the IS (Information System), which enables real-time identification, retrieval, location tracking, monitoring and optimization. Product identification informs the system on the right products, tracking enables the system to detect if the product is lost and to show the right location and the right quantity. Product monitoring ensures the right quality of the product. Every logistic process can be observed and monitored by this data and one can react to unforeseen events in real time. The existence of all these data provides a basis for improving and optimizing business and forming the right price (Ferreira et al., 2010). The presence of smart objects in supply chains is a prerequisite for the realization of the 7R concept.

IoT enables identification, connectivity and mutual communication of different objects in the supply chain. The existence of IoT devices on goods, vehicles, trans-shipment mechanization and other objects ensures information exchange through a wireless network in order to achieve common goals. The application of IoT devices on various objects in logistics and supply chains has led to the existence of smart containers, pallets, packaging, packing, vehicles, shelves, forklifts, infrastructure, ports, terminals, etc. (Radivojević et al., 2017). The most commonly used IoT devices in logistics are: RFID tags, RF readers, wireless sensor networks – WSN, cameras and GPS devices. The basic challenge of the IoT concept is the use of devices that are low-power, inexpensive, networked and compatible with standard communication technologies (Miorandi et al., 2012). RFID tags and WSNs are based on real-time recording of object characteristics and the possibility of communication with superior systems in order to track objects and processes. RF readers enable contactless reading of tags and barcode markings. Different types of cameras in combination with OCR technology (Optical Character Recognition) are used on objects and in business systems for video surveillance and environmental state detection. GPS devices enable real-time location of objects. GPS devices are placed on transport means, pallets, containers and individual goods, industrial and trans-shipment machinery, devices used by employees in business processes, etc.

The expected effects of implementing the IoT concept in logistics and supply chains are as follows (Radivojević et al., 2017):

- Real-time monitoring of transport and trans-shipment means, logistic units, goods and people.
- Measuring resource performance and planning according to current state.
- Logistic controlling of activities and processes, responding to deviation and disruption conditions, and implementing corrective measures in order to ensure realization of set goals.
- Analytics of all data and information in order to analyze the existing state to identify the possibilities of new business promotions.
- Automation of business processes by eliminating manual labour while improving quality and reducing costs.
- Optimization of people, systems and resources and their coordination and integration.

IoT devices are constantly evolving and new devices are emerging on the market specialized for application in specific business processes and activities. Most companies are unable to apply all IoT devices. The choice of IoT devices and their application in supply chains is based on the opinion of experts on their importance and impact on the efficiency of business processes.

3 MODEL FOR EVALUATING THE IMPORTANCE OF IOT DEVICES

The choice of IoT devices that will be implemented in a company depends on a number of factors: the complexity of the business system, organization and cooperation in supply chains, the level of cooperation with business partners, etc. In this paper, a model for the choice of IoT devices has been developed which is based on applying AHP and FAHP methods. The usage of these methods enables decision making based on expert judgement. The model consists of three steps (Radivojević & Popović, 2018):

1. Hierarchical structure of the problem;
2. Application of AHP method, and
3. Application of FAHP method.

3.1. Hierarchical Structure of the Problem

The hierarchical structure of the problem involves examination of all elements and organizing them in hierarchical levels. The basic elements are the goal, the criteria and alternatives. Defining the hierarchical problem structure involves determining the goal, selecting the criteria and potential outcomes of decisions that correspond to the nature of human judgement when solving problems. In this model, the hierarchical structure has three levels shown in Figure 1.

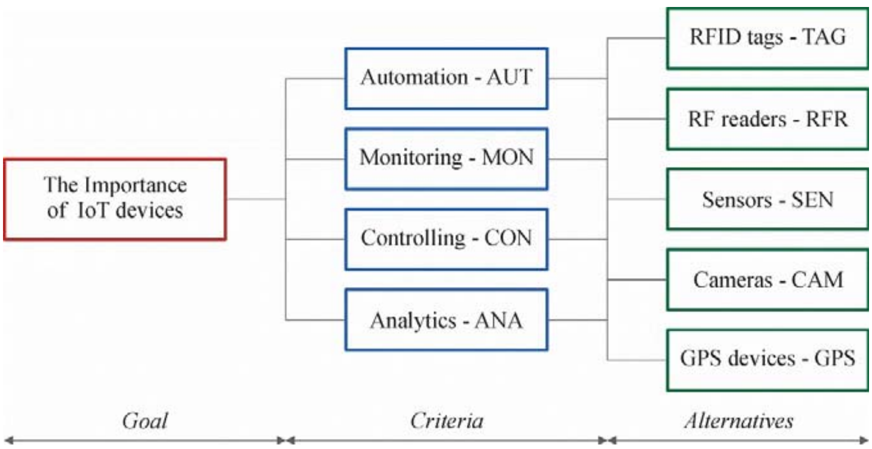


Figure 1. Hierarchical structure of criteria and alternatives

The first level is the goal of the model – determining the importance of IoT devices in logistic processes. The decision criteria are on the second level – automation (AUT), monitoring (MON), controlling (CON) and analytics (ANA). The third level includes alternatives, various IoT devices – RFID tags (TAG), RF readers (RFR), sensors (SEN), cameras (CAM) and GPS devices (GPS).

3.2. The AHP Method

Analytic hierarchy process – AHP is a multicriteria ranking method developed by Saaty (1977). AHP is used to determine priority in decision making. The input data for this method are expert evaluations of pairs of elements at each level of hierarchy in relation to a higher level. AHP enables partial solutions at each level of the hierarchy and consolidation of partial solutions in order to obtain a final solution to the problem. In his papers (1986, 1990), Saaty gave a detailed description and mathematical formulation of the AHP method. The application process can be described through the following steps (Vinod & Ganesh, 1996):

- Comparison of element pairs i and j on each hierarchy level in relation to higher level elements, using the Saaty scale from 1 to 9. Experts determine the value a_{ij} where $a_{ij} = \frac{1}{a_{ij}}, \forall i, j = 1, \dots n$ and $a_{ij} = 1, i = j$.
- Determining the priority of each element in relation to the higher hierarchy level. The priority of the alternative i in relation to criteria j is w_{ij}, where $i = 1, \dots, m$, $j = 1, \dots, n$, m is the number of alternatives and n is the number of criteria.
- Synthesis of all priority values in order to obtain the priority of each element in relation to the goal. W_i is the priority of alternative i and is determined as $W_i = \sum_{j=1}^{n} c_j \cdot w_{ij}$, where c_j is the priority of criterion j, and w_{ij} is the priority of alternative i in relation to criterion j.

When determining priorities at each level, the values for λ_{max} (the principal eigenvalue of matrix), CI (consistency index) and CR (consistency ratio) are determined. The consistency condition is $CI < 10\%$ (Saaty, 1990).

3.3. The FAHP Method

Fuzzy Analytic hierarchy process – FAHP is an expansion of the AHP method and the application of fuzzy sets (Van Laarhoven & Pedrycz, 1983). Fuzzy sets are applied because it is not possible to obtain clear and precise expert evaluations of element comparison in solving certain problems. The use of fuzzy numbers can improve the accuracy of expert judgment and the quality of output results. Expert evaluations of element comparisons are based on the application of fuzzy numbers that correspond to Saaty's scale from 1 to 9. FAHP consists of the following steps (Wang et al., 2008):

- Comparison of element pairs i and j on each hierarchy level in relation to higher level elements, using the fuzzificated Saaty scale from 1 to 9 – determining the value of fuzzy number $\tilde{b}_{ij} = (l_{ij}, m_{ij}, u_{ij})$, $i, j = 1, \dots, n$.
- Determining the value of RS_i by summing the rows of the matrix $\tilde{B} = (\tilde{b}_{ij})$ as per relation $\widetilde{RS}_i = \sum_{j=1}^{n} \tilde{b}_{ij}$. Calculating the value of $\tilde{S}_i$ by normalizing $\widetilde{RS}_i$ according to relation $\tilde{S}_i = \frac{RS_i}{\sum_{j=1}^{n} RS_j}, i = 1, \dots, n$.
- Determining the probability that $\tilde{S}_i \geq \tilde{S}_j$ and the probability that the fuzzy number $\tilde{S}_i$ is greater than other fuzzy numbers according to relation $V(\tilde{S}_i \geq \tilde{S}_j | j = 1, \dots, n; j \neq i) = \min_{j \in \{1,\dots,n\}} V(\tilde{S}_i \geq \tilde{S}_j), i = 1, \dots, n$.
- Determining the priority vector $W = (w_1, \dots, w_n)$ of the comparison matrix $\tilde{B}$ where $w_i = \frac{V(\tilde{S}_i \geq \tilde{S}_j)}{\sum_{k=1}^{n} V(\tilde{S}_k \geq \tilde{S}_j)}, i = 1, \dots, n$.

4 NUMERICAL EXAMPLE

The model for assessing the importance of IoT devices in logistics processes has been tested on an example which included $n = 4$ criteria and $m = 5$ alternatives. The criteria were: automation – AUT, monitoring – MON, controlling – CON and analytics – ANA. The alternatives were RFID tags – TAG, RF readers – RFR, sensors – SEN, cameras – CAM and GPS devices – GPS. A software application was developed for the proposed model in MS Excel VBA environment. The input data – comparison of element pairs, were obtained by surveying logistics experts.

4.1. AHP Method

Table 1 shows the values of the criterion comparison matrix, priority vector W and calculated values λ_{max}, CI and CR. The value CR is less than 10% which means that priority vector W can be accepted. Table 2 shows the values of alternative comparison matrix in relation to chosen criteria and values of local priority w for each criterion. Based on obtained local priority values, global priorities for each alternative are calculated. Table 3 shows the local and global priorities for each IoT device alternative according to criteria. The TAG IoT device has the highest significance of 34.26%, while CAM has the lowest significance of 5.78%. The AUT – automation criteria has the highest influence on the choice of IoT devices with 54.30%, and the lowest influence has ANA – analytics with 7.65%.

Table 1. Comparison matrix and criteria weighting factors

	AUT	MON	CON	ANA	W
AUT	1	3	5	5	0.5430
MON	1/3	1	3	3	0.2445
CON	1/5	1/3	1	3	0.1360
ANA	1/5	1/3	1/3	1	0.0765
λ_{max} = 4.2045		CI = 0.0682		CR = 0.0757	

Table 2. Comparison matrix and weighting factors of IoT devices

		TAG	RFR	SEN	CAM	GPS	W
AUT	TAG	1	1	5	5	3	0.3674
	RFR	1	1	3	3	1	0.2419
	SEN	1/5	1/3	1	3	1/5	0.0895
	CAM	1/5	1/3		1	1/5	0.0566
	GPS	1/3	1	5	5	1	0.2446
	λ_{max} = 5.3484 CI = 0.0871 CR = 0.0778						
MON	TAG	1	1	3	5	5	0.3611
	RFR	1	1	3	5	3	0.3287
	SEN	1/3	1/3	1	3	3	0.1577
	CAM	1/5	1/5	1/3	1	1/3	0.0545
	GPS	1/5	1/3	1/3	3	1	0.0981
	λ_{max} = 5.2216 CI = 0.0554 CR = 0.0495						
CON	TAG	1	1	3	5	1	0.2763
	RFR	1	1	3	5	1/3	0.2319
	SEN	1/3	1/3	1	3	1/3	0.1078
	CAM	1/5	1/5	1/3	1	1/3	0.0590
	GPS	1	3	3	1	1	0.3251
	λ_{max} = 5.2991 CI = 0.0748 CR = 0.0668						
ANA	TAG	1	1	1	3	1	0.2254
	RFR	1	1	1/3	3	1/3	0.1527
	SEN	1	3	1	3	1	0.2734
	CAM	1/3	1/3	1/3	1	1/3	0.0751

	GPS	1	3	1	3	1	0.2734
	$\lambda_{max} = 5.2003$ $CI = 0.0501$ $CR = 0.0447$						

Table 3. Local and global priorities of criteria and alternatives – AHP

	AUT	MON	CON	ANA	
	0.5430	0.2445	0.1360	0.0765	← W ↓
TAG	0.3674	0.3611	0.2763	0.2254	0.3426
RFR	0.2419	0.3287	0.2319	0.1527	0.2549
SEN	0.0895	0.1577	0.1078	0.2734	0.1227
CAM	0.0566	0.0545	0.0590	0.0751	0.0578
GPS	0.2446	0.0981	0.3251	0.2734	0.2219

4.2. FAHP Method

Expert estimations of IoT devices in relation to all criteria presented by fuzzy numbers represent the input data for the FAHP method. The comparison matrix and calculated values of priority vectors for each criterion are presented in Table 4. The AUT – automation criteria of business processes has the highest significance with 44.36%. Table 5 shows the comparison matrices and weighting factors of IoT devices in relation to all criteria. Table 6 shows local and global priorities for each IoT device according to criteria. Based on FAHP method analysis, the highest significance has the TAG IoT device with 30.16%.

Table 4. Fuzzy comparison matrix and criteria weighting factors

	AUT	MON	CON	ANA	W
AUT	(1, 1, 1)	(1, 3, 5)	(3, 5, 7)	(3, 5, 7)	0.4436
MON	(1/5, 1/3, 1)	(1, 1, 1)	(1, 3, 5)	(1, 3, 5)	0.3179
CON	(1/7, 1/5, 1/3)	(1/5, 1/3, 1)	(1, 1, 1)	(1, 3, 5)	0.2070
ANA	(1/7, 1/5, 1/3)	(1/5, 1/3, 1)	(1/5, 1/3, 1)	(1, 1, 1)	0.0315

Table 5. Fuzzy comparison matrix and weighting factors of IoT devices

		TAG	RFR	SEN	CAM	GPS	W
AUT	TAG	(1, 1, 1)	(1, 1, 3)	(3, 5, 7)	(3, 5, 7)	(1, 3, 5)	0.3190
	RFR	(1/3, 1, 1)	(1, 1, 1)	(1, 3, 5)	(1, 3, 5)	(1, 1, 3)	0.2513
	SEN	(1/7, 1/5, 1/3)	(1/5, 1/3, 1)	(1, 1, 1)	(1, 3, 5)	(1/7, 1/5, 1/3)	0.1295
	CAM	(1/7, 1/5, 1/3)	(1/5, 1/3, 1)	(1/5, 1/3, 1)	(1, 1, 1)	(1/7, 1/5, 1/3)	0.0114
	GPS	(1/5, 1/3, 1)	(1/3, 1, 1)	(3, 5, 7)	(3, 5, 7)	(1, 1, 1)	0.2888
MON	TAG	(1, 1, 1)	(1, 1, 3)	(1, 3, 5)	(3, 5, 7)	(3, 5, 7)	0.3116
	RFR	(1/3, 1, 1)	(1, 1, 1)	(1, 3, 5)	(3, 5, 7)	(1, 3, 5)	0.2934
	SEN	(1/5, 1/3, 1)	(1/5, 1/3, 1)	(1, 1, 1)	(1, 3, 5)	(1, 3, 5)	0.2210
	CAM	(1/7, 1/5, 1/3)	(1/7, 1/5, 1/3)	(1/5, 1/3, 1)	(1, 1, 1)	(1/5, 1/3, 1)	0.0235
	GPS	(1/7, 1/5, 1/3)	(1/5, 1/3, 1)	(1/5, 1/3, 1)	(1, 3, 5)	(1, 1, 1)	0.1505
CON	TAG	(1, 1, 1)	(1, 1, 3)	(1, 3, 5)	(3, 5, 7)	(1, 1, 3)	0.2602
	RFR	(1/3, 1, 1)	(1, 1, 1)	(1, 3, 5)	(3, 5, 7)	(1/5, 1/3, 1)	0.2536
	SEN	(1/5, 1/3, 1)	(1/5, 1/3, 1)	(1, 1, 1)	(1, 3, 5)	(1/5, 1/3, 1)	0.1719
	CAM	(1/7, 1/5, 1/3)	(1/7, 1/5, 1/3)	(1/5, 1/3, 1)	(1, 1, 1)	(1/5, 1/3, 1)	0.0542
	GPS	(1, 1, 3)	(1, 3, 5)	(1, 3, 5)	(1, 3, 5)	(1, 1, 1)	0.2602
ANA	TAG	(1, 1, 1)	(1, 1, 3)	(1, 1, 3)	(1, 3, 5)	(1, 1, 3)	0.2284
	RFR	(1/3, 1, 1)	(1, 1, 1)	(1/5, 1/3, 1)	(1, 3, 5)	(1/5, 1/3, 1)	0.2102
	SEN	(1/3, 1, 1)	(1, 3, 5)	(1, 1, 1)	(1, 3, 5)	(1, 1, 3)	0.2284
	CAM	(1/5, 1/3, 1)	(1/5, 1/3, 1)	(1/5, 1/3, 1)	(1, 1, 1)	(1/5, 1/3, 1)	0.1046
	GPS	(1/3, 1, 1)	(1, 3, 5)	(1, 1, 3)	(1, 3, 5)	(1, 1, 1)	0.2284

Table 6. Local and global criteria priorities of criteria and alternatives – FAHP

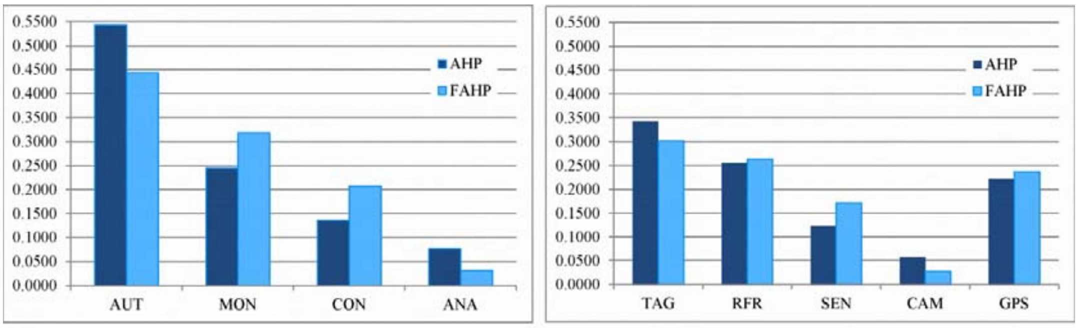
	AUT	MON	CON	ANA	

	0.4436	0.3179	0.2070	0.0315	← W ↓
TAG	0.3190	0.3116	0.2602	0.2284	0.3016
RFR	0.2513	0.2934	0.2536	0.2102	0.2639
SEN	0.1295	0.2210	0.1719	0.2284	0.1705
CAM	0.0114	0.0235	0.0542	0.1046	0.0270
GPS	0.2888	0.1505	0.2602	0.2284	0.2370

4.3. Analysis of Results

The comparative analysis of the results obtained using AHP and FAHP methods is shown in Figure 2 and includes comparisons by (a) criteria and (b) by alternatives. The analysis of obtained priorities shows great similarity. The existing discrepancies are acceptable and are a consequence of the fact that the FAHP method largely acknowledges the subjectivity of experts when comparing criteria and alternatives. The analysis of the results point out to the following conclusions:

- Criteria ranking by significance are as follows: AUT, MON, CON and ANA.
- The criterion of highest significance is AUT – automation of business processes (54.30%, 44.36%), and the least significance has ANA – analytics (7.65%, 3.15%).
- Ranking of alternatives by priority is: TAG, RFR, GPS, SEN and CAM.
- Alternatives TAG (34.26%, 30.16%) and RFR (25.49%, 26.39%) have the highest significance, namely match the chosen criteria the most.



(a) Criterion Comparison (b) Alterantives Comparison

Figure 2. Comparison of the AHP and FAHP methods

5 CONCLUSION

IoT technology greatly improves operations in logistics companies and management of supply chains. Identification technology, locating, tracking and controlling objects are applied in all logistics processes, from procurement processes, product distribution and up to retail facilities. The support of IoT devices enables faster and better decision making, real-time monitoring and controlling of objects. IoT brings economic benefits to logistics companies, saves costs, and improves management and monitoring of objects. This paper describes a model for selecting IoT devices based on application of AHP and FAHP methods. These methods use subjective evaluation, knowledge and experience of experts on IoT devices and logistics business processes. The results obtained in the numerical example show the possibility of applying the model in a logistics company.

In future research, the described model may represent a support basis in making decisions on the significance of introducing and applying IoT devices in logistics processes. An upgrade of the model may be a larger number of criteria, introducing a sub-criteria level, a larger number of alternatives and the formation of a knowledge base of expert estimations on various IoT devices.

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CLUSTER ANALYSIS OF ICT POTENTIAL OF ENTERPRISES WITHIN INDUSTRY 4.0 REQUIREMENTS IN THE CENTRAL AND EASTERN EUROPEAN COUNTRIES

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Abstract

The current paper is devoted to the issue of ICT usage by enterprises within the background of Industry 4.0. The goal of the article is to provide information on the similarity of the Central and Eastern European economies with respect to the ICT utilisation by non-financial enterprises. The analysis is conducted for the years 2010-2018 with application of Eurostat data at the national level. The scientific problem of the paper was treated here as multiple-criteria issue. As a result, the Ward's clustering method was used here in order to group the countries into relatively homogeneous classes. In spite of the high dynamics of improvements obtained by the less developed countries of the region, the conducted research confirmed relatively high stability of the results in time with respect to the heterogeneity between the countries. This factor can indicate that the challenges of transformation towards Industry 4.0 in the case of countries that aim at closing their development gap is much higher that it was believed a decade ago. The current results can be used for looking for examples of best practices within the context of forming the guidelines for national and regional modernisation strategies.

Keywords: *Cluster analysis, Ward's method, Industry 4.0., ICT utilisation, Central and Eastern Europe*

JEL Classification: C38, O30, O14
AMS Classification: 91C20

1 INTRODUCTION

The idea of Industry 4.0 has changed the way of thinking of both managers and policymakers. It has affected the long term development policies in the highly developed countries, but it is also of special importance for the countries that are forced to modernize their economies and close the development traps (Simionescu et al., 2017; Kuc, 2017; Ungerman et al., 2018; Jasinska & Jasinski, 2019; Asim et al., 2020; Mai et al., 2019). At first the concept of Industry 4.0 was mostly related to the technological issues and the problem of more effective application of ICT in the production process. However, currently it is commonly accepted that it is a multi-criteria concept, where not only tangible issues are in the core of interests, but also many intangible factors such as the role of high quality management in the sphere of human capital (Pietrzak & Balcerzak, 2016; Hariharasudan & Kot, 2018; Stankevičienė et al., 2019; Pisar & Bilkova, 2019) or the growing role of entrepreneurs, who would be able to build the structure of Industry 4.0, can determine the development path of the Industry 4.0. In the case of that last factor the role of government in supporting "Industry 4.0 entrepreneurship" can be of basic importance (Pietrzak et al, 2017; Rogalska, 2018; Virglerová, 2018; Bartosik-Purgat & Ratajczak-Mrozek, 2018; Ajaz Khan et al., 2019).

As it has been already mentioned the challenge of taking advantage of the potential related to the Industry 4.0 development can be especially important for Central and Eastern European Union countries, which in spite of the successful transformation process from socialism to market economy, still face the problem of fundamental modernization of their economies (Prause & Atari, 2017; Ślusarczyk, 2018; Nimeczyk et al., 2019; Braja & Gemzik-Salwach, 2019; Bilan et al., 2019; Stopochkin et al., 2020; Keček et al.,2019). Within this context Durana et al. (2019) underline that the Industry 4.0 can help significantly change and improve effectiveness of products and production systems in the European countries concerning processes, operations and services.

Since the emergence of the Industry 4.0 idea in 2011, the concept has been related to the comprehensive digitisation of industrial production that transforms all aspects of production processes based on new technological achievements, including digitalisation and robotisation, artificial intelligence and the Internet of things (IoT), new materials and biotechnology (Krykavskyy et al., 2019; Vasin et al., 2018; Buhr & Stehnken, 2018; Svazas et al., 2019). As a result, the main condition for developing the potential of Industry 4.0 is to build and improve ICT infrastructure (Lechman, 2018), and to provide incentives for enterprises to move all the processes to the digital form. It is obvious that the effective ICT infrastructure is the first condition for application of cyber-physical systems, the internet of things, cloud computing or finally cognitive computing and artificial intelligence. It can also support further development of service sector, which plays a key role in employment growth and improved well-being in OECD countries. In the service industries, knowledge-intensive business services (KIBS) developed within Industry 4.0 concept demonstrate higher levels of efficiency and profitability (Chung & Tseng, 2019).

The main goal of the article is to provide information on the similarity of the Central and Eastern European economies with respect to the ICT utilisation. The research is concentrated on the real economy, so the data for non-financial enterprises was used here. The analysis is conducted for the years 2010-2018 with application of Eurostat data at the national level.

2 RESEARCH METHOD

As it has been already stressed in the introduction the concept of Industry 4.0 is a complex phenomenon. In spite of the fact that in the current research the main attention is given to the problem of ICT utilisation, which can be considered as one aspect related to the issue, still this single factor can be described with many variables. Therefore, in order to analyse it, taxonomic tools can be applied (Pietrzak & Balcerzak, 2017; Szopik-Depczyńska et al., 2018a; 2019b; Liu et al., 2019; Balcerzak, 2020). The aim of the research relates to the problem of classifying (clustering) the object with the objective of creating homogenous subsets (Pietrzak, 2019).

In the literature one can find numerous classification methods, which enable to select groups of objects that are similar to each other based on a set of given factors. In the social sciences the most popular ones are hierarchical clustering methods, where a hierarchy of objects is created due to their similarity. In the case of building a hierarchy there is the lowest level, where all objects are treated as separate. In the next step, from analysed objects groups are created, the number of which decreases as part of transition to a higher level of the hierarchy, due to the fact that the number of objects is increased within groups. At the last level of the hierarchy, all objects form one common group. Among this approach the Ward's method is one of the most commonly applied in the socio-economic research (Rogalska, 2018b). As an example the method has been applied previously in the research on the labour markets and the issue of unemployment (Tatarczak & Boichuk, 2018; Rollnik-Sadowska & Dąbrowska, 2018), the issue of regional economic development and human welfere (Miłek, 2018; Nowak, 2018), the research on the phenomenon of aging of societies (Thalassinos et al., 2019) or fiscal studies (Szymańska, 2018).

In the Ward's method groups (clusters) are determined on the basis of criterion of minimizing the sum of the squares of distance between objects, which are included in the cluster. The obtained results are presented in the form of a dendrogram, where groups of objects are presented, depending on the distance between the groups (Ward, 1963; Rogalska, 2018b).

3 RESULTS

In the study the new member states that joined the UE after the year 2004 were analysed. The classification was done for the three years 2010 and 2018. The research is based on Eurostat data. The set of diagnostic variables was selected after profound literature review (see Balcerzak & Pietrzak, 2017; Ozcan, 2018; Batkovskiy et al. 2019; Pogodina et al., 2019). In the case of selection of diagnostic variables the most important restriction was data availability for the panel of analysed countries. As a result, some important factors such as cloud computing services or applications of social media with use enterprise's blog or microblogs or the development of e-administration could not be included in the study.

The economies of Central and Eastern European countries are significantly dependent on the functions of SMEs, which are responsible for most of the GDP generation and have high share in the employment (Kljucnikov et al. 2016, 2018; 2019; Meyer & Meyer, 2017; Patora-Wysocka, 2018; Haviernikova et al., 2019; De la Hoz-Rosales et al., 2019; Succurro & Costanzo, 2019; Kwasny et al., 2019; Bilan et al., 2019). Therefore, in the current research in the case of data availability the variable for SMEs were included. The current study is devoted to the real economy, therefore, only data on enterprises without financial sector was applied. The set of diagnostic variables is given in table 1. The diagnostic variables were normalized with classic standardization formula.

Table 1. Diagnostic Variables

ICT Usage in enterprises	
X_1	E-commerce purchases - enterprises having purchased via computer mediated networks (Percentage of enterprises)
X_2	E-commerce purchases - small enterprises (10-49 persons employed) having purchased via computer mediated networks (Percentage of enterprises)
X_3	E-commerce sales - Enterprises having received orders via computer mediated networks (Percentage of enterprises)
X_4	E-commerce sales - small enterprises (10-49 persons employed) having received orders via computer mediated networks (Percentage of enterprises)
X_5	Value of e-commerce sales - Enterprises' total turnover from e-commerce (Percentage of turnover)
X_6	Value of e-commerce sales - small enterprises (10-49 persons employed) total turnover from e-commerce (Percentage of turnover)
Employment based on ICT	
X_7	Use of computers and the internet by employees - Persons employed using computers with access to World Wide Web (Percentage of total employment)
Connections to the internet within Industry 4.0 standards	
X_8	Enterprises with broadband access (Percentage of enterprises)*
X_9	Small enterprises (10-49 persons employed) with broadband (Percentage of enterprises) *
Websites and functionalities	
X_{10}	Enterprises with a website (Percentage of enterprises)
Integration of internal processes	

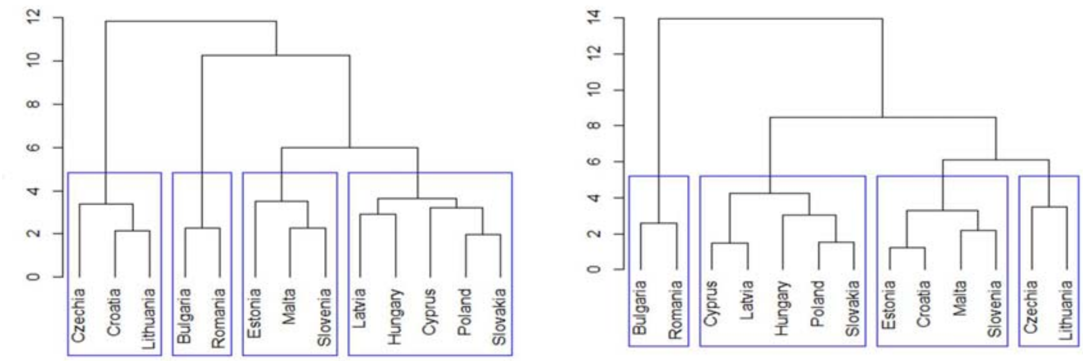
X_{11}	Enterprises who have ERP software package to share information between different functional areas (Percentage of enterprises)

Note: * Data for the year 2017 was used due to availability of data for 2018; ** Data for the year 2019 was used due to availability of data for 2018.
Source: own work.

The dendograms obtained with application of R program for the years 2010 and 2018 with the grouping results are given in figure 1.

Figure 1. Dendogram for the year 2010 and 2018

2010 **2018**



Source: own work based on Eurostat data.

In the case of both years four relatively homogenous clusters were obtained. In the year 2010 the first subset consist of Czech Republic, Croatia and Lithuania. In the second subset the two least developed EU economies can be found Bulgaria and Romania. In the next group there are relatively small economies of Estonia, Malta and Slovenia. The last cluster is the biggest and consists of Latvia, Hungary, Poland and Slovakia. In the year 2018 one can only find one difference, where Croatia moved from the subset grouping Czechia and Lithuania to the ones with Estonia, Croatia, Malta and Slovenia.

The results of the grouping of the countries confirm very high stability of the obtained results. This can indicate that in spite of the optimism related to development of Internet technology, which in the end of XX century was often considered as a great chance for less developed countries, which can provide an opportunity for a quick progress in the technological ladder, the possibilities to use this chance are very difficult to materialize and are restricted with traditional long term technical and institutional barriers.

4 CONCLUSIONS

The objective of the current research was to provide information on the similarity of the Central and Eastern European economies with respect to the ICT utilisation within the Industry 4.0 development context. The research was done with application of the data for non-financial enterprises in the years 2010 and 2018. To fulfil the aim of the article the Ward's method was used, which enabled to group the analysed economies into homogenous subsets.

The current research provides the following new information to the literature: the changes in time with respect to utilisation of ICT technologies within Industry 4.0 paradigm are quite moderate. In spite of the quick technological progress, form the economic perspective the whole phenomenon is characterised with high stability in relation to heterogeneity between the countries. This indicates the pointed heterogeneity is not only present, but it is also very difficult to change, which can affect negativity to potential for closing the development gap.

The current results, combined with other multi-criteria methods such as methods of ordering of objects, can be used for looking for examples of best practices within the context of forming the guidelines for national and regional modernisation strategies.

The author is aware of important limitations of the current paper. The main weaknesses of the research can be related to the general critics concerning the taxonomic research within the context of normative approach of researchers for the choice of diagnostic variables (see Kuc; 2019; Kuc-Czarnecka et al. (2020). Second, the mentioned problem of data availability for the important issues with respect to the latest development of ICT technology affects the obtained results. Therefore, in the case of future studies, which could be restricted only to the latest years, the data on the newest technological solutions should be taken into consideration.

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AN OPTIMISATION OF UNDERGROUND NATURAL GAS STORAGE FACILITIES

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Abstract

Natural gas can be stored in several different ways. The most commonly used way is holding it in underground storage under pressure. Each storage facility has its own physical characteristics and economics. The most important characteristics of an underground storage reservoir are its capacity to hold natural gas for future use and the rate at which gas inventory can be withdrawn (deliverability rate) and refill.

This paper deals with optimisation of usage of an underground natural gas storage. The aim is to determine daily gas volume that will be injected to and withdrawn from the storage, where the objective is to maximise the total profit. The resulting schedule has to follow the injection and withdrawal curves that depend on the current gas volume in the storage. The model is solved using mixed integer linear programming. The injection and withdrawal rates are given as non-linear functions; therefore it was necessary to linearize them.

Keywords: *Underground Natural Gas Storage, Gas Market, Linearization, Mixed Integer Linear Programming*

JEL Classification: C440, C610, Q350
AMS Classification: 90C11, 93B18

1 INTRODUCTION

“Natural gas is the earth's cleanest burning hydrocarbon.” (Natural Gas Facts & Figures | IGU, 2020) Thanks to its wide range of use, it becomes an energy source of the future. The natural gas market is a complex system of activities where each of its sub-cells requires certain optimization techniques. This paper deals with storage of natural gas in underground storage. The aim of this paper is to formulate a mathematical model that will optimise a month schedule of refill and withdraw, where the objective is to maximise the total profit.

First part of the paper focuses on the natural gas, its market, transport and storage. The second part of the paper describes a formulation of mathematical model based on real situation in natural gas storage. Our model optimises the total profit of the storage facilities usage.

2 NATURAL GAS

Natural gas is a mixture of gaseous hydrocarbons (99 % methane; ethane, propane, butane, ethene). Natural gas is located in underground reservoirs all over the world. It is extracted from these reservoirs. The extracted natural gas is further processed (cooled, freed from undesirable components) and in liquid or gaseous state it is transported by pipeline to the countries or regions where it is consumed. However, natural gas consumption during the year is not uniform, but depends on many factors. For end consumers, gas consumption depends mainly on air temperature and wind power. On the other hand, gas extraction is even, for technological reasons. Gas storage facilities are used for storing the extracted gas, especially in summer, when the gas consumption is lower. (Gas market, 2015) From the storage or transmission system, natural gas continues to the distribution pipelines where it is supplied to end consumers. Natural gas is used in many sectors. For example, for power generation, heating in commercial, industrial and residential areas, transport or as a raw material for further processing in the petrochemical industry. The natural gas market is a part of the gas industry value chain, which consists of a network of supply, transport, storage and demand parts.

2.1 Gas trade

The objective of gas trade is no different from trading in other commodities, namely to transport goods (in this case natural gas) from the producer to the consumer. The gas market participants are the manufacturer, the trader, who purchases the gas for resale, either wholesale (trader) or linked to the final consumer (retail supplier). Furthermore, the Distribution System Operator (DSO), which builds on the activities of the Transmission System Operator (TSO) and thanks to its interconnected network of pipelines, ensures the transport of gas from the transmission system to consumers. Market operation is ensured by the market operator (OTE, a.s. in the Czech Republic), which evaluates and clears imbalances, ensures the registration of market participants, operates the short-term gas and electricity market and compiles statistics related to the gas and electricity market. The customer shall arrange for gas supply with the trader. The trader can also provide gas transport and distribution to the customer. The trader purchases gas for his customers at the stock exchange or directly from the manufacturer. Charges for transmission and distribution services are regulated by the Energy Regulatory Office, the price of the gas itself is determined by the trader (like the production and storage of gas, it is an unregulated part of the market). (Gas market, 2015)

The subject of trades is the supplier's obligation to deliver the specified quantity of gas and the customer's obligation to pay or take it for the agreed quantity. The gas supply can be arranged for different periods. Accordingly, long-term and short-term contracts are distinguished. Long-term contracts can be baseload, i.e. supplying and withdrawing the same amount of gas for the duration of the contract, or flexible, where the quantity actually delivered and withdrawn can vary on a daily or annual basis within a pre-agreed flexibility. Short-term contracts are most often negotiated on a daily basis. These contracts are mainly used by traders who own a diverse portfolio of smaller customers whose consumption is difficult to predict due to the large differences. Traders thus offset their positions through the day-ahead market. (Gas market, 2015)

2.2 Underground Gas Storage Facilities

Underground gas storage facilities are an integral part of the gas system. The reasons for storing natural gas in storage tanks are different. Initially, storage tanks were used (and still are used) mainly for seasonal balancing of differences in gas consumption. This means that in the summer months when consumption is lower, gas is injected into the storage tank, and in winter when gas consumption is high, gas is extracted from the storage tanks and transported through the transmission system to the distribution network of gas pipelines connected to end consumers.

On this principle is another way of using gas storage, which is price arbitration. Gas traders take advantage of the fact that in the summer months gas is cheaper than in the winter months when gas demand is high. Thus, they buy “cheap” gas at a period with lower prices, store it in a storage tank, and start storing and selling the stored gas at higher prices in the winter months. However, many factors affect the advantageousness of the whole operation, especially the price for renting capacity in the storage tank and the difference between the price in different periods.

The gas storage facilities further serve to compensate for fluctuations in the transmission system, which must be balanced. Extraction of gas from the reservoir may replace the lack of gas at the entry into the system, while injecting gas into the reservoir may cover excess gas at the entry into the system or insufficient consumption from the system. Tanks can also effectively increase the efficiency of the entire transmission system when they are close to the point of consumption. In such a case, it is possible to reduce the amount of gas needed and thus reduce the load on the transmission system as well as the costs of gas transmission.

The storage facilities are also used to cover unexpectedly high peak consumption, where the rate of gas extraction from individual storage facilities plays a role. Another equally important use of the reservoir is to ensure the security of gas supply, which is discussed in the following section.

2.3 Storage capacity trading

Trading in storage capacity takes the form of auctions. In order to understand the following text, it is necessary to first get acquainted with the parameters' characteristic of individual storages that the operator offers at auction.

The overall capacity of the container is the cushion and the operating capacity of the container. As mentioned above, the cushion in the magazine serves to secure the agreed injection and withdrawal rates. Operating capacity is the volume of gas in the container with which it is possible to operate, extract from the container and inject it into the container. The operating capacity consists of the so-called operating volume of gas, which is the amount of gas that is assigned to the customer after the successful completion of the auction, and the customer can dispose of this volume at will. Other important parameters are injection and withdrawal capacity. This is the volume of gas that the tank operator is able to extract from the tank / inject into the tank every day or hour for the entire duration of the contract due to its technological equipment. According to the injection and withdrawal rates, the tanks are distinguished into so-called "fast" and "slow" as the time required for extraction / injection of the total agreed volume decreases with increasing output. (Pustišek and Karasz, 2017)

The injection and extraction rates depend not only on the technological conditions of the above-ground part of the bunkers, but also on the geological characteristics of the bunker as such. The capacities vary depending on the current state of the gas storage volume in the storage tank, mostly so that as the gas volume in the storage tank decreases, the extraction capacity decreases and the injection capacity increases. This dependence is expressed by injection and extraction curves that are specific to each individual container. (Gas market, 2015)

Products offered at auctions differ especially at the time of booking. Annual contracts are usually concluded for the so-called storage year, which begins on 1.4. and ends on 31.3. the following year (as well as multiannual contracts). However, the booking period may be less than one year. Furthermore, the products differ according to the speed (the ratio of the operating volume to the injection and withdrawal capacity) or the type of injection and withdrawal curves (performance dependence on the actual gas volume in the storage tank). Obviously, the faster the product or the more advantageous the curve, the higher the price for capacity booking. (Gas market, 2015)

3 OPTIMISATION OF NATURAL GAS STORAGE

Optimisation of natural gas storage facilities usage is important nowadays mostly due to gas reserves in Europe supported by increasing gas import and due to lower consumption caused especially by warmer weather. These factors cause that the gassiness of storages increases. This causes the reserved capacity price increase. Hiring capacity in the storage to maximize profit from the summer-winter price differential (the so-called S-W spread) is less beneficial due to rising booking fees and a reduction in S-W spread over time. On the other hand, the use of underground storage facilities is still particularly suitable for the diversification of traders' portfolios or security of supply in the event of a crisis. However, the situation requires careful decision-making when planning injected and extracted volumes, which may not be completely intuitive as the S-W spread decreases, and even a small change in the planned amount may mean a significant profit or loss.

When monitoring prices to determine the advantageous capacity lease (and its use, i.e. injection and gas extraction) in the storage tank, it is necessary to distinguish between the development of prices of individual months and the above-mentioned S-W spread. Spread is very volatile over time, changing in January 2019 by more than one euro. In the past, for example, it was around –0.5 EUR (i.e. the price of the winter season is more expensive only by 0.5 EUR compared to the summer), which could mean that the profit from such a low spread would not cover the cost of storage capacity rental. Especially in case of such unfavourable development it is necessary to optimize the utilization of the container to maximize profit.

In this paper we deal with a practical example of the use of the underground gas storage from the perspective of a natural gas trader in the Czech market. The model maximises profit resulting from the optimal injection or withdrawal schedule and the current S-W spread.

3.1 Data

For the purposes of this paper we use real input data obtained from natural gas trading companies. The model is addressed from the point of view of the natural gas trader, whose objective is to rent the capacity of the underground gas storage facility from the provider. The tank operator offers free capacity in regular multi-round auctions. Before each auction, the offered storage parameters are published – in addition to the rental price, also the total offered storage capacity and maximum injection and extraction capacity.

The input data published in the auction for storage capacity reservation in the storage period from April 1, 2020 to March 31, 2021 are used to calculate the models. The total offered capacity was 5 million MWh with a maximum injection capacity of 50,000 MWh/day and extraction capacity 65,500 MWh/day. More subjects competed for the total capacity; for the calculations it can be assumed that the trader was allocated a capacity of 100,000 MWh and the corresponding proportional part of maximum injection and withdrawal capacity. The input parameters are shown in Tab. 1. Another important parameter is the forward price curve, i.e. the price of each month of the storage period at the time of capacity rent. However, due to company policy this price cannot be published.

Capacity (MWh)	100,000
Max injection rate (MWh/day)	1,000
Max withdrawal rate (MWh/day)	1,310

Table 1 Storage input parameters

Formulating a profit maximizing model based on this data can be quite simple. However, to get the model as close to reality as possible, it is necessary to take the injection and withdrawal curves into consideration. The underground storage facility operator shall publish them in gas storage contracts. The injection and withdrawal curves offered by over provider differ. The curves are non-linear, but they are partially linear (constant) functions. In the Tab. 2 the parameters of both curves are shown.

Injection curve		**Withdrawal curve**	
WGL [%]	**Injection rate [%]**	**WGL [%]**	**Withdrawal rate [%]**
0–8	50	0–3	17.5
8–16	75	3–15	52.5
16–48	100	15–35	77.5
48–75	93	35–72	100
75–82	80	72–85	72.5
82–91	65	85–100	40
91–96	55		
96–100	20		

Table 2 Injection and withdrawal curves parameters (source Innogy)

Both injection and withdrawal rates depend on the working gas level (WGL), i.e. volume of gas that is already in the storage. The rented capacity can be conceived as a virtual part of the storage tank, which is empty at the beginning of the storage period and the trader operates with it during the year, i.e. gradually injects it or withdraw gas from it. Each interval corresponds to a constant percentage level of maximum injection or withdrawal rate.

3.2 Mathematical model

The objective of our analysis is to determine the daily volume of gas that will be injected into the storage during the summer months of the storage period and will be withdrawn from the storage during the winter months of the storage period. Such a schedule of daily gas volumes should maximize the profit resulting from the S-W spread. The final schedule must comply with injection and withdrawal curves depending on the actual volume of gas in the tank, which must not exceed the agreed capacity. Further restrictions arise from the setting of the gas market conditions and the conditions of the contract concluded with the underground gas storage operator. These restrictions will be explained at each constraint in detail.

Let's define all the variables and input parameters of the model:
K total rented storage capacity (MWh),
IR maximal daily injection rate (MWh/day),
WR maximal daily withdrawal rate (MWh/day),
P_i $i = 1, \ldots, n$, natural gas price (CZK),
LI_j $j = 1, \ldots, m$, lower bound of WGL range for injection curve (%),
UI_j $j = 1, \ldots, m$, upper bound of WGL range for injection curve (%),
IC_j $j = 1, \ldots, m$, corresponding injection rate as percentage of IR (%),
LW_k $k = 1, \ldots, p$, lower bound of WGL range for withdrawal curve (%),
UW_k $k = 1, \ldots, p$, upper bound of WGL range for withdrawal curve (%),
WC_k $k = 1, \ldots, p$, corresponding withdrawal rate as percentage of WR (%),
where n is a number of days per year ($n = 365$), m is number of WGL ranges of the injection curve ($m = 8$) and p is a number of WGL ranges of the withdrawal curve ($p = 6$). Values of parameters K, IR and WR can be found in Tab. 1. The bounds are in the Tab. 2. The variables of the model are:
x_i $i = 1, \ldots, n$, the amount of natural gas injected into the storage per day (MWh),
y_i $i = 1, \ldots, n$, the amount of natural gas withdrawn from the storage per day (MWh),
z_i $i = 1, \ldots, n$, working gas level in day i,
u_{ij}, v_{ij}, t_{ij} $i = 1, \ldots, n, j = 1, \ldots, m$, technical binary variables connected with injection curve,
uu_{ik}, vv_{ik}, tt_{ik} $i = 1, \ldots, n$, $k = 1, \ldots, p$, technical binary variables connected with withdrawal curve.

The objective of the model is to maximise the total profit

$$z = \sum_{i=1}^{n} P_i y_i - \sum_{i=1}^{n} P_i x_i \to MAX \tag{1}$$

That is expressed as a difference of revenue and cost based on price coefficients that are the same for injection and withdrawal.

Although products are purchased on a monthly basis, the storage is operated on a daily basis and therefore variables are defined for each day of the storage period. Underground storage operators are reported monthly volumes that the trader requires to inject or withdraw. Daily volumes within each month must therefore be the same. If the model does not include a condition setting the same amount of gas both injected and withdrawn in each month, it could happen that during the month the WGL would skip to an interval that would correspond to another injection or withdrawal rate. After dividing by the number of days, the total monthly volume would not correspond to the curves and would thus not meet the conditions for the use of the underground storage by the operator. The conditions (2) ensure the same amount of injected and withdrawn natural gas within each month defined as part of daily indices.

$$\begin{gathered}
x_i = x_{i+1}, y_i = y_{i+1}, i = 1, \dots, 29, \\
x_i = x_{i+1}, y_i = y_{i+1}, i = 31, \dots, 60, \\
x_i = x_{i+1}, y_i = y_{i+1}, i = 62, \dots, 90, \\
x_i = x_{i+1}, y_i = y_{i+1}, i = 92, \dots, 121, \\
x_i = x_{i+1}, y_i = y_{i+1}, i = 123, \dots, 152, \\
x_i = x_{i+1}, y_i = y_{i+1}, i = 154, \dots, 182, \\
x_i = x_{i+1}, y_i = y_{i+1}, i = 184, \dots, 213, \\
x_i = x_{i+1}, y_i = y_{i+1}, i = 215, \dots, 243, \\
x_i = x_{i+1}, y_i = y_{i+1}, i = 245, \dots, 274, \\
x_i = x_{i+1}, y_i = y_{i+1}, i = 276, \dots, 305, \\
x_i = x_{i+1}, y_i = y_{i+1}, i = 307 \ \dots, 333, \\
x_i = x_{i+1}, y_i = y_{i+1}, i = 335, \dots, 364.
\end{gathered} \tag{2}$$

During the storage period the total amount of withdrawn gas can't be higher than total amount of injected gas; at the beginning of every day WGL has to be lower than total capacity

$$\begin{gathered}
\sum_{i=1}^{n} y_i \le \sum_{i=1}^{n} x_i, \\
z_i \le K, i = 1, \dots, n
\end{gathered} \tag{3}$$

The WGL in the storage is expressed as balance between each pair of days

$$\begin{gathered}
z_i = z_{i-1} + x_{i-1} - y_{i-1}, i = 2, \dots, n, \\
z_1 = 0.
\end{gathered} \tag{4}$$

The following constraints relate to the injection and withdrawal curves. It is necessary to linearize both non-linear curves. The linearization is based on the IF–THEN principle, e.g. if the WGL is between 8 and 16 % of total capacity ($0.08K \le z_i \le 0.16K$), the maximal injection rate is 75 % ($xi \leq 0.75IR$). To express these IF–THEN constraints we use the technical binary variables in the model. The variable u_{ij} is equal 1, if the variable z_i is greater or equal the lower bound LI_j, and 0 otherwise. The variable v_{ij} is equal 1, if the variable z_i is less or equal the upper bound UI_j, and 0 otherwise. The variable t_{ij} is equal 1, if both u_{ij} and v_{ij} are equal 1, and 0 otherwise. The following set of constraints (5) assures, that the gas is injected to the storage with the appropriate maximal injection rate according to the injection curve. Constant M is a big number.

$$
\begin{aligned}
& LI_jK \leq z_i + M(1 - u_{ij}), \\
& LI_jK \geq z_i - Mu_{ij}, \\
& z_i \leq UI_jK + M(1 - v_{ij}), \\
& z_i \geq UI_jK - Mv_{ij}, \\
& u_{ij} + v_{ij} - 2t_{ij} \geq 0, \qquad i = 1, \dots, n, \\
& u_{ij} + v_{ij} - 2t_{ij} \leq 1, \qquad j = 1, \dots, m, \\
& x_i \leq IR \sum_{j=1}^{m} IC_j t_{ij}, \\
& \sum_{j=1}^{m} t_{ij} \leq 1.
\end{aligned}
\tag{5}
$$

Similarly we can express the set of constraint for the withdrawal curve (6) with the technical binary variable uu_{ik}, vv_{ik} and tt_{ik}:

$$
\begin{aligned}
& LW_kK \leq z_i + M(1 - uu_{ik}), \\
& LW_kK \geq z_i - Muu_{ik}, \\
& z_i \leq UW_kK + M(1 - vv_{ik}), \\
& z_i \geq UW_kK - Mvv_{ik}, \\
& uu_{ik} + vv_{ik} - 2tt_{ik} \geq 0, \qquad i = 1, \dots, n, \\
& uu_{ik} + vv_{ik} - 2tt_{ik} \leq 1, \qquad k = 1, \dots, p, \\
& y_i \leq WR \sum_{k=1}^{p} WC_k tt_{ik}, \\
& \sum_{k=1}^{p} tt_{ik} \leq 1.
\end{aligned}
\tag{6}
$$

The variables x_i, y_i and z_i are nonnegative, all other variables are binary:

$$
\begin{aligned}
& x_i \geq 0, y_i \geq 0, z_i \geq 0 \qquad i = 1, \dots, n, \\
& u_{ij}, v_{ij}, t_{ij} = \{0, 1\}, \qquad j = 1, \dots, m, \\
& uu_{ik}, vv_{ik}, tt_{ik} = \{0, 1\}, \qquad k = 1, \dots, p.
\end{aligned}
\tag{7}
$$

Due to the rental contract (seasonal storage) it is possible to inject the gas into storage only from April till September, withdrawal is possible only from October to March, therefore:

$$x_i = 0,\ i = 215, \ldots, 365,$$
$$y_i = 0,\ i = 1, \ldots, 214. \tag{8}$$

4 RESULTS AND CONCLUSIONS

We solved the model with MPL for Windows with Gurobi solver. The final schedule of injection and withdrawal is in the Tab. 3.

	Total MWh/month	
Month	**Injection**	**Withdrawal**
April	1,600.00	0
May	15,500.00	0
June	30,000.00	0
July	28,830.00	0
August	17,050.00	0
September	6,000.00	0
October	1,020.00	0
November	0	15,720.00
December	0	29,442.25
January	0	31,472.75
February	0	19,257.00
March	0	4,108.00

Table 3 Final schedule

The total profit of this schedule will be 6.87 million CZK.

If the trader is allowed to extend the injection schedule by one month until October, it will affect the set of constraints (8). We have to replace them:

$$x_i = 0,\ i = 184, \ldots, 365,$$
$$y_i = 0,\ i = 1, \ldots, 183. \tag{9}$$

The total injected amount will be reduced by 1,020 MWh, which corresponds to the amount injected in October in the original model. In the modified model, there is no activity in October, as it is not financially worthwhile to withdraw and injection is forbidden. The resulting profit is thus more than 20,000 CZK lower.

Furthermore, it is interesting to find out how the resulting dispatch and profit would be affected if the model did not require equal amounts of x_i and y_i during each of the months, i.e. the set of constraints (2) is dropped from the model. In this case the resulting schedule would follow the injection and withdrawal curves. Such an option would be particularly useful in the case of day-to-day modelling of underground storage utilization. Due to the possibility of greater use of price differences between summer and winter months, the resulting dispatch corresponds to a profit of 7.05 million CZK. Although the total agreed capacity of the storage is used, all volumes have been injected into the storage and withdrawn from the storage in a much shorter time.

In future research we can consider the case of the security of supply standard (SSS). SSS ensures the security of gas supply in the event of a disruption of natural gas supplies from transit countries or a failure of the gas system. The traders must prove according to the Decree that they are able to withdraw 30 % of the contracted volumes of protected customers from the storage in winter. (ERÚ, 2015) However, if the trader does not rent the capacity in the storage facility, he must prove that he has the required volume contracted with the intermediary. Thus, the gas storage facility generates so-called SSS certificates, which the trader proves that in the event of a crisis situation it can secure a certain percentage of protected customers with its supplies. However, if he does not have gas from the storage tank, he must purchase these certificates. For our trader the possibility of renting capacity in the storage means an additional profit from purchasing the SSS certificates. In the future, if a trader leased a larger volume of a storage that would generate more certificates than he was required by the Decree, he could sell them, generating additional profit.

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CENTRAL MOMENTS AND RISK-SENSITIVE OPTIMALITY IN CONTINUOUS-TIME MARKOV REWARD PROCESSES

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Abstract In this note we consider continuous-time Markov decision processes with finite state space where the stream of rewards generated by the Markov processes is evaluated by an exponential utility function with a given risk sensitivity coefficient (so-called risk-sensitive models). For the risk-sensitive case, i.e. if the considered risk-sensitivity coefficient is non-zero, we establish explicit formulas for growth rate of expectation of the exponential utility function. Recall that in this case along with the total reward also its higher moments are taken into account. Using Taylor expansion of the utility function we present explicit formulae for calculating variance and higher central moments of the total reward generated by the Markov reward process along with its asymptotic behavior.

Keywords Continuous-time Markov reward chains, exponential utility, moment generating functions, formulae for central moments

JEL Classification C44, C61
AMS Classification 90C40

1 Introduction

The usual optimization criteria examined in the literature on stochastic dynamic programming, such as a total discounted or mean (average) reward structures, may be quite insufficient to characterize robustness of the problem from the point of a decision maker. To this end it may be preferable if not necessary to select more sophisticated criteria that also reflect stability and variability-risk features of the problem. Hence robustness and risk control are also important issues in practical applications. Perhaps the best known approaches stem from the classical work of Markowitz (1952) on mean variance selection rules, i.e. we optimize the weighted sum of average or total reward and its variance, and from the seminal paper titled "Risk-sensitive Markov decision processes" of Howard and Matheson (1972), based on evaluating generated reward by exponential utility functions. Higher moments and variance of cumulative rewards in Markov reward chains have been original studied only for discrete time models. Research in this direction has been initiated in early papers Mandl (1971), Jaquette (1975), Benito (1982) and Sobel (1982). For connections with risk sensitive models see e.g. Cavazos-Cadena and Fernandez-Gaucherand (1999), Cavazos-Cadena and Hernández-Hernández (2005) and Sladký (2005),(2008).

To the best of our knowledge higher moments of cumulative rewards for continuous-time Markov control processes were originally studied by Jaquette (1975). In the paper Van Dijk and Sladký (2006) results for the discrete-time case are extended to continuous-time Markov reward chains. As the essential step is an expression for the variance of the undiscounted cumulative reward and its asymptotic behavior. Limiting average variance for continuous-time models are also studied in Guo and Song (2009) and in Prieto-Rumeau and Hernández-Lerma (2009) (see also the monograph by Guo and Hernández-Lerma (2009)), Wei and Chen (2016) and for discounted models in Guo and Ying (2012). For basic facts on controlled Markov processes, see e.g. Puterman (1994) or Ross (1983).

The present article is a continuous-time version of the author's paper Sladký (2018). The present paper is structured as follows. Section 2 contains notations and summary of basic facts on continuous-time Markov reward processes. Markov models with exponential utility function (called risk-sensitive Markov chains) are studied in section 3 along with the corresponding moment generating functions. Sections 4 and 5 are devoted to explicit formulas of higher moments and higher central models of total rewards generated in continuous-time Markov decision chains.

2 Notations and Preliminaries

In this note we consider Markov decision processes with finite state space $\mathcal{I} = \{1, 2, \ldots, N\}$ evolving in continuous-time. In particular, the development of the considered Markov decision process $X = \{X(t), t \geq 0\}$ (with finite state space $\mathcal{I}$) over time is governed by the transition rates $q(j|i,a)$, for $i, j \in \mathcal{I}$, depending on the selected action $a \in \mathcal{A}_i$. For $j \neq i$ $q(j|i,a)$ is the transition rate from state i to state j, $q(i|i,a) = \sum_{j\in\mathcal{I}, j\neq i} q(j|i,a)$ is the transition rate out of state i. Recall that for sufficiently small δ it holds for transition probabilities $P_{ij}(\cdot)$'s and transition rates $q_{ij}(\cdot)$'s (with $q_{ii}(\cdot) = -q_i(\cdot)$) that

$$P_{ij}(\cdot) = q_{ij}(\cdot) \cdot \delta + o(\delta^2) \ \text{ for } \ i \neq j, \qquad P_{ii}(\cdot) = (1 - q_i(\cdot) \cdot \delta) + o(\delta^2)$$

and similarly for the corresponding one stage rewards we can conclude that

$$\begin{aligned} r_{ij} &:= r(i,j) \ \text{ for } \ i \neq j \ \text{ is the transition reward from state } i \text{ to state } \ j \\ r_i &:= r(i) \ \text{ is the reward rate earned in state } i \end{aligned}$$

Let $\xi(t) := \int_0^t r(X(\tau)\mathrm{d}\tau + \sum_{k=0}^{N(t)} r(X(\tau^-), X(\tau^+))$, obviously $\xi(t)$ is the (random) reward obtained up to time t, where $X(t)$ denotes the state at time t, $X(\tau^-)$, $X(\tau^+)$ is the state just prior and after the kth jump, and $N(t)$ is the number of jumps up to time t. Similarly $\xi(t', t) := \int_{t'}^t r(X(\tau))\mathrm{d}\tau + \sum_{k=N(t')}^{N(t)} r(X(\tau^-), X(\tau^+))$ is the total (random) reward obtained in the time interval $[t', t)$; hence $\xi(t+\Delta) = \xi(\Delta) + \xi(\Delta, t+\Delta)$ or $\xi(t+\Delta) = \xi(t) + \xi(t, t+\Delta)$.

In this note we shall suppose that the obtained random reward, say ξ, is evaluated by an exponential utility function, say $u^\gamma(\cdot)$, i.e. utility functions with constant risk sensitivity depending on the value of the risk sensitivity coefficient γ.

In case that $\gamma > 0$ (the risk seeking case) the utility assigned to the (random) reward ξ is given by $u^\gamma(\xi) := \exp(\gamma\xi)$, if $\gamma < 0$ (the risk averse case) the utility assigned to the (random) reward ξ is given by $u^\gamma(\xi) := -\exp(\gamma\xi)$, for $\gamma = 0$ it holds $u^\gamma(\xi) = \xi$ (risk neutral case). Hence we can write

$$u^\gamma(\xi) = \mathrm{sign}(\gamma)\exp(\gamma\xi) \tag{1}$$

and for the expected utility we have (E is reserved for expectation)

$$\bar{U}^{(\gamma)}(\xi) := \mathsf{E}u^\gamma(\xi) = \mathrm{sign}(\gamma)\mathsf{E}[\exp(\gamma\xi)], \quad \text{where} \quad \mathsf{E}\,[\exp(\gamma\xi)] = \sum_{k=0}^{\infty} \mathsf{E}\,\frac{1}{k!}(\gamma\xi)^k. \tag{2}$$

Then for the corresponding certainty equivalent $Z^\gamma(\xi)$ we have

$$u^\gamma(Z^\gamma(\xi)) = \mathrm{sign}(\gamma)\mathsf{E}[\exp(\gamma\xi)] \Longleftrightarrow Z^\gamma(\xi) = \gamma^{-1}\ln\{\mathsf{E}\,[\exp(\gamma\xi)]\}. \tag{3}$$

From (2),(3) we immediately conclude that

$$Z^{\gamma}(\xi) \approx \mathsf{E}\xi + \frac{\gamma}{2}\mathsf{Var}\,\xi. \tag{4}$$

A (Markovian) policy controlling the decision process is given as a piecewise constant right continuous function of time. In particular, $\pi = f(t)$, is a piecewise constant, right continuous vector function where $f(t) \in \mathcal{F} \equiv \mathcal{A}_1 \times \ldots \times \mathcal{A}_N$, and $f_i(t) \in \mathcal{A}_i$ is the decision (or action) taken at time t if the process $X(t)$ is in state i. Since π is piecewise constant, for each π we can identify the time points $0 < t_1 < t_2 \ldots < t_i < \ldots$ at which the policy switches; we denote by $f^i \in \mathcal{F}$ the decision rule taken in the time interval $(t_{i-1}, t_i]$. Policy which takes at all times the same decision rule, i.e. $\pi \sim f$, is called stationary; $Q(f)$ is the transition rate matrix with elements $q(j|i, f_i)$.

Let for $f \in \mathcal{F}$ $Q(f) = [q_{ij}(f_i)]$ be an $N \times N$ matrix whose ijth element $q_{ij}(f_i) = q(j|i, f_i)$ for $i \neq j$ and for the iith element we set $q_{ii}(f_i) = -q(i|i, f_i)$. The sojourn time of the considered process X in state $i \in \mathcal{I}$ is exponentially distributed with parameter $[q(i|i, f_i)]$. Hence the expected value of the reward obtained in state $i \in \mathcal{I}$ equals $\tilde{r}_i(f_i) = [q(i|i, f_i)]^{-1}\, r(i) + \sum_{j\in\mathcal{I}, j\neq i} q(j|i, f_i)\, r(i, j)$ and $\tilde{r}(f)$ is the (column) vector of reward rates at time t.

Using policy $\pi = f(t)$ means that if the Markov chain X was found to be in state i at time t, the action chosen at this time is $f_i(t)$, i.e. the ith coordinate of $f(t) \in \mathcal{F}$. For any policy $\pi = f(t)$ the accompanying set of transition rate matrices $\{Q(f(t)), t \geq 0\}$ determines a continuous-time (in general, nonstationary) Markov process.

3 Formulas for Higher Moments of Random Reward

Supposing that the obtained random reward up to time t, say $\xi(t)$, is evaluated by an exponential utility function, say $u^{\gamma}(\cdot)$, with the risk sensitivity coefficient γ, let for $\pi \sim (f)$, $U_i^{(\gamma)}(t, f) := \mathsf{E}_i^{\pi}[\exp(\gamma\xi(t))]$ considered as the moment generating function of $\xi(t)$, we can conclude that for $k = 0, 1, 2, \ldots$, $n = 0, 1, 2, \ldots$

$$M_i^{(k,\pi)}(t) := \mathsf{E}_i^{\pi}(\exp(\xi(t)^k) = \frac{\mathrm{d}^k}{\mathrm{d}\gamma^k}\mathsf{E}_i^{\pi}[\exp(\gamma\xi(t))]|_{\gamma=0} \quad \text{is the } k\text{th moment of } \xi(t) \tag{5}$$

and the Taylor expansion around $\gamma = 0$ reads

$$U_i^{(\gamma)}(t, f) = 1 + \sum_{k=1}^{\infty} \frac{\gamma^k}{k!} M_i^{(k,\pi)}(t). \tag{6}$$

Similarly on introducing the moment generating function for the central moments of $\xi(t)$ by

$$\widetilde{U}_i^{(\gamma)}(t, f) := \mathsf{E}_i^{\pi}[\exp(\gamma(\xi(t) - \mathsf{E}_i^{\pi}\xi(t))]^k \quad \text{for all } \ i \in \mathcal{I} \tag{7}$$

for the kth central moments of $\xi(t)$ we have

$$\widetilde{M}_i^{(k,\pi)}(t) := \mathsf{E}_i^{\pi}[\xi(t) - \mathsf{E}_i^{\pi}\xi(t)]^k = \frac{\mathrm{d}^k}{\mathrm{d}\gamma^k}\mathsf{E}_i^{\pi}[\exp(\gamma(\xi(t) - \mathsf{E}_i^{\pi}\xi(t))]_{\gamma=0} \tag{8}$$

and the Taylor expansion around $\gamma = 0$ for sufficiently small γ reads

$$\widetilde{U}_i^{(\gamma)}(t, f) = 1 + \sum_{k=1}^{\infty} \frac{\gamma^k}{k!} \widetilde{M}_i^{(k,\pi)}(t) \tag{9}$$

Let $M^{(k,\pi)}(t)$, $\widetilde{M}^{(k,\pi)}(t)$ be the (column) vector of the k moments, central k moments respectively, with elements $M_i^{(k,\pi)}(t)$, $\widetilde{M}_i^{(k,\pi)}(t)$ respectively.

In particular, if the system starts in state i, the expected total reward earned in state i up to the first exit of state i within time interval $[t, t+\delta)$ is equal to

$$M_i^{(1,\pi)}(t+\delta) = M_i^{(1,\pi)}(t) \cdot [1 - q_i(f_i) \cdot \delta] + \delta \cdot r(i) \cdot [1 - q_i(f_i) \cdot \delta] + o(\delta^2) \tag{10}$$

and for the s-th power of this reward it holds

$$\begin{aligned} M_i^{(s,\pi)}(t+\delta) &= \{M_i^{(1,\pi)}(t)[1 - q_i(\cdot) \cdot \delta] + [r(i) \cdot \delta] \cdot [1 - q_i(f_i) \cdot \delta]\}^s \\ &= M_i^{(s,\pi)}(t) + s \cdot M_i^{(s-1,\pi)}(t) \cdot \delta + o(\delta^2) \end{aligned} \tag{11}$$

(Observe that $M_i^{(s,\pi)}(t) = [M_i^{(1,\pi)}(t)]^s$, $[M_i^{(1,\pi)}(t) + r(i) \cdot \delta]^s = M_i^{(s,\pi)}(t) + s \cdot r(i) \cdot M_i^{(s-1,\pi)}(t) + o(\delta^2)$.)

4 Higher Moments in Continuous-time Models

On inserting from (5),(7),(10),(11) we can conclude that

For $k=0$: $M_i^{(0,\pi)}(t+\delta) = [1 - q_i(f_i) \cdot \delta] + \sum_{j \in \mathcal{I}, j \neq i} q_{ij}(f_i) \cdot \delta \cdot M_j^{(0,\pi)}(t) \quad \Rightarrow M_j^{(0,\pi)}(t) = 1 \;\; \forall j.$ (12)

For $k=1$: $M_i^{(1,\pi)}(t+\delta) = [1 - q_i(f_i) \cdot \delta] \cdot [M_i^{(1,\pi)}(t) + r(i) \cdot \delta] + \sum_{j \in \mathcal{I}, j \neq i} q_{ij}(f_i) \cdot \delta \cdot [r_{ij} + M_j^{(1,\pi)}(t)]$.(13)

For $k=2$:
$$M_i^{(2,\pi)}(t+\delta) = [1 - q_i(f_i) \cdot \delta] \cdot [M_i^{(2,\pi)}(t) + 2 \cdot M_i^{(1,\pi)}(t) \cdot r(i) \cdot \delta] + \sum_{j \in \mathcal{I}, j \neq i} q_{ij}(f_i) \cdot \delta \cdot \Big\{[r_{ij}]^2 + 2 \cdot r_{ij} \cdot M_j^{(1,\pi)}(t) + M_j^{(2,\pi)}(t)\Big\} + o(\delta^2). \tag{14}$$

For $k=3$:
$$M_i^{(3,\pi)}(t+\delta) = [1 - q_i(f_i) \cdot \delta] \cdot [M_i^{(3,\pi)}(t) + 3 \cdot M_i^{(2,\pi)}(t) \cdot r(i) \cdot \delta] + \sum_{j \in \mathcal{I}, j \neq i} q_{ij}(f_i) \cdot \delta \Big\{[r_{ij}]^3 + 3 \cdot [r_{ij}]^2 \cdot M_j^{(1,\pi)}(t) + 3 \cdot [r_{ij}] \cdot M_j^{(2,\pi)}(t) + M_j^{(3,\pi)}(t)\Big\} + o(\delta^2). \tag{15}$$

For $k=4$:
$$M_i^{(4,\pi)}(t+\delta) = [1 - q_i(f_i) \cdot \delta] \cdot [M_i^{(4,\pi)}(t) + 4 \cdot M_i^{(4,\pi)}(t)] + \sum_{j \in \mathcal{I}, j \neq i} q_{ij}(f_i) \cdot \delta \times \times \Big\{[r_{ij}]^4 + 4 \cdot [r_{ij}]^3 \cdot M_j^{(1,\pi)}(t) + 6 \cdot [r_{ij}]^2 \cdot M_j^{(2,\pi)}(t) + 4 \cdot [r_{ij}] \cdot M_j^{(3,\pi)}(t) + M_j^{(4,\pi)}(t)\Big\} + o(\delta^2). \tag{16}$$

$\vdots \qquad \vdots \qquad \vdots$

In general:

$$M_i^{(s,\pi)}(t+\delta) = [1 - q_i(f_i) \cdot \delta] \cdot [M_i^{(s,\pi)}(t) + s \cdot r(i) \cdot \delta] + \sum_{j \in \mathcal{I}, j \neq i} q_{ij}(f_i) \cdot \delta \cdot \left\{\sum_{k=0}^{s} \binom{s}{k} \cdot [r_{ij}]^k \cdot M_j^{(s-k,\pi)}(t)\right\} + o(\delta^2) \tag{17}$$

hence

$$M_i^{(s,\pi)}(t+\delta) - M_i^{(s,\pi)}(t) = s \cdot M_i^{(s-1,\pi)}(t) \cdot r(i) + \sum_{j \in \mathcal{I}, j \neq i} q_{ij}(f_i) \cdot \delta \cdot \left\{\sum_{k=1}^{s} \binom{s}{k} [r_{ij}]^k M_j^{(s-k,\pi)}(t)\right\} + \sum_{j \in \mathcal{I}} q_{ij}(f_i) \cdot \delta \cdot M_i^{(s,\pi)}(t) + o(\delta^2) \tag{18}$$

or equivalently

$$\frac{1}{\delta}\{M_i^{(s,\pi)}(t+\delta) - M_i^{(s,\pi)}(t)\} = s \cdot M_i^{(s-1,\pi)}(t) \cdot r(i) \cdot \delta$$
$$+ \sum_{j\in\mathcal{I}, j\neq i} q_{ij}(f_i) \times \left\{ \sum_{k=1}^{s} \left(\begin{array}{c} s \\ k \end{array} \right) \cdot [r_{ij}]^k \cdot M_j^{(s-k,\pi)}(t) \right\} + \sum_{j\in\mathcal{I}} q_{ij}(f_i) \cdot M_i^{(s,\pi)}(t) + o(\delta) \quad (19)$$

For δ tending to null from the above material we can arrive at the following systems of differential equations.

$$\text{For } k=0: \ \frac{\mathrm{d}}{\mathrm{d}t} M_i^{(0,\pi)}(t) = \sum_{j\in\mathcal{I}} q_{ij}(f_i)\, M_j^{(0,\pi)}(t) \qquad \Rightarrow M_j^{(0,\pi)}(t) = 1 \quad \forall j. \quad (20)$$

$$\text{For } k=1: \ \frac{\mathrm{d}}{\mathrm{d}t} M_i^{(1,\pi)}(t) = r(i) + \sum_{j\in\mathcal{I}, j\neq i} q_{ij}(f_i)\, r_{ij} + \sum_{j\in\mathcal{I}} q_{ij}(f_i)\, M_j^{(1,\pi)}(t). \quad (21)$$

$$\text{For } k=2: \ \frac{\mathrm{d}}{\mathrm{d}t} M_i^{(2,\pi)}(t) = 2 \cdot M_i^{(1,\pi)}(t) \cdot r(i) + \sum_{j\in\mathcal{I}, j\neq i} q_{ij}(f_i) \left\{ [r_{ij}]^2 + 2 \cdot r_{ij} \cdot M_j^{(1,\pi)}(t) \right\}$$
$$+ \sum_{j\in\mathcal{I}} q_{ij}(f_i) \cdot M_j^{(2,\pi)}(t). \quad (22)$$

$$\text{For } k=3: \ \frac{\mathrm{d}}{\mathrm{d}t} M_i^{(3,\pi)}(t) = 3 \cdot M_i^{(2,\pi)}(t) \cdot r(i) + \sum_{j\in\mathcal{I}, j\neq i} q_{ij}(f_i) \times$$
$$\times \left\{ [r_{ij}]^3 + 3 \cdot [r_{ij}]^2 \cdot M_j^{(1,\pi)}(t) + 3 \cdot [r_{ij}] \cdot M_j^{(2,\pi)}(t) \right\} + \sum_{j\in\mathcal{I}} q_{ij}(f_i)\, M_j^{(3,\pi)}(t). \quad (23)$$

$$\text{For } k=4: \ \frac{\mathrm{d}}{\mathrm{d}t} M_i^{(4,\pi)}(t) = 4 \cdot M_i^{(3,\pi)}(t) \cdot r(i) + \sum_{j\in\mathcal{I}, j\neq i} q_{ij}(f_i) \times$$
$$\times \left\{ [r_{ij}]^4 + 4 \cdot [r_{ij}]^3\, M_j^{(1,\pi)}(t) \ + \ 6 \cdot [r_{ij}]^2 \cdot M_j^{(2,\pi)}(t) + \ 4 \cdot [r_{ij}] \cdot M_j^{(3,\pi)}(t) \right\}$$
$$+ \sum_{j\in\mathcal{I}} q_{ij}(f_i) \cdot M_j^{(4,\pi)}(t). \quad (24)$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

In general:

$$\frac{\mathrm{d}}{\mathrm{d}t} M_i^{(s,\pi)}(t) = s \cdot M_i^{(s-1,\pi)}(t) \cdot r(i) + \sum_{j\in\mathcal{I}, j\neq i} q_{ij}(f_i) \left\{ \sum_{k=1}^{s} \left(\begin{array}{c} s \\ k \end{array} \right) \cdot M_i^{(s,\pi)}(t) \cdot \ [r_{ij}]^k\, M_j^{(s-k,\pi)}(t) \right\}$$
$$+ \sum_{j\in\mathcal{I}} q_{ij}(f_i) \cdot \ M_j^{(s,\pi)}(t) \quad (25)$$

5 Higher Central Moments in Continuous-time Models

Supposing that higher moments are known, the corresponding central moments can be easily computed. To this end, on recalling definition central moments, we can easily conclude that the if the system starts in state i and policy π is followed then the nth central moment at time t

$$\widetilde{M}_i^{(n,\pi)}(t) := \mathrm{E}_i^{\pi}[\xi(t) - M_i^{(1,\pi)}(t)]^n \quad (26)$$

Since $M_i^{(j,\pi)}(t) := \mathrm{E}_i^\pi[\xi(t)]^j$, after little algebra we arrive at

$$\widetilde{M}_i^{(n,\pi)}(t) := \sum_{j=0}^{n} \binom{n}{j} \cdot (-1)^{n-j} \cdot M_i^{(j,\pi)}(t) \cdot [M_i^{(1,\pi)}(t)]^{n-j} \quad (27)$$

$$= \sum_{j=0}^{n-2} \binom{n}{j} \cdot (-1)^{n-j} \cdot M_i^{(j,\pi)}(t) \cdot [M_i^{(1,\pi)}(t)]^{n-j} + (-1)^{n-1} \cdot (n-1) \cdot [M_i^{(1,\pi)}(t)]^n \quad (28)$$

In particular,

$$\widetilde{M}_i^{(1,\pi)}(t) = M_i^{(1,\pi)}(t) - M_i^{(1,\pi)}(t) = 0 \quad (29)$$

$$\widetilde{M}_i^{(2,\pi)}(t) = M_i^{(2,\pi)}(t) - [M_i^{(1,\pi)}(t)]^2 \quad (30)$$

$$\widetilde{M}_i^{(3,\pi)}(t) = M_i^{(3,\pi)}(t) - 3 \cdot M_i^{(2,\pi)}(t) \cdot M_i^{(1,\pi)}(t) + 3 \cdot M_i^{(1,\pi)}(t) \cdot [M_i^{(1,\pi)}(t)]^2 - [M_i^{(1,\pi)}(t)]^3$$
$$= M_i^{(3,\pi)}(t) - 3 \cdot M_i^{(2,\pi)}(t) \cdot M_i^{(1,\pi)}(t) + 2 \cdot [M_i^{(1,\pi)}(t)]^3 \quad (31)$$

$$\widetilde{M}_i^{(4,\pi)}(t) = M_i^{(4,\pi)}(t) - 4 \cdot M_i^{(3,\pi)}(t) \cdot [M_i^{(1,\pi)}(t)] + 6 \cdot M_i^{(2,\pi)}(t) \cdot [M_i^{(1,\pi)}(t)]^2$$
$$-4 \cdot M_i^{(1,\pi)}(t) \cdot [M_i^{(1,\pi)}(t)]^3 + [M_i^{(1,\pi)}(t)]^4$$
$$= M_i^{(4,\pi)}(t) - 4 \cdot M_i^{(1,\pi)}(t) \cdot M_i^{(3,\pi)}(t) + 6 \cdot [M_i^{(1,\pi)}(t)]^2 \cdot M_i^{(2,\pi)}(t) - 3 \cdot [M_i^{(1,\pi)}(t)]^3 \quad (32)$$

Since $\widetilde{M}_i^{(1,\pi)}(t) = 0$ we shall consider $\widetilde{M}_i^{(s,\pi)}(t)$ only for $s = 2, 3, \ldots$. From (30)–(32) we immediately obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\widetilde{M}_i^{(2,\pi)}(t) = \frac{\mathrm{d}}{\mathrm{d}t}M_i^{(2,\pi)}(t) - 2 \cdot [M_i^{(1,\pi)}(t)] \cdot \frac{\mathrm{d}}{\mathrm{d}t}[M_i^{(1,\pi)}(t)] \quad (33)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\widetilde{M}_i^{(3,\pi)}(t) = \frac{\mathrm{d}}{\mathrm{d}t}M_i^{(3,\pi)}(t) - 3 \cdot \frac{\mathrm{d}}{\mathrm{d}t}\left\{M_i^{(2,\pi)}(t) \cdot \widetilde{M}_i^{(2,\pi)}(t)\right\} + 6 \cdot [M_i^{(1,\pi)}(t)]^2 \cdot \frac{\mathrm{d}}{\mathrm{d}t}\widetilde{M}_i^{(1,\pi)}(t) \quad (34)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\widetilde{M}_i^{(4,\pi)}(t) = \frac{\mathrm{d}}{\mathrm{d}t}M_i^{(4,\pi)}(t) - 4 \cdot \frac{\mathrm{d}}{\mathrm{d}t}\left\{M_i^{(2,\pi)}(t) \cdot \widetilde{M}_i^{(3,\pi)}(t)\right\} + 6 \cdot \frac{\mathrm{d}}{\mathrm{d}t}\left\{[M_i^{(2,\pi)}(t)]^2 \cdot \widetilde{M}_i^{(2,\pi)}(t)\right\} -$$
$$-9 \cdot [M_i^{(2,\pi)}(t)]^2 \cdot \frac{\mathrm{d}}{\mathrm{d}t}[M_i^{(1,\pi)}(t)] \quad (35)$$

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COMPUTATIONAL ASPECTS OF PRODUCT CLUSTERING BASED ON TRANSACTION INCIDENCE

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Abstract
The categorization of retail products is essential for the business decision-making process. We focus on clustering approach by Holý, Sokol and Černý (Applied Soft Computing, Vol 60, p. 752-762) and study its computational properties. The studied method deals with the clustering using non-linear integer optimization model which is solved using genetic algorithm. The method finds such clustering that minimizes the standardized joint purchases of products from the same cluster. We propose a few improvements to the process of finding the optimal clustering. First, we propose a method to mitigate symmetries in the solution space. Second, we implement a local search heuristic to the genetic algorithm. We study the effect of both improvements on the computational efficiency. The results are promising, indicating that adding local search heuristic allows to find better solutions than basic genetic algorithm.
Keywords: *cluster analysis, incidence matrix, genetic algorithm, local search heuristic.*

JEL Classification: C44, C38
AMS Classification: 90B50, 90-08

1 INTRODUCTION

Categorization in general is important for efficiency in retail business decision-making process as it simplifies the basis for a qualified management decision. Product categorization and customer segmentation belong to the most frequently used methods in transaction data analysis. Especially, the product categorization finds many applications in marketing, e.g., new product development, optimizing placement of retail products on shelves, analysis of cannibalization between products and more general analysis of the affinity between products. Gruca et al. [1] proposed a genetic algorithm to identify optimal new product position. A placement of retail products on shelves was studied by Borin et al. [2]. Finding the right categories is also crucial for sales promotions planning. Cross-category sales promotion effect was studied in detail by Leeflang et al. [3] and Hrushka et al. [4].

However, majority of product categorization studies focus on which products are purchased together. On the other hand, we can also look for groups of similar products, in particular we want to find products consumer sees as the same or similar. Standard approaches include categorization according to their purpose, package properties, e.g., package size, brand and price level. However, there are different approaches to product categorization. For example, Srivastava et al. [5] used hierarchical clustering while Zhang et al. [6] promoted fuzzy clustering. Another interesting possibilistic approach to clustering both customers and products was published by Ammar et al. [7].

An interesting and neglected problem is the aforementioned analysis of similar products instead of complementary, which is still usually solved by expert estimates. But at the same time, it is possible to use transaction data which contain information on the customer's perception of the relationship between products. The problem is therefore the division of products into such groups that products within one group are mutually substitutable using only transaction data.

In this paper, we analyze a purely data-driven approach by Holý, Sokol and Černý [8] from a computational point of view. The applied clustering of products is based exclusively on a customer behavior using transaction data. The model is formulated as an integer optimization problem with the number of clusters being a parameter and is solved by a genetic algorithm.

The method classifies products into clusters according to their common occurrences in the shopping baskets. Sets of products in individual shopping baskets as they were registered by the receipts are the only data used by the method. The method determines product categories under given assumptions of product dependency in the same category. It stems from the assumption that a customer buys only one product per category as the experience shows that customers who buy one product from a given category are generally less likely to buy also another product from the same category.

The method applies a genetic algorithm to transactional data in order to find the best clusters of products based on their joint purchase. For dealing with huge dataset, the clustering using nature-based metaheuristic methods such as genetic algorithms or swarm intelligence is common due to the computationally demanding enumeration, overview of such method can be found in Jose-Garcia et al. [9]. The main shortcomings of method [8] are, however, computational demands and uncertainty whether the genetic algorithm converges to the true optimal solution. The proposed clustering method lasted several hours even for rather small dataset of few hundred thousand baskets. In this paper, we focus on improvements of the genetic algorithm using new heuristic method and other optimization. The original paper [8] used rather basic genetic algorithm.

The paper is structured as follows. In Section 2, we describe the original optimization problem and used genetic algorithm. In Section 3, a data transformation method to disallow symmetric solutions in solution space is introduced and its effect examined. In Section 4, a local search heuristic is proposed and its effect is verified using a simulation study. The paper concludes in Section 5.

2 METHODOLOGY

We stem from a situation in which we have a huge amount of transaction data available in form of receipts. Each receipt consists of set of products purchased together.

For each basket $b = 1, \dots, n_B$ we compute the total number of decisions D_b as

$$D_b = \binom{d_b}{2},$$

where d_b is the number of products in basket b. Then we compute the number of decisions that lead to multiple products within the same cluster based on clustering $\boldsymbol{x}$ as

$$V_b(\boldsymbol{x}) = \sum_{c: v_{b,c(x)}>1} \binom{v_{b,c(\boldsymbol{x})}}{2},$$

where $v_{b,c(\boldsymbol{x})}$ is the number of products in basket b assigned to same cluster c in clustering $\boldsymbol{x}$. $\boldsymbol{x}$ is a vector of elements x_i with length n_P which is the number of products, x_i takes values from 1 to n_C – the number of categories supplied as parameter. The clustering model then consists in minimizing the cost function

$$\min_x \frac{1}{n_B} \sum_{b=1}^{n_B} \frac{V_b(\boldsymbol{x})}{D_b}$$

subject to

$$x_i \leq n_C \text{ for } i = 1, \dots, n_P,$$

$$x_i \in \mathbb{N} \text{ for } i = 1, \dots, n_P.$$

Hence, the cost function equals the average ratio of decisions in which two products from the same cluster are in the same shopping basket. The range of the cost function is from 0 to 1. If there is no basket containing products from the same cluster, then the cost function is 0. On the other hand, if every basket contains only products from the same cluster, then the cost function is 1.

The model has various natural assumptions on the data generating process stemming from the properties of real retail transaction datasets, see [8] for exact description. For finding optimal or at least near-optimal solution we use *GeneticAlg.int* function from R package *gramEvol* [10]. The method is specifically designed to solving integer optimization problems. *GeneticAlg.int* implements evolution algorithm with chromosomes created from integer values in given range, in our case integers from 1 to the given number of clusters n_C.

In order to use genetic algorithm, we need to supply several parameters. The first one is the *size of population*. With bigger population of individuals, more possible clusterings $\boldsymbol{x}$ are explored which can result in finding better solution. We set the population size to 500 individuals. Initial population is generated randomly and then the algorithm iteratively generates new populations. The maximal number of iterations is another parameter -- the *number of generations*. We always use the fixed number of 1 000 generations as simulations show that vast majority of instances converge faster. Each candidate solution is represented by an individual $\boldsymbol{x}$. Properties of candidate solutions are encoded in chromosomes of individuals x_i. At each generation a percentage of individuals with lowest values of cost function (called the elite population) passes to the next generation without any alteration. This ratio is set to 0.1.

Chromosomes of the rest of the new individuals are generated by crossover and mutation operations. For each new individual (called the child) crossover selects two individuals from the last generation (called the parents). The parents are selected randomly with weights according to the sorting by their cost function. The child is then created by combining chromosomes of both parents. We use one-point crossover which means that a random number of chromosomes c is selected. Then the first c chromosomes of the child are taken from one parent while the remaining chromosomes are taken from the other parent. Finally, the mutation operation is performed. Each chromosome of the child has a probability of changing its value to a random one. We set the mutation chance to 0.01 to allow algorithm to concentrate on improving one point while retaining some exploratory ability of mutation. The reason behind the chosen parameters are described in [8]. In the rest of the paper, these are the genetic algorithm parameters we use.

We analyze the effect of our improvements in simulation studies. In the studies, we consider a dataset consisting of 10 000 shopping baskets, each with 4 categories that have one or two products. We have a total of 10 categories with 10 products each. We simulate the behavior of a customer who selects a set of categories he needs (sets of 4 categories are selected with the same probability). Then he selects which products from that category he wants to buy (products are selected with the same probability and there is also a 10 percent chance to buy two products instead of one). This is a same setup as simulation studies in original paper [8].

3 SYMMETRY IN SOLUTION SPACE

The resulting clustering $\boldsymbol{x}$ allows symmetries in the solution space, for example, solution $\boldsymbol{x} = (3, 3, 2, 1, 3)$ is identical to $\boldsymbol{x} = (2, 2, 1, 3, 2)$. Naturally, the question arises as to whether these duplicates cause a problem in chromosome crossing. There is however a simple solution.

This shortcoming can be remedied by introducing a simple rule where cluster numbers are renumbered to start from the smallest based on the first occasion of each cluster, e.g., the solution above would be renumbered to $\boldsymbol{x} = (1, 1, 2, 3, 1)$. This can be achieved by a very simple and fast modification of the algorithm, which does not cause a noticeable slowdown in the iteration time. However, the effect on the time needed to converge is not statistically significant. Even the original algorithm quickly converges to same solution space naturally.

4 LOCAL SEARCH HEURISTIC

We added a local search feature to the original algorithm and discuss suitable values of its parameters. The local search consists in generating all possible *neighbor* individuals, created by exchange of one gene, for each of already selected sets of best individuals in each iteration and checking whether the new individuals improve the cost function. If new individual gives better result than the original one, then the original is replaced by the new one. If the individual has already been checked in the previous iterations, then no local search is executed. We study the optimal number of individuals, LS, which are checked by Algorithm 1.

Let GB is an array of best genomes, LS is a given local search parameter, n_C the number of clusters, n_P the number of products and $cost(\boldsymbol{x})$ is cost function defined as above. The used local search algorithm is following.

Algorithm 1: Local search algorithm.

```
Require: GB; nC; nP; LS; cost(x)
for i∈{1,…,LS} do
 G := GB[I, ]; nCost := cost(G)
 for i∈{1,…,nP} do
 x := G
   for k∈{1,…,nC} do
     x[j] := k
     if cost(x) < nCost then nCost := cost(x);nG := x
    end for
  end for
  GB[i; ] := nG
end for
return GB
```

Because we use the implementation of the genetic algorithm in *R*, we try to avoid *for* cycles that are very slow in the *R* environment. It is much faster to first create a matrix of all adjacent neighbor individuals and then to run the *cost* function in bulk using *apply* function. However, for better clarity, we include the description with *for* cycles in Algorithm 1.

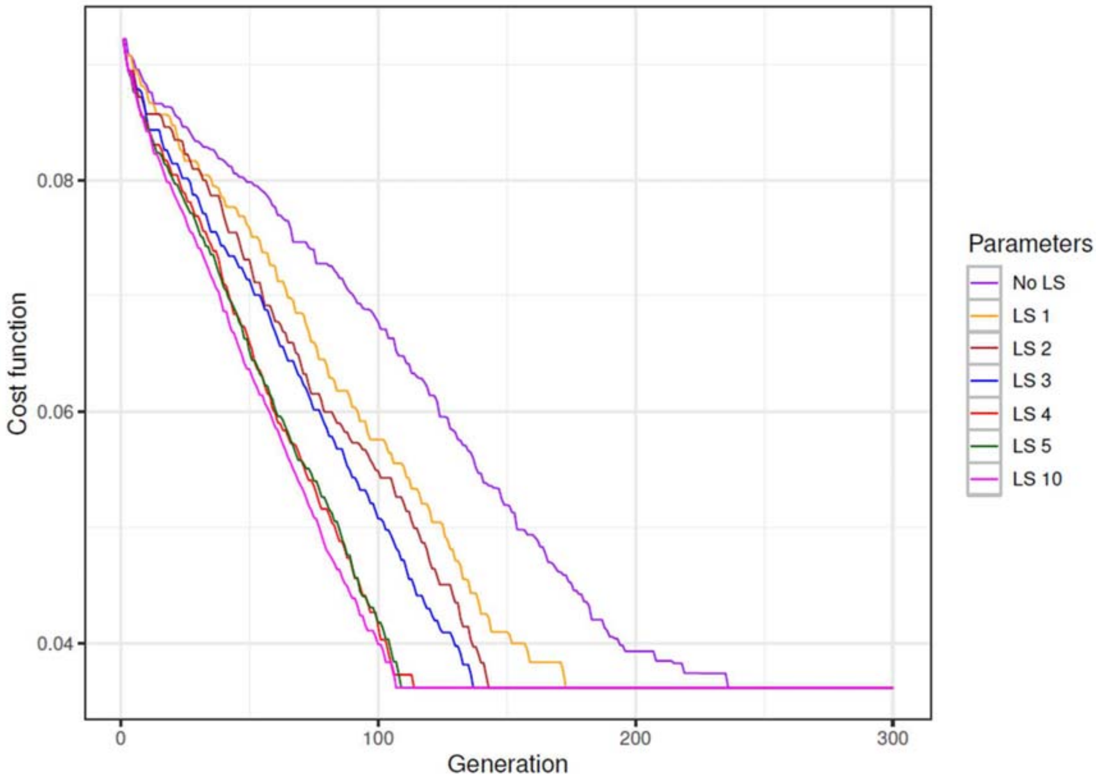

Figure 1: Cost function of best individuals in each generation for different number.

Figure 1 illustrates how the algorithm performs for various numbers of individuals used in the local search method. We show the development of *cost* function for algorithm without local search and for 1, 2, 3, 4, 5 and 10 numbers of checked individuals.

In simulation study, we found that increasing number of checked individuals, quickly decreases the number of iterations needed to find the best solution. On the other hand, the local search method is much more time consuming. In Table 1 we show the results of simulation study based on 100 generated datasets. The parameters for generating datasets and genetic algorithm are the same as in previous sections.

Table 1: Comparison of average results based on local search parameter.

LS	Iterations	Cost	Time
0	322	0.0379	1.00
1	278	0.0366	1.18
2	205	0.0363	1.47
3	164	0.0363	1.76
4	142	0.0363	2.23
5	130	0.0363	2.75

The time values are shown relative to the fastest time needed to converge to the best solution as we run the simulations on two different CPUs.

In some cases, we got apparently lower value of the cost function using the algorithm with local search implementation in comparison to the original algorithm. We also found that in all cases the two checked individuals provided same result as larger number of checked individuals.

The algorithm without local search is the fastest in terms of time. Algorithms with parameters of 1 and 2 checked individuals are 18 and 47 percent slower, and the algorithms with higher parameters are noticeable slower with no benefit. Therefore, we recommend using the local search algorithm with checking two best individuals.

5 CONCLUSIONS

In this paper, we analyzed the improvements to the special case of clustering involving non-linear integer optimization model. The goal is to minimize the standardized joint purchases of products from the same cluster with the only input being transaction data in form of incidence matrix. Two improvements were proposed in compared to the original genetic algorithm. First, we proposed a simple method to mitigate symmetries in the solution space. However, the effect on the efficiency was not statistically significant in both accuracy and time consumption. Second, we implemented a local search heuristic to the genetic algorithm. The results were promising as the solution found was often better than using standard algorithm. However, the time cost was also increased by 47 percent in comparison to the basic algorithm.

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QUEUING SYSTEM WITH IMPATIENT CUSTOMERS - APPLYING SIMULATION TECHNIQUES TO SOLVE A SMALL BUSINESS PROBLEM

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Abstract
The study deals with an analysis and optimization of a queuing system in the service sector. The company under review is a traditional men's hairdresser (barber) whose business model is based on limited offer of services (dry hair cut) with the emphasis on short waiting time and no orders in advance. The analyzed queuing system is characterized by impatient customers in the system. The aim of the study is to design an optimal operating system, which will balance the interests of operator (service owner), service staff, and service customers. Random distributions of system input parameters are modeled on the basis of known information about the number of arriving customers and the time of service. The resulting optimal parameter values were obtained by simulation procedure. The simulation was performed by MS Excel using VBA (Visual Basic for Applications).

Keywords: *queuing system, impatience of customers, optimization, simulation methods*

JEL Classification: C630, L840
AMS Classification: 68U20

1 INTRODUCTION

Queuing theory is a frequently used discipline for modelling and determining important parameters in processes where units (requirements) occur entering the system and waiting until they are served. The occurrence of these units is random. Due to limited number of service lines (service channels) there may be a problem consisting in accumulation of requirements and formation of queues. Such processes may be modelled by means of queuing models. Usually, we need to find such a state of a given model as seen from the point of the number of service lines in which the requirements for the service do not wait too long and, at the same time, the service lines are sufficiently busy.

Some simpler models of queuing systems can be solved analytically. This means that based on known mathematical relations it is possible to calculate the output characteristics that describe the behaviour of such a model. Analytical approaches to the systems of queuing can be found, for example, in Ross (2014). For complex models or models described by means of less common random distributions it is necessary to estimate the output characteristics on the basis of a simulated passage of units through the modelled system. Some selected simulation techniques used in the queuing systems are presented, for example, by Stewart (2009).

The topic related to the application of queuing models for solving diverse issues has been addressed by a number of studies. E.g. De Boer (2000) uses the application of queuing model in the field of telecommunication networks, and, similarly, Madani (2013) in banking. Some studies use queuing models as a tool for solving issues related to improving customer service in the field of retailing (Xing et al. 2015), healthcare (Wang et al. 2019) or general public services (Xian et al. 2016).

The aim of this study is to build on previous work on this topic (Svoboda et al. 2018) and to design an optimal service system in a small business operating in services sector (Vacek et al. 2011), respecting the customer's impatience factor.

Customer impatience manifests itself in queuing systems in two ways. The first way is that the customer does not enter the system if there are already a certain number of customers in the system (so-called a priori impatience). The second way is characterized by the fact that the customer enters the system, queues up and after a certain critical waiting time he/she resigns to be served and leaves the system (so-called a posterior impatience). Basic approaches for customer impatience modeling are presented e.g. in a monograph (Lukáš 2009).

2 CHARACTERISTICS OF THE ANALYZED CASE

The analyzed case is a traditional men´s hairdresser´s with a long history in the centre of town. This location brings both, several advantages as well as some disadvantages. The main advantage is a large number of potential customers moving in its proximity on a daily basis. There are a lot of stops of numerous lines of municipal transport that can also be found near the hairdresser´s. The disadvantage is, on the contrary, the lack of car parking lots in the close vicinity and related high parking rates. The business strategy is based on these factors. The business model is based on a limited offer of services (dry hair cut), on the system of no orders in advance, lower prices and, especially, short waiting times. The opening hours of the hairdresser´s are from 7 a.m. to 8 p.m. only on weekdays, which is once more associated with the location of the establishment.

2.1 Characteristic of the service

There are five service places (hairdressing chairs) in the establishment. The working hours are seven hours, with 6.5 hours of actual operation, and the remaining time is evenly divided into 15 minutes before the start of operation and 15 minutes after the end of the operation. A total of 10 workers (hairdressers) in two-shift operation are required to ensure maximum intensity of service, each with five shift workers, with the first shift serving from 7 a.m. to 1:30 p.m. and the second from 1:30 p.m. to 8 p.m.

The time of serving one customer is understood as the total time which a hairdresser devotes to that particular customer. The time consists of the following activities: calling on a customer whose turn it is, accommodating the customer in the chair, protecting customer´s clothes, agreeing on the type of haircut, the haircut itself and potential corrections in case the customer is not satisfied, removing the protection of clothes and removing the cut off hairs (brushing down), issuing the receipt and receiving the payment (only cash payments are received) and sweeping the work area. The time of serving one customer depends on a lot of factors: the chosen haircut, the density and length of hair, the technique of cutting (scissors or electric trimmer), the speed of the particular hairdresser, corrections of the haircut and possibly also on additional requirements, for example beard trimming. The time of serving one customer is most often 12 minutes, the shortest service time is 9 minutes and the maximum service time is 18 minutes (according the service provider estimates).

2.2 Customers´ entries into the system

The exact times of customers´ entries into the establishment are not known. There is no electronic system in use that would allocate the customer an order number. A daily number of the served customers is known from the accounting documents. The number of customers served in one day may vary from 140 to 200, the average number being around 170 customers.

The distribution of the number of customers during the day is not even. There are three local peak periods during the day: the first one approximately between 7:30 a.m. and 8:30 a.m., the second one between 11:30 a.m. and 12:30 p.m., and the third one between 3 p.m. and 5 p.m. These local peaks can be explained by customers working hours. The first peak represents customers visiting the hairdresser in the morning on the way to work, the second peak is formed by customers who use the lunch break, and the last peak consists of customers after leaving work. For this reason, we can consider the customers as impatient. They enter the hairdresser only if there is no big queue. On the basis of the empirically gained data, the average numbers of entering units in each individual 30 minute interval are displayed on the Figure 1 below.

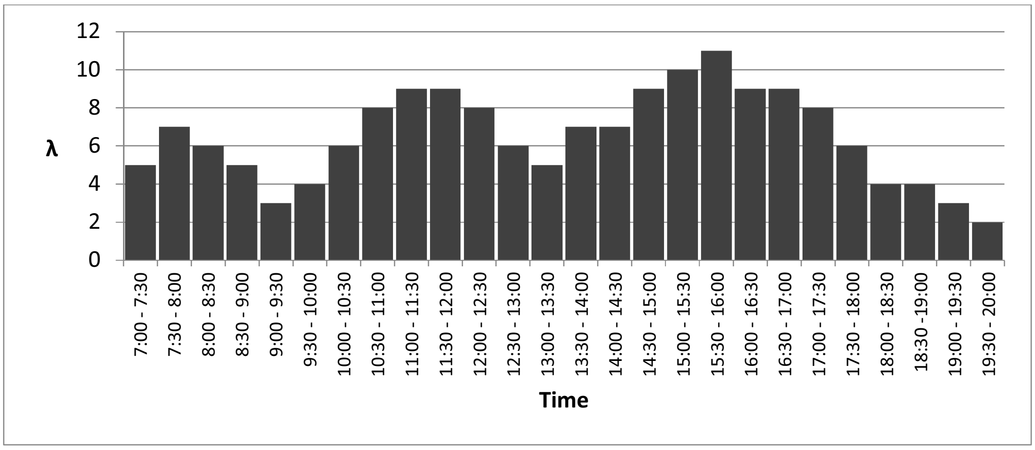

Figure 1: Intensity of customers' entries
Source: author's calculations

3 DESIGN OF THE MODEL

A triangle division is used to model the operating time based on the provided information. The intervals between the arrivals of two customers are modelled by means of exponential division with a variable mean value, which is a standard way of modelling in similar cases. Both the divisions, including their characteristics, are described, for example in (Dlouhý et al. 2007). The triangle division *Tri(a,b,c)* with parameters *a, b, c* is recommended in cases where there are no particular data available but we know the minimum value (parameter *a*), the most frequent value (parameter *b*) and the maximum value (parameter *c*). In our case we have a division with parameters *Tri*(9, 12, 18). The mean value of the triangle division is given by the relation (1):

$$E(X) = \frac{a+b+c}{3} \quad (1)$$

By substituting to relation (1) we will gain the mean value of customer service time which equals 13 min. One operating channel thus serves 4.6 customers in one hour on average. It can be easily calculated that one operating channel could serve, on average, 59.8 customers in a thirteen hours long shift. The exponential division has the only parameter λ which is interpreted as the intensity of entries, i.e. the number of entries over a time unit. The reciprocal value $1/\lambda$ is the mean value of this division, i.e. the average time between the arrivals of two customers. The value λ is considered as a variable dependent on time in our model, according to Figure 1.

The impatience of the customers entering the hairdresser is rather a priori, i.e. the probability of the customer entering the system decreases from a certain number of queued customers. According to information from the service provider, the impatience begins to show when there are already three customers in the queue, and as the queue grows, the probability of entering another customer decreases relatively quickly. The amount of queued customers rarely exceeds the number 6. Based on this information, we used the relationship (2) to model the probability of customer entry into the system:

$$\begin{aligned} q \le 2 \quad & P(q) = 1 \\ q > 2 \quad & P(q) = e^{-\alpha \cdot q}, \qquad \alpha = 0.7 \end{aligned} \tag{2}$$

where

q is the number of customers in the queue,

$P(q)$ is the probability of entering the system of a new incoming customer if there are exactly q customers in the queue, and

α is an optional parameter that affects the impatience.

In our model, $\alpha = 0.7$. The Table 1 shows the probability a new customer coming into the hairdresser stays here (he/she queues) when q customers are queued already.

Table 1: Probability a new customer queues when q other customers are queued

q	0	1	2	3	4	5	6	7	8
$P(q)$	1.000	1.000	1.000	0.497	0.247	0.122	0.061	0.030	0.015

Source: author's calculations

It is necessary to know the cost of service and the revenue for serving the customer for economic evaluation of the model. The basic and most used service is hair cutting, whether by scissors or electric shears, for a single price of CZK 139. Service costs consist of labor costs per worker (hairdresser) that consist of the hairdresser's salary (gross wage), and health and social insurance contributions counted in our model count as 0.35 times the gross wage. The gross wage of the hairdresser consists of a fixed component of CZK 80 per hour (i.e. CZK 108 in labor costs) and CZK 40 (CZK 54 in labor costs) for each served customer. Hairdressers usually get tips from the vast majority of customers, which they keep, usually CZK 11 or CZK 21. We calculate with a conservative estimate of CZK 11 in our model.

Owing to the above mentioned characteristic of the variability of the intensity of inputs we will solve the task by means of simulation. The simulation model was programmed in the environment of MS Excel by means of VBA language. One simulation run models (simulates) the course of one working day in the establishment. This simulation has a finite time horizon. The simulation run is started at 7 a.m. and finished at 8 p.m.

4 COMPUTATIONAL RESULTS

Overall 6 different models of the above described system of queuing were analyzed. The first model is a model of maximal service, i.e. all service points (channels) are used throughout the working hours. Such a service system requires 10 workers (hairdressers). In each subsequent model, one worker is gradually reduced. As mentioned earlier, each worker works 7 hours. Decrease in staff by one worker reduces the operator's labor costs by 756, - CZK (7x108). On the other hand, the operator loses CZK 85 (hair cutting price minus hairdresser´s bonus for served customer) with every customer who does not enter the hairdresser because of the large queue.

A total of 200 simulations were performed for each model. We are interested in the following characteristics of the realized simulation: number of entering customers, number of non-entering (impatient) customers, average waiting time for a queued customer, and total waiting time of all service lines for a customer. From these characteristics we can easily calculate:

service line vacancy (in %), total labor costs, one employee's wage, and operator's income. The operator's income is represented by revenues reduced by staff costs. The operator's income is not the profit, the operator must pay rent, energies, and other operating and administrative costs that are fixed and cannot be changed in our model. Only the number of workers can be changed and thus labor costs influenced. The number of open service lines for each model is shown in Table 2. Operating times were determined with respect to customers' entries intensity as shown in Figure 1.

Table 2: Operating times on individual channels (service lines)

	n	channel 1	channel 2	channel 3	channel 4	channel 5
Model 1	10	07:00-20:00	07:00-20:00	07:00-20:00	07:00-20:00	07:00-20:00
Model 2	9	07:00-20:00	07:00-20:00	07:00-20:00	07:00-20:00	11:00-17:30
Model 3	8	07:00-20:00	07:00-20:00	07:00-20:00	10:30-17:00	11:00-17:30
Model 4	7	07:00-20:00	07:00-20:00	10:00-16:30	10:30-17:00	11:00-17:30
Model 5	6	07:00-20:00	07:00-20:00	10:30-17:00	11:00-17:30	x
Model 6	5	07:00-20:00	07:00-20:00	11:00-17:30	x	x

Source: author's calculations

The results for each model (mean values) are shown in Table 3. In addition to the above variant, the simulation for Model 6 was also realized in case the first channel was available throughout the working hours, the second channel was available from 10:00 to 16:30, the third from 10:30 to 17:00, and the fourth from 11:00 to 17:30. However, the characteristics of this variant showed worse results than the original one and therefore we do not list them.

Table 3: Simulation results for individual models (one-day average values)

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Number of entrants [persons]	166.7	166.2	163.7	156.8	150.9	136.0
Number of non-entrants [persons]	3.3	4.0	6.0	12.7	18.4	33.8
Average waiting time [hour:min:s]	0:01:01	0:01:18	0:02:04	0:04:20	0:05:47	0:09:09
Total time of service lines vacancy	27:09:42	21:31:08	16:09:01	11:33:49	6:19:58	3:00:05
Service lines vacancy [%]	41.8%	36.8%	31.1%	25.4%	16.2%	9.2%
Labor costs [CZK]	17,102.-	16,265.-	15,321.-	14,138.-	13,008.-	11,396.-
Worker's wage [CZK/worker]	1,266.-	1,338.-	1,418.-	1,496.-	1,605.-	1,620.-
Operator's income [CZK]	**6,070.-**	**6,838.-**	**7,436.-**	**7,659.-**	**7,965.-**	**7,513.-**

Source: author's calculations

The results show that, up to model 5, the operator's income increases with service lines (channels) reductions, and wage savings outweigh the losses of the non-entering customers. In Model 6, the losses from non-entering customers outweighed the labor cost savings.

In terms of operator's income, the least profitable is Model 1. In this model, the daily takings are on average CZK 23,171; after deduction of labor costs, the operator remains CZK 6,070, i.e. CZK 121,390 per month (at 20 working days per month). This amount is large enough to cover the monthly rent, energies, operating and administrative costs, annual remunerations, and reasonable profit for the operator. From the labor costs after deduction of insurance contributions, we calculate an employee's gross daily income of CZK 1,266 for seven-hour working time, which corresponds to a gross monthly wage of CZK 25,336. The tip can be added to this wage that can be estimated to about CZK 3,674.- per month net (at 16.7 customers served per day on average). The model is very favorable to customers, service lines vacancy is 41.8%, the number of non-entrants is only 3.3 per day on average, and the customer’s average waiting time is only 1:01 (1.016 minutes). The calculation of the above characteristics for other models is left to the reader. A typical time frame with non-entering (impatient) customers for each model is shown in Figure 2.

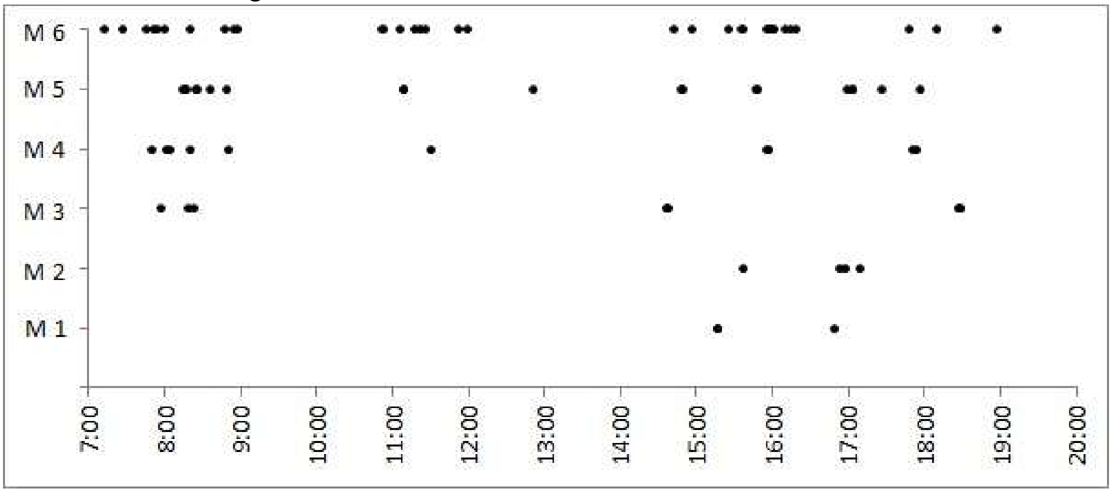

Figure 2: Time frame with non-entering customers

Source: author's calculations

5 CONCLUSIONS

All simulated models are profitable for the operator. It depends on the marketing strategy whether the operator wants to maximize the profit or customer's comfort. From a long-term point of view and from the perspective of all three entities: customers, workers and operator, it seems best to have 9 workers. Workers may be ill, take leave, etc. Taking into account that a worker is entitled to 4 weeks of leave, i.e. 36 weeks in total for 9 workers, plus illness or care for sick children, it can be assumed that total absence at the workplace can easily reach 52 weeks, i.e. there will be 8 workers a day in the workplace on average. If the operator employs 9 workers, he/she can react very flexibly to the absence of the worker at the workplace and switch from Model 2 (all workers present) to Model 3 (1 worker absent) or Model 4 (2 workers absent). Assuming, these three models will be used evenly, the operator's monthly income would be CZK 146,219. In such a case, the average gross monthly wage of employees is CZK 28,355 and tipping is CZK 4,497. On average, customers wait in the queue for 2 minutes and 34 seconds and the total number of non-entering customers per day is 7.6. These parameters can be considered balanced and the entire service system sustainable.

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IS THERE ROCKETS AND FEATHERS EFFECT BETWEEN CRUDE OIL PRICES AND PORT FUEL PRICES OR BETWEEN PORT FUEL PRICES AND RETAIL FUEL PRICES?

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Abstract
Numerous studies have dealt with the transmission of crude oil prices to retail gasoline prices and indicate that retail gasoline prices respond more quickly when crude oil prices rise than when they decrease. This kind of asymmetric retail fuel pricing is also confirmed in our study based on deriving reaction equations of the price-makers from a linear exponential form of the adjustment costs (a linex approach). Asymmetric pricing can occur at different stages of fuel production and sale. We have data on crude oil spot prices, fuel traded in US ports spot prices, and US retail fuel prices. Using the linex approach and these data, the asymmetric reactions of port fuel spot prices to changes in crude oil spot prices are not confirmed, but the asymmetric reactions of retail fuel prices to changes in port fuel spot prices are confirmed.

Keywords: *Rockets and feathers effect, Crude oil spot prices, Port fuel spot prices, Retail fuel prices, Linex approach*

JEL Classification: C26, C51, Q41
AMS Classification: 62P20

1 INTRODUCTION

Numerous studies dealing with the transmission of crude oil prices to retail fuel (gasoline and diesel) prices indicate that the retail fuel prices respond more quickly when the crude oil price rises rather than when it decreases; e.g. Radchenko (2005), Grasso and Manera (2007), Honarvar (2009), Meyler (2009), Liu et al. (2010), Rahman (2016) and Sun et al. (2019). Bacon (1991) called this asymmetric retail gasoline price adjustment as ”rockets and feathers” effect. His study was followed by a paper of Borenstein et al. (1997) who provide strong evidence of asymmetry in the US market between 1986 and 1992 in different stages of the production and distribution of gasoline.

Several theoretical explanations of the asymmetry have been proposed and tested. A short review is provided by Brown and Yücel (2000) and also Radchenko (2005). Borenstein et al. (1997) suggest three possible explanations for the asymmetric response of gasoline prices:

- the oligopolistic coordination theory,
- the production and inventory cost of adjustment, and
- the search theory.

A traditional approach to test asymmetric response of retail gasoline and diesel prices on changes in crude oil prices is the use of error correction models (ECM) and vector error correction models (VECM); e.g. Radchenko (2005), Grasso and Manera (2007), Honarvar (2009), Liu et al. (2010) and Szomolányi et al. (2020a).

A novel approach is based on the linear exponential (linex) adjustment costs formulation (Szomolányi et al., 2020b). The nonlinear reaction function of fuel prices is derived from the linex adjustment costs function form. Estimating the coefficients of the reaction function, one can verify the asymmetric reactions of fuel prices on price changes in crude oil. The approach allows to estimate an average weekly gasoline or diesel price bias per litre of gasoline or fuel. By the traditional approach, Szomolányi et al. (2020b) could not confirm the asymmetric reactions of fuel prices in Slovak market. However, by the linex approach, the asymmetries were confirmed. We prefer this approach, because in our opinion the character of fuel price making is rather discretionary and it better corresponds to the process of minimizing price adjustment costs.

The aim of this paper is to respond the following questions:

- Do U.S. spot port fuel prices react asymmetrically on changes in crude oil prices?
- Do U.S. regional retail fuel prices react asymmetrically on changes in spot port fuel prices?

The gasoline and diesel prices are considered to be fuel prices. By the linex approach we negatively answer the first question and positively the second.

2 DATA

Our analysis focuses on the U.S. fuel markets. We consider the relations between the spot price of West Texas Intermediate (WTI) light crude oil and spot port fuel prices as well as the relations between the spot fuel prices and the regional retail fuel prices. The spot fuel prices consist of:

- spot price of New York Harbour Conventional Gasoline,
- spot price of New York Harbor Ultra-Low Sulfur No 2 Diesel,
- spot price of U.S. Gulf Coast Conventional Gasoline,
- spot price of U.S. Gulf Coast Ultra-Low Sulfur No 2 Diesel,
- spot price of Los Angeles Reformulated RBOB Regular Gasoline (LA) and
- spot price of Los Angeles, CA Ultra-Low Sulfur CARB Diesel.

The regional retail fuel prices consist of:

- retail price of East Coast Regular All Formulations Gasoline,
- retail price of East Coast No 2 Diesel,
- retail price of Midwest Regular All Formulations Gasoline,
- retail price of Midwest No 2 Diesel,
- retail price of Gulf Coast Regular All Formulations Gasoline,
- retail price of Gulf Coast No 2 Diesel
- retail price of Rocky Mountain Regular All Formulations Gasoline,
- retail price of Rocky Mountain No 2 Diesel
- retail price of West Coast Regular All Formulations Gasoline,
- retail price of West Coast No 2 Diesel

All price series are obtained from the U.S. Energy Information Administration website. Crude oil and gasoline spot prices are collected at daily sampling frequency, while retail gasoline and diesel prices are available only at weekly frequency. The spot and retail prices of petroleum products are denominated in dollars per gallon, while the spot price of oil is expressed in dollars per barrel. According to the U.S. Energy Information Administration website, the retail fuel prices are collected every Monday, therefore the daily spot prices are aggregated to weekly to match their Monday spot values. A description of the dataset is presented in Table 1.

Table 1: Dataset

no	Data Series	Type	Dollars per	Geo	Freq	Material	No.Obs.	1 st Obs.
1	Cushing, OK WTI Spot Price FOB	Spot	Barrel	Aggregate	Daily	Crude Oil	8452	08/20/1990
2	New York Harbor Conventional Gasoline Regular Spot Price FOB	Spot	Gallon	---	Daily	Gasoline	8333	08/20/1990
3	U.S. Gulf Coast Conventional Gasoline Regular Spot Price FOB	Spot	Gallon	---	Daily	Gasoline	8330	08/20/1990
4	Los Angeles Reformulated RBOB Regular Gasoline Spot Price	Spot	Gallon	---	Daily	Gasoline	4108	03/10/2003
5	New York Harbor Ultra-Low Sulfur No 2 Diesel Spot Price	Spot	Gallon	---	Daily	Diesel	3292	06/12/2006
6	U.S. Gulf Coast Ultra-Low Sulfur No 2 Diesel Spot Price	Spot	Gallon	---	Daily	Diesel	3292	06/12/2006
7	Los Angeles, CA Ultra-Low Sulfur CARB Diesel Spot Price	Spot	Gallon	---	Daily	Diesel	5836	04/15/1996
8	East Coast Regular All Formulations Retail Gasoline Prices	Retail	Gallon	Regional	Weekly	Gasoline	1419	05/11/1992
9	Midwest Regular All Formulations Retail Gasoline Prices	Retail	Gallon	Regional	Weekly	Gasoline	1419	05/11/1993
10	Gulf Coast Regular All Formulations Retail Gasoline Prices	Retail	Gallon	Regional	Weekly	Gasoline	1419	05/11/1994
11	Rocky Mountain Regular All Formulations Retail Gasoline Prices	Retail	Gallon	Regional	Weekly	Gasoline	1419	05/11/1995
12	West Coast Regular All Formulations Retail Gasoline Prices	Retail	Gallon	Regional	Weekly	Gasoline	1419	05/11/1996
13	East Coast No 2 Diesel Retail Prices	Retail	Gallon	Regional	Weekly	Diesel	1322	03/21/1994
14	Midwest No 2 Diesel Retail Prices	Retail	Gallon	Regional	Weekly	Diesel	1322	03/21/1994
15	Gulf Coast No 2 Diesel Retail Prices	Retail	Gallon	Regional	Weekly	Diesel	1322	03/21/1994
16	Rocky Mountain No 2 Diesel Retail Prices	Retail	Gallon	Regional	Weekly	Diesel	1322	03/21/1994
17	West Coast No 2 Diesel Retail Prices	Retail	Gallon	Regional	Weekly	Diesel	1322	03/21/1994
Note: The last observation: 07/15/2019.								

3 METHODOLOGY

The problem of firm minimizing linex price adjustment costs function produces the following non-linear price reaction function (Szomolányi et al. 2020b):

$$\Delta p_t = k\Delta c_t - \frac{1}{2}\gamma\Delta\left[\left(p_t - kc_t\right)^2\right] + u_t \tag{1}$$

One-unit price of given commodity *p* reacts on a change in a one-unit price of another commodity *c*. The technology coefficient is *k*, one can persuade that $\mathrm{d}p_t / \mathrm{d}c_t = k$. The asymmetry coefficient is γ and by symbol *u* is denoted the remainder of the Taylor expansion of first-order firm's optimality condition. A negative value of the coefficient γ implies that a negative value of the difference $p_t - kc_t$ causes higher costs to the price-maker than it would if γ were positive.

Further, if the changes in the crude oil prices Δc_t are normally distributed process with zero mean and variance σ^2, the average gasoline or diesel price bias caused by the "rockets and feathers" effect, $\gamma < 0$, is in the form:

$$E\left(\Delta p_t\right) = -\frac{k^2\gamma}{2}\sigma^2 \tag{2}$$

The orthogonality conditions implied by the rational expectation hypothesis makes the GMM a natural candidate to estimate equation (1). As the residuals of the equations are statistically significantly correlated, we estimate the equation (1) with the system GMM method. The standard errors are computed with the procedure of Newey and West. The most important feature of the procedures explained by Newey and West (1987) is their consistency in the presence of both heteroskedasticity and the autocorrelation of unknown forms.

4 RESULTS

The rockets and feathers effect is verified in two stages of the production and distribution of fuel. Firstly, we estimate the (1) equation considering c to be the WTI crude oil price and p to be the spot harbour gasoline and diesel prices. We then have system of six equations for each spot harbour fuel price and daily data (see Table 1). However, we are not able to estimate statistically significant the coefficients of (1) with the system GMM method and the results are unpublishable.

Secondly, we estimate the (1) equation considering c to be the spot harbour gasoline (or diesel) prices and p to be the retail regional gasoline (or diesel) prices. We then have three systems for each U.S. harbour (New York, U.S. Gulf and Los Angeles) of ten equations for each retail regional fuel price and (after time aggregating spot harbour prices; see the second section) weekly data (see Table 1). Only two of them are publishable.

Table 2: Asymmetric reactions of regional retail fuel prices on changes in spot New York harbour fuel price.

Region	*Equation*	*k*	*γ*	*Bias*
East Coast	Gasoline	0.589	-0.614 **	0.066
	(std.err.)	(0.372)	(0.312)	
	Diesel	0.967 ***	-1.073 ***	0.309
	(std.err.)	(0.089)	(0.248)	
Midwest	Gasoline	0.906 ***	-1.092 **	0.275
	(std.err.)	(0.188)	(0.483)	
	Diesel	0.953 ***	-1.077 ***	0.301
	(std.err.)	(0.078)	(0.216)	
Gulf Coast	Gasoline	0.637 **	-0.730 *	0.091
	(std.err.)	(0.321)	(0.379)	
	Diesel	0.993 ***	-1.278 ***	0.387
	(std.err.)	(0.083)	(0.319)	
Rocky Mountain	Gasoline	-0.151	-0.315 **	0.002
	(std.err.)	(0.618)	(0.137)	
	Diesel	2.537 ***	0.217	-0.430
	(std.err.)	(1.210)	(0.178)	
West Coast	Gasoline	0.602 **	-0.522 ***	0.058
	(std.err.)	(0.292)	(0.172)	
	Diesel	0.872 ***	-0.759 ***	0.178
	(std.err.)	(0.081)	(0.116)	

Note: Asterisks denote statistically significance at 10%, 5% and 1% level. Average weekly biases (2) are in eurocents per litre of gasoline or diesel. The instruments are lagged first differences of New York Harbour gasoline and diesel spot prices.

The estimates of coefficients of (2), considering c to be spot New York harbour gasoline (diesel) price (Table 2), U.S. Gulf harbour gasoline (diesel) price (Table 3) and p to be retail regional gasoline (diesel) prices (in rows) are in the Table 2 and Table 3. Standard errors in parenthesis of the tables are computed with the procedure Newey and West. The statistical significance of the coefficients are denoted by asterisks; statistical significance at 1% level is denoted by three asterisks, at 5% level by two asterisks and at 10% level by one asterisk.

Table 3: Asymmetric reactions of regional retail fuel prices on changes in spot U.S. Gulf harbour fuel price.

Region	***Equation***	**k**		**gamma**		***Bias***
East Coast	Gasoline	0.517	***	-0.556	***	0.080
	(std.err.)	(0.051)		(0.046)		
	Diesel	1.182	***	-2.309		0.378
	(std.err.)	(0.126)		(1.871)		
Midwest	Gasoline	0.666	***	-0.699	***	0.338
	(std.err.)	(0.042)		(0.053)		
	Diesel	1.069	***	-1.459	***	0.369
	(std.err.)	(0.059)		(0.340)		
Gulf Coast	Gasoline	0.597	***	-0.681	***	0.112
	(std.err.)	(0.043)		(0.058)		
	Diesel	1.147	***	-2.340	*	0.475
	(std.err.)	(0.085)		(1.270)		
Rocky Mountain	Gasoline	0.055		-0.364	***	0.003
	(std.err.)	(0.070)		(0.018)		
	Diesel	-0.306		-0.237	***	-0.527
	(std.err.)	(0.418)		(0.067)		
West Coast	Gasoline	-0.049		-0.301	***	0.071
	(std.err.)	(0.110)		(0.021)		
	Diesel	0.798	***	-0.659	***	0.218
	(std.err.)	(0.078)		(0.092)		

Note: Asterisks denote statistically significance at 10%, 5% and 1% level. Average weekle biases (2) are in eurocents per litre of gasoline or diesel. The instruments are lagged first differences of U.S. Gulf Harbour gasoline and diesel spot prices.

5 CONCLUSION

The rockets and feather effect is confirmed in final stages of the fuel distribution. This cognition may be useful for the theoretical discussion studying sources of asymmetries in retail fuel price-making process. It may also be useful for explanation of price rigidities, currently vehemently used in the New Keynesian models of business cycles.

It is worth to focus in the paper of Douglas and Herrera (2010). According to the authors, asymmetric retail gasoline price reactions to crude oil price changes can be addressed by the theory of strategic interactions between a firm and its consumers; e.g. Okun (1981). The theory of strategic interactions between a firm and its consumers is one of three theories explaining the price stickiness hypothesis which is used in New Keynesian monetary models. Douglas and Herrera argue that a good testing ground for various theories of price stickiness is the dataset describing price adjustment of gasoline sellers. Particularly, the theory predicting price adjustment asymmetry in the retail gasoline market is the theory of strategic interactions between a firm and its consumers. The state that asymmetries come only in the retail stage of fuel distribution is another indication supporting the theory of strategic interactions between a firm and its consumers.

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OF APPROACH TO MINIMIZE THE NUMBER OF SHUNTING TRACKS AT TURNING LOOPS

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Abstract
Each transport mode needs a specific infrastructure to be operated. The same applies to tram transport. An integral part of the tram infrastructure is formed by shunting tracks at turning loops. The turning loops can have a variety of configurations (shapes). The shape depends primarily on the tram traffic volume.
The construction of the tram infrastructure (including the turning loops) and their subsequent operation is very expensive for each tram transport operator. In addition, to the need for building a higher number of shunting tracks, the number of switches included at the turning loop etc. is higher. In addition to the construction costs, the maintenance costs for the turning loop increase proportionally as well.
The article deals with the possibility of reducing the number of the shunting tracks. One of the ways how to reduce the number of necessary shunting tracks at the turning loop is to coordinate the arrivals and departures of individual tram connections. Mathematical programming can be used to deal with the coordination problem. Presenting a simple example, the article shows a basic principle of the positive impact of coordination on the necessary number of the shunting tracks at the turning loops.

Keywords: *Optimisation, Tram transport, Turning loop, Tram turnarounds, Shunting tracks*

JEL Classification: C610
AMS Classification: 90C05

1 INTRODUCTION – OUR MOTIVATION TO DEAL WITH THE PROBLEM

For means of public transport which convey passengers on pre-defined routes so called turning loops (or balloon loops) must be located at final stops of the individual public transport routes. The turning loops can be a part of regular transport infrastructure or they can be designed as special infrastructure elements. The turning loops fulfil two basic functions:

- They enable turnarounds of the public transport vehicles in order to serve the route in the opposite direction.
- The vehicles can be parked at the turning loops when it is necessary to wait for serving a consecutive connection or when drivers are taking his/her break.

Because the presented paper is focused on tram transport, we limit ourselves to the tram turning loops in the article. In practice, we can see different layouts of the turning loops. But in general, we can say that to speed up turnarounds of trams it is necessary not to use dead-end tracks at the turning loops because marshalling of trams is more time-consuming and more difficult to organize when done using the dead-end tracks.

Tram turnarounds are usually done at:

- Tram depots containing the turning loops.
- The individually built turning loops.

The turning loops can be divided based on their permanency into:

- The permanent turning loops – Figure 1 presents an example of such turning loop.
- The temporary turning loops – see Figure 2.

In the article we will focus on optimisation of the permanent turning loops.

Figure 1: The permanent turning loop [12]

Figure 2: The temporary turning loop [13]

When traffic volume increases (the number of connections that ends and consequently begins at the turning loop), then it is often necessary to extend the turning loop. By the extension of the turning loop we mean increasing the number of the shunting tracks forming the turning loop.

The most adverse situation (from the point of view of the tracks that form the turning loop) occurs in cases when the individual connections enter the turning loop during a short time interval and, moreover, different turnaround times are defined for the lines which share the same shunting track at the turning loop. In contrary to bus and trolleybus transport, changing the order of the trams located on the same track is limited or even excluded (an example is shown in Figure 1 – there is no possibility to change the order of the trams). Therefore, it is necessary to assign the individual connections of the same tram line to different turning loop tracks so that the tram traffic fluency is not disrupted or is disrupted only in case of emergency operational situations.

In order to decrease the number of the turning loop tracks we can take some measures. Such measures are:

- Continuous changeover of the trams (or their sets) among the different lines – this is possible when the individual trams are mutually replaceable regarding their capacity.
- Increasing the level of time coordination of arrivals and departures of the individual connections at the turning loop.

When assuming that the individual trams (or their sets) are not interchangeable, then the only way to reduce the number of the turning loop tracks is increasing the level of time coordination. In the following text we will present a mathematical programming approach to time coordination of the connections of the individual tram lines in order to reduce the number of the turning loop tracks. Mathematical programming was chosen because we expect in the future an increase in the extent and complexity of the given coordination problem. In addition, these types of optimisation problems are often solved by mathematical programming methods. Please note that the optimisation problem arises when the number of the turning loop tracks is less than the number of the tram lines which use the turning loop.

As regards publications devoted to optimisation of the number of the shunting tracks at the turning loops, we can say that we have not been able to find any articles. However, there are many articles dealing with different problems related to tram transport. The articles are most often aimed at solving different technical issues such as tram network maintenance planning (Kiefer, Schilde and Doerner, 2018), noise and vibration elimination problems (Czechyra, Tomaszewski and Nowakowski, 2015), reducing emissions (Vogiatzis and Cards, 2005), interaction problems between trams and tram tracks (Hauser et al., 2017), energetical problems of tram transport (Yan and Min, 2018) and comparison of tram and bus transport from the different point of views (Prud'homme, Koning and Kopp, 2011). Regarding technological issues of tram transport, the following problems are dealt with – tram network capacity (Molecki and Gąska, 2012), tram transport preference at intersections (Sermpis, Papadakos and Fousekis, 2012), design of tram transport timetables (Galkin et al., 2019) and assigning suitable tram vehicles to tram lines (Teichmann, Dorda and Olivkova, 2013). Another important problem that is discussed in publications is devoted to safety of tram transport users – see for example (Kazhaev, Almetova, Shepelev and Shubenkova, 2018).

2 IMPACT OF TIME COORDINATION ON THE NUMBER OF TRACKS

Let us consider that the arrivals and departures of the different tram lines that are served by the same trams are scheduled as shown in Figure 3. The turning loop is depicted by the horizontal line (the same holds for Figures 4 and 5).

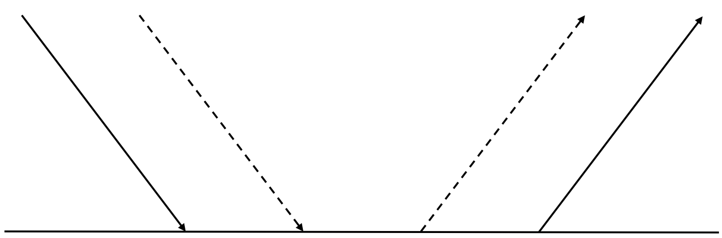

Figure 3: A situation requesting two shunting tracks for the tram turnarounds

The individual symbols in the figure represents:

- ↘ the arrival time of a connection of line 1 at the turning loop according to the timetable,
- ↗ the departure time of the consecutive connection of line 1, which is served by the same tram, from the turning loop according to the timetable,
- ⇢↘ the arrival time of a connection of line 2 at the turning loop according to the timetable,
- ⇢↗ the departure time of the consecutive connection of line 2, which is served by the same tram, from the turning loop according to the timetable.

One can see in Figure 3 that the trams of both lines need two shunting tracks for their turnarounds (the same situation occurs when the connection of line 2 enters the turning loop before the arrival of the connection of line 1 and departs after the departure of the connection of line 1). If the arrival and departures times of the individual connections were coordinated in a better manner, then it would be possible to reduce the number of the tracks necessary to the turnarounds – see Figures 4 and 5. Please note that the possibility of moving the individual connections in time is a necessary condition for the time coordination process.

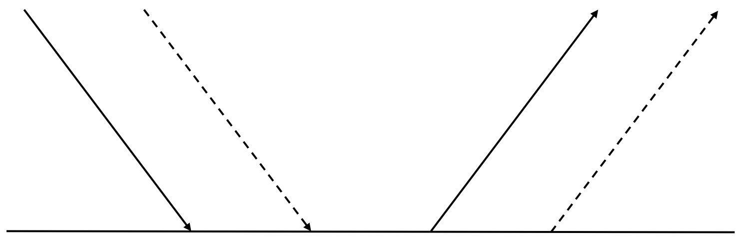

Figure 4: A situation requesting a shunting track for the tram turnarounds – example 1

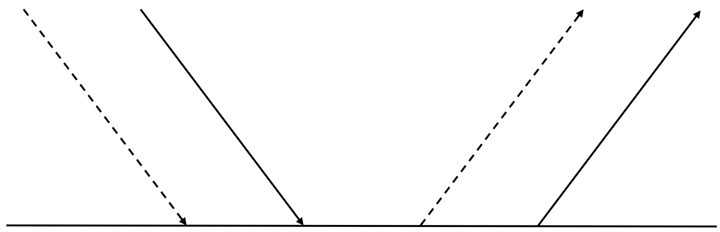

Figure 5: A situation requesting a shunting track for the tram turnarounds – example 2

In the first example (see Figure 4) the departure times of lines 1 and 2 are interchanged (in Figure 3 the connection of line 1 arrives as first and departs as second and the connection of line 2 arrives as second and departs as first). In Figure 4 the connection of line 1 arrives and departs as first and the connection of line 2 arrives and departs as second. Reducing the number of the tracks was achieved by interchanging the departure times of the connections of lines 1 and 2. In the second example, which is depicted in Figure 5, the order of the connections of line 1 and 2 was changed in comparison with Figure 3.

3 PROBLEM FORMULATION AND ITS MATHEMATICAL MODEL

In the section we focus on description of the basic principle of time coordination and its impact on the number of the turning loop tracks. The principle is applied on an example of the turning loop with two shunting tracks and two tram lines which use the turning loop.

3.1 Formulation of optimisation problem

Let us consider a turning loop for tram transport equipped with two shunting tracks. Let us assume that the turning loop is used by the trams that serve two tram lines and the trams assigned to the individual lines are not interchangeable because of their capacity. That means the trams cannot be assigned to different lines within their schedule.

For each tram line $i = 1, 2$ arrivals t_i^{act} and departures T_i^{act} of the individual connections and time intervals which form the boundary of possible changes in the arrivals and departures (so called coordination time intervals) are known. The coordination time intervals are labelled as $\langle \tau_i^{arr-}, \tau_i^{arr+} \rangle$ for the arrivals of the connections and $\langle \tau_i^{dep-}, \tau_i^{dep+} \rangle$ for the departures of the connections. It holds for the values τ_i^{arr-}, τ_i^{arr+}, τ_i^{dep-} a τ_i^{dep+} that all of them are nonnegative.

Let us further assume that between each arrival and departure time of the tram which serves the connections of line i a minimal turnaround time (the minimal time the tram must spend at the turning loop) must elapse. Our goal is to by means of time coordination find out the arrivals and departures of the individual connections so that the number of the shunting tracks at the turning loop necessary for the turnarounds of the trams is as minimal as possible (for the turning loop equipped with two shunting tracks it means that only 1 track is necessary).

If it holds for the coordination time interval $\langle \tau_i^{arr-}, \tau_i^{arr+} \rangle$ that $\tau_i^{arr-} = 0$ and $\tau_i^{arr+} = 0$, then the arrival time of the connection of line i cannot be changed (is fixed). The same also holds for the departure times and their coordination time intervals $\langle \tau_i^{dep-}, \tau_i^{dep+} \rangle$. If it holds for both coordination time intervals at the same time, then the current solution cannot be improved (no possibility of optimisation).

3.2 Design of optimisation model

Before presenting a mathematical model let us modify some pieces of input data:

- The current arrival and departure times of both lines will be transformed into the set of real numbers with a time unit which is equal to 1 minute and value 0 corresponds to the midnight of a day. That means for example if the real arrival time of the connection of line i is 6:30 then the arrival time in the model will be represented by the value $t_i^{act} = 6 \cdot 60 + 30 = 390$ min.
- All the arriving and departing connections will be rescheduled to their earliest possible time positions. By rescheduling, the earliest possible arrival times t_i^{ear} and the earliest possible departure times T_i^{ear} of line i are created. For example, if the arrival time of line i is originally scheduled at 6:30 and the corresponding coordination time interval is $\langle 5; 10 \rangle$ minutes then the earliest possible arrival time equals to $t_i^{ear} = t_i^{act} - \tau_i^{arr-} = 390 - 5 = 385$ minutes.

Let us define the following variables necessary for the optimisation task:

x_i a time shift of the arrival of the connection of line $i = 1{,}2$, the time shift is related to the earliest possible arrival time,

y_i a time shift of the departure of the connection of line $i = 1{,}2$ the time shift is related to the earliest possible departure time,

u_{ij} a binary variable for modelling mutual positions of the arrivals of the connections of lines i and j, where $j \neq i$. If the arrival time of the connection of line i (after its potential time shift, the time is related to the earliest possible arrival time) is lower than the arrival time of the connection of line j (after its potential time shift, the time is related to the earliest possible arrival time as well), then $u_{ij} = 1$, otherwise $u_{ij} = 0$,

v_{ij} a binary variable for modelling mutual positions of the departures of the connections of lines i and j, where $j \neq i$. If the departure time of the connection of line i (after its potential time shift, the time is related to the earliest possible departure time) is lower than the departure time of the connection of line j (after its potential time shift, the time is related to the earliest possible departure time as well), then $v_{ij} = 1$, otherwise $v_{ij} = 0$,

z a binary variable related to the number of the shunting tracks used for the turnarounds of the trams. If both tracks are used, then $z = 1$, otherwise $z = 0$.

The mathematical model can be defined in the following form:

$$min\, f(x, y, u, v, z) = z \quad (1)$$

subject to:

$$t_j^{ear} + x_j - (t_i^{ear} + x_i) \leq M \cdot u_{ij} \quad i, j = 1,2 \quad (2)$$

$$T_j^{ear} + y_j - (T_i^{ear} + y_i) \leq M \cdot v_{ij} \quad i, j = 1,2 \quad (3)$$

$$u_{ij} + v_{ji} \leq 1 + z \quad i, j = 1,2 \quad (4)$$

$$u_{ij} + u_{ji} \leq 1 \quad i, j = 1,2;\ i < j \quad (5)$$

$$v_{ij} + v_{ji} \leq 1 \quad i, j = 1,2;\ i < j \quad (6)$$

$$t_i^{ear} + x_i + \bar{T}_i \leq T_i^{ear} + y_i \quad i = 1,2 \quad (7)$$

$$x_i \leq \tau_i^{arr-} + \tau_i^{arr+} \quad i = 1,2 \quad (8)$$

$$y_i \leq \tau_i^{dep-} + \tau_i^{dep+} \quad i = 1,2 \quad (9)$$

$$x_i \in R_0^+ \quad i = 1,2 \quad (10)$$

$$y_i \in R_0^+ \quad i = 1,2 \quad (11)$$

$$u_{ij} \in \{0,1\} \quad i = 1,2;\ j \neq i \quad (12)$$

$$v_{ij} \in \{0,1\} \quad i = 1,2;\ j \neq i \quad (13)$$

$$z \in \{0,1\} \quad (14)$$

Function (1) represents an optimisation criterion of the mathematical model. If the right side of function (1) is positive, then both shunting tracks of the turning loops are used for the turnarounds of the trams. If the right side of function (1) is not positive, then only single shunting track is necessary for the turnarounds of the trams.

Constraints (2) ensure that if the connection of line i arrives (after its potential time shift, counted from the earliest possible arrival time) before the connection of line j (after its potential time shift, counted from the earliest possible arrival time), then $u_{ij} = 1$. Constraints (3) assure that if the connection of line i departs (after its potential time shift, counted from the earliest possible departure time) before the connection of line j (after its potential time shift, counted from the earliest possible departure time), then $v_{ij} = 1$.

Constraints (4) ensure that if the connection of line i arrives (after its potential time shift, counted from the earliest possible arrival time) before the connection of line j (after its potential time shift, counted from the earliest possible arrival time) and concurrently the connection of line j departs (after its potential time shift, counted from the earliest possible departure time) before the connection of line i (after its potential time shift, counted from the earliest possible departure time), then it is necessary to use both tracks of the turning loop for the turnarounds of the connections of both lines. Inequality $\leq$ is used to assure the model is working in cases when the connections of both lines which are served by the same vehicle arrive and depart at the same time in the timetable.

Constraints (5) guarantee that if the connection of line 1 arrives before the arrival of the connection of line 2, then the inverse variable u_{21} representing the opposite case takes value 0 and vice versa (that means if $u_{21} = 1$, then $u_{12} = 0$). Constraints (6) ensure that if the connection of line 1 departs before the departure of the connection of line 2, then the variable v_{21} takes value 0 and vice versa (that means if $v_{21} = 1$, then $v_{12} = O$). Without constraints (5) and (6) it may happen that u_{12} = 1 and u_{21} = 1 concurrently which leads to incorrect behaviour of the mathematical model.

Constraints (7) model that any tram cannot depart from the turning loop before its arrival at the turning loop. Constraints (8) and (9) ensure that the time shifts of the arrivals and departures of the individual connections are within the given limits. Constraints (10) up to (14) define domains of definition for the variables used in the model. Symbol M used in the model represents a large enough constant.

4 CALCULATION EXPERIMENTS WITH THE MATHEMATICAL MODEL

Let us consider the turning loop equipped with two tracks for the turnarounds of the trams serving two lines labelled as 1 and 2. A summary of all necessary pieces of input data for the optimisation experiments is presented in Table 1. One can see that currently both tracks are used. Real situations may differ in the coordination intervals defining the allowable time shifts. In some cases, it is possible to reduce the number of the tracks necessary for the tram turnarounds whilst such real situations may occur for which reducing is not possible. The minimal turnaround time for both lines is $\bar{T}_i = 5$ minutes. And finally, for the large enough constant M it holds that $M = 1000$ in the presented experiments.

Table 1: Summary of input data

Situation	Line no. i	t_i^{act}	T_i^{act}	τ_i^{arr-}	τ_i^{arr+}	τ_i^{dep-}	τ_i^{dep+}	Current number of shunting tracks
1	1	6:30	6:40	0	0	0	0	2
	2	6:32	6:38	0	0	0	0	
2	1	6:32	6:38	0	0	0	0	2
	2	6:30	6:40	0	0	0	0	
3	1	6:30	6:40	0	0	0	0	2
	2	6:32	6:38	2	3	4	2	
4	1	6:30	6:40	2	3	4	2	2
	2	6:32	6:38	0	0	0	0	
5	1	6:32	6:38	0	0	0	0	2
	2	6:30	6:40	2	3	4	2	
6	1	6:32	6:38	2	3	4	2	2
	2	6:30	6:40	0	0	0	0	
7	1	6:30	6:40	0	0	6	7	2
	2	6:32	6:38	0	0	2	5	

8	1	6:30	6:40	6	7	0	0	2
	2	6:32	6:38	2	5	0	0	
9	1	6:32	6:38	0	0	6	7	2
	2	6:30	6:40	0	0	2	5	
10	1	6:32	6:38	6	7	0	0	2
	2	6:30	6:40	2	5	0	0	
11	1	6:30	6:46	1	2	2	3	2
	2	6:35	6:40	2	1	2	2	
12	1	6:35	6:40	2	1	2	2	2
	2	6:30	6:46	1	2	2	3	

Modified data – the inputs to the model (the arrival and departure times expressed in minutes, the beginning 0 corresponds to 6:00) are listed in Table 2.

Table 2: Summary of modified data

Situation	Line no. i	t_i^{ear}	T_i^{ear}	$\tau_i^{arr-} + \tau_i^{arr+}$	$\tau_i^{dep-} + \tau_i^{dep+}$
1	1	30	40	0	0
	2	32	38	0	0
2	1	32	38	0	0
	2	30	40	0	0
3	1	30	40	0	0
	2	30	34	5	6
4	1	28	36	5	6
	2	32	38	0	0
5	1	32	38	0	0
	2	28	36	5	6
6	1	30	34	5	6
	2	30	40	0	0
7	1	30	34	0	13
	2	32	36	0	7
8	1	24	40	13	0
	2	30	38	7	0
9	1	32	32	0	13
	2	30	38	0	7
10	1	26	38	13	0
	2	28	40	7	0
11	1	29	44	3	5
	2	33	39	3	4
12	1	33	39	3	4
	2	29	44	3	5

All calculation experiments were carried out using optimisation software Xpress-IVE. Results – outputs from the model (the time shifts of the individual connections, the modified arrival and departure times, the order of the arrivals and departures of the connections of the individual lines which are served by the same vehicle and the number of the tracks necessary for the tram turnarounds) are presented in Table 3. Please note that it holds in the table $i = 1$ and $j = 2$.

Table 3: Summary of optimisation results

Situation	x_i	x_j	y_i	y_j	$t_i^{ear} + x_i$	$t_j^{ear} + x_j$	$T_i^{ear} + y_i$	$T_j^{ear} + y_j$	u_{ij}	u_{ji}	v_{ij}	v_{ji}	z
1	0	0	0	0	30	32	40	38	1	0	0	1	1
2	0	0	0	0	32	30	38	40	0	1	1	0	1
3	0	0	0	6	30	30	40	40	0	0	0	0	0
4	4	0	2	0	32	32	38	38	0	0	0	0	0
5	0	4	0	2	32	32	38	38	0	0	0	0	0
6	0	0	6	0	30	30	40	40	0	0	0	0	0
7	0	0	9	7	30	32	43	43	1	0	0	0	0
8	9	3	0	0	33	33	40	38	0	0	0	1	0
9	0	0	13	7	32	30	45	45	0	1	0	0	0
10	2	0	0	0	28	28	38	40	0	0	1	0	0
11	3	0	0	4	32	33	44	43	1	0	0	1	1
12	0	3	4	0	33	32	43	44	0	1	1	0	1

If $z = 1$, then the trams need both turning loop tracks for their turnarounds, if $z = 0$, then one track is enough for the turnarounds.

5 CONCLUSIONS

In the article we presented that time coordination of the public transport connections may have an essential impact on transport system effectivity. Its impact is currently limited only on positive effects which are brought to users of the public transport systems. The article has shown that time coordination may positively affect the costs of the public tram transport system. For a higher number of the tram lines that use the same turning loop, the public transport operator can save money by reducing the extent of necessary infrastructure at the design stage, while in operation it can increase the number of tram lines that can use the turning loop at the same time.

The article presented the mathematical approach which we would like to further develop in the future. The mathematical model can now coordinate the connections of two tram lines at the turning loops equipped with two shunting tracks. The results indicated that time coordination of the arrival and departure times of the individual connections can reduce in some cases (defined by the coordination intervals which limit the possible time shifts of the connections) the number of the shunting tracks.

The second track, which is unused, may be available for the turnarounds of the trams (or their sets) of other tram lines or for necessary maintenance without the necessity of limiting tram traffic and its replacement by bus transport. In the future, we plan to propose a coordination model for higher numbers of the shunting tracks at the turning loop and the tram lines that will use them.

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INTERDEPENDENCIES ON THE CZECH CAPITAL MARKET IN THE CONTEXT OF ACCESSION TO THE STRUCTURES OF THE EUROPEAN UNION

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Abstract

The Czech Republic began official accession negotiations with the European Union in 1998. As a result of the reforms and structural changes carried out in the country, the Czech Republic joined the structures of the European Union in 2004. Undoubtedly, the development of international organizations, including the European Union and participation in these institutional structures, has contributed to the increase of links and interrelations between member states. This phenomenon is also an important determinant of the increase of interdependence between capital markets of the member countries. An important issue is the analysis of changes in the strength of links between markets over time, the results of which allow for more effective management of financial risk related to the functioning of capital markets. The main objective of the article is to provide information on the interrelations between the Czech capital market and leading capital markets in Europe, the capital market of Germany and Great Britain. The analysis was carried out in the period 1998-2019, in which the Czech economy underwent numerous institutional changes related to membership in the European Union. The DCC-GARCH model with conditional t-student distribution was used to identify changes in interrelationships over time.

Keywords: *institutional economics, capital market, DCC-GARCH model, conditional variance and correlation, Czech Republic, European Union*

JEL Classification: G15, C58
AMS Classification: 91B30, 91B84

1 INTRODUCTION

The main scientific problem of the article concerns possible changes in the interdependencies between the capital market in the Czech Republic and selected leading markets in Europe. At the beginning of the 1990s, the Czech Republic began a system transformation with the aim of switching from a centrally planned to a market oriented economy. To obtain this objective state policy has been focused on a number of institutional reforms that have allowed for the fundamental structural transformation of the whole economic system. One of the major goals of changes in the functioning of the economy was the creation of the capital market, where the Prague Stock Exchange began operating already in 1992. In the following years, the Czech Republic developed rules for the functioning of the capital market within the entire financial system.

In parallel with the reforms mentioned, the Czech Republic, as part of the Visegrad Group, implemented efforts to become a member of the European Union and was officially recognized as a candidate country in 1998. Since then, the Czech Republic has introduced a number of further reforms aimed at bringing the economic institutions to those functioning in the EU

(Balcerzak, 2020; Kozubikova et al., 2019), which translated into a significant improvement not only in the quality of institutional order in the sphere of finance, but also influenced positively the quality of the broadly understood business environment, competitiveness, innovation, international convergence (Cheba, 2015; Cheba & Szopik-Depczyńska, 2017; Kuc, 2017a, 2017b; Szopik-Depczyńska et al., 2017, 2018a, 2018b; Jankiewicz, 2018a, 2018b; Korcsmáros & Šimova, 2018; Kiseľáková et al., 2019; Dvorský et al., 2019; Kuc-Czarnecka et al., 2020; Dima et al., 2018). The risk aspects of the changes in the business environment are portrayed in the studies of Emerling and Wojcik-Jurkiewicz (2018) or Abakumova and Primierova (2018). These factors, supported with the external financing of economy form European Union funds, have influenced the phenomena of fiscal, nominal and real convergence of the Czech economy with the European Union countries (Balcerzak et al., 2016; Balcerzak & Rogalska, 2016; Rogalska et al., 2017; Kuc, 2017; Melecký, 2018). All these processes had an impact on the functioning of the capital market in the Czech Republic (Skalicka et al., 2019a, 2019b; Meluzin et al., 2018a, 2018b; Strýčková, 2017; Vychytilova, 2018; Zaremba & Nikorowski, 2019; Onofrei et al., 2019). The adaptation process of the Czech economy to this new environment lasted until 2004, when the country was admitted to become a member of the European Union. Joining the EU structures has allowed for a significant intensification of links with other member countries, including the increase of interdependence between capital markets (see Schmukler & Abraham, 2017; Wajda-Lichy & Kawa, 2018; Shkolnyk et al., 2019). It should be emphasized that progressive globalization processes has also contributed to the dynamic development of interrelations between capital markets (see Hadaś-Dyduch, 2016, 2017; Vukovic & Prosin, 2018; Skare & Porada-Rochoń, 2019a, 2019b; Gospodarchuk & Suchkova, 2019; Stavytskyy et al., 2019; Sánchez-López et al., 2019; Thalassinos and Thalassinos, 2006; 2015; Thalassinos and Stamatopoulos, 2015; Mishchuk et al., 2019). Therefore, it is important to analyze changes in the strength of the links between the Czech capital market and leading capital markets in Europe.

Therefore, the purpose of the proposed article is to analyze the interdependence between the Czech capital market and the most important capital markets in Europe. The capital markets of Germany and Great Britain were selected for this analysis. The UK capital market is considered as one of the global financial centers. The German capital market was chosen because of its geographical proximity and importance for the developing markets of Central and Eastern Europe. The study was conducted in the period 1998-2019, in which the Czech economy underwent numerous institutional changes related to membership of the European Union.

The goal will be implemented by interpreting conditional correlations between the main stock indices of selected capital markets, which will be measured based on application of the DCC-GARCH model with conditional t-student distribution. As part of the research, it was assumed that institutional reforms related to the Czech Republic's accession to the European Union have had a positive impact on the functioning and effectiveness of the capital market and it translated into growing links between this market and the European Union's financial system. From the perspective of practical value added of the current analysis, establishing a trend regarding the strength of the interdependences between the Czech market and the EU's leading capital markets is important for economic policy and microeconomic risk management (see also: Bekaert & Wu, 2000; Pericoli & Sbracia, 2003; Baur, 2003; Corsetti, et al., 2005; Prasad et al., 2007; Gaigaliene et al., 2018; Dvorský, et al., 2019).

2 RESEARCH ON THE RELATIONSHIP BETWEEN THE CZECH, UK AND GERMAN CAPITAL MARKET

In the study, the DCC-GARCH model was used, whose design allows the analysis of interrelationships between markets by estimating the conditional correlation variable for subsequent pairs of markets. The introduction of DCC-GARCH models enabled modelling of both conditional variance for individual assets and conditional correlation for selected financial asset pairs. This was the result of many empirical studies, which pointed to the need to take into account the interrelationships between many potential markets studied (see Engle, 2002, 2009).

In accordance with the adopted objective of the article, the DCC-GARCH model specification was first established for three selected capital markets. The PX index was selected for the Czech capital market, the FTSE100 index was selected for the UK capital market, and the DAX index for the German capital market. In the empirical analysis carried out, the rates of return of three selected stock indexes from the period 1998-2019 were used. The diagnostic tests carried out regarding the properties of the rates of return and the squares of the rates of return allowed the DCC-GARCH model specification to be adopted, where the equation for the conditional mean value was described with application of the AR(1) model, and single equations of the conditional variance were described using the GARCH(1 1) model.

To estimate the parameters of the DCC-GARCH model, a two-stage estimation method was used (see Engle 2002, 2009), where the maximum likelihood method with the conditional t-student distribution was used to estimate the model parameters. In the first stage, estimation of parameters of equations of conditional variances and means for individual indexes was done in accordance with the adopted specifications of the AR(1) and GARCH(1,1) models. In the second stage, however, assessments of the parameters of the equation for conditional correlation was obtained, which allowed to assess the interdependencies between selected capital markets.

Table 1 presents the results of the obtained estimation of parameters of the DCC-GARCH model. For all indexes, the parameters of the AR(1) model (conditional mean equations) turned out to be statistically significant at a 5% significance level. Also all parameters for the individual GARCH models (conditional variance equations) and parameters of the conditional correlation equation (α and β parameters) proved to be statistically significant. It should be noted that the obtained estimates of parameters of the degrees of freedom of t-student distribution indicate the occurrence of thick tails of rates of return of the studied stock indices.

Table 1. The results of estimation of the parameters of the DCC-GARCH model

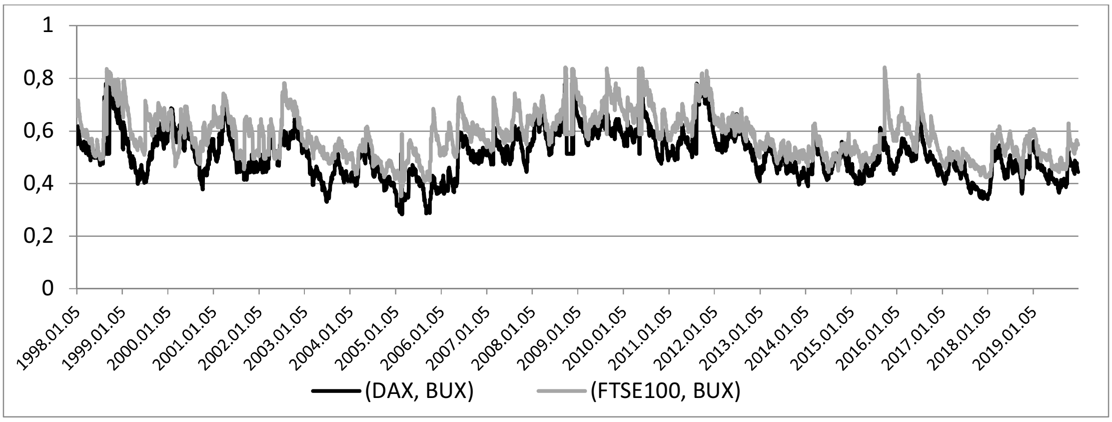
Equation of conditional mean and conditional variance for index PX				**Equation of conditional mean and conditional variance for index PX**			
Model	Parameter	Estimate	p-value	Model	Parameter	Estimate	p-value
AR(1)	α_0	0.008	~0.00	AR(1)	α_0	0.005	~0.00
	α_1	-0.06	0.034		α_1	0.043	0.012
GARCH(1,1)	ω_0	0.002	0.63	GARCH(1,1)	ω_0		~0.00
	β_1	0.087	~0.00		β_1	0.174	~0.00
	γ_1	0.907	~0.00		γ_1	0.816	~0.00
	v	8.545	~0.00		v	6.278	~0.00
Equation of conditional mean and conditional variance for index FTSE100				**Equation of conditional correlation**			
Model	Parameter	Estimate	p-value		Parameter	Estimate	p-value
AR(1)	α_0	0.006	~0.00		α	0.027	~0.00
	α_1	0.112	~0.01		β	0.968	~0.00
GARCH(1,1)	ω_0	0.001	~0.00		v	7.352	~0.00
	β_1	0.133	~0.00		Log-Likelihood		59919.238

	γ_1	0.852	~0.00		Akaike criteria		-20.037
	v	6.08	~0.00		Bayes criteria		-20.009

Source: own estimation based on the data form http://www.finance.yahoo.com.

The obtained assessments of the DCC-GARCH model parameters allowed the determination of conditional correlation values for subsequent pairs of indexes, the course of which is presented in Figure 1. The conditional correlation values for a pair of indexes (DAX, PX) and for a pair of indexes (FTSE100, PX) are presented. The evolution of correlation values for a specific pair of indices over time indicates the strength of the relationship between the two capital markets. Based on the course of conditional correlation in the adopted time-frame, it is possible to determine development trends for interrelationships between the markets. In accordance with the stated goal of the article, the analysis of conditional correlation values allowed to assess the functioning and effectiveness of the capital market in the Czech Republic. The improvement in the functioning of the capital market in the Czech Republic should be reflected in a systematic decrease in the correlation between the specific indexes, which can indicate that the avaialble information is not directly duplicated between the markets, but the given mareket is matured enough to be able to filter the value of the specific inforamtion.

Figure 1. The conditional correlation between the PX, DAX and the FTSE 100 index.



Source: own estimation based on the results from the DCC-GARCH model.

An analysis of the time series of conditional correlations between two pairs of indexes shows that in 1998 the capital market in the Czech Republic was characterized by a high level of interdependence with leading capital markets of the European Union. During the period of institutional reforms and the time of intense preparations of the Czech Republic for joining the EU (1998-2004), a systematic decrease in the strength of interdependence with both the German and Great Britain capital markets is visible. Since the dependence on external information from other capital markets in the valuation of assets on the Czech stock exchange has been systematically falling, this phenomenon indicates an improvement in the quality of financial institutions and the functioning of the capital market in the Czech Republic during this period.

Next, the largest decrease in the correlation between the studied capital markets could be noted in the first period of Czech Republic's membership in the EU in 2005-2007. Unfortunately, towards the end of 2007, the correlation between the Czech capital market and leading EU markets increased due to the global financial crisis. The financial crisis has contributed to maintaining a high level of interdependence between the capital markets over the world until 2012. In 2012, the situation calmed down in most of the financial markets in the world. This fact is also visible in the case of the capital market in the Czech Republic, where after 2012 a systematic decrease in the interdependence between the studied markets is visible. In the period 2012-2019, individual shock events occurred. However, these events, apart from a short-term increase in volatility in capital markets, did not manage to disrupt their functioning and cause the contagion effect, as was the case during the global financial crisis.

The analysis showed that in the entire 1998-2019 period examined, trends in changes in conditional correlation value are similar between the Czech capital market and selected leading markets in Europe. It should also be noted that in spite of the geographical proximity the capital market in the Czech Republic was more strongly correlated with the British market compared to the German market. Establishing a trend of a systematic decrease in the interdependence between the Czech capital market and leading capital markets in Europe allows verification of the research hypothesis about the positive impact of Czech accession to the structures of the European Union on the functioning and effectiveness of the Czech capital market.

3 CONCLUSIONS

The main subject discussed in the article concerned the analysis of interdependencies between capital markets of selected European Union member states, here Czech Republic, Germany and Great Britain. Interdependencies between markets are constantly increasing along with the progressing globalization and functioning of international communities. The aim of the article was to examine the links between the Czech capital market and mentioned leading markets in Europe. The study made it possible to assess the interrelationships between the markets, as well as to assess changes in these relationships over time.

The established interdependencies between the Czech capital market and the capital markets of Germany and Great Britain developed similarly throughout the period considered. Despite the fact that the capital markets of individual European countries operate in a specific way, they react similarly to external information from leading capital markets in the European Union. It can be stated that the shock events on the neighbouring capital market of Germany and the strongest in Europe Great Britain are shifting in a similar way to the stock exchange in the Czech Republic. Establishing correlation changes as a result of research may allow for establishing a strategy of operation during the crisis and developing system tools, which can be applied for improving the functioning of financial markets.

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DYNAMIC DEA MODELS WITH SHORT-TERM MEMORY

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Abstract
This paper deals with dynamic efficiency analyses. The efficiency analyses are based on data envelopment analysis models. The aim is to find a functional DEA model with time series, which solved the efficiency of single decision-making unit in one stage. One of the possibilities is a dynamic DEA model with short-term memory, which is originally deduced in this paper. This dynamic model computes the overall efficiency score for every decision-making unit. Results of this model are compared with results of dynamic DEA models with window analysis, which computed efficiency score for every decision-making unit in every time. Then the one efficiency for every unit is computed. Both models are illustrated on the real dataset – German NUTS (Nomenclature of Units for Territorial Statistics) 2 regions. The data set contains information about the NUTS 2 regions for nine following years starting by 2008. There are used two inputs – employment (thousand hours worked) and gross fixed capital formation (million euro) and one output – gross domestic product (million euro). All calculations are realised using original procedures written in the LINGO modelling language.

Keywords: *DEA models, time series, efficiency analyses*

JEL Classification: C44, C61
AMS Classification: 90C05,90C39, 90C90

1 INTRODUCTION

This paper deals with dynamic efficiency analyses based on Data Envelopment Analysis. The efficiency evaluation of the set of decision-making units (DMUs) is an assignment which can be solved in several ways. An application of data envelopment analysis (DEA) is one of them. DEA models have been first developed by (Charnes, Cooper and Rhodes, 1979) based on the concept introduced by (Farrell, 1957). Using DEA models, the efficiency scores of decision-making units are computed. These models classified decision-making units into two subsets – efficient and inefficient. Inefficient units can be easily ranked according to efficiency scores, but efficient ones cannot be ranked due to their identical maximum efficiency scores. These models do not cover the influence of time, that means these models are static. If we want to cover the impact of time in the counting of efficiency scores, we have to use dynamic DEA models. Firstly were dynamic data envelopment analysis showed by (Sengupta, 1995) and then (Färe and Grosskopf,1996). Also (Tone and Tsutsui, 2010) showed dynamic slack-based measure DEA models. These models but differ from the model proposed in this paper. These dynamic models can be divided into two groups – the first group, where the efficiency score for every year is computed, and the second group, where one value of overall efficiency scores is calculated. This article is focusing on the one overall efficiency score for every decision-making unit. An original dynamic data envelopment model is derived.

The paper is organised as follows: The next section presents definitions of data envelopment models generally. The following section discusses dynamic data envelopment models and the author's dynamic DEA model. The following section contains numerical illustration modelled on the real data set – dynamic analysis of German NUTS (Nomenclature of Units for Territorial Statistics) 2 regions and comparison between static analyses used in (Zýková, 2019). German NUTS 2 units have been studied from these points of view: inputs: labour - employment in thousands of hours and capital – gross fixed capital in millions of euros. And one output: GDP in millions of euros. The final section of the paper concludes the results of models and discusses future research.

2 DEA MODELS

DEA models are a general tool for efficiency and performance evaluation of the set of homogenous DMUs that spend multiple (w) inputs and transform them into multiple (t) outputs. The measure of efficiency (efficiency score) of this transformation is one of the main results of the application of DEA models. Let us denote $\mathbf{Y} = \left(y_{rj},\ r = 1,\dots,t,\ j = 1,\dots n\right)$ a non-negative matrix of outputs and $\mathbf{X} = \left(x_{kj}, k = 1,\dots,w, j = 1,\dots n\right)$ a non-negative matrix of inputs. The efficiency score of the unit under evaluation DMU_{j_0} is derived as follows:
Maximise

$$U_{j_0} = \frac{\sum_{r=1}^{t} u_r y_{rj_0}}{\sum_{k=1}^{w} v_k x_{kj_0}}$$

subject to

$$\frac{\sum_{r=1}^{t} u_r y_{rj}}{\sum_{k=1}^{w} v_k x_{kj}} \le 1, \quad j = 1,\dots,n, \tag{1}$$

$$u_r \ge \varepsilon, \quad r = 1,\dots,t,$$

$$v_k \ge \varepsilon, \quad k = 1,\dots,w,$$

where u_r is a positive weight of the r-th output, v_k is a positive weight of the k-th input, and ε is an infinitesimal constant. Model (1) is not linear in its objective function but may easily be transformed into a linear program. The linearised version of the input-oriented CCR (Charnes, Cooper, Rhodes) is as follows:
Maximise

$$U_{j_0} = \sum_{r=1}^{t} u_r y_{rj_0}$$

subject to

$$\sum_{k=1}^{w} v_k x_{kj_0} = 1,$$

$$\sum_{r=1}^{t} u_r y_{rj} - \sum_{k=1}^{w} v_k x_{kj} \le 0, \quad j = 1,\dots,n, \tag{2}$$

$$u_r \ge \varepsilon, \quad r = 1,\dots,t,$$

$$v_k \ge \varepsilon, \quad k = 1,\dots,w.$$

Above mentioned DEA models analyse efficiency score of decision-making units only in one time period.

3 DYNAMIC DEA MODELS

Dynamic data envelopment models are divided into two groups. In the first group, the efficiency score is computed for every year or every group of years (windows). Window analysis is one example of these dynamic models. Windows analysis application was studied in (Zýková, 2019). In the second group, the one overall efficiency score is computed for every decision-making unit. The overall efficiency score is one value summarising all years and the influence of all other units in all years. That means that all inputs and outputs overall years have some impact. Now, one model from the second group would be shown. It is the original model from the author of this article.

Let us denote $\mathbf{Y} = \left(y_{rj}^{\tau}, \tau = 1,\ldots,T,\ r = 1,\ldots,t, j = 1,\ldots n\right)$ a non-negative three-dimensional matrix of outputs and $\mathbf{X} = \left(x_{kj}^{\tau}, \tau = 1,\ldots,T, k = 1,\ldots,w, j = 1,\ldots n\right)$ a non-negative three-dimensional matrix of inputs. The overall efficiency score of the unit under evaluation DMU_{j_0} is derived as follows:
Maximise

$$AR_{j_0} = \sum_{\tau=1}^{T} \phi_{j_0}^{\tau} U_{j_0}^{\tau}$$

$$\text{subject to} \quad \begin{array}{llll}
\phi_j^{\tau} & \geq \delta, & \tau = 1,\ldots,T, & j = 1,\ldots,n, \\
\sum_{\tau=1}^{T} \phi_j^{\tau} U_j^{\tau} & \leq 1, & j = 1,\ldots,n, & \\
\phi_j^{\tau} - \phi_j^{\tau-1} & \geq \delta, & \tau = 1,\ldots,T-1, & j = 1,\ldots,n, \\
\sum_{r=1}^{\tau} u_r^{\tau} y_{rj_0}^{\tau} & = U_{j_0}^{\tau}, & \tau = 1,\ldots,T, & \\
\sum_{k=1}^{w} v_k^{\tau} x_{kj_0}^{\tau} & = 1, & \tau = 1,\ldots,T, & \\
\sum_{r=1}^{t} u_r^{\tau} y_{rj}^{\tau} - \sum_{k=1}^{w} v_k^{\tau} x_{kj}^{\tau} & \leq 0, & \tau = 1,\ldots,T, & j = 1,\ldots,n, \\
u_r^{\tau} & \geq \varepsilon, & \tau = 1,\ldots,T, & r = 1,\ldots,t, \\
v_k^{\tau} & \geq \varepsilon, & \tau = 1,\ldots,T, & k = 1,\ldots,w,
\end{array} \qquad (3)$$

where AR_{j_0} is the overall efficiency score for j_0-th unit under evaluation, ϕ_j^{τ} is time weight of τ-th year and j-th unit, U_j^{τ} is a year efficiency in τ-th year and for j-th unit, δ is a small number, where u_r^{τ} is a positive weight of the r-th output in a year τ, v_k^{τ} is a positive weight of the k-th input in year τ and T is number of years. The first set of constraints assures that every year has an impact on the overall efficiency score AR_{j_0}. The second set of constraints $\sum_{\tau=1}^{T} \phi_j^{\tau} U_j^{\tau} \leq 1$ guarantees that the overall efficiency score AR_j is smaller or equal to one, and also, it means that the j_0-th decision-making unit is efficient if AR_{j_0} is equal to one. The set of constraints $\phi_j^{\tau} - \phi_j^{\tau-1} \geq \delta$ says that the time weights ϕ_j^{τ} are decreasing with time. It signifies that the model has a short-term memory. The rest constraints are usual as CCR input-oriented DEA model.

The model (3) contains three indexes, one for the years (or generally index of time series), the second index is for the decision-making unit, and the third index is for input or output. It means that the model counted with a three-dimensional matrix. This occasion is problematic by solving the model (3). It is necessary to transfer data set in this case in MS Excel, which is only two-dimensional to the three-dimensional **X** and **Y** matrixes. This was procured by three-cycle in LINGO program. The code of the model (3) is not simple and the creation of the code spent quite enough time.

4 NUMERICAL ILLUSTRATION

The use of the model (3) is illustrated in Germany. Germany consists of 16 federal countries. Federal countries are NUTS 1. Further, Germany is divided into 38 regions level NUTS 2. In this analysis, we are investigating efficiency scores of these NUTS 2 regions for nine following years from the year 2008 to 2016. We have used these inputs: labour - employment in thousands of hours and capital – gross fixed capital in millions of euros. And one output: GDP in millions of euros. The data was obtained from (Eurostat, 2016).
There are results of the model (3) in the second column in Table 1 and Table 2, and the results of the model (2) are in the rest columns in Table 1 and Table 2.

Table 1: Efficiency scores of the model (3) and (2) for every German NUTS 2 region.

	model (3)	2008	2009	2010	2011	2012	2013	2014	2015	2016
Stuttgart	1.00	0.90	0.87	0.94	0.95	0.94	0.92	0.94	0.96	0.95
Oberbayern	0.94	0.93	0.92	0.95	0.99	1.00	0.98	0.99	0.98	0.99
Düsseldorf	0.92	1.00	1.00	1.00	1.00	1.00	1.00	0.98	1.00	1.00
Darmstadt	0.92	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Hamburg	0.86	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Köln	0.82	0.94	0.97	0.97	0.96	0.98	0.97	0.95	0.97	0.98
Arnsberg	0.79	0.91	0.90	0.91	0.92	1.00	0.93	0.95	0.97	0.97
Schl-Holstein	0.69	0.79	0.80	0.76	0.76	0.78	0.82	0.78	0.80	0.78
Karlsruhe	0.62	0.88	0.87	0.90	0.91	0.88	0.90	0.88	0.88	0.90
Berlin	0.62	0.86	0.86	0.84	0.88	0.83	0.83	0.81	0.82	0.82
Rheinh-Pfalz	0.61	0.84	0.83	0.85	0.87	0.88	0.87	0.85	0.86	0.87
Sach-Anhalt	0.53	0.75	0.67	0.71	0.78	0.74	0.75	0.79	0.82	0.79
Weser-Ems	0.49	0.80	0.74	0.76	0.77	0.75	0.75	0.75	0.76	0.78
Münster	0.47	0.86	0.86	0.87	0.87	0.86	0.86	0.86	0.86	0.87
Hannover	0.47	0.94	0.87	0.88	0.90	0.88	0.88	0.88	0.88	0.90
Freiburg	0.46	0.79	0.78	0.81	0.83	0.79	0.80	0.80	0.79	0.81
Dresden	0.46	0.64	0.65	0.62	0.63	0.64	0.68	0.69	0.74	0.71
Detmold	0.43	0.85	0.84	0.83	0.87	0.89	0.90	0.87	0.89	0.89
Tübingen	0.43	0.81	0.79	0.83	0.85	0.82	0.83	0.84	0.84	0.86
Thüringen	0.42	0.59	0.65	0.72	0.69	0.71	0.73	0.72	0.80	0.73
Koblenz	0.41	0.79	0.78	0.83	0.84	0.83	0.83	0.83	0.84	0.85
Mittelfranken	0.39	0.77	0.79	0.79	0.80	0.81	0.80	0.80	0.81	0.81

Source: author's calculations

Table 2: Efficiency scores of the model (3) and the model (2) for every German NUTS 2 region.

	model (3)	2008	2009	2010	2011	2012	2013	2014	2015	2016
Braunschweig	0.39	0.87	0.80	0.86	0.89	0.88	0.87	0.89	0.83	0.94
Schwaben	0.38	0.76	0.78	0.78	0.80	0.80	0.81	0.80	0.79	0.80
Saarland	0.38	0.88	0.96	0.84	0.88	0.89	0.86	0.86	0.87	0.87
Brandenburg	0.37	0.67	0.69	0.69	0.72	0.70	0.72	0.72	0.72	0.71
Chemnitz	0.35	0.66	0.74	0.71	0.69	0.73	0.75	0.69	0.81	0.75
Lüneburg	0.29	0.75	0.74	0.75	0.75	0.75	0.76	0.76	0.77	0.76
Unterfranken	0.28	0.81	0.85	0.92	0.89	0.88	0.89	0.89	0.87	0.90
Leipzig	0.27	0.75	0.74	0.71	0.72	0.71	0.70	0.73	0.83	0.75
Kassel	0.27	0.79	0.82	0.79	0.80	0.80	0.83	0.83	0.84	0.85
Mec-Vorpom	0.27	0.62	0.69	0.64	0.64	0.65	0.70	0.68	0.68	0.67
Niederbayern	0.25	0.76	0.81	0.89	0.86	0.81	0.82	0.82	0.82	0.81
Oberpfalz	0.24	0.74	0.79	0.77	0.80	0.79	0.84	0.81	0.80	0.79
Gießen	0.22	0.83	0.84	0.82	0.81	0.83	0.82	0.82	0.85	0.84
Oberfranken	0.22	0.74	0.82	0.80	0.82	0.81	0.81	0.80	0.81	0.84
Bremen	0.19	0.97	1.00	0.98	1.00	0.91	0.95	1.00	0.95	0.98
Trier	0.16	0.71	0.68	0.67	0.70	0.69	0.72	0.72	0.74	0.76

Source: author's calculations

The efficiency scores in Table 1 and Table 2 are sorted according to the overall efficiency score computed by the model (3). The efficiency scores in Table 1 and Table 2 are colour highlighted, the most efficient German NUTS 2 regions are red, the less efficient German NUTS 2 regions are blue, and regions with efficiency scores between are green. According to the model (3), the most efficient German NUTS 2 region is Stuttgart, after Stuttgart is Oberbayern, Düsseldorf, Darmstadt, Hamburg und Köln. The red highlighted cells in Table 1 and Table 2 for the model (2) correspond with the red cells for the model (3) for the most efficient regions except for region Bremen. It is not sure why region Bremen is efficient according to single years in the model (2) and is not efficient by solving model (3). It could be studied in further research.

5 CONCLUSIONS

This paper dealt with dynamic efficiency analyses based on data envelopment analysis. The efficiency evaluation of the set of decision-making units (DMUs) can be solved in several ways. An application of data envelopment analysis (DEA) is one of them. These models split decision-making units into two subsets – efficient and inefficient. Basic DEA models do not deal with time series, due to dynamic DEA models are proposed. Development of efficiency score in time is a significant factor. These dynamic models can be divided into two groups – the first group, where the efficiency score for every year is computed, and the second group, where one value of overall efficiency scores is calculated. This article was focusing on the one overall efficiency score for every decision-making unit. An original dynamic data envelopment model was derived. This dynamic DEA model was tested on a dataset of German NUTS 2 regions between years 2008 and 2016. German NUTS 2 units have been studied from these points of view: inputs: labour - employment in thousands of hours and capital – gross fixed capital in millions of euros. And one output: GDP in millions of euros. The results were compared with static results, where the efficiency score for every decision-making unit in every year was computed. The results are in Table 1 and Table 2, and they are detail described in the previous section of the article. The findings by the dynamic model are nearby the results by static CCR input-oriented model, but definitely not the same. There is obviously the covering of time impact. Concretely in this proposed model, the time impact has a short-term memory, which means that the influence of recent years has a bigger impact than the impact of earlier years.

The dynamic models contain three indexes, one for the years, one for the decision-making unit, and the last index was for input or output. The model has been counted with a three-dimensional matrix. This occasion caused problems by solving the model. It was necessary to transfer the two-dimensional data set to three-dimensional matrixes of inputs and outputs. This was procured by utilisation of three for cycles in the LINGO modelling language. The code of the model was not simple, and the creation of the code spent quite enough time.
Dynamic DEA models are an intriguing subject for further research. In our future research, we plan to extend the influence of time in DEA models to use long-term memory model.

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