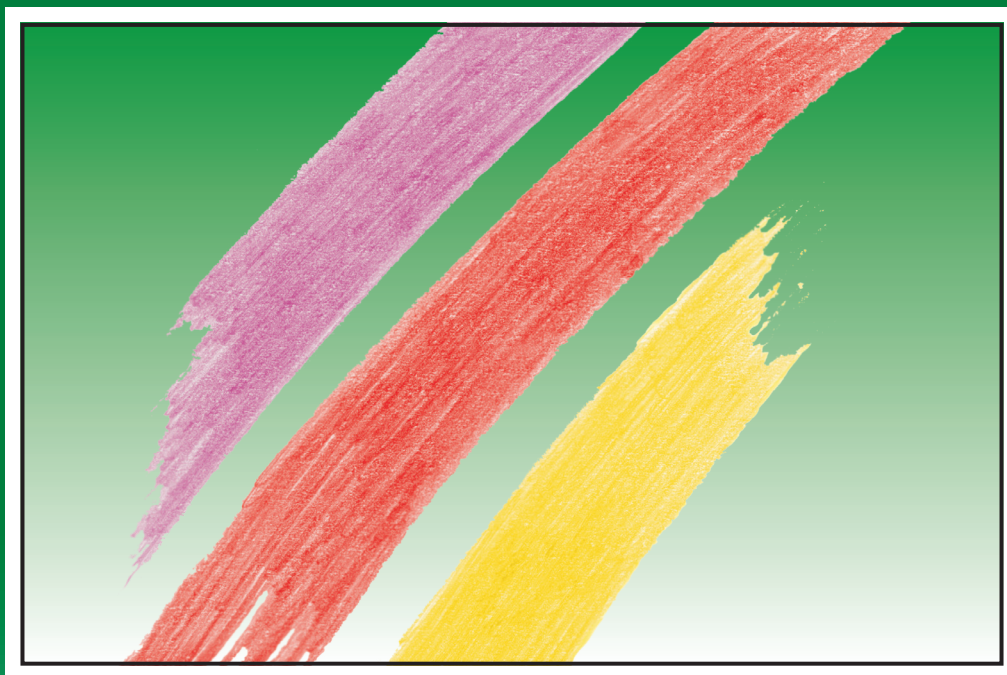


QUANTITATIVE METHODS IN ECONOMICS

Multiple Criteria Decision Making XXIII



Proceedings of the International Scientific Conference
27th May - 29th May 2026
Bratislava, Slovakia

The Slovak Society for Operations Research
Department of Operations Research and Econometrics
Faculty of Economic Informatics, Bratislava University of Economics and
Business



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23rd International Scientific Conference QUANTITATIVE METHODS IN ECONOMICS
Hosted by Bratislava University of Economics and Business,
Dolnozemska cesta 1, 852 35 Bratislava, Slovak Republic

**27th May - 29th May 2026
Bratislava, Slovakia**

Technical Editor: Pavel Gežík, Bratislava University of Economics and Business
Web Editor: Pavel Gežík, Bratislava University of Economics and Business
Web: <https://fhi.euba.sk/katedry/katedra-operacneho-vyskumu-a-ekonometrie/konferencia-qme#proceedings>

The proceedings were not subject to language editing.

Publisher: Vydavateľstvo EKONÓM,
Dolnozemska cesta 1, 852 35 Bratislava, Slovak Republic
Publication year: 2026

ISBN: 978-80-225-5329-2

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COMPLEX SURVEY DESIGN IN ORDERED RESPONSE MODELS: EVIDENCE FROM EU SURVEY DATA

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Abstract

This paper examines the econometric implications of complex survey design on estimation and inference in nonlinear ordered response models. Using pooled microdata from five waves over the period 2002–2018 of the Health Behaviour in School-aged Children (HBSC) study, we analyse a sample of approximately 400,000 observations from selected European Union member states. Adolescent body weight status is modelled as an ordered outcome driven by socioeconomic and behavioural determinants. Three estimation strategies are compared: (i) naïve maximum likelihood, (ii) probability-weighted estimation, and (iii) fully design-consistent inference accounting for clustering, stratification, and sampling weights. While coefficient estimates remain stable, survey-adjusted standard errors are systematically larger, with implied design effects ranging from 1.17 to 1.72. The results demonstrate that ignoring complex sampling structures leads to spurious precision and underestimated variance in nonlinear econometric models. These findings highlight the critical importance of design-consistent inference for robust parameter identification in large-scale survey data and provide a methodological framework for policy-oriented empirical research.

Keywords: *Complex survey design, Design-based inference, Ordered logit model, Parameter identification, Socioeconomic determinants*

JEL Classification: C25, C83, I14

AMS Classification: 62D05, 62J12, 62P20

1 INTRODUCTION

Large-scale survey datasets have become an important empirical foundation for research in applied economics and the social sciences. Many internationally comparable microdata sources are collected using complex multistage sampling designs that combine stratification, clustering and unequal selection probabilities. While such designs improve the representativeness of survey data and reduce collection costs, they also complicate econometric inference because standard regression models typically assume independently and identically distributed observations. When this assumption is violated, conventional variance estimators may underestimate statistical uncertainty and lead to misleading inference.

The implications of complex survey design for statistical estimation have been extensively discussed in the methodological literature. Early contributions demonstrated that estimators derived under standard likelihood assumptions remain consistent under complex sampling but require design-based variance estimation to produce valid inference (Binder, 1983; Pfeffermann, 1993). When observations are clustered within primary sampling units, intra-cluster correlation reduces the effective sample size relative to simple random sampling and inflates parameter variance. Ignoring such clustering therefore leads to underestimated standard errors and spurious precision (Cameron and Miller, 2015; Heeringa, West and Berglund, 2017). Design-consistent inference consequently requires explicit incorporation of sampling weights, clustering and stratification in regression models estimated from survey data. These issues are particularly relevant in nonlinear econometric models widely used in empirical

economics. Ordered response models, such as the ordered logit specification, are frequently applied when economic and social outcomes are measured on an ordinal scale. Despite their widespread use, empirical applications often rely either on naïve estimation strategies that ignore the survey design or on probability-weighted estimators that incorporate sampling weights but neglect clustering and stratification. However, weighting alone does not fully correct the inferential distortions created by multistage sampling, because correlated observations within clusters affect the variance of parameter estimates. Correct inference therefore requires estimation procedures that account for the full sampling structure underlying the data (Heeringa, West and Berglund, 2017; Lumley and Scott, 2017).

This paper examines the implications of alternative estimation strategies for ordered response models applied to large multistage survey datasets. The central methodological and econometric question concerns whether accounting for the complex sampling design affects only the estimated precision of coefficients or also alters substantive conclusions. The analysis compares three ordered logit specifications: a naïve model assuming independent observations, a probability-weighted model incorporating sampling weights, and a fully survey-adjusted model that accounts simultaneously for weights, clustering and stratification.

The empirical application uses a subset of pooled data from five waves of the Health Behaviour in School-aged Children (HBSC) survey collected over the period 2002–2018 (Inchley et al, 2020), restricted to selected European Union member states. Adolescent body weight status, measured using BMI-based categories, serves as an ordered outcome variable driven by socioeconomic and behavioural determinants. While the substantive context relates to adolescent obesity, the primary contribution of the paper is econometric, focusing on the consistent identification and inference of parameters in nonlinear models. By comparing alternative estimation strategies within a large international survey dataset, the analysis illustrates how complex survey design affects econometric inference in nonlinear econometric models.

The remainder of the paper is organised as follows. Section 2 describes the data and the survey design structure. Section 3 outlines the econometric specification and estimation strategies. Section 4 presents the empirical results and compares the inferential implications of alternative estimators. Section 5 concludes with methodological implications for applied econometric research using large-scale survey data.

2 DATA AND METHODS

2.1 Data and Sample Characteristics

The empirical analysis utilizes pooled microdata from five waves (2002, 2006, 2010, 2014, and 2018) of the Health Behaviour in School-aged Children (HBSC) study. The HBSC is an internationally coordinated cross-national survey conducted in collaboration with the World Health Organization, targeting adolescents aged 11, 13, and 15 years. This study restricts the sample to selected European Union (EU) member states participating in the relevant waves to ensure a comparable socio-political context. The original pooled file contained approximately 411,000 observations. A complete-case analytical sample was created for comparability across all model specifications, excluding observations with missing or non-positive sampling weights, missing school identifiers, or missing values in the regression covariates. The final analytical sample contains 404,871 observations, representing approximately 98.5% of the original EU-restricted dataset.

The dependent variable is adolescent weight status derived from body mass index (BMI), calculated from self-reported height and weight. BMI values were standardized using the WHO

BMI-for-age growth reference for children and adolescents (de Onis et al, 2007). To ensure model stability, the five original WHO categories were collapsed into three ordered outcomes: (1) normal weight or underweight, ($z \leq 1$), (2) overweight ($1 < z \leq 2$), and (3) obesity, including severe obesity ($z > 2$).

The primary socioeconomic predictor is a continuous scaled Socioeconomic Status (SES) index. This was derived from the Family Affluence Scale (FAS), a validated proxy for household material wealth. The index is based on a principal component analysis of four items: family car ownership, individual bedroom availability, the number of household computers, and the frequency of family holidays. The resulting factor scores were normalized to a 0–1 scale, where 1 represents the highest level of affluence. Health behaviour variables include physical activity (days per week with ≥ 60 minutes) and breakfast frequency (every school day versus otherwise). Demographic controls include sex, age group, and EU membership group (distinguishing between long-term members and 2004/2007 accession states).

HBSC employs a stratified multistage cluster sampling design where schools serve as primary sampling units (PSUs) and students are sampled within schools. To account for this structure in the pooled file, unique PSU identifiers were constructed by combining country, wave, and school codes. Regional strata were incorporated to reflect the geographic stratification of the survey. The dataset contains 19,445 PSUs across 142 strata, providing 19,303 design degrees of freedom for variance estimation (Table 1).

Table 1 Descriptive and survey-design characteristics of the analytical sample

Indicator	Value	Indicator	Value
Analytical sample size	404,871	Number of PSUs	19,445
Normal weight or lower	81.53%	Design degrees of freedom	19,303
Overweight	14.42%	Singleton-strata observations	58
Obesity	4.05%	Mean SES index	0.636
Number of strata	142	Mean physical activity (days)	4.05

Note: Percentages are weighted. PSU denotes primary sampling unit.

2.2 Methods

Adolescent weight status is modelled using an ordered logistic regression model, which is appropriate for ordinal outcomes (McCullagh, 1980; Long and Freese, 2014). Let y_i^* denote the latent propensity of individual i to belong to a higher BMI category. The latent variable depends on a vector of explanatory variables $\mathbf{X}_i$ and a logistic error term ε_i defined as (1).

$$y_i^* = \mathbf{X}_i\boldsymbol{\beta} + \varepsilon_i \quad (1)$$

The observed outcome y_i is determined by threshold parameters (cut-points) κ_j that partition the latent variable into ordered categories. The cumulative probability that the observed outcome falls in category j or below is defined as (2).

$$P(y_i \leq j) = \frac{1}{1 + \exp [-(\kappa_j - \mathbf{X}_i\boldsymbol{\beta})]} \quad (2)$$

To evaluate the implications of complex survey design for econometric inference, three estimation strategies are compared. The first specification estimates a naïve ordered logit model under the assumption that observations are independently and identically distributed. The second specification estimates a probability-weighted ordered logit model that incorporates sampling weights to adjust for unequal selection probabilities (Pfeffermann, 1993; Solon, Haider and Wooldridge, 2015). The third specification estimates a design-consistent ordered logit model that explicitly incorporates the complex sampling structure by accounting for clustering, stratification, and sampling weights.

In this case, the variance estimator reflects the multistage sampling design using a sandwich estimator (Binder, 1983; Heeringa, West and Berglund, 2017) (3).

$$\text{Var}(\boldsymbol{\beta}) = \mathbf{H}^{-1}\mathbf{V}\mathbf{H}^{-1} \quad (3)$$

In this estimator, $\mathbf{H}$ represents the Hessian matrix of the log-likelihood function and $\mathbf{V}$ captures the covariance structure induced by clustering at the primary sampling unit (PSU) level and regional stratification. Variance estimation is based on Taylor linearization, which produces design-consistent standard errors for nonlinear estimators in complex samples (Skinner, Holt and Smith, 1989; Skinner and Wakefield, 2017).

In the pooled dataset, unique PSU identifiers were constructed by combining country, survey wave, and school identifiers to avoid cluster overlap. Strata containing a single primary sampling unit were handled using the centered adjustment for singleton units. The empirical comparison focuses on the precision of parameter estimates across these strategies using the Design Effect metric (Kish, 1965) (4).

$$\text{DesignEffect} = \left(\frac{\text{SE}_{\text{survey}}}{\text{SE}_{\text{naive}}} \right)^2 \quad (4)$$

As a robustness check, the proportional-odds assumption was evaluated using a design-consistent generalized ordered logit model via the `gologit2` procedure (Williams, 2016). This specification relaxes the parallel-lines restriction and allows coefficients to vary across thresholds when the assumption is statistically rejected, ensuring that findings are not biased by structural misspecification common in large-scale survey data (Fuller, 2009; Lumley, 2010). All econometric estimations and survey-design adjustments were performed using Stata/MP 19 (StataCorp, 2025).

3 RESULTS AND DISCUSSION

The pooled complete-case sample contains 404,871 observations, corresponding to 79.4% of the EU-restricted HBS dataset. Retention was relatively stable across survey waves, ranging from 75.7% to 82.1%, which suggests no pronounced wave-specific selection. Survey design diagnostics indicate a well-defined multistage structure, with 142 strata, 19,445 primary sampling units and 19,303 design degrees of freedom. Only 58 observations belonged to singleton strata, and their treatment through the centred adjustment is unlikely to have any material effect on the results.

The weighted distribution of the dependent variable indicates that 81.53% of adolescents are classified as normal weight or lower, 14.42% as overweight, and 4.05% as obese. The analytical

sample is balanced across survey waves and demographic groups, with a mean SES index of 0.636 (SD = 0.238) and average physical activity of 4.05 days per week.

The core econometric comparison, summarized in Table 2, reveals that across all predictors, point estimates are almost identical in the naive, probability-weighted, and survey-adjusted models. By contrast, standard errors (SE) increase systematically once the full survey design is incorporated. For example, the standard error for the 15-years category rises from 0.010 in the naive model to 0.012 in the survey-adjusted model, while the 2018 survey-year indicator exhibits an increase from 0.015 to 0.019. This pattern confirms that the sampling design primarily affects the precision of econometric inference rather than the magnitude of the coefficients.

The observed increase in standard errors is consistent with established survey econometrics literature. When observations are clustered within primary sampling units (PSUs), intra-cluster correlation reduces the effective sample size relative to simple random sampling, thereby inflating parameter variance (Heeringa, West and Berglund, 2017). The implied design effects (DEFF) range from 1.17 to 1.72, indicating that naive models underestimate sampling variance by up to 72%. These findings illustrate that ignoring the sampling structure does not necessarily bias point estimates but does severely distort the precision of inference (Pfeffermann, 1993).

An additional insight concerns the limited corrective role of probability weights when used without cluster adjustment. Although weights correct for unequal selection probabilities, they do not address the correlation structure generated by clustered sampling. The empirical results confirm this: the probability-weighted model produces results nearly indistinguishable from the naive specification in terms of standard errors. Only the design-adjusted estimator, which simultaneously incorporates weights, clustering, and stratification, yields design-consistent variance estimates.

Table 2 Estimated coefficients, standard errors, and design effects

Variable	Naive (SE)	Survey-adjusted (SE)	Survey-adjusted OR	Design Effect
Age 13 years	-0.251*** (0.010)	-0.246*** (0.012)	0.782***	1.44
Age 15 years	-0.496*** (0.010)	-0.494*** (0.012)	0.610***	1.45
Male	0.719*** (0.008)	0.716*** (0.009)	2.047***	1.27
SES index	-0.361*** (0.019)	-0.361*** (0.020)	0.697***	1.17
EU new	-0.084*** (0.009)	-0.084*** (0.011)	0.920***	1.72
2006	0.171*** (0.014)	0.168*** (0.019)	1.182***	1.72
2018	0.451*** (0.015)	0.446*** (0.019)	1.562***	1.60

Note: Standard errors in parentheses. Design effect = $(SE_{\text{survey}}/SE_{\text{naive}})^2$. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Substantively, the survey-adjusted model identifies a significant long-term upward trend in obesity risk. Relative to 2002, the odds of being in a higher weight category increased steadily over time, reaching a 56.2% increase by 2018 (OR = 1.562). This growing temporal risk corresponds to widely observed epidemiological patterns in high-income countries (World Health Organization, 2022). Furthermore, the strong sex differential (OR = 2.047 for males) and the protective effect of higher socioeconomic status (OR = 0.697) align with cross-national evidence documenting a persistent socioeconomic gradient in adolescent health in Europe (Elgar et al, 2015). Behavioural factors also exhibit expected relationships: regular breakfast consumption and higher levels of physical activity are associated with lower obesity risk

A survey-weighted generalized ordered logit model was used to evaluate the proportional-odds assumption. An adjusted Wald test ($p = 0.077$) failed to reject the restriction at the 5% level, confirming that the standard ordered logit remains the most parsimonious and appropriate specification. These results illustrate that while substantive patterns are robust, design-consistent variance estimation is essential for credible parameter identification and uncertainty quantification when analysing pooled international survey data.

4 CONCLUSIONS

This paper examined the implications of complex survey design for inference in ordered response models using pooled microdata from selected EU member states in the HBSC study over the period 2002–2018. Three estimation strategies were compared: a naïve model, a probability-weighted model, and a design-consistent specification accounting for weights, school-level clustering, and stratification.

The results show that while coefficient estimates remained virtually identical across all specifications, standard errors increased systematically under full survey adjustment. This reflects the intra-cluster correlation inherent in the multistage sampling design. With design effects ranging from 1.17 to 1.72, the naïve model underestimated sampling variance by as much as 72%. Crucially, the probability-weighted model alone did not correct this bias, confirming that econometric inference requires the simultaneous treatment of clustering and stratification. Furthermore, a generalized ordered logit model confirmed the validity of the proportional odds assumption, supporting the choice of the standard ordered logit specification.

Substantively, the design-consistent model identifies a persistent and increasing trend in adolescent obesity risk. After controlling for socioeconomic and behavioural determinants, the odds of an adolescent belonging to a higher weight category rose by 56% ($OR = 1.562$) between 2002 and 2018. This trend, alongside the significant risks found among boys and lower-affluence households, highlights the growing challenge of the obesogenic environment in Europe.

In conclusion, although the large scale of the pooled dataset provides immense statistical power, it also risks generating spurious precision if the survey design is ignored. The widening of confidence intervals under the design-consistent model provides a more rigorous and realistic quantification of uncertainty. For nonlinear models applied to multistage survey data, design-based variance estimation is an essential requirement for ensuring that observed trends are statistically robust and serve as a reliable foundation for parameter identification and evidence-based policy.

Acknowledgements: This work was supported by the Slovak Research and Development Agency under Contract No. APVV-24-0412 Sustainable and Healthy Food Consumption: Effects of Socio-economic Factors and Policies on Eating Behaviour, Obesity, and Food System.

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SERVICE ORDER CONSERVATION EFFECT ON THE MAKESPAN OF THE AVS/RS WITH SIMULTANEOUS EXECUTION OF STORAGE AND RETRIEVAL TASKS

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Abstract

Application of automated vehicle storage and retrieval systems (AVS/RS), particularly those based on shuttle technologies, has in recent decades become one of the key approaches used by logistics and manufacturing companies to cope with increasing pressure to deliver small quantities of goods within ever shorter timeframes while simultaneously managing a growing product assortment. The operation and control of such systems represent a highly challenging task. One important aspect of this control is the determination of the service sequence for a known set of tasks within a given planning period, usually within the shortest possible time, i.e. the makespan. The objective of this paper is to investigate the effects of omitting service order conservation constraint, which implies that the order of task service on shuttle service must be in accordance with the lift service order, from the mathematical formulation of the problem. The results of the study indicate that by removing the aforementioned constraint, the makespan time could be reduced by a few percent on average without violating the assumptions regarding the operation of the system.

Keywords: Warehousing, AVS/RS, Shuttles, MILP, Scheduling.

JEL Classification: C610, L870, L910.

AMS Classification: 90B06, 90C11.

1 INTRODUCTION AND LITERATURE REVIEW

The rapid expansion of e-commerce, coupled with increasing expectations for fast and reliable order fulfillment, has significantly amplified the importance of efficient warehouse operations. To address high order volumes and a growing variety of products, many companies have turned to automated warehousing solutions. In this context, automated storage and retrieval systems (AS/RS), along with their more flexible counterparts, such as autonomous vehicle storage and retrieval systems (AVS/RS) and shuttle-based storage and retrieval systems (SBS/RS), have attracted considerable research and practical interest. Compared to conventional storage systems, these technologies offer higher storage density, reduced response times, and enhanced operational flexibility.

In contrast to traditional AS/RS, where horizontal and vertical movements are typically executed simultaneously by a single device, AVS/RS systems decouple these movements across different resources. Horizontal transport within individual tiers is carried out by shuttle vehicles, while vertical movement between tiers and the input/output (I/O) station is handled by lifts, as illustrated in Figure 1. Such a modular architecture enables parallel operation of multiple shuttles, thereby increasing system throughput relative to crane-based solutions. Nevertheless, the overall system performance critically depends on effective coordination and synchronization between lifts and shuttle vehicles.

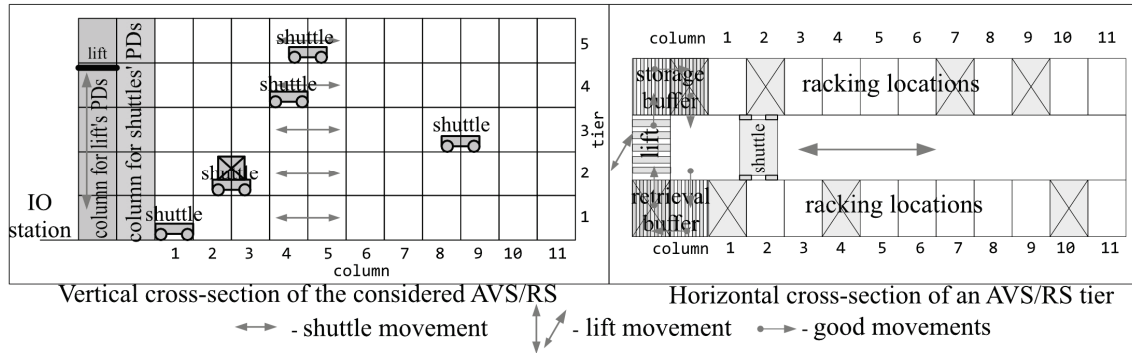


Figure 1. Vertical and horizontal cross-section of AVS/RS's operations

Due to the inherent complexity of AVS/RS, existing literature addresses decision making problems across multiple planning levels, ranging from system design to real-time operational control. At the strategic and tactical levels, a significant body of research focuses on performance evaluation and configuration decisions, including the determination of the number of vehicles and lifts, rack design parameters, and buffer capacities. In this context, analytical performance modeling based on queueing networks has emerged as a widely adopted approach. For instance, (Heragu *et al.*, 2011) and (Ekren *et al.*, 2013) developed analytical and stochastic models that represent AVS/RS as (semi)open queueing networks to estimate key performance indicators such as system throughput, resource utilization, and expected waiting times. These types of models enable rapid evaluation of alternative system configurations and facilitate the analysis of design tradeoffs prior to implementation. Similarly, (Zhao *et al.*, 2020) proposed a semi-open queueing network model for shuttle-based storage systems, aimed at assessing lift and shuttle utilization. On the other side, (Manzini *et al.*, 2016) and (Kosanić *et al.*, 2024) developed methods for estimating shuttle task cycle times under varying operational and structural conditions.

Another important research stream focuses on operational decision-making problems, where the system configuration is assumed to be fixed and the objective is to optimize the execution of storage and retrieval tasks. In real world warehouse environments, such requests arise dynamically from inbound processes, replenishment activities, and order picking operations. Efficient scheduling of these tasks is therefore critical to maintaining high throughput and preventing congestion within material handling subsystems. This challenge is particularly pronounced in goods-to-person order picking systems, where AVS/RS technologies are commonly used to supply picking stations with items stored in totes, bins, or pallets. In such settings, retrieval tasks are driven by picking demands, while storage tasks originate from replenishment or the return of items after order fulfillment. Consequently, both types of tasks occur simultaneously and must be carefully coordinated to ensure smooth system operation. Delays in executing storage tasks may lead to congestion in conveyor systems or inefficient utilization of transport resources, further highlighting the importance of effective coordination between storage and retrieval processes.

One promising approach to improving operational efficiency involves the use of dual command cycles without pairing constraints, where storage and retrieval tasks are executed within the same operational cycle without a predefined alternating cyclic structure. In such scenarios, transport resources, either lifts or shuttles, can process tasks in a sequence that best satisfies the chosen objective function. However, the effectiveness of this approach strongly depends on the sequencing of tasks. Despite its practical relevance, this problem has received limited attention

in the literature. To the best of our knowledge, it has been explicitly addressed only by (Yu *et al.*, 2025), who formulated the storage and retrieval sequencing problem as a mixed-integer linear programming model aimed at minimizing the makespan for a given set of tasks. In addition, the authors proposed both an exact branch-and-bound method and a hybrid heuristic combining beam search with adaptive large neighborhood search to efficiently solve larger instances.

The remainder of the paper is structured as follows. Section 2 provides a detailed description of the considered problem. Section 3 presents the numerical experiments and provides the obtained results. Finally, concluding remarks and brief discussion of results are given in Section 4.

2 PROBLEM DESCRIPTION

The problem considered in this study involves the execution of a predefined set of storage (S) and retrieval (R) tasks that are available at the beginning of the planning horizon and assigned to one of several lifts operating within the system. It is assumed that any required sequence of storage tasks is possible and available at the input/output (I/O) station. A storage task is performed such that the lift picks up the load unit, transports it to the designated tier, and unloads it into the corresponding tier buffer. After completing the unloading operation, the lift proceeds to the next task in its service sequence. The load unit placed in the buffer remains there until it is picked up by the shuttle permanently assigned to that tier (tier captive AVS/RS), which then transports it to the designated storage location's column and completes the task. In alignment with the lift's behavior, the shuttle continues executing its assigned tasks from the task sequence. Retrieval tasks are carried out in the reverse manner: the shuttle first collects the load unit from its storage location, delivers it to the tier buffer, and the lift subsequently transports it to the I/O station, where the task is completed.

The efficiency of such a system is highly dependent on the synchronization between lift and shuttle operations. In particular, if one resource arrives at the tier buffer before the other has completed its preceding operation, unnecessary waiting times occur. This issue is especially critical for the lift, as it serves all tiers and thus represents a key resource whose idle time significantly affects overall system performance. Consequently, determining an optimal lift service sequence that minimizes a chosen objective function is a central focus of the mathematical modeling of these systems. To address this problem, (Yu *et al.*, 2025) proposed a mixed-integer linear programming (MILP) model. Due to space limitations, the full formulation is not reproduced here, and the reader is referred to Appendix B of (Yu *et al.*, 2025) for detailed model specifications.

Within the proposed MILP framework, shuttle service sequences can be directly derived from the lift service sequence through a specific set of constraints (B.6), which enforce the conservation of task service order at each tier. This implies that the sequence in which tasks are processed by the lift must be preserved by the corresponding shuttle serving that tier. In other words, if tasks i and j are executed in positions u and v at the shuttle's task service sequence, and at positions k and h in the lift's sequence, then their relative order must remain unchanged. Mathematically, this condition is expressed as follows:

$$(k - h) - (v - u) \geq 1 \quad (1)$$

However, bearing in mind the sensitivity of the systems efficiency on the synchronization of the lift and shuttle devices we noticed the possibility for the improvement in system's efficiency

if the order conservation constraint is not obeyed. The main idea is to allow the shuttle vehicle to prepare R tasks for the lift prior or while it is working on S task of the same tier. In order to illustrate the constrained possibility let's consider the following example. In the AVS/RS whose loading and unloading times are one time unit, as well as time a transportation devices requires to transfer load for one tier or one column, there is two tasks that have to be served. 1st requires storage at the 3rd tier's 2nd column, while 2nd task requires to be retrieved from the 3rd column of the same 3rd tier. At the beginning of the planning period all shuttles are at the tiers buffer, and the lift is at the IO station. As it is illustrated at Figure 2a, if lift's task order sequence is 1→2, and in the case when the order conservation constraint exists, the process is executed in a way that load is picked up by the lift at $t=1$. At $t=3$ lift arrives at the 3rd tier and unloads the load at $t=4$. At time $t=5$ load is at the shuttle device and starts its transportation towards the destination column, where it arrives at $t=7$. Task 1 is unloaded to its final destination at $t=8$. In that moment shuttle starts execution of the next tasks from its task service sequence, meaning that it comes to the column 3 at $t=9$ and loads the load at $t=10$. It comes to the tier buffer at $t=13$, where it unloads load at $t=14$. Lift loads the 2nd task at $t=15$, comes to the IO station at $t=17$ and unloads task 2 at $t=18$.

In the case when order conservation constraint is not obeyed shuttle can start by firstly serving task 2 and avoid unnecessary waiting of the lift for the task 2. Precisely, as it is illustrated at Figure 2b, task 1 is unloaded to the tier buffer at $t=4$, while task 2's load is picked up at the exact moment to the shuttle device. Shuttle comes to the buffer at $t=7$, and unloads load at $t=8$. At $t=9$, lift loads task 2 and shuttle loads task 1. At $t=11$ both, shuttle and lift reach their final destinations, and at $t=12$ both tasks are finished.

Although various objective functions may be considered, makespan minimization is the most commonly adopted criterion in the literature. In the given example, relaxing the order conservation constraint leads to a reduction in makespan of 33% (from 18 to 12 time units). This observation motivates a more comprehensive investigation of the impact of removing such constraints within the MILP framework. To this end, numerical experiments were conducted, as described in the following section.

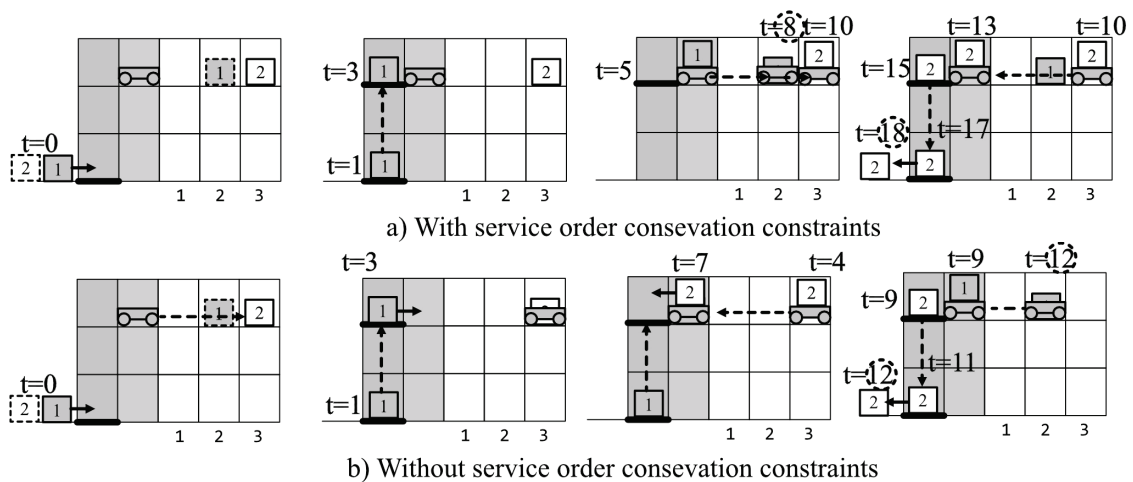


Figure 2. Time-frame realization of one storage and one retrieval tasks

3 NUMERICAL EXPERIMENTS

To evaluate the impact of omitting the order conservation constraint from the MILP model, the same set of problem instances as in (Yu *et al.*, 2025) was employed. While the authors in (Yu *et al.*, 2025) report results for instances with up to 10 tasks, this study extends the analysis to all instance sizes given from the authors of mentioned paper, including those with up to 16 tasks, in order to provide a more comprehensive assessment.

The numerical experiments were conducted using GUROBI 13 with default parameter settings. All models were implemented and solved on a system equipped with a four-thread Intel Core i5-3470 CPU at 3.2 GHz, 8 GB of RAM, and a 64-bit Windows 11 operating system. The results of the experiments are summarized in Table 1. Each instance is identified by a code consisting of the letter “T” followed by a two-digit number indicating the number of tasks and a two-digit instance index. The column “obj_w” reports the objective values obtained with the MILP model with the order conservation constraint, while “obj_wo” presents the corresponding values without this constraint. The column “Δobj” shows the relative difference between these two values, calculated as $\Delta\text{obj} = 100(\text{obj}_w - \text{obj}_{wo})/\text{obj}_w$. Finally, the last column reports the difference in computational time between the two model variants.

It should be noted that slight discrepancies exist between the objective values reported in the “obj_w” column and those presented for the same instances in (Yu *et al.*, 2025). These differences are likely attributable to minor corrections made to the original MILP formulation.

Table 1. Results of numerical experiments

inst	obj_w	obj_wo	Δobj [%]	Δt [s]	inst	obj_w	obj_wo	Δobj [%]	Δt [s]
T0401	24	24	0.00	-0.06	T1019	58	56	-3.45	4055.17
T0402	32	31	-3.13	-0.17	T1020	48	48	0.00	2428.20
T0403	28	28	0.00	-0.16	T1221	62	60	-3.23	-75.89
T0404	32	32	0.00	-0.08	T1222	68	64	-5.88	-972.38
T0405	30	30	0.00	-0.08	T1223	64	62	-3.12	420.34
T0606	30	28	-6.67	-1.31	T1224	70	64	-8.57	1184.12
T0607	38	36	-5.26	-1.53	T1225	62	60	-3.23	238.66
T0608	32	32	0.00	-2.41	T1426	74	72	-2.70	-1034
T0609	41	37	-9.76	-2.03	T1427	74	70	-5.41	1449.55
T0610	36	36	0.00	-1.05	T1428	70	66	-5.71	761.34
T0811	42	40	-4.76	43.88	T1439	76	74	-2.63	-10618
T0812	50	46	-8.00	52.22	T1430	68	66	-2.94	722.89
T0813	38	36	-5.26	71.33	T1631	88	86	-2.27	126.64
T0814	52	50	-3.85	26.81	T1632	80	78	-2.50	-951.53
T0815	42	42	0.00	-9.44	T1633	80	78	-2.50	-10837
T1016	50	46	-8.00	2338.25	T1634	82	78	-4.88	-605.84
T1017	56	52	-7.14	1561.66	T1635	78	76	-2.56	940.59
T1018	50	48	-4.00	4787.17					

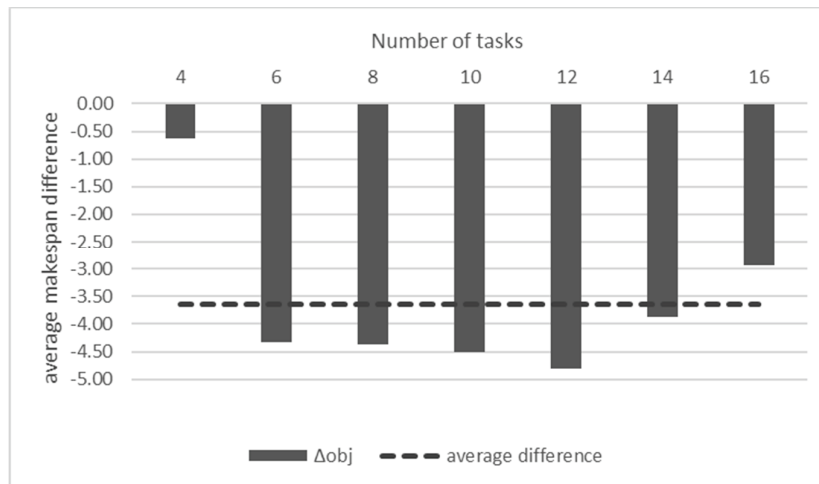


Figure 3. Overview of eliminating precedence constraints on test instances regarding the number of tasks

4 CONCLUSIONS

Based on the conducted analysis and numerical experiments, it can be concluded that the service order conservation constraint between lifts and shuttle vehicles, although intuitively contributing to simpler coordination, may unnecessarily limit the performance of AVS/RS systems. The results obtained using the MILP model on a set of test instances show that removing this constraint leads to a reduction in makespan in a significant number of cases, with improvements typically amounting to a few percent, i.e. 3.6% on average (Figure 3), and in some instances even more pronounced. These improvements stem from increased flexibility in task sequencing, which enables better synchronization between horizontal and vertical transport resources and reduces idle times, particularly for the lift as a critical system component.

Furthermore, the results indicate that omitting this constraint does not violate the fundamental operational assumptions of the system. On the contrary, it allows more efficient resource utilization by enabling anticipatory execution of tasks, such as preparing retrieval requests while the lift is performing storage operations on the same tier. Although the impact on computational time is not uniform and varies across instances, i.e. it may be considered as a matter of chance, the overall findings clearly support the consideration of relaxing this constraint in operational planning models.

In conclusion, the study demonstrates that strict service order conservation is not a necessary condition for efficient AVS/RS operation. Its removal represents a simple yet effective way to improve system performance, opening additional possibilities for future research focused on developing advanced models and heuristics that further exploit sequencing flexibility to enhance the throughput of considered systems.

Acknowledgements: The authors gratefully acknowledge Professor Yunfeng Ma for his exceptionally prompt and kind provision of the test instances upon our request.

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EVALUATING THE FINANCIAL HEALTH OF COMPANIES USING A COMPOSITE INDICATOR WITH WEIGHTS DERIVED FROM PRINCIPAL COMPONENTS

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Abstract

Because firm financial health is multidimensional and relevant to many stakeholders, relying on a single ratio can misrepresent overall performance. This paper constructs a composite indicator to compare a data-driven statistical weighting derived from principal component analysis (PCA) with a transparent equal-weight benchmark. The analysis uses 2022 data for 12,452 Slovak hospitality firms and combines nine financial ratios covering profitability, liquidity, indebtedness, and efficiency. The ratios were transformed by desirability normalization with expert-defined lower and upper thresholds and then aggregated into two bounded financial health scores: a PCA weighted score and an equal-weight score. Under the Kaiser criterion, PCA retained three components, which together explained 67.46% of total variance in the desirability-normalized indicators. Relative to the equal-weight score, the PCA weighted score had a slightly lower mean and median, but a more dispersed distribution, with variance higher by 8.71%. At the same time, the two scores remained very strongly associated ($r = 0.984819$), indicating that the weighting rule did not fundamentally change the overall pattern of firm evaluation, but it did change the distribution of final scores.

Keywords: *Financial health, Composite indicators, PCA, Weighting*

JEL Classification: C38, C43, L25

AMS Classification: 62H25, 62P20

1 INTRODUCTION

Assessing the financial health of companies is a practical problem for several groups that make different but related decisions, such as creditors when evaluating repayment capacity, financing terms, and exposure to risk, or investors when comparing firms within an industry and to distinguish stronger positions from weaker ones, and managers rely on the same information for internal monitoring, and support for decisions. These users do not assess only one aspect of performance, because a firm may report strong performance in one indicator while remaining weak in another. Financial health is therefore multidimensional, spanning through profitability, liquidity, indebtedness, and operating efficiency, so a summary measure that combines several ratios can provide a more balanced and interpretable view than any one indicator alone.

Composite indicators combine multiple individual variables into one summary measure intended to represent a broader concept. They are widely used in economics, policy analysis, and comparative assessment because they compress multidimensional information into a form that is easier to communicate, compare, and rank across units (Commissione Europea and OECD, 2008). Their usefulness, however, does not arise automatically from the act of aggregation. If poorly constructed, composite indicators can also send misleading messages about the phenomenon they are meant to summarize (Commissione Europea and OECD, 2008). The meaning of a composite score depends on how the underlying variables are prepared before they are merged into a single value. Decisions about normalization, weighting, and aggregation influence the contribution of each input and can affect the interpretation and validity of the resulting index (Gan et al., 2017). For this paper, the key design issue is weighting, because the chosen weighting rule determines

whether the final financial health score is a simple benchmark or a data-driven measure based on the statistical structure of the sample.

Selecting weights is one of the most difficult and most debated steps in composite-indicator construction. Equal weighting remains common because it is simple, transparent, and easy to communicate: every indicator enters the composite with the same formal importance, and users can immediately see how the index is built. Equal weighting is not invalid by definition, but it embeds a strong assumption that all indicators should contribute identically to the final assessment (Perisic, 2015). Perisic also shows why this decision matters, since alternative weighting schemes can produce meaningfully different composite outcomes. Reviews of weighting practice therefore emphasize that no single method is universally appropriate and that the choice of weighting procedure depends on the purpose of the analysis and the properties of the data being summarized (Gan et al., 2017). Data-driven approaches are attractive in this context because they use information contained in the sample rather than assigning identical contributions by default. Among them, principal component analysis (PCA) is one of the common statistical techniques for reducing dimensionality and summarizing common variation in multidimensional data, and it can be used to derive a weighting rule for aggregation. In this role, PCA derived weights reflect the covariance or correlation structure of standardized indicators, so they capture statistical contribution to common variance of the data rather than normative economic importance. Therefore, they should not be interpreted as the true economic importance of particular financial ratios. Moreover, they may place more emphasis on indicators that are more strongly correlated with others in the dataset (Becker, 2021). Comparing PCA derived weights with equal weights is methodologically informative because it shows how sensitive the resulting financial health evaluation is to the choice between simple equal weighting and data-driven weighting.

Other weighting approaches could also be considered. Methods based on expert opinion, such as budget allocation or the Analytic Hierarchy Process, can incorporate judgment, but depend on the selected experts and weighting procedure. Regression or validation-based weighting would require an external benchmark of financial health, such as default, insolvency, or an established distress score. PCA was therefore used as a reproducible data-driven approach and compared with equal weighting, while its interpretation remains limited to statistical common variation rather than economic or financial importance (Gan et al., 2017; Becker, 2021).

PCA and FA based weighting has also been used in composite indicators for water quality index assessment, territorial quality of life in European regions, and health-related multidimensional indices (Tripathi and Singal, 2019; Ustaoglu et al., 2025; Chao and Wu, 2017). These examples show that data-driven weighting is used across different domains, while the presented paper applies the logic to firm financial health.

The aim of this paper is to construct a composite indicator of financial health for firms from one industry and to compare scores obtained from PCA derived weights with scores obtained from equal weighting. The paper does not seek to identify universally correct weights for financial ratios and it rather examines how two defensible aggregation rules shape the same multidimensional assessment in a comparable sectoral setting.

2 METHODOLOGY

This section describes the data used in the empirical analysis and the methodological steps applied to construct the financial health scores. It first introduces the dataset and selected indicators, and then explains the normalization, weighting, and aggregation procedures used in the study.

2.1 Data

The empirical analysis uses 2022 firm-level data for 12,452 companies from the hospitality industry in Slovakia, that were obtained from the official Register of Financial Statements of the Slovak Republic and the dataset was compiled by FinStat.sk and is publicly available online (FinStat.sk). Each firm in the final analysis sample is uniquely identified by Ico. Only complete observations for the nine selected indicators were retained, so the input data would contain no missing values.

Nine financial ratios were selected to represent profitability, liquidity, indebtedness, and efficiency or management performance. The indicator set covers return on equity (ROE), return on investment (ROI), return on sales (ROS), cash return on assets (CASH/TA), the current ratio (CA/CL), the debt ratio (TL/TA), long-term debt to total assets (NCL/TA), asset turnover (SAL/TA), and days payable outstanding (DPO). Due to the nature of the ratios, some indicators were treated as desirable when larger, whereas others were treated as desirable when smaller, as shown in Table 1. The retained ratios were chosen because they could be computed consistently for all firms in the final sample, had a clear financial interpretation, and together covered the main dimensions needed for a financial health indicator, namely profitability, liquidity, indebtedness, and efficiency. Therefore, the selected ratios are not intended to represent a theoretically optimal list of financial health indicators, but a transparent set of available ratios suitable for demonstrating the proposed composite-indicator procedure.

Table 1. Selected financial ratios, preferred direction, and desirability-normalization thresholds.

Category	Indicator	Preferred direction	Lower threshold	Upper threshold
Profitability	Return on Equity (ROE)	higher is better	0.00	0.30
	Return on Investment (ROI)	higher is better	0.00	0.20
	Return on Sales (ROS)	higher is better	0.00	0.15
	Cash Return on Assets Ratio (CASH/TA)	higher is better	0.08	0.70
Liquidity	Current Ratio (CA/CL)	higher is better	1.00	2.50
Debt Indicators	Debt Ratio (TL/TA)	lower is better	0.00	0.70
	Long-Term Debt to Total Assets Ratio (NCL/TA)	lower is better	0.00	0.10
Efficiency and Management	Asset Turnover (SAL/TA)	higher is better	0.30	0.90
	Days Payable Outstanding (DPO)	lower is better	50.00	440.00

2.2 Desirability Normalization

Before aggregation, the raw financial ratios were transformed using expert-defined desirability normalization with capped thresholds because the dataset contained a large number of outlier observations and the indicators were measured on different scales. Putting indicators on a common scale before weighting and aggregation is a standard requirement in composite-indicator construction (Commissione Europea and OECD, 2008). The lower and upper reference values were specified before aggregation based on common financial interpretations of healthy ratio ranges, the author's interpretation of the indicators, and a consultation with a domain expert in accounting and auditing. The indicator thresholds used in the analysis are reported in Table 1.

For indicators where higher values imply better financial health, the transformation increases with the raw value of the ratio. For indicators where lower values imply better financial health, the transformation is reversed so that lower raw values receive larger desirability scores. Values outside the reference interval were capped at 0 and 1, respectively, so the procedure reduced the influence of extreme observations. This procedure differs from classical winsorization because the capping thresholds were defined separately for each indicator on substantive financial grounds rather than by symmetric tail replacement above and below particular percentile based on sample order statistics.

Let r_{ij} denote the raw value of indicator j for firm i , a_j the lower threshold, b_j the upper threshold, and x_{ij} the resulting desirability score. For an indicator where higher is better,

$$x_{ij} = \begin{cases} 0, & r_{ij} \leq a_j, \\ \frac{r_{ij} - a_j}{b_j - a_j}, & a_j < r_{ij} < b_j, \\ 1, & r_{ij} \geq b_j. \end{cases} \quad (1)$$

For an indicator where lower is better,

$$x_{ij} = \begin{cases} 1, & r_{ij} \leq a_j, \\ \frac{b_j - r_{ij}}{b_j - a_j}, & a_j < r_{ij} < b_j, \\ 0, & r_{ij} \geq b_j. \end{cases} \quad (2)$$

This transformation yields a matrix of desirability scores $X = (x_{ij})$ with all entries bounded in $[0, 1]$.

2.3 PCA-Based Weighting

Because the indicators normalized by desirability functions had different empirical distributions, PCA was applied to their standardized values rather than directly to the raw desirability scores. Standardization was performed using the sample mean and sample standard deviation of each normalized indicator. Principal component analysis was then performed on the standardized desirability indicators, and the number of retained components was determined using the Kaiser criterion. Because the first three eigenvalues exceeded 1 (3.092731, 1.517420, and 1.461571) and the fourth eigenvalue was below 1 (0.947238), three components were retained. The retained components together explained 67.46% of the total variance.

Let L_{jk} denote the loading of indicator j on retained principal component k . Furthermore, q_j denotes the total contribution of financial indicator j across the principal components, computed as the sum of squared loadings across the K retained components. The nonnegative PCA weights were constructed from retained principal component loadings as

$$q_j = \sum_{k=1}^K L_{jk}^2 \quad (3)$$

$$w_j = \frac{q_j}{\sum_{h=1}^p q_h} \quad (4)$$

where the denominator sums the contributions across all p indicators. Thus, the weights are nonnegative, sum to one, and assign larger values to indicators with larger overall squared loadings. The resulting PCA-derived weights should be interpreted as statistical contributions across the retained principal components, not as direct measures of the economic or financial importance of the ratios (Gan et al., 2017; Becker, 2021).

2.4 Financial Health Scores

The PCA weighted score of financial health index for firm i , denoted by FH_i^{PCA} , was defined as

$$FH_i^{\text{PCA}} = \sum_{j=1}^p x_{ij} w_j \quad (5)$$

Because the desirability-normalized inputs lie in $[0, 1]$, and the weights are nonnegative and sum to one, the PCA weighted financial health score also lies in $[0, 1]$. The equal-weight financial health benchmark, denoted by FH_i^{EW} , was defined as

$$FH_i^{\text{EW}} = \frac{1}{p} \sum_{j=1}^p x_{ij} \quad (6)$$

Because it is the arithmetic mean of the same desirability scores calculated from the raw data, the equal-weight financial health benchmark also lies in $[0, 1]$. The equal-weight financial health score was used as a transparent benchmark for comparing the consequences of a statistical weighting with a uniform aggregation rule. The comparison between the two scoring schemes focused on summary distribution statistics and their Pearson correlation. Both scores were computed on the same set of 12,452 firms. All data processing and statistical computations were performed in Python using the scikit-learn library.

3 RESULTS

This section presents the main empirical results of the PCA-based weighting procedure and compares them with the equal-weight benchmark. It first reports the retained components and resulting indicator weights, and then evaluates how the PCA-weighted financial health scores differ from the benchmark distribution.

3.1 PCA Output and Weights

According to the Kaiser criterion, as briefly mentioned in the methodological section, three principal components were retained because the first three eigenvalues (3.092731, 1.517420, and 1.461571) exceeded 1, whereas the fourth eigenvalue (0.947238) did not. The retained components explained 34.36%, 16.86%, and 16.24% of total variance, respectively, for a cumulative share of 67.46%. This retained variance suggests that most of the common variation in the nine indicators normalized by desirability was captured by a reduced three component structure, but not by only one principal component. Therefore, the retained PCA structure was not concentrated in a single dominant component, because the second and third components still contributed substantial variance. Table 2 reports the evidence for all the retained components, while Table 3 summarizes the final PCA derived weights used to construct FH_i^{PCA} .

Table 2. Retained principal components under the Kaiser criterion.

Retained component	Eigenvalue	Explained variance	Cumulative share
PC1	3.092731	0.343637	0.343637
PC2	1.517420	0.168602	0.512239
PC3	1.461571	0.162397	0.674636

Table 3. PCA-derived weights of the nine financial indicators

Indicator	PCA derived weight
ROS	0.140797
SAL/TA	0.140636
ROI	0.137812
DPO	0.131798
TL/TA	0.130324
CA/CL	0.113094
ROE	0.106578
CASH/TA	0.071744
NCL/TA	0.027217

Table 3 shows that the PCA weighting differentiated across the nine indicators, but not extremely. The largest statistical weights in this sample were assigned to return on sales (0.140797), asset turnover (0.140636), and return on investment (0.137812), while the smallest statistical weight in this sample was assigned to long-term debt to total assets (0.027217). Among the middle-ranked indicators, days payable outstanding (0.131798) and total liabilities to total assets (0.130324) also received substantial weights, whereas the current ratio (0.113094) and return on equity (0.106578) received more moderate weights. The five indicators with the highest weights accounted for 0.681367 of the total weight summed to 1. At the lower end, cash return on assets and long-term debt to total assets together accounted for 0.098961, which indicates that the variables with lower weights still remained as a contribution in the composite indicator rather than being eliminated from it completely. The resulting weights combine profitability, efficiency, liquidity, and indebtedness variables rather than attributing almost all influence to a single part of the indicator set.

3.2 Comparison with Equal Weighting

Table 4 compares the empirical distributions of the PCA weighted score FH_i^{PCA} and the equal-weight benchmark FH_i^{EW} . Both aggregation schemes produced bounded scores over almost the full $[0, 1]$ interval, with minima at 0 and maxima close to 1 in both cases. However, the PCA weighted score distribution was shifted slightly downward relative to the benchmark with equal weights, with a lower mean (0.441751 for PCA weights versus 0.471210 for equal weights) and a lower median (0.403738 versus 0.440378). The lower center of FH with the PCA derived weights was also visible in the quartiles, because its 25th percentile (0.275459) and 75th percentile (0.594501) were below the corresponding quartiles of FH with equal weights (0.306883 for 25th and 0.623403 75th percentile). Although both scoring methods produced values over almost the same range, PCA weighting shifted the score distribution slightly downward.

Table 4. Summary statistics for the PCA-weighted and equal-weighted composite scores.

Score	Mean	Std. dev.	Min	25%	Median	75%	Max	Variance
FH_i^{EW}	0.471210	0.224963	0.000000	0.306883	0.440378	0.623403	0.999760	0.050608
FH_i^{PCA}	0.441751	0.234556	0.000000	0.275459	0.403738	0.594501	0.999719	0.055016

At the same time, PCA weighting produced a slightly more dispersed score distribution than equal weighting, as indicated by the higher standard deviation (0.234556 compared to 0.224963) and variance (0.055016 for PCA weights and 0.050608 for equal weights), which corresponds to an 8.71% increase in variance under PCA weighting relative to the equal-weight benchmark. Despite these distributional differences, the two composite scores remained very strongly linearly associated, with a Pearson correlation of 0.984819. This suggests that PCA weighting did not substantially change the overall pattern of firm scores, although it did affect the spacing between observations compared to the benchmark with equal weights. The two composite scores based on different weighting methods were therefore similar, but not identical, and the high Pearson correlation does not by itself imply identical firm rankings.

4 CONCLUSION

This paper constructed a composite indicator of financial health for firms and compared scores obtained from PCA derived weights with scores obtained from equal weighting. The comparison was carried out on 2022 data for more than 12 thousand Slovak hospitality firms using nine financial ratios normalized using desirability function.

The PCA procedure retained three components that together explained 67.46% of total variance. The PCA weighted and equal-weighted scores were very strongly linearly associated, but PCA weighting produced a score distribution with about 8.71% higher variance than the equal-weight benchmark, along with slightly lower mean and median values.

These results suggest that the choice of weighting did not fundamentally alter the overall pattern of firm assessment, but it did change the distribution of the final scores, so the two weighting methods did not produce identical composite indicators. The interpretation of the results is limited by the use of expert defined thresholds and by the fact that the analysis covers one industry in one time period. Future research can extend the comparison to other sectors, time periods, and alternative weighting methods, include sensitivity analysis of the selected thresholds, and validate the resulting score against an external established benchmark such as the Altman Z-score.

Acknowledgements: This work was supported by VEGA 1/0435/26 Investment Effectiveness and Spatial Spillover Effects in the EU Green Transformation: A Spatial Econometric and Machine Learning Approach, by The Grant Agency of Slovak Republic, grant no. 1/0052/24, "Estimation of Key Structural Parameters of Current Macroeconomic Models" and as part of the grant tasks for young teachers, researchers, and doctoral students within project A-25-103/3020-08 Examining Interdisciplinary Strategies for Effective Environmental Management and Sustainable Development.

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LOCATION-ALLOCATION PROBLEM IN A COMPETITIVE ENVIRONMENT: A GAME-THEORY APPROACH BASED ON BIMATRIX GAMES

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Abstract

Location-allocation problems are widely used in planning service systems, where traditional coverage models aim to maximize serviced demand under given constraints and assume centralized decision-making. In many real-world situations, however, service providers operate as independent entities whose decisions are mutually interdependent. This paper proposes a game-theory approach to the location-allocation problem by formulating it as a bimatrix game. Players represent competing service providers, and their payoffs correspond to the demand they capture under distance-based accessibility constraints. The resulting Nash equilibrium solutions are compared with those obtained from the classical maximal covering model. A numerical example shows that while both approaches may achieve the same level of total coverage, they can lead to different location decisions. In particular, equilibrium solutions provide stable outcomes in which no player has an incentive to deviate, offering a more realistic perspective on decentralized spatial decision-making.

Keywords: *Location-allocation problem, Maximal covering model, Game theory, Bimatrix game, Nash equilibrium, Spatial competition*

JEL Classification: C72, R12, C61

AMS Classification: 91A10, 90B80, 90C27

1 INTRODUCTION

Location-allocation problems belong to the classical problems of operations research and are widely applied in the planning of both public and private service infrastructure (Daskin, 2013; Hakimi, 1964). These problems primarily involve decisions on the placement of service facilities (such as healthcare centers, logistics hubs, or recycling facilities) in order to ensure the highest possible accessibility of services for the population under given capacity or budget constraints. Traditional approaches to solving these problems are mainly based on coverage models, which aim to maximize the amount of serviced demand for a given number of facilities and a specified level of accessibility (Church and ReVelle, 1974).

However, these models implicitly assume the existence of a central planner who determines the location of facilities with the objective of maximizing the overall efficiency of the system. In many real-world situations, individual service providers do not act as coordinated entities, but rather as independent agents pursuing their own objectives, which may lead to competitive interactions in space (Čičková, Reiff and Holzerová, 2022). This situation is particularly evident in cases where individual facilities are evaluated based on their performance, or where multiple service providers operate within the same spatial environment (Tirole, 1988; Osborne, 2004).

The aim of this paper is to present an alternative perspective on the location-allocation problem that incorporates the strategic behavior of individual actors. This approach is based on game theory, where the problem of facility location is formulated as a bimatrix game between two players representing competing entities. Within this framework, it is possible to analyze not

only optimal solutions in terms of total coverage, but also equilibrium configurations in which no player has an incentive to unilaterally change their decision. The proposed model provides a more realistic representation of situations involving decentralized decision-making and offers a complementary perspective to traditional optimization approaches. At the same time, it creates a framework for further investigation of the consequences of such decisions in terms of system stability and the spatial distribution of services. The integration of location-allocation models with game theory has recently been addressed, for example, in (Čičková and Sequeira Lopez, 2025).

The paper is structured as follows. The first section presents the classical coverage model for the location-allocation problem. This is followed by the formulation of an alternative approach based on game theory and a discussion of its solution properties. Finally, the differences between the approaches are illustrated using a numerical example, and their implications for decision-making in practice are discussed.

2 CLASSICAL MAXIMAL COVERING LOCATION MODEL

In this section, we focus on the formulation of the classical location-allocation model based on the concept of coverage, which serves as a starting framework for further analysis. The objective of the model is to determine the location of service facilities in such a way that the total covered demand is maximized for a given number of facilities, while respecting a maximum allowable distance between the service location and the demand point.

We consider a discrete space represented by a fully connected graph, where nodes correspond to potential demand locations as well as possible locations of service facilities. Distances between individual nodes are expressed by a distance matrix, which may represent geographical, temporal, or other relevant accessibility metrics. The model is based on the assumption of centralized decision-making, where the objective is to optimize the total coverage of the population subject to a constraint on the maximum number of facilities to be established. Such a formulation corresponds to standard maximal covering models and provides a reference solution that will be compared in the following section with the approach based on game theory.

In the following, we specify the sets, parameters, and decision variables of the model, as well as its mathematical structure.

Let us consider a fully connected graph $\bar{G} = (V, \bar{H})$, where $V = \{1, 2, \dots, n\}$, $n \in \mathbb{Z}^+$ denotes the set of nodes, and $\bar{H}$ represents the set of edges between each pair of nodes v_i and v_j , and the cost of each edge corresponds to the minimum distance (e.g., geographical or temporal) between nodes v_i and v_j . This evaluation can be represented by the distance matrix $\mathbf{D}_{n \times n} = \{d_{ij}\}$, where $i, j \in V$.

Sets and parameters:

$n \in \mathbb{Z}^+$ – number of nodes in the graph

$V = \{1, 2, \dots, n\}$ – set of all nodes

p – maximum (predefined) number of service facilities

K – maximum allowable distance (which may not be sufficient to cover the entire graph)

$b_i, i \in V$ – demand at i -th node (e.g., number of inhabitants)

$a_{ij}, i, j \in V$ where $a_{ij} = 0$, if $d_{ij} > K$ and $a_{ij} = 1$, if $d_{ij} \leq K$ – parameters representing whether i -th node is covered by a facility located at j -th node within distance K .

Variables:

$y_j, y_j \in \{0,1\}, j \in V$ - binary variable indicating whether j -th node is covered by at least one facility ($y_j = 1$) or not ($y_j = 0$)

$x_i, x_i \in \{0,1\}, i, j \in V$ - binary variable indicating whether a facility is located at node i -th ($x_i = 1$), or not ($x_i = 0$)

The mathematical model can be formulated as follows:

$$f(\mathbf{x}, \mathbf{y}) = \sum_{i \in V} b_i y_i \rightarrow \max \quad (1)$$

$$\sum_{i \in V} x_i \leq p \quad (2)$$

$$\sum_{j \in V} a_{ij} x_j - y_i \leq 0, i \in V \quad (3)$$

Objective function (1) ensures the maximum possible coverage of the graph, i.e., the largest possible covered demand (or population). Constraint (2) guarantees that the number of service facilities does not exceed the predefined maximum. Constraint (3) ensures that if i -th node is within the coverage range of a facility located at j -th node, then i -th node is considered covered by that facility.

Although the above model provides an efficient solution in terms of maximizing total coverage, its fundamental assumption is the existence of a central decision-maker who determines the location of service facilities in order to optimize the overall system performance. In practice, however, such assumptions may not always hold. Individual service facilities or providers may act as autonomous agents pursuing their own objectives, while their decisions are interconnected through the shared spatial structure and demand. In such situations, it is appropriate to consider an alternative approach that takes into account strategic interactions among the actors. The spatial decision-making process regarding facility location can then be interpreted as a conflict of interests between entities competing for demand in individual locations. Consequently, the resulting system configuration is not determined by centralized optimization, but rather emerges as an outcome of their mutual interaction.

3 GAME-THEORY APPROACH TO THE LOCATION-ALLOCATION PROBLEM

In this section, we present an approach that, in the case of a duopoly, allows the location-allocation problem to be formulated as a game in normal form. We then focus on its modeling using a bimatrix game for two players (duopoly) and show that the resulting equilibrium solutions may provide a different perspective on the location of service facilities compared to the classical coverage model. At the same time, we discuss their properties in terms of stability and decision-making under conditions of uncoordinated behavior.

We focus on a simplified case of two players (duopoly), each of whom can locate their service facility at any node of the considered graph. Each combination of their decisions leads to a particular allocation of demand between the players, depending on the distances to individual nodes and the maximum allowable accessibility. In this way, a value representing the serviced demand can be assigned to each pair of strategies, which allows the problem to be formulated as a bimatrix game in normal form. The model defined in this manner enables the analysis of resulting system configurations through the concept of Nash equilibrium. In contrast to the classical optimization approach, where the solution is determined by maximizing a global objective function, the game-theory approach focuses on stable configurations in which no player has an incentive to unilaterally change their strategy. Thus, this approach provides a complementary perspective on the facility location problem under conditions of uncoordinated decision-making.

We now present the formal construction of the payoff matrices and specify the conditions under which equilibrium strategies of the players can be identified. We consider a duopoly consisting of two players (P_1 and P_2). Each player can locate their service facility at any node of the graph G . At each node, demand is assumed to be equal to the number of inhabitants, and a maximum allowable distance K between the service location and the demand point is considered. This situation can be formulated as a bimatrix game (i.e., a non-zero-sum game), where the payoff matrix $\mathbf{A}$ for player P_1 and the payoff matrix $\mathbf{B}$ for player P_2 are both of dimension $(n \times n)$. The elements a_{ij} , $i, j \in V$, represent the total demand captured when player P_1 locates at node $i \in V$ and player P_2 locates at node $j \in V$.

We further assume that the demand of each node is served by the closer facility, provided that the distance to that facility does not exceed K ; otherwise, the node remains uncovered. If player P_1 locates at i -th node and player P_2 at j -th node, then player P_1 captures the demand from k -th node $k \in V$ if $d_{ki} < d_{kj}$. If $d_{ki} = d_{kj}$, the demand of k -th node is shared equally between the two players. Otherwise, node k is served by player P_2 . The elements of the payoff matrix ($\mathbf{A} = \{a_{ij}\}$, $i, j \in V$), can then be defined as follows:

```

LET  $V = \{1, 2, \dots, n\}$ ,  $\mathbf{D}_{n \times n} = \{d_{ij}\}$ ,  $\mathbf{b} = \{b_k\}$ ,  $K$ 
LOOP ( $i, j \in V$ ) DO
   $a_{ij} = 0$ ;
  LOOP ( $k, i, j \in V$ ) DO
    IF  $d_{ki} < d_{kj}$  and  $d_{ki} \leq K$  DO  $a_{ij} = a_{ij} + b_k$ ;
    ELSEIF  $d_{ki} = d_{kj}$  and  $d_{ki} \leq K$  DO  $a_{ij} = a_{ij} + 0,5b_k$ ;
  ENDIF

```

It is evident that under these assumptions, the payoff matrix of player P_2 is identical to that of player P_1 , i.e., $\mathbf{B} = \mathbf{A}$. The game is therefore symmetric. The solution of a bimatrix game consists in identifying equilibrium strategies. An equilibrium strategy is one that is optimal for a player given the strategy of the opponent. In other words, if a player adopts an equilibrium strategy, they cannot improve their payoff by unilaterally deviating, assuming that the opponent behaves rationally. In this context, such strategies correspond to the concept of Nash equilibrium.

In accordance with the formulation of the location-allocation problem, we restrict our analysis to the existence of solutions in pure strategies. A pair of pure strategies (i_0, j_0) is considered an equilibrium if, for all $i, j \in V$, $a_{i_0 j_0} \geq a_{ij}$ and $b_{j_0 i_0} \geq b_{ji}$. In other words, a pair of strategies constitutes a Nash equilibrium if each player's strategy is a best response to the strategy of the other player. Under these conditions, the following cases may arise:

1. A single equilibrium point is identified, which uniquely determines the optimal pure strategies of the players.
2. Multiple equilibrium points are identified, but only one is dominant (i.e., more advantageous for both players); this dominant point represents the solution.
3. Multiple equilibrium points are identified, with at least two of them not being dominated; in this case, no unique equilibrium in pure strategies exists and it is necessary to consider mixed strategies.
4. No equilibrium point exists; therefore, the solution must be sought in mixed strategies.

In the following analysis, we restrict our attention to pure strategies.

The solution of the location-allocation problem can be interpreted as the solution maximizing total payoff (i.e., total covered demand), obtained as $\max\{a_{ij} + b_{ji}\}$. To highlight the differences between the classical coverage model and the game-theory approach, we now present a numerical example.

4 NUMERICAL ILLUSTRATION

To illustrate the theoretical considerations, we present a numerical example in which we compare the solution of the classical coverage model with the equilibrium solution of the bimatrix game. The example shows that even with the same level of total coverage, the two approaches may lead to different and differently interpretable results.

We consider the following input parameters:

Number of nodes: $n = 5$

Set of nodes: $V = \{1, 2, \dots, 5\}$

Maximum (predefined) number of facilities: $p = 2$

Maximum allowable distance: $K = 10$

Demand at each node: $\mathbf{b} = (20 \quad 120 \quad 30 \quad 50 \quad 100)^T$

Distance matrix: $\mathbf{D} = \begin{bmatrix} 0 & 7 & 12 & 7 & 11 \\ 7 & 0 & 6 & 14 & 11 \\ 12 & 6 & 0 & 11 & 5 \\ 7 & 14 & 11 & 0 & 13 \\ 11 & 11 & 5 & 13 & 0 \end{bmatrix}$

Based on the above assumptions, we construct the bimatrix of payoffs for both players (with elements $\{a_{ij}; b_{ji}\}$), which captures the distribution of demand for each combination of their location decisions. The Nash equilibrium is highlighted in bold.

$$\begin{bmatrix} 95.95 & 70.140 & 70.240 & 140.50 & 190.120 \\ 140.70 & 80.80 & \mathbf{140.120} & 150.60 & 140.120 \\ 240.70 & \mathbf{120.140} & 120.120 & 240.70 & 140.100 \\ 50.140 & 60.150 & 70.240 & 35.35 & 70.120 \\ 120.190 & 120.140 & 100.140 & 120.70 & 60.60 \end{bmatrix}$$

For comparison with the classical location-allocation model, we also construct a matrix of total serviced demand, whose elements represent the sum of the payoffs of both players for each combination of their strategies. The solution corresponding to the classical approach is then obtained by maximizing this matrix, i.e., by finding $\max\{a_{ij} + b_{ji}\}$.

$$\begin{bmatrix} 190 & 210 & 310 & 190 & 310 \\ 210 & 160 & 260 & 210 & 260 \\ 310 & 260 & 240 & 310 & 240 \\ 190 & 210 & 310 & 70 & 190 \\ 310 & 260 & 240 & 190 & 120 \end{bmatrix}$$

It can be seen that for $K = 10$, the maximum coverage of the network is 310. By applying model (1)–(3), we obtain the following solution (the problem was solved in GAMS using the CPLEX 12.10.0.0 solver):

$$\begin{aligned} \mathbf{x} &= (1 \quad 0 \quad 0 \quad 0 \quad 1)^T \\ \mathbf{y} &= (1 \quad 1 \quad 1 \quad 1 \quad 1)^T \\ f(\mathbf{x}, \mathbf{y}) &= 310 \end{aligned}$$

Although the value of the objective function in this solution is also 310, from the perspective of system stability, it may be preferable for management to select the locations at nodes 2 and

3, where the Nash equilibrium is achieved. In the case of the solution obtained from the classical coverage model, one of the players may have an incentive to change their decision in order to increase their share of the serviced demand. In contrast, the Nash equilibrium represents a stable configuration in which no player has an incentive to unilaterally deviate from their strategy, which may be preferable from a practical point of view. The example clearly shows that both the classical coverage model and the game-theory approach lead to the same value of the objective function, but result in different location decisions. In the case of the classical model, the solution is based on centralized optimization and does not take into account the individual incentives of the actors.

This highlights the instability of the solution obtained from the classical coverage model, as one of the players has an incentive to unilaterally change their strategy. Similar conclusions regarding the importance of equilibrium solutions in location-allocation problems formulated as bimatrix games are also reported by Čičková and Sequeira Lopez (2025). From a practical perspective, this suggests that although both approaches yield the same level of total coverage, the equilibrium solution may be more appropriate in environments where individual agents make decisions independently and are not centrally coordinated.

5 CONCLUSION

The location of service facilities is a problem addressed by many companies, local authorities, as well as governments. It may involve the placement of central warehouses, healthcare facilities, or recycling centers. Models based on the concept of coverage, commonly referred to as location-allocation models, have been at the center of research interest for several decades. This paper compares two approaches to solving the location-allocation problem: the classical coverage model and a game-theory approach. While the traditional model makes it possible to determine the optimal location of service facilities in terms of maximizing total coverage, its main limitation lies in the assumption of centralized decision-making. The alternative approach, formulated as a bimatrix game, allows for the modeling of situations in which individual entities act independently and engage in competitive interactions. In such an environment, the resulting system configuration does not arise as a global optimum, but rather as an equilibrium state that is stable from the perspective of individual decision-making. The illustrative example has shown that these two approaches may lead to different interpretations of appropriate facility location decisions, with the equilibrium solution being preferable in terms of stability. From a practical perspective, the proposed approach may be useful in situations where service facilities are operated by independent entities and where centralized coordination is limited or absent. In such settings, equilibrium-based solutions may provide more stable location patterns than solutions derived solely from centralized optimization.

The obtained results suggest that, when analyzing location-allocation problems, it is important to consider not only optimization criteria but also the nature of the decision-making environment, particularly the degree of coordination among individual actors. The game-theory approach thus represents a valuable complement to classical models and enables a better understanding of the consequences of uncoordinated decisions in spatial settings. Possible extensions of the presented approach include the analysis of a larger number of players, the consideration of mixed strategies, as well as the integration with empirical data in order to quantify the effects of different decision-making regimes on the spatial distribution of services. These research directions open up opportunities for further investigation into the efficiency and stability of resource allocation in real-world conditions.

Acknowledgements: This work was supported by VEGA 1/0435/26 “Investment Effectiveness and Spatial Spillover Effects in the EU Green Transformation: A Spatial Econometric and Machine Learning Approach” and by VEGA 1/0052/24, "Estimation of Key Structural Parameters of Current Macroeconomic Models".

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MODELING PUBLIC PROCUREMENT AUCTIONS AND THE EFFECTS OF LOBBYING

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Abstract

Public procurement contracts are allocated in large volumes to interested parties through auctions. Auctions are a market mechanism for balancing supply and demand. There are many types of auctions. Reverse combinatorial auctions are suitable for modeling the allocation of public contracts. Reverse auctions are oriented toward purchasing by a single buyer and a larger number of sellers. Combinatorial auctions focus on auctions of combinations of objects. Buyers try to purchase the desired combination of objects from suppliers at minimum cost. An integer linear programming model for the allocation of public contracts is proposed. The effects of lobbying on the efficiency of these auctions are demonstrated.

Keywords: *Public procurement, Reverse combinatorial auctions, Lobbying*

JEL Classification: L22, C70

AMS Classification: 93A30, 91A40

1 INTRODUCTION

Public procurement is the competitive process by which the government and public institutions purchase goods, services, or construction work using public funds. This process should meet the criterion of value for money, meaning the best results for public funds. Auctions are a market mechanism for balancing supply and demand (Fiala, 2012). Auctions appear to be an effective tool for awarding contracts (Decarolis, 2018).

There are many types of auctions. This will not be an auction for a single property, but rather an auction for multiple properties, as the overall contract may consist of a number of individual contracts. Combinatorial auctions will be used, in which the buyer requests multiple items simultaneously (Cramton et al., 2006, de Vries and Vohra, 2003). Auctions are further classified into forward and reverse auctions. Traditional forward auctions are sales-oriented, with one seller and multiple buyers. Reverse auctions are purchase-oriented, with one buyer and multiple sellers. Reverse auctions are well-suited for modeling public procurement, where the buyer is a public authority and the sellers are the suppliers of individual contracts (Lorentziadis, 2023). The overall model is thus a reverse combinatorial auction, which can be formulated as a linear binary programming problem (Elmaghraby and Keskinocak, 2004).

Lobbying is the effort to influence the rules and decisions of public authorities (Burguet and Sákovics, 2022, Chassang and Ortner, 2019). The article analyzes the effects of lobbying on changes to public procurement rules and their impact on cost-effectiveness. The effects of these interventions are illustrated using a simple numerical example.

The rest of the paper is organized as follows. Section 2 presents basics of modelling public procurement using reverse combinatorial auctions. Section 3 is devoted to analyzing the effects of lobbying as a modification of the original model. A simple example of public procurement involving lobbying is presented in Section 4. Section 5 presents conclusions.

2 PUBLIC PROCUREMENT AUCTIONS

Public contracts are allocated in large volumes to bidders through auctions. Reverse combinatorial auctions are suitable for modeling the allocation of public contracts to bidders and modeling the impact of lobbying. This problem is characterized by a single buyer and a large number of suppliers (Cramton et al., 2006, Pekeć and Rothkopf, 2003). The buyer seeks to purchase the desired set of items from suppliers at minimum cost. Suppose that m potential suppliers $D_1, D_2, \dots, D_m$ offer a set N of n items, $j = 1, 2, \dots, n$, to a single buyer K (Fig. 1).

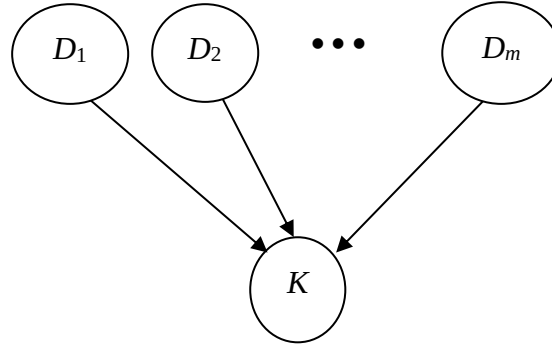


Figure 1 Public procurement auctions

A bid made by supplier D_i , $i = 1, 2, \dots, m$, is defined as

$$b_i = \{C, c_i(C)\}, \quad (1)$$

where

$C \subseteq N$ is the offered combination of items,

$c_i(C)$ is the price offered by supplier D_i for the combination of items C .

The goal is to minimize the buyer's costs given the suppliers' bids. The constraints specify the conditions for purchasing the required set of all items. Binary variables are used to formulate the problem:

$y_i(C)$ is a binary variable indicating whether combination C is purchased from supplier D_i ($y_i(R) = 1$).

The reverse combinatorial auction problem can be formulated as follows

$$\sum_{i=1}^m \sum_{C \subseteq N} c_i(C) y_i(C) \rightarrow \min \quad (2)$$

under constraints

$$\sum_{i=1}^m \sum_{C \subseteq N} y_i(C) \geq 1, \quad \forall j \in N, \quad (3)$$

$$y_i(C) \in \{0, 1\}, \quad \forall C \subseteq N, \quad \forall i, i = 1, 2, \dots, m. \quad (4)$$

The objective function expresses the goal, i.e., minimizing the buyer's costs. The constraints ensure that the required set of items is purchased.

3 EFFECTS OF LOBBYING

We will not consider gross manipulations of the auction that could completely distort it. Let's look at simple rule changes and their effects, which we will demonstrate using a simple example.

Possible lobbying interventions:

1. *A single supplier wins the entire contract.*

Constraint (4) is replaced by constraint (5), i_0 is the winner.

$$y_i(C) \in \{0, 1\}, \quad \forall C \subseteq N, i = i_0. \quad (5)$$

2. *Some suppliers are excluded a priori for failing to meet certain conditions.*

Constraint (4) is replaced by constraint (6), the subset of non-excluded suppliers I_0 .

$$y_i(C) \in \{0, 1\}, \quad \forall C \subseteq N, \quad \forall i, i \in I_0 \quad (6)$$

3. *Conversely, all suppliers must be awarded some part of the contract.*

Constraint (7) is added to the model

$$\sum_{C \subseteq N} y_i(C) \geq 1, \quad \forall i = 1, 2, \dots, m. \quad (7)$$

4. *Each supplier delivers at most one item.*

Constraint (8) is added to the model

$$\sum_{C \subseteq N} y_i(C) \leq 1, \quad \forall i = 1, 2, \dots, m. \quad (8)$$

5. *Certain combinations of items cannot be supplied by a single supplier.*

The subset C_0 is a subset of all permissible combinations. In the basic model (2) – (4), the set C is replaced by the subset C_0 .

6. *Supplier collusion is allowed, enabling them to agree on higher prices.*

When optimizing the objective function (2), the min operator is replaced with the max operator. The model is then supplemented with a surplus-sharing agreement.

Analysis of these interventions can also be viewed positively: lobbying ensures that these possibilities are prohibited, resulting in a fair auction.

4 EXAMPLE

Using a simple example, we will examine interference with public procurement auction rules, such as through lobbying. The buyer wishes to acquire three items: A, B, and C. Let us consider three potential suppliers—D1, D2, and D3—who offer combinations of the three requested items. The suppliers' bids, in the form of item combinations and requested prices, are shown in Table 1.

	A	B	C	AB	AC	BC	ABC
D_1	5	10	15	12	18	23	28
D_2	6	9	12	14	17	20	26
D_3	7	8	16	14	20	22	29

Table 1 Bids from suppliers

The optimal solution to the reverse combinatorial auction problem is highlighted in bold in the table and means that the buyer purchases items A and B from supplier D_1 and item C from supplier D_2 . Supplier D_3 was not selected. The buyer obtains items A, B, and C at a cost of 24.

Possible lobbying interventions:

1. *A single supplier wins the entire contract:*

The buyer purchases items A, B, and C from supplier D_2 at a cost of 26.

2. *Some suppliers are excluded a priori for failing to meet certain conditions:*

For example, supplier D_1 is excluded. Then the best solution is for the buyer to purchase item B from supplier D_2 and items A and C from supplier D_3 . The buyer obtains items A, B, and C at a cost of 25.

3. *Conversely, all suppliers must supply some part of the contract:*

In this example, each supplier supplies exactly one item. The buyer purchases item A from supplier D_1 , item C from supplier D_2 , and item C from supplier D_3 . The buyer obtains items A, B, and C at a cost of 25.

4. *Each supplier delivers at most one item:*

In this example, each supplier again delivers exactly one item, and the solution is the same as in the previous case. The buyer purchases items A from supplier D_1 , item C from supplier D_2 , and item B from supplier D_3 . The buyer obtains items A, B, and C at a cost of 25.

5. *Some combinations of items cannot be supplied by a single supplier.*

For example, the combination of A and B cannot be supplied by a single supplier. In this case, there are several equally good solutions, e.g., the buyer purchases items A from supplier D_1 , item C from supplier D_2 , and item B from supplier D_3 . The buyer obtains items A, B, and C at a cost of 25.

6. *Supplier collusion is allowed:*

For example, suppliers agree that each will offer exactly one item, but at the highest price among the original bids. For example, the buyer purchases item A from supplier D_1 for 7, item B from supplier D_2 for 10, and item C from supplier D_3 for 16. The buyer acquires items A, B, and C at a cost of 33.

In all cases, the buyer pays more for the items than in the original auction.

5 CONCLUSION

This article proposes modeling public procurement as a reverse combinatorial auction. This model is a linear binary programming problem that can be used to present and analyze the selection of appropriate public contracts for a cost-optimal allocation of contracts to achieve a specified objective. There are efficient methods for solving this model, but they are not the

subject of this article. There are also software packages available for handling combinatorial auctions (Fiala et al., 2010).

The aim of this article is to analyze the effects of lobbying on public procurement solutions. Simple effects of lobbying are modeled as changes to the original public procurement model. The interventions are demonstrated using a simple example, and the resulting changes in the solution are illustrated. These changes lead to higher costs compared to the model without lobbying.

In subsequent research, we intend to focus on analyzing more complex lobbying interventions in public procurement and identifying rules and measures to prevent public procurement costs from rising.

Acknowledgements: This work was supported by the grant No. IGA F4/18/2024, Faculty of Informatics and Statistics, Prague University of Economics and Business, Prague.

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SIMULATION-BASED EVALUATION OF INTERSECTION IMPROVEMENT STRATEGIES

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Abstract

Traffic-light-controlled intersections are critical elements of the road network, where traffic jams and vehicle delays are common, particularly during peak hours. This paper evaluates selected traffic management measures and infrastructure modifications using microsimulation modelling. The analysed traffic section consists of two interconnected signal-controlled intersections with high traffic volumes. The simulation model was created in the AnyLogic environment using empirical traffic data obtained through field measurements. As part of the experiments, scenarios were designed to adjust signal phase durations and extend turning lanes. The simulation results showed that changes to signal plans only yielded local improvements in traffic performance. These results confirm the effectiveness of using microsimulation models to support the design and evaluation of traffic management measures at signal-controlled intersections.

Keywords: *simulation, traffic, Anylogic*

JEL Classification: R41, C63

AMS Classification: 90B20, 90B06

1 INTRODUCTION

The rapid increase in the number of vehicles and growing traffic demand, coupled with the increasing complexity of transport systems, places high demands on the effective management of traffic flows. Intersections are often critical points in the transport network where congestion, delays and potential accidents occur. For this reason, it is essential to study their behaviour and find effective solutions. Simulation models have long been recognised as an effective decision-support tool for transportation systems, including signal-controlled intersections. Microsimulation models allow for the detailed evaluation of various optimisation alternatives for such systems at low time and financial cost. This article aims to evaluate the impact of changes to signal plans and infrastructure interventions using a microsimulation model.

2 RELATED WORK

Over the past decade, microscopic traffic simulation has become a vital tool for designing and evaluating strategies to enhance the capacity of signal-controlled intersections. Modern approaches lead to agent-based modelling, which allows for a detailed capture of individual interactions between vehicles and the stochastic nature of traffic flows in the AnyLogic environment [7]. Malim et al. [6] used AnyLogic and the OptQuest optimisation tool in their study to minimise the average time spent at a node, achieving a delay reduction. Similarly, Pop

[8] demonstrated the effectiveness of simulation experiments in determining optimal green phase times, which were found to directly correlate with a reduction in average travel time through the network. In the Slovak context [9], artificial intelligence and deep reinforcement learning methods were incorporated into simulation models in Sabinov, significantly improving traffic flow compared to fixed signal schedules. In addition to traffic control strategies, the focus of research is the impact of infrastructure changes. The authors [3] analysed the impact of various intersection configurations and confirmed that, while signal optimisation helps significantly, physical modifications are essential when traffic reaches critical saturation levels. The findings [2], that investigated hybrid modelling, also confirm this. Studies at intersections in Bratislava [5] emphasise that extending turning lanes is often more effective than phase tuning alone when congestion is caused by the blocking of through traffic. In their research, Drliciak et al. [4] identified a relationship between congestion cluster formation and a 30% decrease in speed when capacity is exceeded. This allows models to be calibrated according to real-world spatial conditions. Zhen et al. [10] examine how to increase the efficiency of transit hubs and urban corridors by integrating multimodal flows and adaptive strategies. Baza et al. [1] compare commercial simulators with open-source tools such as SUMO, demonstrating the level of detail in commercial solutions for engineering practice.

Existing research highlights the usefulness of simulation models in finding solutions to problems in transportation systems. However, it is important to note that different sections of the road network have different characteristics. This article therefore focuses on one such section and its simulation model, highlighting the potential applications of such a model in addressing the specific challenges of this section.

3 CASE STUDY

This article focuses on the section of the transport network shown in Figure 1. As can be seen, it consists of two signal-controlled, interconnected intersections: the D3 highway feeder road intersection (ID3) and the Radol'a village intersection (IR). Three roads lead from the ID3 intersection: one towards the highway feeder road (HF), one towards Žilina (ZA) and one towards Čadca (CA). Four roads lead from the IR intersection: one towards Žilina (ZA), one towards Čadca (CA), one towards Kysucké Nové Mesto (KM) and one towards the village of Radol'a (RA).

The selected road segment is characterized by increased traffic load during peak hours, which results in the formation of vehicle queues and reduced traffic flow efficiency. The analyzed section consists of two signalized intersections connected by a road segment, where the signal control at both intersections is operated using fixed-time signal phases. Traffic control is implemented through signal groups representing individual traffic movements, while each lane is equipped with stop lines enabling the modeling of vehicle responses to traffic signal indications. Such a system configuration allows for a realistic representation of traffic flow behavior under signalized intersection conditions.

The traffic load of the analyzed road segment was determined based on data obtained from field traffic measurements. The data collection process also included the identification of directional traffic flow distribution at individual intersection approaches. The use of empirical traffic data

enables a realistic representation of the operating conditions of the analyzed road segment and increases the credibility of the simulation experiment results. Based on these data, a microscopic simulation model was developed to serve as an experimental environment for comparing alternative measures aimed at improving the operational characteristics of the studied traffic system.

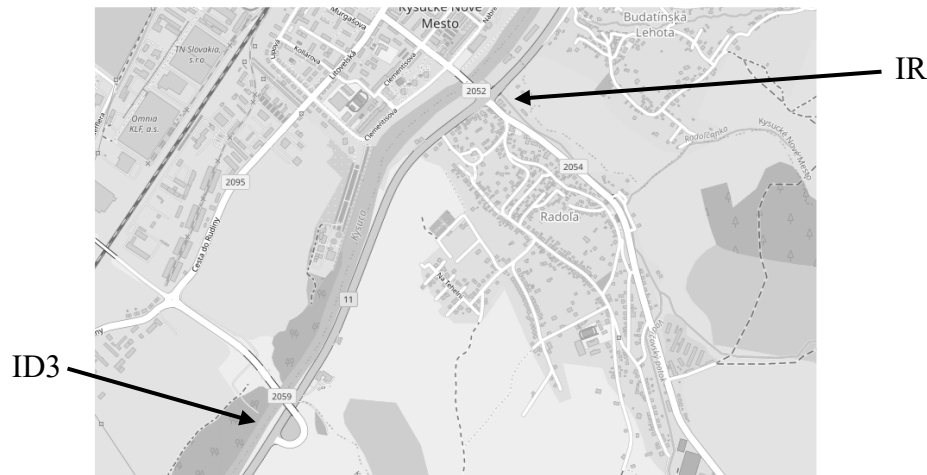


Fig. 1 Map of analyzed segment

4 SIMULATION MODEL

The simulation model of the analyzed road segment was developed in the AnyLogic environment, which enables microscopic modeling of traffic flows at the level of individual vehicles. The model was implemented as an agent-based system in which each vehicle represents an individual agent responding to the current traffic situation and the state of traffic signal control. The objective of the model was to create an experimental environment enabling the testing of alternative traffic control strategies and infrastructure modifications while maintaining realistic traffic flow behavior. The simulation model is based on a microscopic representation of the traffic system in which individual lanes are modeled as separate traffic objects with defined directional connections.

Vehicle arrivals in the model were generated based on probability distributions derived from empirical traffic data obtained from video recordings. The model distinguishes several vehicle categories (passenger cars, trucks, and buses) and incorporates directional movement probabilities at individual intersection approaches. Traffic control was modeled using fixed-time signal control plans, allowing experimental modification of signal phase durations and analysis of their impact on the operational characteristics of the intersection. The model does not consider extraordinary traffic events, and the input parameters are assumed to be representative of the analyzed time period.

The validation of the simulation model was performed by comparing selected operational indicators of the simulated system (average travel times from individual approaches to the nearest intersection) with values obtained from real traffic conditions using a t-test. The comparison confirmed that the model reproduces the behavior of the analyzed traffic system

with sufficient accuracy and is therefore suitable for conducting simulation experiments aimed at evaluating alternative traffic control strategies.



Fig. 2 Animation of the simulation model

5 EXPERIMENTS

The objective of the experimental study was to compare the impact of selected traffic control measures and infrastructure modifications on the operational characteristics of the analyzed road segment. The simulation experiments were designed to enable a systematic comparison of alternative traffic organization variants while maintaining identical input traffic conditions. The experimental design consisted of four scenarios—a reference scenario representing the current state of the traffic system and three experimental scenarios representing proposed measures aimed at improving traffic flow efficiency.

The reference scenario represents the existing state of signal control and intersection geometry. The first experimental scenario focused on modifying the signal control plan by extending the duration of selected signal phases. The objective of this measure was to increase the capacity of traffic movements with higher traffic demand and to reduce queue formation at the main intersection approaches. The extension of signal phases was implemented by increasing the green time for selected traffic movements while maintaining the basic structure of the signal cycle. The expected effect of this modification was a reduction in the average vehicle delay and an increase in the capacity of the dominant traffic movements.

The second experimental scenario focused on modifying the signal control plan by shortening the duration of selected signal phases. This variant represents an alternative approach to traffic control optimization based on balancing the duration of individual signal intervals among traffic movements.

The third experimental scenario represents an infrastructure measure consisting of extending the turning lane at a selected intersection approach. The objective of this modification was to increase the capacity of the traffic movement with a high proportion of turning vehicles and to reduce the blocking of the through traffic stream. Extending the turning lane enables a more

efficient separation of traffic movements and reduces the likelihood of congestion caused by interactions between through and turning traffic flows. This scenario makes it possible to compare the effectiveness of infrastructure-based measures with measures based on adjustments to signal control parameters.

The performance of the individual scenarios was evaluated using the average travel time through the analyzed road segment, which represents an appropriate indicator of the overall efficiency of traffic control. This indicator makes it possible to compare the impact of the individual measures on the operational conditions of the traffic system. A total of 30 simulation replications were performed for each scenario, and the results were evaluated using confidence intervals. The results of the individual experiments are presented in Tables 1, 2, and 3.

Table 1. Results of 1. experiment

Intersection	Direction	Original	10% extension of the phases	20% extension of the phases	30% extension of the phases
ID3	ZA, CA, HF	(77; 82)	(78; 85)	(78; 83)	(82; 88)
ID3	CA	(21; 27)	(21; 27)	(20; 23)	(19; 21)
ID3	HF	(57; 61)	(62; 66)	(62; 65)	(65; 68)
ID3	ZA	(127; 135)	(131; 140)	(131; 139)	(138; 151)
IR	ZA, CA, KM, RA	(119; 137)	(139; 146)	(142; 149)	(138; 151)
IR	CA	(210; 242)	(235; 247)	(219; 234)	(202; 222)
IR	KM	(163; 204)	(218; 233)	(260; 272)	(265; 299)
IR	RA	(125; 158)	(145; 190)	(179; 220)	(221; 281)
IR	ZA	(14; 15)	(15; 16)	(15; 16)	(14; 16)

Table 2. Results of 2. experiment

Intersection	Direction	Original	10% reduction of the phases	20% reduction of the phases	30% reduction of the phases
ID3	ZA, CA, HF	(61; 62)	(60; 61)	(61; 61)	(60; 61)
ID3	CA	(14; 14)	(13; 14)	(13; 13)	(12; 12)
ID3	HF	(52; 55)	(49; 52)	(48; 51)	(44; 47)
ID3	ZA	(106; 107)	(106; 107)	(107; 108)	(106; 107)
IR	ZA, CA, KM, RA	(71; 77)	(77; 84)	(74; 80)	(70; 76)
IR	CA	(94; 95)	(93; 94)	(91; 92)	(89; 90)
IR	KM	(119; 146)	(97; 113)	(120; 135)	(127; 144)
IR	RA	(95; 113)	(212; 271)	(150; 195)	(129; 160)
IR	ZA	(13; 14)	(13; 13)	(12; 12)	(10; 11)

Table 3. Results of 3. experiments

Intersection	Direction	Original	Extension of lanes
ID3	ZA, CA, HF	(61; 62)	(61; 63)
ID3	CA	(14; 14)	(14; 14)

Intersection	Direction	Original	Extension of lanes
ID3	HF	(52; 55)	(53; 57)
ID3	ZA	(106; 107)	(107; 107)
IR	ZA, CA, KM, RA	(71; 77)	(56; 58)
IR	CA	(94; 95)	(94; 95)
IR	KM	(119; 146)	(69; 74)
IR	RA	(95; 113)	(70; 75)
IR	ZA	(13; 14)	(14; 15)

The experimental results showed that modifications to signal phase durations do not have a uniformly positive impact on all traffic movements. Both the extension and shortening of signal phases resulted only in local improvements, while in some directions transit times even increased, for example on movement ID3–HF. The most significant effect was achieved by extending the turning lanes, where the transit time was substantially reduced, particularly on movements IR–KM and IR–RA. Improvements were also observed at the level of the entire analyzed road segment. The results confirm that infrastructure measures may be more effective than signal timing optimization alone under existing capacity constraints. Based on the conducted experiments, it can be concluded that signal timing optimization represents an appropriate tool for fine-tuning traffic control, whereas infrastructure modifications have the potential to produce more substantial improvements in operational performance, especially under existing capacity limitations. The simulation results thus confirm the suitability of microscopic simulation models as a decision-support tool for the design and evaluation of traffic management measures at urban intersections.

6 CONCLUSION

The paper addressed the evaluation of selected measures aimed at improving traffic conditions at a signalized intersection through microscopic simulation modeling in the AnyLogic environment. In the experimental part, three types of interventions were analyzed: extension of signal phases, shortening of signal phases, and extension of turning lanes, while the individual variants were compared based on the average vehicle transit time. The simulation results showed that modifications to signal phase durations did not lead to a consistent improvement and that their effect was strongly dependent on the traffic load of individual traffic streams. The most significant positive effect was achieved in the scenario involving the extension of turning lanes, which confirms the importance of infrastructure measures, particularly under existing capacity constraints. The obtained results also confirm the suitability of microscopic simulation models as a decision-support tool for the design of traffic management measures. The simulation environment makes it possible to test different solution variants before their implementation in real conditions and to identify measures with the highest potential for improving traffic performance.

Future research may extend the model by incorporating additional traffic performance indicators, such as queue lengths, average vehicle delay, or travel time variability, as well as by implementing adaptive signal control strategies. Such extensions would enable a more comprehensive evaluation of the effectiveness of the proposed measures and improve the accuracy of decision-making processes in transport infrastructure planning.

Acknowledgements: This work was supported by the research grant VEGA 1/0696/26 "Predictions of delays and vehicle occupancies to inform the planning of public transport" and VEGA 1/0516/26 "Optimal design of charging infrastructure for electric vehicles and its efficient operation in urban mobility conditions"

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MODELING RETURNS OF PRODUCTS/PACKAGING FOR REUSE

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Abstract

This paper deals with the modelling of product and package returns for reuse in reverse logistics. The increasing importance of recycling and repeated material use creates a need for practical methods that can describe how return flows develop over time. Existing approaches make it possible to capture the growing return of products back to the producer, but their practical applicability differs. Some of them require a more detailed specification of return behaviour across several periods, which may be difficult to estimate in real business conditions. The paper therefore focuses on a simpler and more practically applicable approach based on a single return estimate and an explicit consideration of waste. The original model is further adjusted so that one of its key values is no longer fixed in advance, but derived from assumptions about return and waste. The proposed adjustment is illustrated by simulation on several return scenarios. The results show that the modified approach preserves the logic of the original model while providing a more realistic and flexible basis for practical applications in reverse logistics.

Keywords: *Reverse logistics, Return of Products/Packaging, Reuse*

JEL **Classification:** *L91,* *M11,* *C61*
AMS Classification: *90B06, 90C90*

1 INTRODUCTION

Reverse logistics is an important area of contemporary logistics and production decision-making, particularly in connection with recycling, material reuse, and the reduction of dependence on new raw materials. When a used product is returned to the manufacturer, it can become a source of material for further production and thus influence the purchase of new stock, production planning, and inventory management. For this reason, it is important to estimate how many products will return to the system over time.

Along with the emergence of reverse logistics as a distinct concept, attention also turned to extending the product life cycle into its subsequent phases and to the further reuse of materials that had already been used once before. New programmes in the field of reverse logistics were developed (Stock, 1998), and numerous examples of practical applications followed, including comprehensive questionnaires designed for different industrial sectors (Rogers and Tibben-Lembke, 1999; Guide, 2000). The broadest overview of possible applications of reverse logistics appeared in 2004, when more than 60 case studies were analysed through content analysis (de Brito, Dekker and Flapper, 2004). All of these works shared one common element: the return of the product to the manufacturer.

Against this background, modeling the return of products or packaging back to the manufacturer is an important part of reverse logistics models. Product return modeling was also addressed in “Inventory Management Model with Return” (Brezina, Gežík and Pekár, 2010) and “Modeling of Return in Reverse Logistics” (Brezina, Gežík, 2012), where product returns were formalized through probabilistic relationships.

This paper presents a general approach to modeling, but its examples, and indeed its central idea, are framed around the replacement of plastic packaging with glass packaging in mineral-water production. Although plastic bottles are formally returnable in Slovakia, they are not returned directly to the manufacturer; instead, they are processed back into primary raw materials. Glass bottles, by contrast, are returned directly to the water producer.

2 GROWTH OF RETURNS OVER TIME

The collection of returned products or packaging is naturally conditional upon these items having first been sold to the final consumer and then returned after fulfilling the purpose for which they were purchased. In the initial periods after a product is introduced into circulation, the volume of returns is low because the products are still in distribution, on sale, or in use. Only after some time do products begin to flow back, and the return volume starts to increase. This phenomenon represents the basic logic of growing returns over time.

If a company initially produces only from new materials, products gradually accumulate at the consumer end and only later begin to return to the system. Returns therefore do not arise solely from sales in the immediately preceding period; instead, they are spread across several subsequent periods. As a result, returns increase over time. This growth has a direct impact on the need to purchase new materials (glass bottles), because in the initial periods the company depends primarily on new material, while in later periods an increasing share of production can be covered by material obtained from returned products or packaging. These products or packages thus accumulate with consumers, who gradually return them. The development of returns over time, including the gradual accumulation of returned material, is illustrated in *Figure 1* (Schulz and Ferretti, 2008).

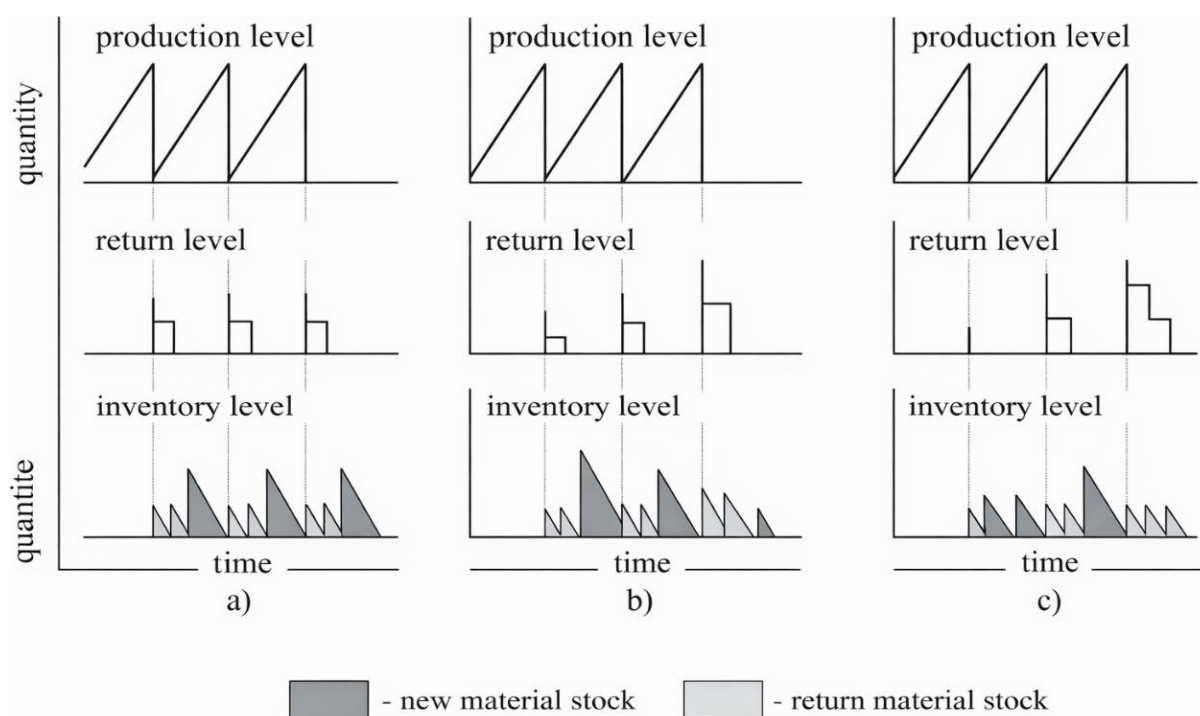


Figure 1 Growth of returns over time

Source: own processing based on Schulz and Ferretti (2008).

Part a) of the figure shows regular returns from only one previous period. By contrast, parts b) and c) already reflect increasing returns. The underlying assumption is that over several periods enough products enter consumption that, with a time lag, the amount returned, or the amount of material obtained from returned products, can exceed the amount of new material. The difference between parts b) and c) lies either in the level of the returns or in the intensity of the return process; in both cases, however, they describe growth in returns over time.

3 CALCULATION OF GROWING RETURNS

If returns come only from one previous period, as illustrated for example in part a) of the figure, their calculation is relatively straightforward. The quantity of returned products in period t is determined by the probability $PR(R)_t$, which indicates what share of the products sold in the previous period $t-1$ will be returned in period t .

The product return (R_t) can then be expressed by the relationship $R_t = PR(R)_t D_{(t-1)}$, for $t = 1, 2, \dots, T$, where D_{t-1} denotes product sales in period $t-1$. In the case of growing returns, however, the calculation is not based only on sales in period $t-1$ (D_{t-1}), but on sales in several periods preceding period t .

Variant 1

There are several ways to calculate such growing returns. Probably the simplest is the variant based on the total quantity sold in the relevant period, to which the return probability is assigned. The original $PR(R)_t$ thus becomes $PR(Rx)_t$, where x represents the number of periods that the product has remained with the consumer; for example, $PR(R3)_t$ expresses the probability that the product will be returned in the third period after its sale.

The value of x lies in the interval from 1 to X , $x \in \langle 1; X \rangle$, where the upper value X represents a threshold after which the product is assumed no longer to return to the manufacturer and to end up as waste. The return-probability distribution has a decreasing character, since most returns are assumed to occur in the period immediately following the sale, after which the probability of return declines sharply.

In this variant, the sum of the probabilities is lower than one, and it is therefore assumed that a certain percentage of products will never be returned to the manufacturer and will instead become waste. This percentage is given by the difference between the sum of the probabilities and the value 1, since the probabilities are calculated from the total quantity sold. The return can therefore be written as $R_t = PR(R1)_t D_{(t-1)} + PR(R2)_t D_{(t-2)} + \dots + PR(RX)_t D_{(t-X)}$, for $t = 1, 2, \dots, T$, where X is the number of return periods.

Example 1:

Example 1 follows the first variant and demonstrates the return calculation using random values fluctuating around 1000. The upper value of X is assumed to be five, which means that after the fifth period the product will no longer be returned to the manufacturer and will instead become waste. The probabilities used in the example are $PR(R1) = 0,5$; $PR(R2) = 0,2$; $PR(R3) = 0,05$; $PR(R4) = 0,025$; $PR(R5) = 0,025$. The study assumes that 20% of products never return to the manufacturer and therefore become waste. It is presented for one collection point and one product type. The return is calculated as follows, for $t = 1, 2, \dots, T$ and $X = 5$. The numerical calculation of returns under the first variant is presented in *Table 1*.

1st per.	2nd per.	3rd per.	4th per.	5th per.	6th per.	7th per.	8th per.	9th per.	10th per.
	1000	500	200	50	25	25	0	0	0
		980	490	196	49	24,5	24,5	0	0
	0		1020	510	204	51	25,5	25,5	0
		500		1010	505	202	50,5	25,25	25,25
			690		970	485	194	48,5	24,25
				756		1000	500	200	50
					783		1030	515	206
						787,5		1010	505
							794,5		990
								814,25	
									810,5

xxx – quantity sold in the period

xxx – returns in the period

Table 1 Example 1
Source: own calculations.

Variant 2

The second variant keeps the probability $PR(R)_t$ unchanged, but modifies the return calculation itself. Unlike the previous variant, the return is not calculated from the total quantity of products sold in the individual periods, but only from the quantity that remained with the consumer in the given period. This method is more complex, but also more realistic, because it no longer counts products that were already returned in previous periods and therefore provides more accurate results.

This means that the probability applies only to the residual quantity of products that remained with the consumer from the X previous periods, together with all products sold in the immediately preceding period. This variant also works with a threshold X , after which the product is assumed no longer to return to the manufacturer and to become waste. The probability $PR(R)_t$ must always be less than one; this ensures that after X periods a certain quantity still remains with the consumer as waste. This way of defining returns can be written as follows: $R_t = PR(R)_t D_{(t-1)} + PR(R)_t (1 - PR(R)_t) D_{(t-2)} + PR(R)_t (1 - PR(R)_t)^2 D_{(t-3)} + \dots + PR(R)_t (1 - PR(R)_t)^{X-1} D_{(t-X)}$, for $t = 1, 2, \dots, T$, where X is the number of return periods.

Example 2:

Example 2 is based on the same assumptions as the previous one. It illustrates the return calculation using random numbers fluctuating around 1000 and assumes that the upper value of X is five, which means that after the fifth period the product will no longer be returned to the manufacturer and will instead become waste. The study is presented for a single collection point and a single product type. The return probability $PR(R) = 0,3$, and the return is calculated as follows: $R_t = PR(R) D_{t-1} + PR(R)(1 - PR(R)) D_{t-2} + PR(R)(1 - PR(R))^2 D_{t-3} + PR(R)(1 - PR(R))^3 D_{t-4} + PR(R)(1 - PR(R))^4 D_{t-5}$, for $t = 1, 2, \dots, T$ and $X = 5$. The corresponding calculation for the second variant is shown in Table 2.

1st per.	2nd per.	3rd per.	4th per.	5th per.	6th per.	7th per.	8th per.	9th per.	10th per.
	1000	300	210	147	102,9	72,03	0	0	0
		980	294	205,8	144,06	100,842	70,5894	0	0
0			1020	306	214,2	149,94	104,958	73,4706	214,2
	300			1010	303	212,1	148,47	103,929	72,7503
		504			970	291	203,7	142,59	99,813
				658,8		1000	300	210	147
					764,16		1030	309	216,3
						825,912		1010	303
							827,717		990
								838,989	
									838,863

xxx – quantity sold in the period

xxx – returns in the period

Table 2 Example 2
Source: own calculations.

4 ADAPTATION OF THE SECOND VARIANT FOR PRACTICAL USE

From a theoretical point of view, the above variants for calculating growing product returns represent interesting modeling alternatives; in practice, however, their use is relatively demanding. Both variants require several return rates to be specified for individual periods, that is, separately for returns in the first, second, third, and subsequent periods after sale. Such an approach assumes that the company has sufficiently detailed data on consumer behaviour and the timing of returns. In real operating conditions, however, companies usually do not have such detailed data, and obtaining it would require considerable effort in recording, processing, and evaluation. For this reason, the second variant appears more suitable for practical use, because it requires only one return rate and one value of X , that is, the number of periods during which the product may still be returned to the manufacturer. At the same time, the second variant is more realistic in terms of the return logic itself, because it does not work with returns from the entire originally sold quantity, but only from the remainder that stayed with the consumer in the given period. In each period, the product return is therefore calculated only from the quantity that has not yet been returned in previous periods.

However, the application of the second variant revealed a weakness in the fixed specification of X and in the absence of an explicit waste rate. In the first variant, the waste share can be derived relatively easily; in the second variant, by contrast, the waste level depends directly on X . In theoretical examples, the value of X can be chosen directly, but in business practice such a procedure is problematic. A company can usually estimate much more easily what proportion of products will be returned on average and what proportion will never return at all, that is, what proportion will become waste. By contrast, the number of periods after which the remaining product is regarded as waste is not usually a quantity that a company tracks directly or can determine reliably without additional calculation. For this reason, the second variant was modified so that the value of X is not entered as a fixed parameter, but is instead calculated from the return rate R and the waste rate W . This follows the logic of the second variant, according to which only a certain share of the remainder left with the consumer is returned in each period. If a share R is returned in one period, then after the first period a share $(1 - R)$ of the original quantity remains with the consumer. After two periods, $(1 - R)^2$ remains; after three periods, $(1 - R)^3$ remains; and in general, after X periods, the share $(1 - R)^X$ remains. This remainder represents the quantity of products that no longer returns to the system and can therefore be treated as waste. If the waste rate is denoted by W , then the following relationship holds: $W = (1 - R)^X$.

The above relationship can then be rearranged to express the value of $X = \frac{\ln(W)}{\ln(1-R)}$. Since the number of return periods must be an integer in practical applications, the value of X should be rounded up $X = \left\lceil \frac{\ln(W)}{\ln(1-R)} \right\rceil$.

This modified second variant changes the original treatment of the parameter X . While X was an input parameter in the original formulation, in the modified formulation it becomes a derived quantity. The practical importance of this modification lies in the fact that the company no longer needs to determine the number of return periods directly, but can work only with two parameters that are more naturally estimable from a managerial point of view, namely the return rate R and the waste rate W .

The proposed modification was then verified using an illustrative example. Sales were generated as a time series fluctuating around 1000 units with a deviation of 10%, and the number of observed periods was set to 15. Instead of a fixed value of X , a common waste rate $W = 0,1$ was specified, meaning that approximately 10% of products are assumed not to return to the system once the return process is completed. Five different return rates R were then examined, namely 0,1; 0,2; 0,3; 0,4 and 0,5. For each of these values, the corresponding value of X was calculated using the above relationship. The resulting values were subsequently used in the simulation of the second variant. The simulation results for the modified second variant, where X is calculated from the assumed return and waste rates, are summarized in *Table 3*.

R	X	Total D	Total returns	Total waste after X	Simulated W
0,1	16	15109	7846,30	0,00	0,00
0,2	8	15109	10505,09	1343,35	0,09
0,3	5	15109	11413,53	1826,92	0,12
0,4	4	15109	12326,93	1550,79	0,10
0,5	3	15109	12697,38	1626,63	0,11

Table 3 Example from 15 periods with calculated X
Source: own simulation.

The simulation results showed that the proposed method for determining the parameter X preserves the logic of the original second variant while making it more suitable for practical use. Lower return-rate values lead to higher values of X , meaning that products remain in circulation longer before their remainder is treated as waste. By contrast, higher return-rate values lead to lower values of X , because a larger share of the product returns earlier and the return process closes over a shorter time horizon. Under the original approach, with a fixed value of X , the waste values for the simulated 15 periods were inconsistent and, in practical terms, unrealistic. This comparison is shown in *Table 4*.

R	X	Total D	Total returns	Total waste after X	Simulated W
0,1	5	15109	5416,30	6418,63	0,42
0,2	5	15109	9053,66	3561,88	0,24
0,3	5	15109	11413,53	1826,92	0,12
0,4	5	15109	12890,44	845,25	0,06
0,5	5	15109	13784,94	339,69	0,02

Table 4 Example from 15 periods with fixed X
Source: own simulation.

A practical benefit of this modification is that a company can simulate different situations while keeping the same waste rate constant and simultaneously observe how product returns, waste volume, and the stock remaining with consumers change under different return-rate values. The second variant thus becomes not only a tool for calculating returns, but also a tool for experimenting with different behavioural scenarios of the system.

It can therefore be concluded that the modified second variant represents a compromise between theoretical simplicity and practical usability. Unlike the first and third variants, it does not require several return rates to be specified, while at the same time it removes the weakness of the original fixed value of X . The value of X thus changes from an exogenously specified parameter into a parameter derived from realistic assumptions about product returns and waste. The development of returns over the simulated 15 periods for different return-rate values is illustrated in *Figure 2*.

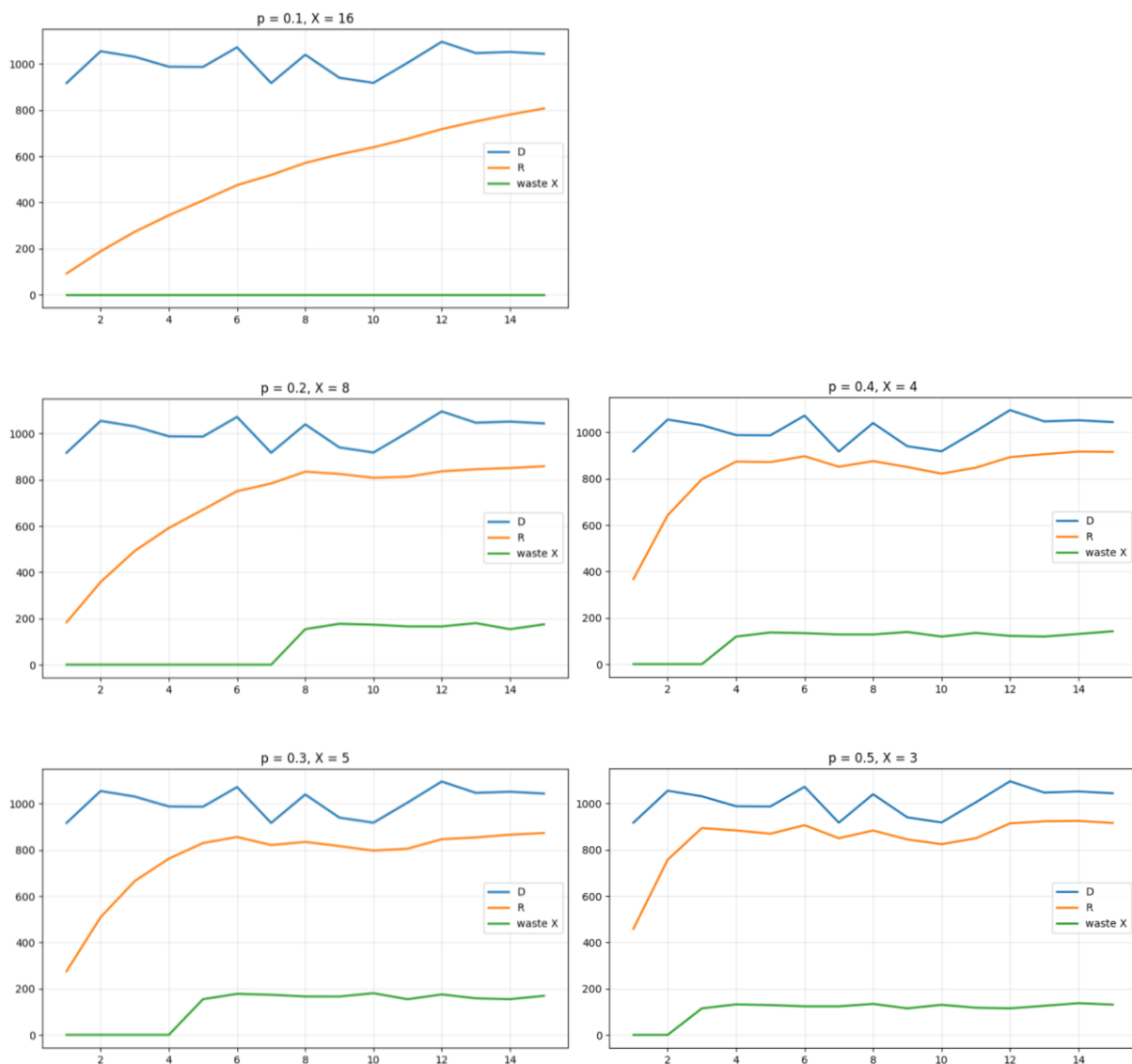


Figure 2 Growth of returns in 15 periods for different rates of return
Source: own simulation.

5 CONCLUSIONS

Existing approaches make it possible to model the growth of product returns over time; however, not all of them are equally suitable for business practice. The first variant requires several return-rate inputs, whereas the second variant is simpler because it works with only one return rate and the parameter X , which expresses the number of return periods. In practice, however, fixing the value of X proves problematic. It is more natural to assume that a company can estimate the return rate and the waste rate, while the number of return periods is a quantity that should be derived.

This paper therefore addressed a modification of the second variant for calculating product returns in reverse logistics. From a practical point of view, the original second variant is more suitable than the first one because it requires only a single return rate. Its weakness, however, lies in the fixed specification of X . The proposed modification is based on the assumption that, in practice, it is more natural to estimate the return rate and the waste rate and then calculate X from them. This changes X from a fixed input into a derived quantity. The simulation confirmed that this approach leads to logical results and makes it possible to observe different return and waste patterns under different return-rate values.

The main contribution of the paper lies in offering a more practical parameterization of the second variant of return modeling. The model modified in this way can serve as a basis for further simulation and decision-making applications in reverse logistics.

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EGARCH MODELLING OF STOCK MARKET VOLATILITY: EVIDENCE FROM S&P 500, DAX, AND FTSE 100 INDICES

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Abstract

This paper analyses the volatility of daily logarithmic returns of the S&P 500, DAX, and FTSE 100 indices over the period January 1, 2020, to March 30, 2026, using EGARCH(1,1) models with a skewed Student's t-distribution. The study captures key stylized facts of financial time series, including volatility clustering, asymmetry, and heavy tails. The empirical results indicate high volatility persistence and significant asymmetric effects across all indices. Diagnostic tests confirm the overall adequacy of the model specification. In addition, volatility forecasts are examined to illustrate differences between static and dynamic approaches. The findings support the suitability of EGARCH models for modelling and forecasting financial market volatility.

Keywords: *EGARCH model, volatility, stock market indices, asymmetry, financial time series*

JEL Classification: C22, C58, G17

AMS Classification: 62M10, 91G70

1 INTRODUCTION

The analysis of financial time series represents a fundamental area of empirical research in economics and finance, as it provides essential insights into the behaviour of financial markets, risk dynamics, and investment decision-making (Reiff, Brezina and Pekár, 2018). In particular, the study of stock index returns and their volatility is of primary importance, since volatility is widely interpreted as a measure of uncertainty and risk associated with financial assets (Tsay, 2010; Hull, 2018).

Financial time series, such as stock prices or exchange rates, are typically observed at relatively high frequencies, most commonly on a daily basis. Unlike many macroeconomic time series, these data exhibit several distinctive features, including non-normality, volatility clustering, leverage effects, and time-varying variance (Cont, 2001). Due to these characteristics, traditional linear models assuming constant variance are often inadequate for capturing the underlying dynamics of financial returns.

A major breakthrough in volatility modelling was achieved with the introduction of the Autoregressive Conditional Heteroskedasticity (ARCH) model by Engle (1982), which allows the conditional variance to depend on past shocks. This framework was later generalized by Bollerslev (1986) into the Generalized ARCH (GARCH) model, which incorporates both past shocks and past variances. These models successfully capture volatility clustering and have become widely used in empirical finance (Brooks, 2019).

Nevertheless, standard GARCH models assume a symmetric response of volatility to positive and negative shocks, which is often inconsistent with empirical evidence. Financial markets tend to exhibit asymmetry, where negative shocks (bad news) have a stronger impact on volatility than positive shocks (good news) of the same magnitude. To address this issue, Nelson (1991) proposed the Exponential GARCH (EGARCH) model, which allows for asymmetric effects and does not impose non-negativity constraints on the parameters. As a result, EGARCH models are particularly suitable for capturing the leverage effect observed in financial markets.

The aim of this paper is to analyse the volatility of daily logarithmic returns of selected major stock indices, namely the S&P 500, DAX, and FTSE 100, over the period from January 1, 2020, to March 30, 2026. The empirical analysis is based on the estimation of EGARCH-type models, which allow for a flexible representation of time-varying and asymmetric volatility dynamics. All models are estimated using the *rugarch* package in the R software environment (Ghalanos, 2026), which provides a comprehensive framework for specifying, estimating, and forecasting a wide range of GARCH family models. The estimation is performed via maximum likelihood methods, and model adequacy is assessed using standard diagnostic tools. In addition to modelling volatility, the paper also aims to demonstrate the suitability and effectiveness of the EGARCH model for forecasting volatility in financial time series.

The remainder of the paper is organized as follows. The next section presents the methodological framework and describes the econometric models used in the analysis. This is followed by a section presenting the data and empirical results. The final section summarizes the main findings.

2 METHODOLOGY

Let P_t denote the adjusted closing price of a given stock market index at time t . The continuously compounded (logarithmic) return r_t is as follows:

$$r_t = \ln(P_t/P_{t-1}) \quad (1)$$

The use of logarithmic returns is standard in financial econometrics due to their desirable statistical properties (Tsay, 2010). In general, the dynamics of financial return series can be decomposed into two components: the conditional mean and the conditional variance. The important role plays also the choice of the error terms distribution. The conditional mean is often modelled using ARMA (Autoregressive Moving Average) models introduced by Box and Jenkins (1970). However, the empirical analysis conducted in this study indicated that the return series do not exhibit significant autocorrelation, and thus the conditional mean can be adequately specified as a constant μ , corresponding to an ARMA(0,0) model. This implies that the primary source of dependence in the data arises from the conditional variance rather than the mean process. As mentioned above, an important aspect of modelling is the choice of distribution of the error terms. Although the normal distribution is often assumed in classical models, empirical evidence suggests that financial return series exhibit excess kurtosis and skewness. Therefore, assuming normality may lead to biased estimates and inaccurate inference (Tsay, 2010). In this study, we employ the skewed Student's t-distribution (*sstd*), which accommodates both heavy tails and asymmetry, offering a more flexible and realistic representation of financial return distributions. The following formulas will therefore be presented under this distribution assumption.

Thus, the conditional mean equation is as follows:

$$r_t = \mu + \varepsilon_t \quad (2)$$

where r_t denotes returns, μ is a constant and ε_t is an error term/shock/innovation.

A typical stylized fact of financial returns is the presence of volatility clustering, meaning that large changes in returns tend to be followed by large changes (of either sign), and small changes tend to be followed by small changes, we employ models from the GARCH family. The standard GARCH model specifies the conditional variance as a function of past squared innovations and past conditional variances. However, this model assumes a symmetric response of volatility to positive and negative shocks. This assumption is often violated in financial

markets, where negative shocks tend to have a larger impact on volatility than positive shocks of the same magnitude. To account for this asymmetry, the EGARCH(1,1) model is employed. The EGARCH(1,1) model is specified as¹:

$$\ln(\sigma_t^2) = \omega + \alpha_1(\varepsilon_{t-1}/\sigma_{t-1}) + \beta_1 \ln(\sigma_{t-1}^2) + \gamma_1(|\varepsilon_{t-1}/\sigma_{t-1}| - E|z_t|) \quad (3)$$

where symbol σ_t^2 represents the conditional variance, $z_t = \varepsilon_{t-1}/\sigma_{t-1}$ are standardized residuals/shocks $z_t \sim sstd(skew, shape)$, γ_1 is magnitude of the reaction to shocks, β_1 denotes the volatility persistence and α_1 measures the asymmetry (leverage effect: $\alpha_1 < 0$).

3 DATA AND EMPIRICAL RESULTS

The empirical analysis is based on daily closing prices of three major stock market indices: the S&P 500, DAX, and FTSE 100, covering the period from January 1, 2020 to March 30, 2026. After removing missing observations, the dataset consists of 1,524 price observations, corresponding to 1,523 continuously compounded returns. The data was retrieved through the R package “*quantmod*” (Ryan and Ulrich, 2025) from the Yahoo Finance (2026). The use of daily data allows for capturing short-term dynamics and volatility patterns typical for financial markets.

The analysed period is particularly interesting, as it includes several major global events that significantly influenced financial markets. These include the COVID-19 pandemic and its economic consequences in 2020–2021, the subsequent recovery period, rising inflation and tightening monetary policy by central banks in 2022–2023, and ongoing geopolitical tensions, including the war in Ukraine and instability in energy markets. These events contributed to increased uncertainty and are reflected in the heightened, time-varying volatility of stock returns (see Figure 1)². Descriptive analysis of the return series confirms the presence of typical stylized facts of financial time series. The return distributions exhibit excess kurtosis and asymmetry, indicating deviations from normality (see Table 1 and Figure 2). Furthermore, visual inspection and statistical testing confirm the absence of significant autocorrelation in returns, while strong evidence of conditional heteroscedasticity is detected³. These findings justify the use of GARCH-type models for volatility modelling.

The volatility dynamics of the analysed indices were estimated using the EGARCH(1,1) model with a skewed Student’s t-distribution (Ghalanos, 2026). This specification allows for capturing both heavy tails and asymmetry in the distribution of returns, as well as the leverage effect in volatility⁴.

Figure 3 presents the estimated conditional standard deviation obtained from the EGARCH model (3). The results clearly indicate that volatility is not constant over time but exhibits pronounced clustering. Periods of elevated volatility are followed by further turbulent periods, while calmer market conditions tend to persist.

A significant spike in volatility occurred at the start of the sample period, coinciding with the outbreak of the pandemic in early 2020. This period was characterised by extreme uncertainty and sharp stock market declines worldwide. Subsequently, volatility decreased gradually during the recovery phase, although it remained sensitive to new information and policy responses.

¹ This is the formulation from the *rugarch* package in R (Ghalanos, 2026).

² S&P 500, DAX, and FTSE 100 are in R outputs denoted as GSPC, GDAXI, and FTSE.

³ Results are available from the author upon request.

⁴ Based on the values of Akaike, Bayes and Hannan-Quinn information criteria, the EGARCH model specification was preferred to the widely used GJR-GARCH model (Glosten, Jagannathan and Runkle, 1993).

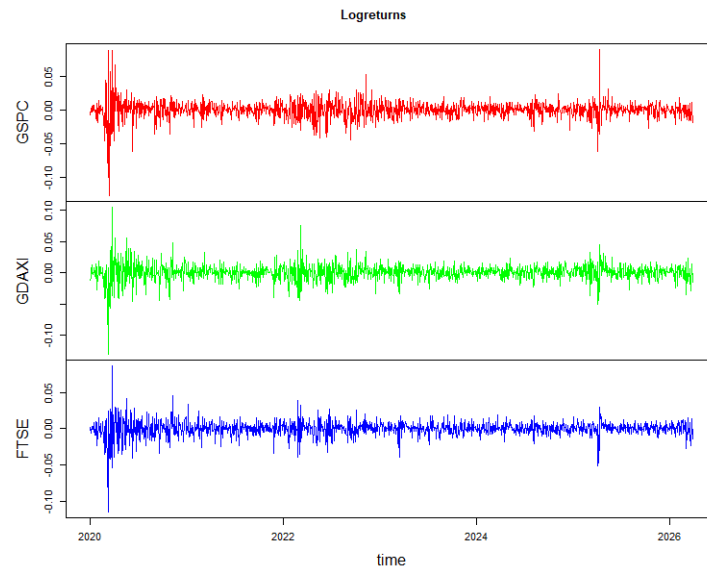


Figure 1 Returns of S&P 500, DAX and FTSE 100

Source: author's own calculations in R

Table 1 Descriptive statistics for analysed returns

GSPC		GDAXI		FTSE	
Min.	:-0.1276522	Min.	:-0.1305486	Min.	:-0.1151171
1st Qu.	:-0.0050620	1st Qu.	:-0.0051225	1st Qu.	:-0.0040037
Median	: 0.0009160	Median	: 0.0006914	Median	: 0.0007116
Mean	: 0.0004401	Mean	: 0.0003351	Mean	: 0.0001777
3rd Qu.	: 0.0068693	3rd Qu.	: 0.0068443	3rd Qu.	: 0.0051832
Max.	: 0.0908949	Max.	: 0.1041429	Max.	: 0.0866642

Source: author's own calculations in R

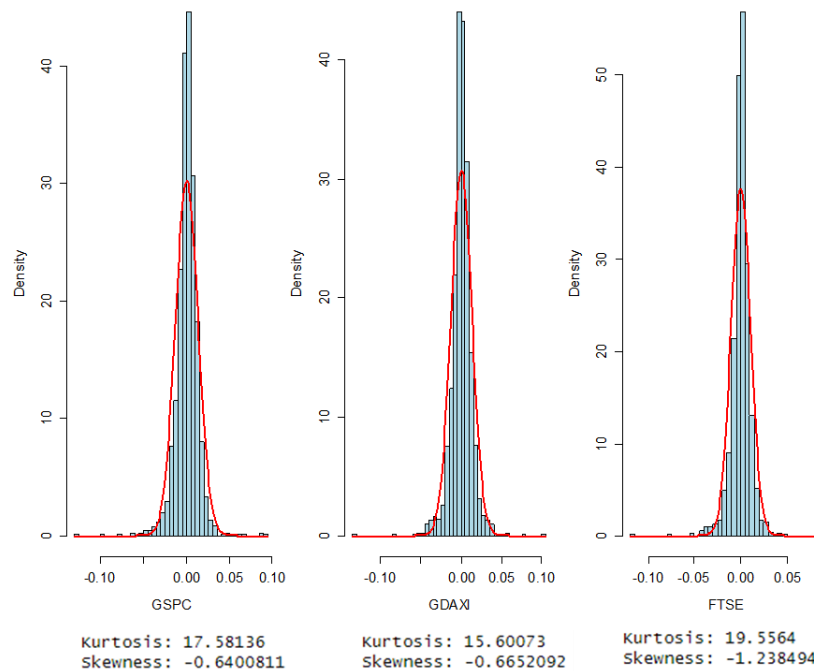


Figure 2 Histograms of returns with fitted normal density curve, kurtosis and skewness

Source: author's own calculations in R

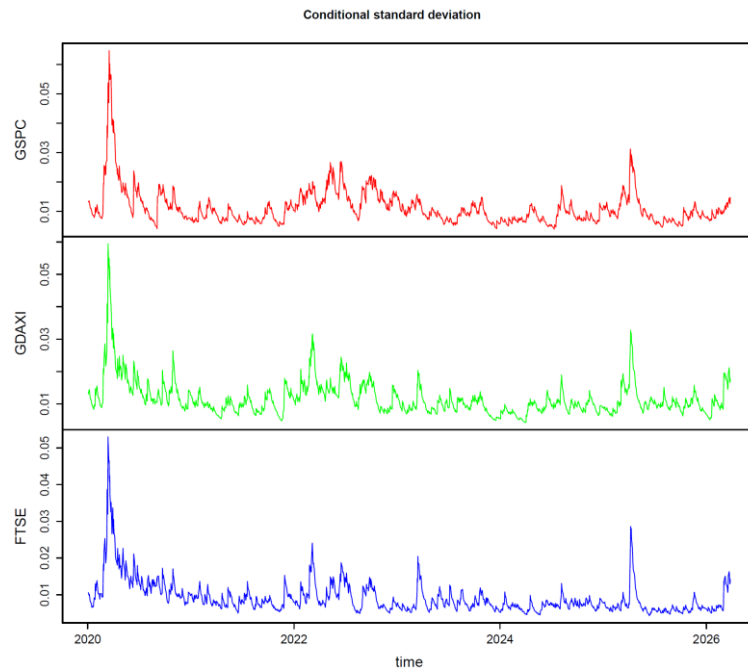


Figure 3 Conditional standard deviations of returns (whole period)

Source: author's own calculations in R

Another notable increase in volatility occurred during the post-pandemic period, when monetary policy was tightened and inflation rose. Central banks, particularly the Federal Reserve and the European Central Bank, implemented interest rate hikes, contributing to increased market uncertainty and fluctuations in asset prices. Additionally, geopolitical tensions, particularly the war in Ukraine, further amplified volatility, especially in European markets such as the DAX and FTSE 100.

The estimation results of the EGARCH models together with the values of the corresponding information criteria are presented in Table 2.

Table 2 EGARCH(1,1) Estimation Results

Parameter	GSPC	p-value	GDAXI	p-value	FTSE	p-value
μ	0.000328	0.0195	0.000301	0.4396	0.000077	0.6874
ω	-0.2401	0.0000	-0.3429	0.0000	-0.4048	0.0000
α_1	-0.1768	0.0000	-0.1581	0.0000	-0.1519	0.0000
β_1	0.9738	0.0000	0.9621	0.0000	0.9573	0.0000
γ_1	0.1227	0.0000	0.1864	0.0000	0.2012	0.0000
skew	0.7852	0.0000	0.8670	0.0000	0.8571	0.0000
shape	7.1502	0.0000	5.2814	0.0000	5.3916	0.0000
Information Criteria						
AIC	-6.4278	-	-6.3065	-	-6.8052	-
BIC	-6.4033	-	-6.2820	-	-6.7807	-
H-Q	-6.4187	-	-6.2974	-	-6.7961	-

Source: author's own calculations in R

where *skew* and *shape* are parameters of the skewed Student's t-distribution and the other symbols were presented above.

The estimation results of the EGARCH(1,1) models indicate that volatility dynamics are highly persistent across all three indices. This is evident from the high and statistically significant values of the parameter β_1 (0.9738 for GSPC, 0.9621 for GDAXI, and 0.9573 for FTSE), which are close to unity. This suggests that volatility shocks decay slowly over time, confirming the presence of strong volatility clustering.

The parameter γ_1 , which captures the magnitude effect of shocks, is positive and statistically significant in all cases (0.1227 for GSPC, 0.1864 for GDAXI, and 0.2012 for FTSE). This reflects that larger innovations (regardless of their sign) have a stronger impact on current volatility, confirming the relevance of modelling conditional heteroscedasticity.

The asymmetry parameter α_1 is negative and highly significant for all indices (-0.1768, -0.1581, and -0.1519, respectively), indicating the presence of asymmetric volatility effects. Specifically, negative shocks increase future volatility more than positive shocks of the same magnitude. This finding is consistent with the traditional leverage effect commonly observed in equity markets.

The estimated skewness and shape parameters confirm that the skewed Student's t-distribution is appropriate. The *skew* parameter differs significantly from unity, indicating asymmetry in return distributions. Meanwhile, the relatively low values of the *shape* parameter point to heavy tails, i.e. the presence of extreme observations.

Regarding the conditional mean, only the S&P 500 index exhibits a statistically significant mean return. For the DAX and FTSE 100 indices, however, the mean is not statistically different from zero. This is consistent with the efficient market hypothesis (Fama, 1970).

The adequacy of the estimated models is evaluated using several diagnostic tests applied to standardised residuals and their squares. These tests confirm the overall adequacy of the estimated EGARCH models. The Ljung–Box and ARCH LM tests indicate no significant serial correlation or residual ARCH effects, and the sign bias test generally yields satisfactory results.⁵ Overall, the skewed Student's t-distribution provides a reasonable fit to the data.

Furthermore, the estimated EGARCH models were tested for forecasting purposes, with a forecasting exercise being conducted. Specifically, the models were re-estimated using a shortened sample that excluded the last 242 observations; these observations were then used for forecasting evaluation. Following standard volatility modelling practice (Brooks, 2019; Wichmann and Brooks, 2019), two types of forecast were generated: static (rolling one-step-ahead) and dynamic (multi-step-ahead).

Figure 4 shows how the forecasted conditional standard deviations for returns evolve, including both static and dynamic forecasts. Forecasting performance was evaluated using RMSE (Root Mean Squared Error), MAE (Mean Absolute Error) and QLIKE (Quasi-Likelihood) loss functions.⁶ The results are reported in Table 3 separately for static and dynamic forecasts for all three indices: S&P 500, DAX and FTSE 100. Lower values of RMSE, MAE and QLIKE indicate better forecasting performance.

The results highlight the clear differences between dynamic and static forecasts. Dynamic forecasts follow a smooth path, converging to the long-run volatility level due to mean reversion. In contrast, static forecasts are more variable because they incorporate new information at each stage, enabling them to better capture short-term fluctuations. These findings are consistent with the theory that dynamic forecasts reflect model persistence, while

⁵ Results are available from the author upon request.

⁶ The QLIKE loss function was additionally employed because it is commonly recommended for evaluating volatility forecasts in GARCH-type models.

static forecasts use real-time data. However, fixed model parameters may limit adaptability during structural changes.

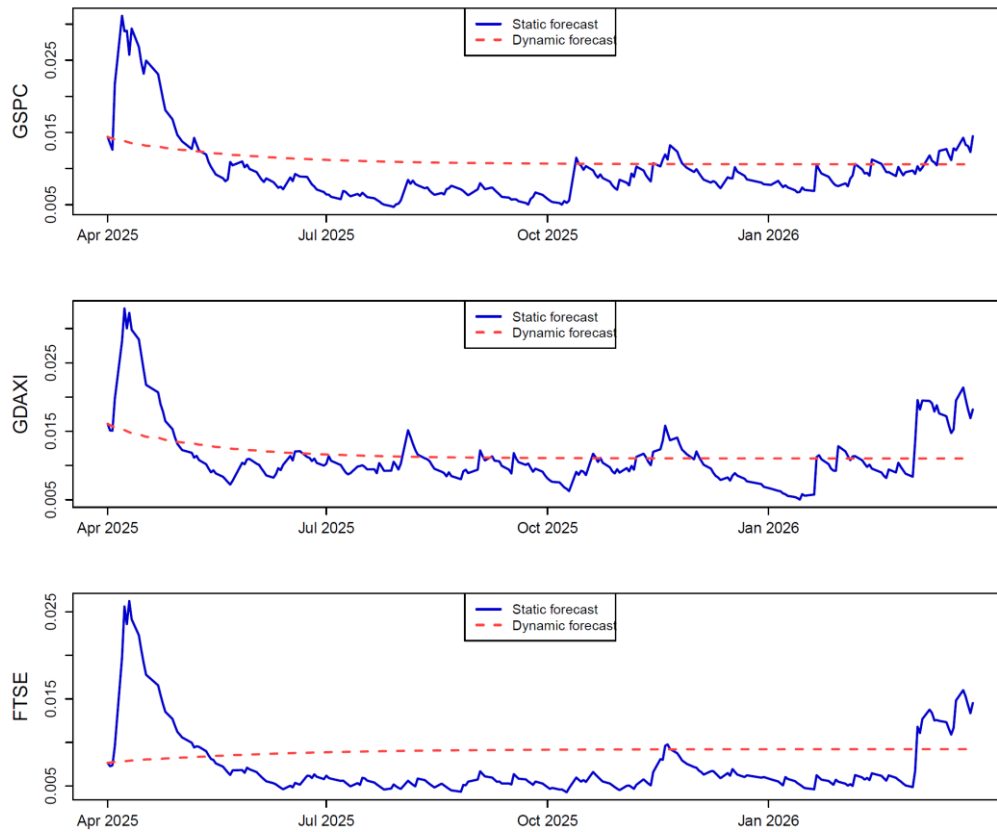


Figure 4 Conditional standard deviations of returns (static vs. dynamic forecasts)

Source: author's own calculations in R

Table 3 Forecast accuracy measures for static and dynamic EGARCH forecasts

Index	Forecast	RMSE	MAE	QLIKE
GSPC	Static	0.000567	0.000145	-8.515118
GSPC	Dynamic	0.000605	0.000167	-8.155108
GDAXI	Static	0.000249	0.000135	-8.212118
GDAXI	Dynamic	0.000279	0.000150	-8.092296
FTSE	Static	0.000214	0.000071	-8.938688
FTSE	Dynamic	0.000239	0.000094	-8.436992

Source: author's own calculations in R

4 CONCLUSION

In conclusion, the results demonstrate that EGARCH models provide a robust framework for capturing the key characteristics of financial return volatility, including persistence, asymmetry and heavy tails. Diagnostic tests largely confirm the adequacy of the model specification, with only minor limitations observed in specific cases. The differences between static and dynamic forecasts highlight the trade-off between short-term responsiveness and long-term stability. While static forecasts better reflect recent market movements, dynamic forecasts provide a

smoother view of volatility convergence. Overall, the findings are consistent with theoretical expectations and empirical evidence.

Acknowledgements: This work was supported by the Grant Agency of Slovak Republic – VEGA 1/0435/26 “Investment Effectiveness and Spatial Spillover Effects in the EU Green Transformation: A Spatial Econometric and Machine Learning Approach” and VEGA 1/0052/24 “Estimation of key structural parameters of current macroeconomic models.”

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CURRENT ADVANCES IN TWO-STAGE DATA ENVELOPMENT ANALYSIS NETWORK MODELS

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Abstract

The paper deals with analysis of efficiency and performance in two-stage serial data envelopment analysis (DEA) models. The common approach for analysis of this class of network processes is Kao and Hwang (2008) model. Its disadvantage is that, in a typical case, it does not specify any decision making unit (DMU) as fully efficient. Not only this, but other models were designed for the same purpose eliminating the mentioned shortcoming. The aim of the paper is to review the main models for two-stage serial network processes, and, where possible, extend the models by a super-efficiency option to allow fully ranking of the DMUs. The presented models are illustrated on an evaluation of 22 DMUs with two inputs, two intermediate variables and two outputs.

Keywords: *two-stage network model, data envelopment analysis, efficiency, ranking*

JEL Classification: C44

AMS Classification: 90C08

1. INTRODUCTION

Data envelopment analysis (DEA) models are widely applied tools for efficiency and performance evaluation of a set of homogeneous decision-making units (DMUs). Each unit produces several outputs (effects) and consumes several inputs (resources) to produce them. DEA models are based on solving an optimization problem that maximizes the efficiency score of the unit under evaluation under the assumption that the efficiency scores of all other units of the set do not exceed 1. First model of this category was introduced by Charnes et al. (1978). This model can be formulated as follows:

$$\begin{aligned}
 &\text{Maximize} \quad \phi_q = \frac{\sum_{k=1}^r u_k y_{qk}}{\sum_{j=1}^m v_j x_{qj}}, \\
 &\text{subject to} \quad \frac{\sum_{k=1}^r u_k y_{ik}}{\sum_{j=1}^m v_j x_{ij}} \leq 1, \quad i = 1, \dots, n, \\
 &\quad \quad \quad v_j, u_k \geq \varepsilon, \quad j = 1, \dots, m, k = 1, \dots, r,
 \end{aligned} \tag{1}$$

where n is the number of DMUs, m the number of inputs and r the number of outputs. x_{ij} is the value of the j -th input for the DMU $_i$. and y_{ik} is the value of the k -th outputs for the same unit. u_k , $k = 1, \dots, r$, and v_j , $j = 1, \dots, m$, are the weights of the DMUs. They are variables of the model that maximizes the efficiency score ϕ_q of the unit under evaluation DMU $_q$. Model (1) is not linear in its objective function but can be linearized using Charnes and Cooper

transformation quite easily. The result is a multiplier version of the model that either maximizes the numerator of the objective function or minimizes its denominator. This leads either to an input-oriented or output-oriented DEA multiplier model. From the computational points of view, it may be better to solve dual models that are called DEA envelopment models. An output oriented DEA envelopment model follows:

$$\begin{aligned}
 &\text{Maximize} && \phi_q \\
 &\text{subject to} && \\
 &&& \sum_{i=1}^n x_{ij} \lambda_i \leq x_{qj}, \quad j = 1, \dots, m, \\
 &&& \sum_{i=1}^n y_{ir} \lambda_i \geq \phi_q y_{qr}, \quad r = 1, \dots, s, \\
 &&& \lambda_i \geq 0, \quad i = 1, \dots, n.
 \end{aligned} \tag{2}$$

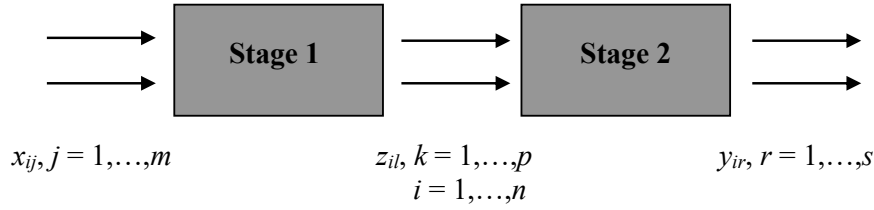
According to Model (2) and other traditional DEA models, the efficiency scores of efficient units are all the same (equal to 1), which does not allow for their complete ranking. Therefore, various models have been proposed that do allow for such ranking. An important category of this type are super-efficiency models. The application of these models leads to the calculation of super-efficiency scores that are greater than 1. The main representatives of these models are the Andersen and Petersen (1993) model and the slack-based super-efficiency model - Tone (2002).

The aim of this paper is to deal with other important category of DEA models – network DEA models. In line with reality, these models view the production process as a network rather than merely as a relationship between multiple inputs and multiple outputs. A nice review of basic network DEA models is given in Kao (2014). We will focus on the simplest case, which is two-stage serial DEA models. We will review the latest advances in this category of models and also examine the potential applications of super-efficiency models in this field.

In this paper, we will deal with two-stage serial DEA models. Next section contains formulation of the most important models of this class, and discussion about their advantages and shortcomings. Section 3 contains an illustrative example and a comparison of the results of previously formulated models. The last section points out the main findings and discusses future research.

2. METHODOLOGY

Traditional DEA models evaluate efficiency of the units in one stage – multiple inputs are transformed into multiple outputs. However, production processes can be more complex. Outputs of a stage may enter as inputs to the next stages. Then we talk about network production systems. The efficiency and performance of such systems can be analyzed using network DEA models. The simplest case of network production process is two-stage serial system. It is illustrated on Figure 1. Stage 1 considers m -inputs and produces p -outputs. In our simplified case, all outputs of the first stage enter as inputs of the stage 2. Second stage produces s -final outputs of the production process.

Figure 1: *Two-stage serial production process.*

Let us denote x_{ij} , $i = 1, \dots, n$, $j = 1, \dots, m$, the value of the j -th input for the DMU, z_{ik} , $i = 1, \dots, n$, $k = 1, \dots, p$, the value of the k -th intermediate variable for the DMU $_i$, and y_{ir} , $i = 1, \dots, n$, $r = 1, \dots, s$, the value of the r -th final output. Two-stage and multi-stage serial DEA models are frequently discussed in the literature, and several models addressing them have been formulated in the past. This is an interesting and complex problem because improving the efficiency of one stage (for example, by increasing outputs) leads to worsening the performance at the next stage, provided that the outputs of the preceding stage serve as inputs to the following one. Among the best-known two-stage serial DEA models is the model formulated by Kao and Hwang (2008). Output-oriented version of this model that evaluates the efficiency of the DMU $_q$ is formulated as follows:

$$\begin{aligned}
 &\text{Maximize} && \phi_q \\
 &\text{subject to} && \\
 &\sum_{i=1}^n x_{ij} \lambda_i \leq x_{qj}, && j = 1, \dots, m, \\
 &\sum_{i=1}^n z_{ik} \lambda_i - \sum_{i=1}^n z_{ik} \mu_i \geq 0, && k = 1, \dots, p, \\
 &\sum_{i=1}^n y_{ir} \mu_i \geq \phi_q y_{qr}, && r = 1, \dots, s, \\
 &\lambda_i \geq 0, \mu_i \geq 0, && i = 1, \dots, n,
 \end{aligned} \tag{3}$$

where λ_i and μ_i , $i = 1, \dots, n$ are the weights of the DMUs in the first and second stages respectively, and ϕ_q is the efficiency score of the unit under evaluation. The efficiency score obtained by model (3) is always greater or equal to 1. For this reason, reciprocal values are often used, as they are easier to explain—a higher value indicates a higher level of efficiency. One of the major criticisms of Model (3) is that the efficiency scores obtained using this model are very low and essentially never reach the maximum value of 1. In this context, no unit is identified as efficient. In this section, we will further present alternative approaches trying to reduce this shortcoming.

Despotis et al. (2025) proposed an approach that eliminates the mentioned shortcoming. That is, their model ensures that at least one of the evaluated units will be efficient and will have an efficiency score of 1. The efficiency score of a unit under evaluation is derived as a geometric average of two sub-efficiency scores obtained by the following two models (4) and (5) – let us suppose an output-oriented version of radial DEA models.

Maximize
subject to

$$\begin{aligned} \phi_q^{1*} &= \phi_q^1 \\ \sum_{i=1}^n x_{ij} \lambda_i &\leq x_{qj}, \quad j = 1, \dots, m, \\ \sum_{i=1}^n z_{ik} \lambda_i &\geq \phi_q^1 z_{qk}, \quad k = 1, \dots, p, \\ \sum_{i=1}^n y_{ir} \lambda_i &\geq \phi_q^1 y_{qr}, \quad r = 1, \dots, s, \\ \lambda_i &\geq 0, \quad i = 1, \dots, n. \end{aligned} \quad (4)$$

and

Maximize
subject to

$$\begin{aligned} \phi_q^{2*} &= \phi_q^2 \\ \sum_{i=1}^n x_{ij} \lambda_i &\leq x_{qj}, \quad j = 1, \dots, m, \\ \sum_{i=1}^n z_{ik} \lambda_i &\leq z_{qk}, \quad k = 1, \dots, p, \\ \sum_{i=1}^n y_{ir} \lambda_i &\geq \phi_q^2 y_{qr}, \quad r = 1, \dots, s, \\ \lambda_i &\geq 0, \quad i = 1, \dots, n. \end{aligned} \quad (5)$$

Model (4) considers inputs of the first stage as factors that influence the outputs of both the first and second stages simultaneously. In the similar way, model (5) assumes that the final outputs of the second stage are influenced by the inputs of both stages. Since models (4) and (5) are output-oriented, we consider the reciprocal value of their efficiency scores, and the final efficiency score of the entire two-stage system for the DMU_q is

$$\varphi_q = \left(\frac{1}{\phi_q^{1*} \phi_q^{2*}} \right)^{1/2}. \quad (6)$$

A DMU is efficient if it is efficient in both models (4) and (5), i.e. the overall efficiency score $\varphi_q = 1$. When this model is applied, there may be multiple efficient units. If the user requires a unique ranking of units, one of the super-efficiency models or another tool for distinguishing between efficient units can be used. The objection to this model is that, for inefficient units, it is not possible to obtain target values for projection onto the efficient frontier.

Khezrimotlagh and Zhu (2023) offer an approach that consists of two steps. The first step is identical to the previous method, i.e., when evaluating DMU_q, model (4) is solved. The target values defined by this model is $(x_{qj}, \phi_q^{1*} z_{qk}, \phi_q^{1*} y_{qr})$ are added as the $(n+1)$ -th DMU to the set of units and the following model is solved:

Maximize
subject to

$$\begin{aligned} \phi_q^{2*} &= \phi_q^2 \\ \sum_{i=1}^{n+1} x_{ij} \lambda_i &\leq x_{qj}, \quad j = 1, \dots, m, \end{aligned} \quad (7)$$

$$\begin{aligned} \sum_{i=1}^{n+1} z_{ik} \lambda_i &\leq \phi_q^{1*} z_{qk}, & k=1, \dots, p, \\ \sum_{i=1}^{n+1} y_{ir} \lambda_i &\geq \phi_q^2 \phi_q^{1*} y_{qr}, & r=1, \dots, s, \\ \lambda_i &\geq 0. & i=1, \dots, n. \end{aligned}$$

The overall efficiency score may be computed as a geometric average (6) or a simple product of the optimal objective functions of models (4) and (7). Like the Kao and Hwang (2008) model, this model does not guarantee that the maximum efficiency¹ will be achieved for at least one unit in the set.

As a last option for evaluating the efficiency of two-stage serial systems, three efficiency scores using standard DEA model (2) can be calculated. The first of them can be derived from the first stage, the second one from the second stage, and the last one by omitting the intermediate variables. Final efficiency score may be computed again as a product or geometric average of all three sub-scores.

3. RESULTS

The results of all presented models will be illustrated on an example that originates from Despotis et al. (2025). In this example, there are considered 2 inputs, 2 intermediate variables and two outputs that are used for description of 22 automotive companies. They are expressed as follows:

Table 1 - *Dataset*

	X1	X2	Z1	Z2	Y1	Y2
Toyota	370871	476451	277269	22477	168557	5.97
Volkswagen	671206	579730	282903	20558	97256	29.81
Daimler	298656	338677	193444	6415	59147	2.49
Ford	190001	258538	155901	550	37861	0.01
Fiat	191753	110055	122964	6610	23216	2.04
BMW	133779	270620	116697	8099	53964	8.37
Hyundai	122218	166864	90716	3103	27072	9.72
Peugeot	208781	78126	83686	7082	21345	4.01
AUDI	90641	77582	62352	4363	38522	100.26
Suzuki	67722	31209	35516	2976	20729	3.63
Mazda	49999	26399	32697	756	7156	0.92
Subaru	34201	29178	28954	1748	17744	1.69
Mitsubishi	31315	18442	23068	1026	8029	0.82
Isuzu	37264	19549	19716	1622	9838	1.38
Nissan	4033	2476	5531	72	1192	0.38
Sanyang	2302	1331	1081	15	566	0.09
IFAD	1050	389	132	20	238	0.06
GM	164001	228038	137238	8394	52051	4.82
VOLVO	103986	55409	28986	1513	35424	1.87
KIA	52579	47478	49881	1725	15237	0.01
SAIC	216361	122957	122045	1659	40339	0.32
Honda	219723	187318	145757	6664	48345	3.17

X1 – the number of employees,

X2 – total assets,

Z1 – revenues,

Z2 – earnings before interest and taxes,
 Y1 – market capitalization, and
 Y2 – earnings per share.

The efficiency scores of the models discussed above are included in Table 2. In more detail, this table contains:

- (1) A geometric average of two partial efficiency scores computed by traditional CCR model under the assumption of constant returns to scale. Both scores are derived from first and second stages independently.
- (2) A geometric average of three partial efficiency scores. The first two comes from both stages as in the previous case, the third one considers all inputs of the first stage and all outputs of the second stage only. "
- (3) Efficiency scores obtained by Kao and Hwang (2008) – model (3).
- (4) Efficiency scores computed by models (4) and (5) and their geometric average as proposed by Despotis et al. (2025).
- (5) Results of models (4) and (7) and their geometric average in accordance with the methodology introduced by Khezrimotlagh and Zhu (2023).

Table 2 - *Results*

	XZ/ZY	XZ/XY/ZY	Kao	Despotis	Zhu
DMUs	(1)	(2)	(3)	(4)	(5)
Toyota	0.6611	0.7261	0.3935	0.9653	0.9653
Volkswagen	0.4069	0.3612	0.1471	0.4742	0.5540
Daimler	0.4486	0.4251	0.1738	0.5519	0.6623
Ford	0.7735	0.6125	0.1714	0.7898	0.7898
Fiat	0.3638	0.3520	0.1316	0.5305	0.5747
BMW	0.5993	0.6548	0.3530	0.8841	0.8841
Hyundai	0.5043	0.4807	0.2200	0.6175	0.7064
Peugeot	0.4200	0.4174	0.1639	0.6276	0.6422
AUDI	0.9662	0.9774	0.9336	1.5818	13.2916
Suzuki	0.6512	0.7513	0.3961	1.0616	1.1477
Mazda	0.4487	0.4382	0.1644	0.5193	0.6506
Subaru	0.6982	0.7870	0.4773	1.1042	1.2235
Mitsubishi	0.5354	0.5806	0.2778	0.7732	0.8277
Isuzu	0.5886	0.6451	0.3058	0.8647	0.8809
Nissan	0.7144	0.7331	0.3498	0.9189	0.9189
Sanyang	0.6065	0.6256	0.2759	0.8255	0.8255
IFAD	0.7343	0.7940	0.3746	0.9635	0.9635
GM	0.5332	0.5582	0.2778	0.7638	0.7821
VOLVO	0.6121	0.7178	0.3747	0.9935	0.9935
KIA	0.5445	0.5492	0.2597	0.7681	0.8002
SAIC	0.5356	0.5275	0.1998	0.6495	0.7590
Honda	0.4551	0.4446	0.1994	0.6019	0.6737

As can be seen, according to the first three models, no DMU is identified as efficient. According to the remaining two models, the following three units are consistently identified as efficient: AUDI, Suzuki, and Subaru. It is worth noting that these models operate in two phases, and the resulting efficiency score is the geometric mean of the scores derived from both phases. These units were therefore the only ones that were efficient in both phases. To distinguish them, we used the Andersen and Petersen model for the model by Khezrimotlagh and Zhu (2023) , and

SBM model for the model by Despotis et. al. (2025). They consistently ranks AUDI first for both models, followed by Subaru and Suzuki. It is clear that these two models lead to very similar results, with the model by Khezrimotlagh and Zhu (2023) consistently achieving a higher or equal level of efficiency compared to the model by Despotis et al. (2025). Table 2 also shows that the ranking of DMUs by efficiency scores is quite similar. This can be seen from the data in Table 3. This table contains a sum of differences (in absolute values) of rankings over all DMUs. All values are very low, with the lowest values for the last two models. We do not present the exact ranking of the DMUs, but it is easy to derive this. Andersen and Petersen model has serious disadvantages. One of them is that this model overestimates the super-efficiency scores in certain cases. In our example it is for AUDI, and it is given by extremely high the second final output in comparison with other units. The results derived by SBM model looks much better.

Table 3 – *Differences in ranking of DMUs*

	XZ/ZY	XZ/XY/ZY	Kao&Hw	Despotis	Zhu
	(1)	(2)	(3)	(4)	(5)
XZ/ZY	0	30	61	53	49
XZ/XY/ZY		0	35	29	27
Kao&Hwang			0	38	24
Despotis				0	16
Zhu					0

4. CONCLUSIONS

The problem of evaluation of efficiency of a set of DMUs within network production systems is widely discussed in the literature. This article provides an overview of the current models for evaluation of efficiency in two-stage serial production systems which is the simplest case of network structures. All presented models have their advantages but also shortages. If the decision maker requires rather a ranking of the DMUs it is possible to use all approaches as they lead to very similar results. For detailed analysis of results, models by Despotis et al. (2025) or Khezrimotlagh and Zhu (2023) are the best options.

Acknowledgements: The research is supported by the Faculty of Informatics and Statistics. Prague University of Economics and Business, project no. F4/46/2025.

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A Memetic Algorithm for Charging Infrastructure Location Planning in Electric Bus Systems

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Abstract

As the application of electric buses is becoming more common in public transport, a few challenges arise to keep their operation sustainable and efficient. The main challenge of electric buses is their need for charging, which is connected to a charging infrastructure location problem. In this paper we propose an algorithm based on a Memetic algorithm metaheuristic to solve the problem of locating the charging stations and compare it with a Genetic algorithm in order to find potential improvements, which comes from a local search stage. The numerical experiments were performed on real-world data from the city of Žilina. The results show that the local search used in Memetic algorithms can highly improve the obtained results.

Keywords: *Memetic algorithm, electric bus, charging infrastructure, genetic algorithm, local search*

JEL Classification: *JEL C61*

AMS Classification: *AMS 90C27*

1 INTRODUCTION

The electrification of urban public transport has accelerated in recent years as cities seek to reduce emissions, noise, and environmental impacts while improving energy efficiency. Battery electric buses have emerged as a leading solution, supported by advances in battery technology, charging power, and vehicle reliability, prompting many operators to move from pilot projects to large-scale fleet deployment. This transition, however, introduces planning and operational

challenges that differ fundamentally from conventional diesel-bus systems, most notably the design of charging infrastructure. Charging stations entail significant investment, and inadequate placement or capacity can lead to infeasible schedules, increased operational costs, or reduced service quality. Because electric buses operate under fixed timetables with limited route flexibility, charging opportunities must be carefully synchronized with vehicle schedules, making the charging infrastructure location problem inherently coupled with scheduling and energy feasibility considerations.

Early simulation-based studies already demonstrated that ignoring detailed operational constraints may lead to misleading infrastructure designs. For instance, Vasilovský et al. verified candidate charging station locations for electric buses using simulation-based feasibility analysis, showing that charging conflicts and battery depletion may arise even in seemingly well-designed infrastructure layouts if schedule interactions are neglected [6]. Their work highlighted the importance of realistic simulation in evaluating infrastructure decisions, a principle that remains highly relevant in current research.

In recent years, research has increasingly moved toward integrated optimization models that explicitly consider charging infrastructure location, charger configuration, and vehicle scheduling. Liu et al. developed a comprehensive optimization model capturing charger-battery power matching and seasonal variations in energy consumption, demonstrating that

environmental conditions can significantly affect infrastructure requirements [3]. Similarly, He et al. proposed a joint optimization framework for charging infrastructure planning, vehicle scheduling, and charging management, showing that integrated approaches can substantially reduce total system costs compared to sequential planning methods [2]. Recent review papers confirm that such integrated formulations are central to electric bus system design research, particularly for transit networks with dense schedules and high vehicle utilization [1].

Despite their advantages, integrated mathematical optimization models often suffer from scalability issues when applied to large, realistic instances. This leads to usage of heuristic and metaheuristic methods such as evolutionary algorithms. Genetic algorithms have been widely applied to electric bus charging and scheduling. However, purely population-based approaches may struggle to effectively exploit promising regions of the search space. Therefore, recent studies increasingly adopt hybrid metaheuristics that combine global evolutionary search with problem-specific local improvement strategies. Memetic algorithms represent a particularly powerful class of such hybrid methods, integrating evolutionary operators with local search procedures. Recent applications demonstrate their effectiveness in transportation and energy-constrained problems, including electric vehicle routing with charging decisions [5], charging-synchronized feeder services [4], and integrated charging and energy storage scheduling in transit systems [7]. These studies consistently report substantial improvements in solution quality compared to standard genetic algorithms, albeit at the cost of increased computational effort. To address the gaps in studies that combine terminal-based opportunity charging, explicit modeling of charging point capacity constraints, and simulation-based feasibility evaluation within a memetic optimization framework, we developed a memetic algorithm to charging infrastructure location problem for electric buses with fixed schedules. The proposed approach is validated on real-world public transport data from the city of Žilina under multiple seasonal scenarios. To quantify the contribution of local search, the memetic algorithm is compared with a genetic algorithm using the same representation and evaluation framework.

2 PROBLEM DESCRIPTION

We are dealing with a charging infrastructure location problem for electric buses. The infrastructure used is the charging stations, which can be located on specified terminals. Each charger can have multiple charging points, meaning that multiple electric buses can be charged at the same time on the same charger. The core decision in this problem is where to build the charging stations and how many charging points are needed on each charger. The decisions are connected to the schedules of electric buses. In our problem we assume that the schedules are fixed and charging can be done only during time between service trips. To increase the efficiency of charging, we only allow charging after the end of service trip, and it can be done only on a charger located on the last terminal station of the service trip. We do not allow detours to other chargers.

As we are using electric buses, we need to address problems connected to their limited driving range. This means, we need to keep track of the energy state of each electric bus. If the energy state drops below minimal state (usually empty battery) the schedule is infeasible. The main objective is to minimize the number of charging stations needed as well as the number of charging points at each charging station. The solution should also satisfy these constraints:

- All schedules should be feasible – battery does not drop below minimal state, while each bus starts the day with full battery.
- At each charger, multiple electric buses can be charged but are limited by a number of available charging points.
- During charging the maximal battery capacity cannot be exceeded.

3 METHODS

As this problem is quite complex the use of a heuristic algorithm is justified. We decided to develop a heuristic algorithm based on one of the standard metaheuristic algorithms – Memetic algorithm. This algorithm is one of the evolutionary metaheuristics and is very similar to the genetic algorithm with the addition of local search procedure, which can help to create better solutions in shorter time and therefore improve the solving efficiency.

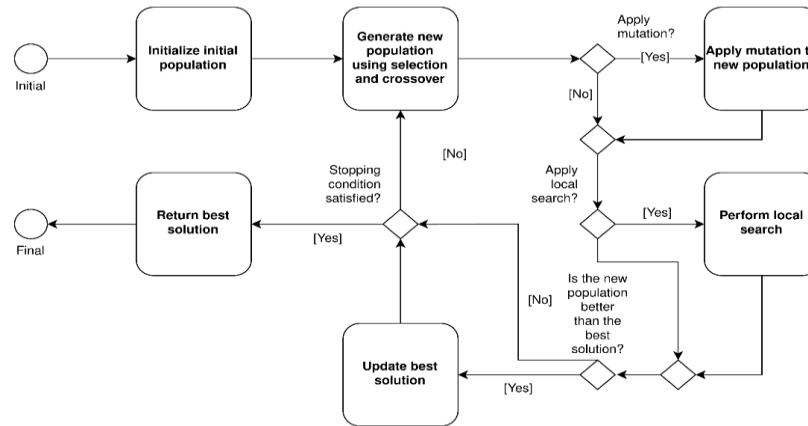


Figure 1. Memetic algorithm scheme

The memetic algorithm consists of three parts (Figure 1). The first part is the creation of initial population of solutions. The second part is the main loop where new solutions are created and the populations are exchanged. This part is the main process of finding new better solutions. The new solutions are mainly created by crossover operator, which combines parts of the selected parent solutions to create new ones. The next operator is mutation, which is trying to improve the diversity of the newly created solutions. The addition of the memetic algorithm is a local search operator, which is usually some local search heuristic. It is trying to find good solutions in the near neighborhood of the current solution, which improves the exploitation phase of the metaheuristic, while the crossover and mutation are mainly used for exploration of new potential parts of solution space. The third part of the algorithm is the final evaluation part, which checks whether the termination criterion is met. If yes, it returns the best- found solution, otherwise the algorithm continues. The termination criterion was defined as a number of iterations without change of the best-found solution.

3.1 Solution representation and fitness

The solution to the problem is defined by locations of the charging stations and the number of charging points at each station. We use a vector of integers in our algorithm as the solution representation. Therefore, the vector has a length of number of potential charging locations, where 0 is representing state, when at the location no charger is built and a positive integer indicates a built charger with specified number of charging points. The fitness function is directly connected to the objective function and is evaluated as a sum of costs for building charging stations and charging points. During the process of creating new solutions an infeasible solution can be obtained. Therefore, we apply a penalty function to these infeasible solutions to worsen their fitness. The penalty for infeasibility is a multiple of the charging station cost for each unfinished electric bus schedule. The initialization of the initial population is made by a heuristic, which randomly chooses charging locations and number of charging points at the charging station. After that a local search is applied to improve the initial population.

3.2 Operators

The crossover starts with a selection mechanism, which chooses the parent solutions. The selection is made via a Roulette wheel algorithm. In this algorithm each solution has a probability of being chosen as a parent solution proportional to their quality represented by a fitness function. Due to fact we are minimizing the fitness we use a reciprocal of fitness function. The crossover is performed by a randomly created binary mask vector. If the cell of the mask contains 0 the child solution copies the cell from first parent solution and if the cell contains 1, the child copies the cell from the second parent solution. The mutation is performed as a slight change of solution. We randomly choose a charging location and add or remove a charging point. After that we check whether we created a charging station without charging points and if yes, this charging station is cancelled (the solution vector cell for this location is set to 0).

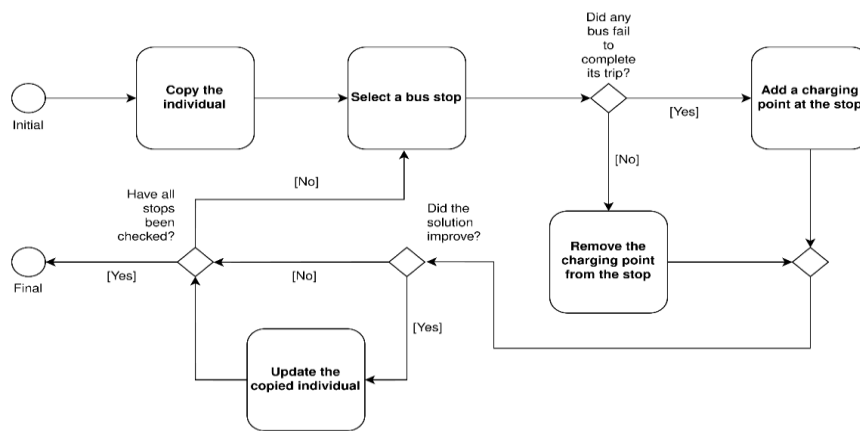


Figure 2. Scheme of the local search operator

The local search is performed as follows. We create a copy of the individual solution. After that we go through all charging locations, where we add or remove charging points. The decision whether we add or remove the charging point is based on the feasibility evaluation. If all the buses performed their schedules without dropping their energy state below minimum, we remove the charging point. Otherwise, we add the charging point. Then we run the feasibility evaluation once more and check if the fitness of the new solution is better than the previous one. If so, we update the solution and continue until we run through all the charging locations. A basic scheme of the local search is shown at Figure 2.

3.3 Solution feasibility simulation

To check whether the solution is feasible and to determine the penalty function we propose a discrete event simulation. This simulation can be described by a diagram at Figure 3. The simulation operates in parallel with all electric bus schedules. Each schedule is seen as a list of events - serving a service trip, transfer and charging. The events are then processed based on their starting time. During the service trips and transfers the energy is consumed, while during charging the energy is charged to the battery. During the simulation, we specifically check each electric bus battery state. If the battery state dropped below minimum state, we report this fact and end the schedule with a failure. The number of failed schedules determines the penalty for infeasibility. When the electric bus is at the charging station with an available charging point after finishing the service trip, it charges for maximum available charging time. This time is determined by the transfer time to next service trip in the schedule.

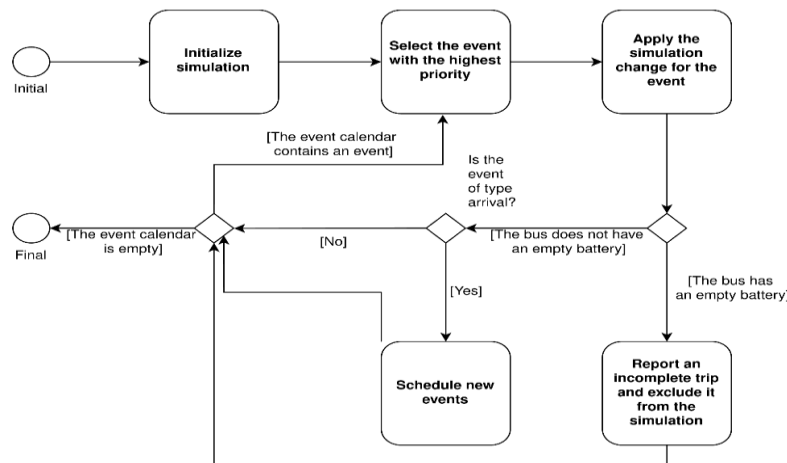


Figure 3. Activity diagram of the solution feasibility discrete event simulation

4 NUMERICAL EXPERIMENTS

To test the efficiency of the proposed method and to evaluate the impact of the local search, we performed numerical experiments on real world data provided by the public transport operator in the city of Žilina. The testing benchmark contained actual schedules of 82 buses and trolleybuses with 1729 service trips scheduled. We considered a situation where all the vehicles were exchanged for electric buses. As for the number of charging locations, we considered every terminal station, where service trips were finished as well as the depot location, meaning 17 available charging locations. The maximum number of charging points at each location was limited to 3, while each charger had a charging power of 80kW.

Besides that, we prepared three different scenarios regarding the battery capacity and the energy consumption of the electric buses. The first scenario is denoted Basic. It is connected to the spring and autumn seasons, which does not have many factors influencing consumption and capacity. The second scenario is denoted Summer, connected to summer season, when energy consumption is increased due to higher outdoor temperatures and need for running air-conditioning system. The last scenario is denoted Winter, connected to winter season, when the outdoor temperatures are usually lower and drivers run the heating system. This increases the energy consumption as well as decreases the maximal battery capacity due to low temperature. A summary of the values used in each scenario is in Table 1.

Table 1. Seasonal scenario description

Scenario	Energy consumption	Battery capacity
Basic	0.8 kWh/km	140 kWh
Summer	1.0 kWh/km	140 kWh
Winter	1.0 kWh/km	112 kWh

As for the parameter regarding the cost of each charging station: the construction cost of charging station was set to 50 000€ and construction of a single point on already constructed station was set for 2 500€. This setting was chosen to mainly minimize the number of charging stations needed. The parameters of the algorithm were set after a tuning process as follows: Population size: 50, Termination criteria: 25 iterations without change of best solution, Probability of mutation: 0.15, Probability of local search: 0.35, Penalty for failed schedule: 100 000€. The experiments were performed on a machine with an Intel Core i7-4790, 3.6 GHz processor and 16GB of RAM.

4.1 Results

Table 2. Numerical experiments results

Algorithm	Scenario	Computation time (s)		Incomplete schedules		Fitness	
		Best	Average	Best	Average	Best	Average
Genetic (GA)	Basic	7,67	8,14	0	0	485000	650250
	Summer	7,64	8,14	4	4	647500	947750
	Winter	7,65	8,11	10	10,1	915000	1047250
Memetic (MA)	Basic	125,84	128,65	0	0	327500	366250
	Summer	160,25	176,36	4	4	537500	579500
	Winter	154,13	168,94	5	5	657500	784500

We can see the results of the algorithm run in Table 2. We list also the average values as we run each algorithm on each scenario 10 times to better evaluate the performance and eliminate outliers. We can see that both the Memetic and Genetic algorithms achieved quite good results in a short time on all scenarios. We can also see that the local search procedure highly increases the solution time, which is over 15 times higher than in case of genetic algorithm without local search. However, the fitness was much better in the case of Memetic algorithm than in case of Genetic algorithm. This shows that the local search operator has a high impact on the quality of the solution. This is also supported by a metric counting a number of incomplete schedules. In case of Basic and Summer scenarios, the results are similar. But the Winter scenario shows high difference in number of incomplete schedules, where Memetic algorithm produces much better solution.

5 CONCLUSIONS

This paper proposes a memetic algorithm for the charging infrastructure location problem and evaluates its performance against a genetic algorithm using real-world data across three scenarios. The results demonstrate that incorporating a local search operator significantly improves solution quality, achieving improvements of over 20% in all scenarios, albeit at the cost of substantially increased computation time. Future research should focus on benchmarking the proposed algorithms against optimal solutions where available and identifying factors that influence the efficiency of the solution process across diverse problem instances.

Acknowledgements: This work was supported in part by the Scientific Grant Agency of the Ministry of Education, Research, Development and Youth of the Slovak Republic under the project VEGA 1/0516/26 “Optimal design of charging infrastructure for electric vehicles and its efficient operation in urban mobility conditions”.

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MULTIPLE IMPUTATION AND INFERENCE IN COMPLEX SURVEY DATA: EVIDENCE FROM HBSC

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Abstract

Missing data are a persistent challenge in large-scale survey research and may influence econometric inference, particularly when complex sampling designs are used. This paper examines the impact of multiple imputation on parameter estimation and statistical precision in survey-weighted ordered logistic regression models. Using pooled data from the Health Behaviour in School-aged Children (HBSC) study across selected European Union member states, inference based on complete-case estimation is compared with results obtained after multiple imputation. The analysis focuses on differences in coefficient estimates, standard errors, statistical significance, and the Fraction of Missing Information as an indicator of uncertainty due to missing values. The results show that coefficient estimates remain largely stable across models, while multiple imputation moderately improves estimation precision by restoring observations lost due to item nonresponse and thereby reducing standard errors. These findings highlight the importance of properly addressing missing data in complex survey analyses and demonstrate the benefits of multiple imputation for improving the efficiency of econometric inference.

Keywords: *Complex survey design, Missing data, Multiple imputation, Ordered logit, HBSC*

JEL Classification: C25, C81, C83

AMS Classification: 62F30, 62J12, 62D05

1 INTRODUCTION

Missing data are a common feature of large-scale survey datasets and represent an important challenge for empirical quantitative research. Item nonresponse may arise from respondent fatigue, sensitive questions, recall errors, or survey routing procedures, potentially reducing the effective sample size and distorting statistical inference. When missing values are ignored in econometric estimation, the resulting analysis relies only on observations with complete information for all variables included in the model. This may reduce efficiency and lead to biased results if missingness is systematically related to observed or unobserved characteristics (Little and Rubin, 2019; Enders, 2010).

The issue becomes particularly important when analysing data obtained from complex sampling designs. Large international surveys typically employ stratification, clustering and unequal sampling probabilities, which require the use of sampling weights and design-based variance estimation to obtain unbiased population inference (Binder, 1983; Heeringa, West and Berglund, 2017). The presence of missing data in such settings complicates statistical analysis because both sampling uncertainty and imputation uncertainty must be properly accounted for. Multiple imputation (MI) has become one of the most widely recommended approaches for handling missing data in applied research. Instead of replacing missing values with a single estimate, MI generates several plausible values drawn from an imputation model and combines the results across multiple completed datasets using Rubin's rules (Rubin, 1987). Among practical implementations, multiple imputation by chained equations (MICE) is particularly popular because it can handle variables with different distributions and complex dependence

structures (Azur et al, 2011; van Buuren, 2018).

Despite the widespread use of MI, empirical studies often provide limited evidence on how imputation affects inference in econometric models estimated using complex survey data. The purpose of this study is therefore to examine the impact of multiple imputation on parameter estimates and statistical precision in survey-weighted ordered logistic regression models. The empirical application uses pooled data from the Health Behaviour in School-aged Children (HBSC) study across several European Union member states. The analysis focuses on differences in coefficient estimates, standard errors and the Fraction of Missing Information as indicators of uncertainty attributable to missing data.

2 DATA AND METHODS

This study uses pooled data from the Health Behaviour in School-aged Children (HBSC) survey, an international cross-sectional study conducted in collaboration with the World Health Organization that collects harmonised information on health behaviours and social determinants of health among adolescents aged 11, 13 and 15 years (Inchley et al, 2020). The analysis uses data from four survey waves (2006, 2010, 2014 and 2018) across selected European Union member states. After harmonisation, the dataset contains 439,466 observations. The estimation sample used in the complete-case analysis is smaller because observations with missing values in the variables included in the model are excluded.

HBSC employs a complex multistage sampling design with stratification and clustering. Schools represent primary sampling units (PSUs), sampling strata correspond to geographical or administrative regions used in the sampling design, and sampling weights adjust for unequal selection probabilities and survey nonresponse to ensure population-representative estimates. In order to obtain valid population inference, all models in this study are estimated using survey-adjusted procedures that incorporate sampling weights, stratification and clustering, with variance estimation based on Taylor linearisation (Binder, 1983; Heeringa, West and Berglund, 2017).

As the dependent variable, we use BMI (body mass index) categories derived from self-reported height and weight. For the purposes of this study, the five BMI categories defined by the WHO child growth reference based on BMI-for-age z-scores (de Onis et al, 2007) are aggregated into three ordered categories: 0 – normal weight or underweight, 1 – pre-obese (overweight), and 2 – obese (including severe obesity).

The explanatory variables include demographic, socioeconomic and behavioural characteristics. Demographic variables comprise survey year, age category (11, 13 and 15 years) and sex. Socioeconomic status is represented by the father's employment status and by an SES index constructed from the Family Affluence Scale using principal component analysis based on the number of cars, number of computers and family holidays. Behavioural variables include weekday breakfast consumption and the number of days with at least 60 minutes of physical activity during the previous week.

Missing values occur in several socioeconomic and behavioural variables. To address missingness, we apply multiple imputation by chained equations (MICE). Recent methodological research emphasises that when multiple imputation is applied to complex survey data, the imputation procedure should account for survey design features such as sampling weights and auxiliary information to ensure design-consistent inference (Kalpourtzi, Carpenter and Touloumi, 2024; Xu and Reiter, 2024).

Let the dataset contain observed values Y_{obs} and missing values Y_{mis} . The imputation procedure generates M completed datasets by drawing plausible values from conditional distributions given by (1).

$$P(Y_j | Y_{-j}, \theta_j) \quad (1)$$

where: Y_{-j} denotes all remaining variables in the dataset and θ_j represents the parameters of the imputation model (van Buuren, 2018).

After generating M imputed datasets, the substantive econometric model is estimated separately for each dataset and the results are combined using Rubin's rules. The pooled estimator is given by (2).

$$\bar{\beta} = \frac{1}{M} \sum_{m=1}^M \hat{\beta}_m \quad (2)$$

while the total variance is given by (3).

$$T = \bar{U} + (1 + \frac{1}{M})B \quad (3)$$

where $\bar{U}$ denotes the within-imputation variance and B denotes the between-imputation variance (Rubin, 1987).

The outcome variable is modelled using a survey-weighted ordered logistic regression model. The model assumes the existence of a latent variable y_i^* defined as (4).

$$y_i^* = \mathbf{X}_i \boldsymbol{\beta} + \varepsilon_i \quad (4)$$

where: $\mathbf{X}_i$ is a vector of explanatory variables, $\boldsymbol{\beta}$ is a vector of parameters and ε_i follows a logistic distribution (Greene, 2018). The observed outcome y_i is determined by threshold parameters separating BMI categories.

To evaluate the impact of missing data treatment on econometric inference, results obtained using complete-case estimation based on observed data are compared with estimates obtained after multiple imputation. The comparison focuses on changes in coefficient estimates, differences in standard errors and the Fraction of Missing Information (FMI), defined as (5).

$$\text{FMI} = \frac{(1 + \frac{1}{M})B}{T} \quad (5)$$

where: M is the number of imputations, B is the between-imputation variance and T is the total variance of the pooled estimator.

The relative variance increase (RVI), reported in the estimation output, measures the proportional increase in variance attributable to missing data (Rubin, 1987) and is defined as (6).

$$\text{RVI} = \frac{(1 + \frac{1}{M})B}{\bar{U}} \quad (6)$$

All data preparation, multiple imputation procedures and econometric estimations were performed using Stata 19 (StataCorp, 2025).

3 RESULTS

The survey-weighted ordered logistic regression model estimated using complete-case observations included 336,217 observations, 6,739 primary sampling units (PSUs), and 72 strata. The results indicate that later survey waves are associated with a higher probability of belonging to higher BMI categories. Relative to the baseline year 2006, the coefficients for 2010 ($\beta = 0.150$), 2014 ($\beta = 0.210$), and 2018 ($\beta = 0.280$) suggest an increasing trend in adolescent overweight and obesity over time.

Age shows a negative association with BMI categories. Adolescents aged 13 ($\beta \approx -0.249$) and 15 ($\beta \approx -0.478$) are less likely to belong to higher BMI categories compared with younger adolescents aged 11. Gender differences are also substantial, with males exhibiting a significantly higher probability of belonging to higher BMI categories ($\beta \approx 0.709$). Socioeconomic status measured through the SES index has a protective effect, while behavioural variables such as regular breakfast consumption and higher levels of physical activity are associated with a lower likelihood of overweight and obesity.

After applying multiple imputation with 20 imputations, the estimation sample increased to 439,466 observations, with 6,931 PSUs and 78 strata. The re-estimated survey-weighted ordered logistic regression produced results that are highly consistent with those obtained from the model estimated using the complete-case observations.

Table 1 presents a comparison of coefficient estimates and standard errors obtained from both models. The differences in coefficient estimates between the models are very small. For most parameters, the difference between the estimates obtained before and after imputation is below 0.01, indicating that the treatment of missing data does not substantially alter the substantive relationships between BMI categories and the explanatory variables. For instance, the coefficient associated with the survey year 2010 changes only slightly from $\beta = 0.150$ to $\beta = 0.149$, while the coefficient for physical activity changes from $\beta = -0.0869$ to $\beta = -0.0870$. Similar stability is observed for demographic variables such as age and gender, suggesting that the underlying relationships captured by the ordered logistic specification are robust to the treatment of missing data.

Table 1 Comparison of Coefficients and Standard Errors for CC and MI Models

Variable	β (Complete-Case)	SE (CC)	β (MI)	SE (MI)	$\Delta\beta$	SE Ratio
Survey Year 2010	0.150	0.017	0.149	0.016	-0.001	0.93
Survey Year 2014	0.210	0.017	0.232	0.015	0.022	0.89
Survey Year 2018	0.280	0.017	0.281	0.016	0.001	0.93
Age 13	-0.249	0.013	-0.248	0.011	0.001	0.88
Age 15	-0.478	0.014	-0.485	0.012	-0.007	0.89
Male	0.709	0.010	0.703	0.009	-0.006	0.92
SES	-0.100	0.005	-0.098	0.005	0.002	0.94
Physical Activity	-0.087	0.0025	-0.087	0.0023	-0.0001	0.92

Note: $\Delta\beta$ represents the difference between coefficient estimates after and before imputation. SE ratio represents the ratio of standard errors (MI / complete-case estimates).

At the same time, the results reveal a systematic reduction in standard errors after multiple

imputation, which indicates improved estimation precision due to the increased effective sample size obtained through imputation. For example, the standard error for the 2010 survey year coefficient decreases from 0.017 to 0.016, while similar reductions are observed for most other parameters. The ratio of standard errors between the imputed and original models is therefore typically below one, suggesting that the inclusion of imputed observations increases the efficiency of estimation.

The Fraction of Missing Information (FMI) indicates that missing data contribute moderately to estimation uncertainty. The average Relative Variance Increase (RVI) is approximately 0.25, meaning that missing data increase the variance of the parameter estimates by roughly 25% relative to the variance that would be obtained with fully observed data. The largest FMI reaches approximately 0.35, implying that for some parameters about one third of the total estimator variance is attributable to missing information. These values indicate that missing data represent a non-negligible source of estimation uncertainty and highlight the importance of addressing missingness when conducting inference in complex survey data.

Figure 1 further illustrates the differences in coefficients between the two models. The values cluster closely around zero, confirming that the estimated relationships remain stable after imputation while precision improves.

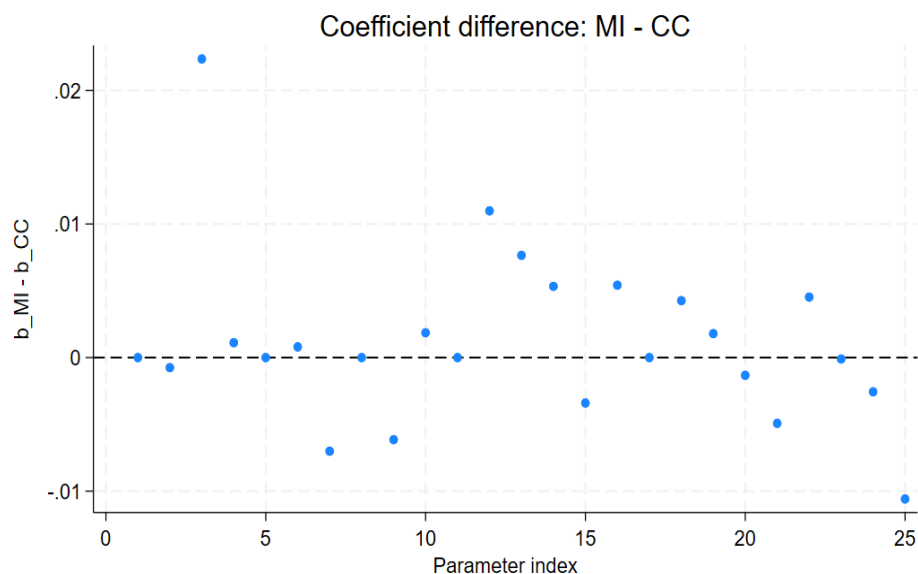


Figure 1 Coefficient differences between models estimated before and after multiple imputation.

The estimated coefficients also provide a brief overview of the determinants of adolescent obesity in the analysed European Union sample. Later survey waves are associated with higher probabilities of belonging to higher BMI categories, indicating an increasing trend in adolescent overweight and obesity over time. Male adolescents show significantly higher odds of belonging to higher BMI categories compared with females. Socioeconomic status has a protective effect, as higher SES is associated with a lower likelihood of overweight and obesity. Behavioural factors also play an important role: more frequent breakfast consumption and higher levels of physical activity are associated with lower probabilities of belonging to higher BMI categories. Importantly, these relationships remain consistent across both estimation approaches, indicating that the treatment of missing data does not alter the substantive interpretation of the determinants of adolescent obesity. These findings are broadly consistent with previous studies on adolescent health in Europe, which similarly

document increasing obesity prevalence and highlight the role of socioeconomic conditions and lifestyle behaviours such as diet and physical activity (Inchley et al, 2020; Ahrens et al, 2014).

4 DISCUSSION AND CONCLUSIONS

The results demonstrate that multiple imputation improves the statistical precision of survey-weighted econometric estimates without substantially altering substantive conclusions. While coefficient estimates remain highly stable across models, the reduction in standard errors indicates that the use of imputed data increases the efficiency of estimation by incorporating information from observations with partially missing data. Similar findings have been reported in recent methodological research emphasizing the advantages of MI in survey data analysis (van Buuren, 2018; Woods et al, 2024).

The observed FMI values further confirm that missing data contribute meaningfully to estimation uncertainty. This supports recommendations from recent literature that emphasise the importance of using formal imputation procedures instead of relying solely on observed cases in survey-based research (White, Royston and Wood, 2011; Kalpourtzi, Carpenter and Touloumi, 2024; Xu and Reiter, 2024).

This study examined the impact of multiple imputation on econometric inference in survey-weighted ordered logistic regression models using pooled HBSC data. The results show that coefficient estimates remain stable across complete-case and multiply imputed models, while standard errors are generally smaller in the imputed models.

These findings suggest that multiple imputation improves estimation efficiency without altering substantive conclusions. The results therefore support the use of multiple imputation as a recommended approach for handling missing data in econometric analyses based on complex survey datasets.

Acknowledgements: This study was supported by the Slovak Research and Development Agency under Contract No. APVV-24-0412 Sustainable and Healthy Food Consumption: Effects of Socio-economic Factors and Policies on Eating Behaviour, Obesity, and Food System.

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GENETIC ALGORITHM FOR COMBINED CHARGING INFRA-STRUCTURE DESIGN PROBLEM

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Abstract

Transportation has major influence on greenhouse gasses production. Reduction of emissions and associated costs can be achieved by the support of public transport and alternative fuels such as electric buses. Major limitation of current electric buses is mostly in their driving range that is much smaller in comparison to the combustion engine buses. This problem can be solved by continuous charging during breaks between trips or overnight charging in the depot and in-motion charging.

We propose an optimization approach to design a charging infrastructure for a fleet of electric buses in urban areas with the respect to current schedules of the vehicles that must be fulfilled. We are dealing with the location of the charging stations for continuous charging of electric buses in the terminal stops of the lines and charging sections on the infrastructure for in-motion charging. We formulate a mathematical model that combines location and scheduling decisions to ensure that requirements of electric buses can be satisfied and propose a solution method based on genetic algorithm.

Keywords: *charging infrastructure, hybrid trolleybuses, in-motion charging, genetic algorithm*

JEL Classification: C61

AMS Classification: 90C27

1 INTRODUCTION

Transportation has major influence on greenhouse gasses production. Reduction of emissions and associated costs can be achieved by the support of public transport and alternative fuels such as electric buses. Major limitation of current electric buses is mostly in their driving range that is much smaller in comparison to the combustion engine buses. This problem can be solved by continuous charging during breaks between trips or overnight charging in the depot and in-motion charging.

There are several charging technologies, in this paper we will focus on charging at selected stops or charging stations and continuous charging while the vehicle is driving (so-called in-motion charging). In [1], the authors developed a non-linear mathematical model for charging station placement and charging scheduling in an urban area and tested it on bus route data in Salt Lake City. In paper [6], the authors present a mathematical model that minimizes the number of used charging stations and simulation verification of the proposed solution. The simulation approach was also chosen by the authors in [7], where they compared the possibilities of different types of charging within continuous charging and their economic benefits. In the case of the issue of building charging sections for continuous vehicle charging, there are few available resources. The authors in [1] and [2] published findings from observations of in-motion charging vehicle operation in the city of Gdynia. Authors of the

article [5] created a linear mathematical model of the problem and solved the problem through heuristics.

2 MATHEMATICAL MODEL OF THE PROBLEM

Our goal is to design an efficient charging infrastructure which consists of charging sections and charging stations. When solving this problem, we will focus on minimizing the costs of building the infrastructure. The formulation of the mathematical model ensures compliance with the capacity limitations of the batteries of all vehicles. Specifically, the model guarantees that the battery charge level will not be lower than the specified minimum limit and will not fall below this limit during operation. The model also includes a simplification consisting in the assumption that the built charging station will be equipped with enough charging stands to serve all vehicles charging at a given time. The mathematical model of the problem was originally described in [8]. Decision variables in the model can be defined as follows:

- $y_i \in \{0, 1\}$ – Decision variable that determines whether a charging line will be built on section i ($y_i = 1$) or not ($y_i = 0$).
- $z_k \in \{0, 1\}$ – Decision variable that determines whether a charging station will be built at a possible location $k \in K$ ($z_k = 1$) or not ($z_k = 0$).
- $x_{r,j+1}$ – The state of charge of the battery of vehicle r after performing activity j .
- $h_{r,j}$ – Charging station index, if the vehicle is at a charging station during activity j .

Constants in the model can be defined as follows:

- B_{max} – maximal allowed level of the state of charge of the battery in kWh.
- B_{min} – minimal allowed level of the state of charge of the battery in kWh.
- $b_{r,j} \in \{0, 1\}$ – determines the type of activity within the vehicle rotation:
 - $b_{r,j} = 1$: movement on the line,
 - $b_{r,j} = 0$: movement in terminus,
- m – number of vehicles.
- D_i – length of the section i .
- $a_{r,j}$ – index of the j -th activity on the route of the vehicle r .
- $q(r)$ – number of activities of the vehicle r .
- C_o – energy consumption (in kWh).
- C_h – amount of the energy charged on the section with overhead wires (in kWh).
- $CN_{r,j}$ – amount of the energy charged on charging station during the activity j (in kWh).
- E – cost per one meter of charging section.
- F – cost per one charging station.

Mathematical model of the problem can be defined as follows:

$$\min \sum_{i=1}^n E * D_i * y_i + \sum_{k \in K} F * z_k \quad (1)$$

$$\text{s. t. } x_{r,1} = B_{max} - B_{min} \quad \text{for } r = 1, \dots, m \quad (2)$$

$$\begin{aligned} x_{r,j+1} - x_{r,j} \leq & -CO * D(a_{r,j}) * (1 - y(a_{r,j})) + CH & \text{for } r = 1, \dots, m \\ & * D(a_{r,j}) * y(a_{r,j}) + B_{max} * (1 & \text{for } j = 1, \dots, \\ & - b_{r,j}) & q(r) \end{aligned} \quad (3)$$

$$x_{r,j+1} - x_{r,j} \leq CN_{r,j} * z(h_{r,j}) + B_{max} * b_{r,j} \quad \text{for } r = 1, \dots, m \quad (4)$$

$$\begin{aligned}
 x_{r,j+1} &\leq B_{max} - B_{min} & \text{for } j = 1, \dots, q(r) & \quad (5) \\
 y_i &\in \{0,1\} & \text{for } i = 1, \dots, n & \quad (6) \\
 z_k &\in \{0,1\} & \text{for } k \in K & \quad (7) \\
 x_{r,j} &\geq 0 & \text{for } j = 1, \dots, q(r) & \quad (8)
 \end{aligned}$$

The objective function (1) minimizes the cost of building charging sections and charging stations. Constraints (2) provide an initial battery charge level for the start of the shift for each vehicle r . Constraints (3) determine the maximum amount of energy to charge at section j for vehicle r . Constraints (4) the maximum amount of energy to charge at the charging station during the j -th activity. Constraints (5) determine the maximum amount of energy that vehicle r can recharge during charging during activity j .

3 SOLUTION APPROACH

Genetic algorithm is a metaheuristic inspired by the biological process of evolution. The basic principle is based on combining and improving the properties of already existing good solutions [9]. The basic scheme of a genetic algorithm can be described by the following steps:

Step 1. At the beginning of the genetic algorithm run, an initial population is created, which is the basis for further evolution. The population is composed of randomly generated chromosomes that represent solutions to the problem.

Step 2. A fitness value is expressed for each chromosome contained in the population.

Step 3. The stopping criterion is checked. The most common stopping criterion is reaching the maximum number of generations or the expiration of the allocated time window for the algorithm. If the criterion is evaluated positively, the algorithm reaches point 6.

Step 4. Creation of a new population with the same number of individuals as the original population. To create a whole new population from an existing one, the operations explained below are used, but the following steps are required:

- a. We select two chromosomes by selection, which will be the parents for the next, new chromosome. We then apply the crossover operations and then mutations to them.
- b. New chromosomes are added to the new population.
- c. Then points a and b are repeated until the new population is full. The new population will be the next generation.

Step 5. Replacing the current population with the new generation created in the previous point and then moving to point 2.

Step 6. Returning the best solution found, which appeared in all generations during the algorithm. End of the algorithm process.

3.1 Adaptation of genetic algorithm for combined charging infrastructure problem

In the solved problem, we make decisions on the location of charging sections and decisions on the location of charging stations, including determining the exact number of stands at charging station in a possible location. Thanks to this, we can determine that the final solution consists of two main parts - decisions on the selected sections and decisions on the number of charging stations. However, it is also necessary to ensure the possibility of building more than one stand in one charging station. To describe the solution in our approach we have selected two different vectors – separate vectors would contribute to clarity and unambiguity of the connection to the real solution. The disadvantage remains the slightly more complex formulation of appropriate operations of the genetic algorithm.

As part of the adaptation of the genetic algorithm to the solved problem, we decided to adapt the mutation and crossover operations in the genetic algorithm. Below we will explain how we will adapt these operations with respect to the chosen representation of an individual using two separate vectors.

Mutation: Representing an individual using two separate vectors brings various challenges, especially when performing the mutation operation. The application of the mutation operation will be performed on each vector separately. This will ensure that the integrity of the given vectors is maintained. In our approach we used the creep mutation. It allows for fine-tuning of chromosome values. Its application can be performed on both real and integer numbers. It adjusts the value of a selected gene by a selected value or a value generated from a given interval by adding or subtracting from the original value.

Crossover: Like the mutation operation, it is necessary to approach individual vectors separately when performing this operation with the crossover operation. Each vector is a separate part of the solution and therefore it is necessary for the individual parts to be independent of each other. This avoids mixing genes between vectors, which could lead to infeasible solutions. In our approach we used the uniform crossover.

Selection: In our approach we used the tournament selection that consists of randomly selecting k individuals from a given population P . Individuals from the selected subset then participate in a tournament T where the best individual B is selected based on the fitness value. If greater efficiency is required, the tournament can be played in multiple stages. The tournament size is a variable value k , and should satisfy $1 < k < m$, where m is the population size. The entire population should not be selected for the tournament, because the selection would always select the best solution and would result in a local optimum.

4 NUMERICAL EXPERIMENTS

Numerical experiments are designed to verify the correctness of the implementation and at the same time serve to examine various options for setting the parameters of the genetic algorithm. All experiments were carried out using the Java language in JDK version 22. A personal computer with the Windows 11 Home operating system, an Intel(R) Core (TM) i7-1065G7 CPU @ 1.30GHz 1.50 GHz processor and 16GB of installed RAM was used for their implementation. For each numerical experiment, 10 runs of the genetic algorithm were performed, and the obtained values were subsequently recorded and processed.

In this work, we used data provided by the Žilina City Transport Company for the purposes of testing the genetic algorithm. The data used contains transparent data from real-life public transport operations in the city of Žilina. Dataset contains 7 days in October 2016 (Monday to Sunday) with 82 vehicles with approx. 2000 trips.

The ŠKODA 26TrA Solaris vehicle was chosen for the purposes of our research, namely a hybrid trolleybus, which is part of the fleet of the transport company of the city of Žilina. This type of vehicle was chosen due to its real-world use and the combination of trolleybus and battery drive, which is consistent with the subject of the task. The selected vehicle has a battery with a capacity of 45.40 kWh and, under operating conditions and in each period, has a minimum range of 12 km. [3] For the purposes of the simulation, some parameters were set as constant, based on the real possibilities of the infrastructure and the technical specifications of the selected electric vehicle. The charging performance parameter at the charging stations is set to 1.667 kWh/min and the charging speed parameter on the charging section has a value of 0.75 kWh/min. The minimum charging time was chosen as constant, namely 3 minutes, which is based primarily on practical constraints.

The genetic algorithm also evaluates the value of the objective function, i.e. the fitness of the given solution. This value expresses the total required amount for building the necessary infrastructure that the given solution represents. Infrastructure includes building charging sections, charging stations and stands. The simulation sets the costs for building one kilometre of charging section at 1 000 000 €. Building one charging station costs 500 000 € and 50 000 € is the cost required for building one charging stand at the charging station. These three parameters are constant for each experiment performed.

The experimentation is carried out using three scenarios. Each scenario is aimed at simulating representative climate changes occurring in individual seasons. Climate changes mainly affect the level of discharge and capacity of the vehicle battery. The level of discharge was calculated from the driving range of the simulated vehicle in the most demanding scenario on the capacity and level of battery discharge. The starting battery level was set to 80% of its maximum capacity in all scenarios, while the absolute value in kWh varies depending on the total battery capacity for a given scenario. The spring scenario represents ideal climatic conditions without significant limitations. During this period, the battery capacity is highest, and the discharge level is lowest, as the vehicle does not require excessive heating or air conditioning. The summer scenario differs from the spring scenario primarily in increased energy consumption, which is approximately 25% higher due to the vehicle's air conditioning in the hot summer months. However, the battery capacity remains the same. The winter scenario has the greatest negative impact on the vehicle's parameters. The battery discharge level is approximately 25% higher compared to the spring scenario, due to the intensive heating of the vehicle. At the same time, the maximum available battery capacity decreases by approximately 20% in the winter months, which is caused by the influence of low temperatures.

These three scenarios were deliberately chosen for their ability to cover the range of possible real-world operating conditions of an electric vehicle throughout the year, thus allowing a more

thorough assessment of their impact on the overall efficiency of operation. Based on previous experiments the initial parameter setting for all experiments is listed in Table 1.

Tab. 1 Initial parameter setting

	Summer	Spring	winter
Population size	50	50	50
Number of generations	35	25	35
Elite set size (%)	0	5	5
Mutation probability	0,75	0,75	0,75

4.1 Population size

In this part of the experiment, the effect of population size was analysed. The assumption was that a larger population size would contribute to increased diversity and thus to the discovery of better-quality solutions. The same population sizes were the values: 15, 25, 35 and 50.

Tab. 2 Population size

Scenario	Population size	Best time (sec.)	Total time (sec.)	Average objective (mil. €)	Best objective (mil. €)
Summer	15	53	568	180,93	167,45
	25	87	886	177,25	172,81
	35	113	1 255	176,72	170, 89
	50	171	1 992	174,54	170,29
Spring	15	29	340	182,97	174,80
	25	52	555	177,26	171,83
	35	75	779	176,94	170,60
	50	127	1 518	174,84	170,43
Winter	15	46	488	192,13	187,25
	25	82	865	186,24	180,58
	35	126	1 270	185,22	178,34
	50	173	1 953	183,37	177,54

Based on the results in the Table 2, in all three scenarios, the best results were again achieved with the largest tested population (50 individuals). It can also be observed that despite a significant increase in computational time, the lowest objective function values were achieved when using the largest population.

4.2 Number of generations

The experiment focusing on the number of generations with improved parameters was performed similarly to the first phase with values of 15, 25 and 35 generations.

Tab. 3 Population size

Scenario	Number of generations	Best time (sec.)	Total time (sec.)	Average objective (mil. €)	Best objective (mil. €)
Summer	15	80	991	194,80	188,35
	25	136	1417	181,29	175,35
	35	171	1992	174,54	170,29
Spring	15	69	708	190,56	186,73
	25	127	1518	174,84	170,43
	35	178	1813	166,14	161,78
Winter	15	76	794	206,87	199,46
	25	132	1366	190,89	184,35
	35	173	1953	183,37	177,54

The obtained results in the Table 3 for all three scenarios achieved the best values of the objective function at the parameter value of 35. Here it is possible to observe a gradual increase in the computational time, which also brings with it better solutions. The obtained results confirm the importance of the number of generations performed in the genetic algorithm, with a higher value of their number ending in a higher quality solution obtained.

4.3 Mutation probability

Experimentation with the mutation probability was carried out using the probability parameter value equal to 0.5 and 0.75. At the same time, testing of the parameter was also carried out for the interval 0.3 to 0.7.

Tab. 4 Mutation probability

Scenario	Mutation probability	Best time (sec.)	Total time (sec.)	Average objective (mil. €)	Best objective (mil. €)
Summer	0,5	156	1636	173,10	169,18
	0,75	171	1992	174,54	170,29
	0,3 - 0,7	150	1606	166,66	159,81
Spring	0,5	93	1001	176,31	169,89
	0,75	127	1518	174,84	170,43
	0,3 - 0,7	95	997	170,51	165,30
Winter	0,5	146	1517	184,90	178,25
	0,75	173	1953	183,37	177,54
	0,3 - 0,7	147	1500	176,95	173,72

The summarized results in Table 4 clearly show the dominance of the adaptive mutation probability, where the interval ranges between 0.3 and 0.7 in all the scenarios performed. Increasing the population size and the number of generations led to a larger number of solutions,

which highlighted the importance of maintaining quality solutions. Adaptive mutation proved to be effective in this regard, as it reduces the mutation probability for quality solutions, while it increases this probability for lower quality solutions, thus contributing to a better optimization process.

5 CONCLUSIONS

The aim of this paper was to design and implement an effective solution for the combined charging infrastructure problem in urban public transport, specifically for hybrid trolleybuses. Based on the definition of the problem, a mathematical model was presented. We proposed the solution approach based on the genetic algorithm. An important part was the implementation of a simulator to verify the admissibility of the created solutions based on real data provided by the Transport Enterprise of the Žilina Municipality. The vehicle in the simulator had to meet the minimum battery charge level during the entire operation time and at the same time was not allowed to exceed the maximum permitted daily distance travelled solely on the battery. The practical value of the proposed solution was also confirmed by numerical experiments, which were focused on optimizing the parameters of the genetic algorithm. Three different scenarios were simulated, representing different seasons, which allowed for a comparison of the behaviour of the algorithm and the results of optimization in different climatic conditions. Based on the experiments performed, suitable algorithm parameters were selected to maximize the efficiency and robustness of the solution.

Acknowledgements: This work was supported by the VEGA grant 1/0516/26 „Optimal design of charging infrastructure for electric vehicles and its efficient operation in urban mobility conditions“.

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AIR HANDLING UNIT PRODUCTION PROCESS IMPROVEMENT USING DISCRETE EVENT SIMULATION MODEL IN SIMUL8

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Abstract

Simulation modelling represents an effective tool for analysing and improving production processes without direct intervention in real systems. It enables testing of various scenarios and evaluation of their impact on system performance and efficiency. This paper focuses on the production process of Air Handling Unit (AHU) panels in JANKA Engineering. A discrete event simulation model was developed in SIMUL8 based on real production data. Several simulation experiments were conducted to identify bottlenecks and evaluate potential improvements. The results indicate that implementing a more powerful Trumatic machine and adjusting workforce allocation in selected operations can significantly enhance production performance, without necessarily increasing the total number of employees.

Keywords: *simulation modeling, discrete event simulation, production optimization, SIMUL8, air handling unit*

JEL Classification: C44, C63, L23

AMS Classification: 90B30, 37M05

1 INTRODUCTION

Simulation modelling has become an important tool for analysing complex production systems where analytical approaches are not applicable, supporting decision-making without direct intervention in the production environment (Banks, 1998). As part of operations research and management science, simulation modelling integrates statistical and computational methods to evaluate system performance under varying conditions (Rotaru et al., 2014). In production systems with stochastic parameters, discrete event simulation (DES) is commonly used (Brailsford, 2014), as it allows modelling of dynamic processes with random variables, such as processing times and resource availability. The choice of modelling approach depends on the nature of the problem, software availability, and user requirements (Dlouhý et al., 2007, Robinson, 2025). Huynh et al. (2020) presents a case study from Model Factory at the Advanced Remanufacturing and Technology Centre Singapore where simulated DES data feed a production database to generate KPIs for performance monitoring. It also demonstrates how DES can be applied to optimize process and design parameters in the production flow. The study of Bajjou and Chafi (2021) demonstrates the evaluation of the impact of lean construction principles on production performance. Their model shows that applying lean principles increases productivity by 41%, boosts efficiency by 14%, and reduces cycle time by 17%.

This paper addresses a manufacturing problem in JANKA Engineering, a Czech company specializing in ventilation and air-conditioning systems. Despite its long tradition, the company faces challenges related to outdated technology, inefficient layout, and material flow.(JANKA.CZ, 2021). The study focuses on the production process of air-handling unit (AHU) panels. The objective is to analyse the current production system and evaluate improvement scenarios using simulation experiments. A DES model was developed and tested

using SIMUL8 software. Compared to existing studies, this paper focuses primarily on practical implementation and evaluation of improvement scenarios in a real industrial environment, rather than on methodological development or optimization techniques.

2 DATA AND METHODOLOGY

Simulation modelling is commonly applied to production systems with stochastic behaviour and complex interactions. DES models production systems as networks of queues and activities, where entities compete for limited resources (Dlouhý et al., 2007). The simulation time advances based on the occurrence of events, allowing efficient analysis of system performance and identification of bottlenecks. Typical applications of DES include evaluation of alternative system configurations, testing of “what-if” scenarios, and optimization of resource utilization (Jackson et al., 2016). The development of a simulation model generally follows several steps: problem formulation, conceptual modelling, data collection and analysis, model implementation, verification and validation, experimentation, and evaluation of results (Rotaru et al., 2014, Dlouhý et al., 2007, Wainer, 2017). The selection of modelling tools depends on the problem characteristics and the required level of detail. In this study, the SIMUL8 software was used due to its suitability for process-oriented simulation and ease of model development.

2.1 Case study: AHU panel production

The analysed system is the production of air-handling unit (AHU) panels in JANKA Engineering (JANKA.CZ, 2021). The company faced challenges related to outdated equipment, inefficient layout, and suboptimal material flow, which negatively affect production performance. The objective of the study is to develop a simulation model of the existing production system and to evaluate potential improvements. The proposed changes focus on increasing production capacity through better allocation of workers and the introduction of a more advanced machine (Poništ, 2018).

2.1.1 Conceptual Model

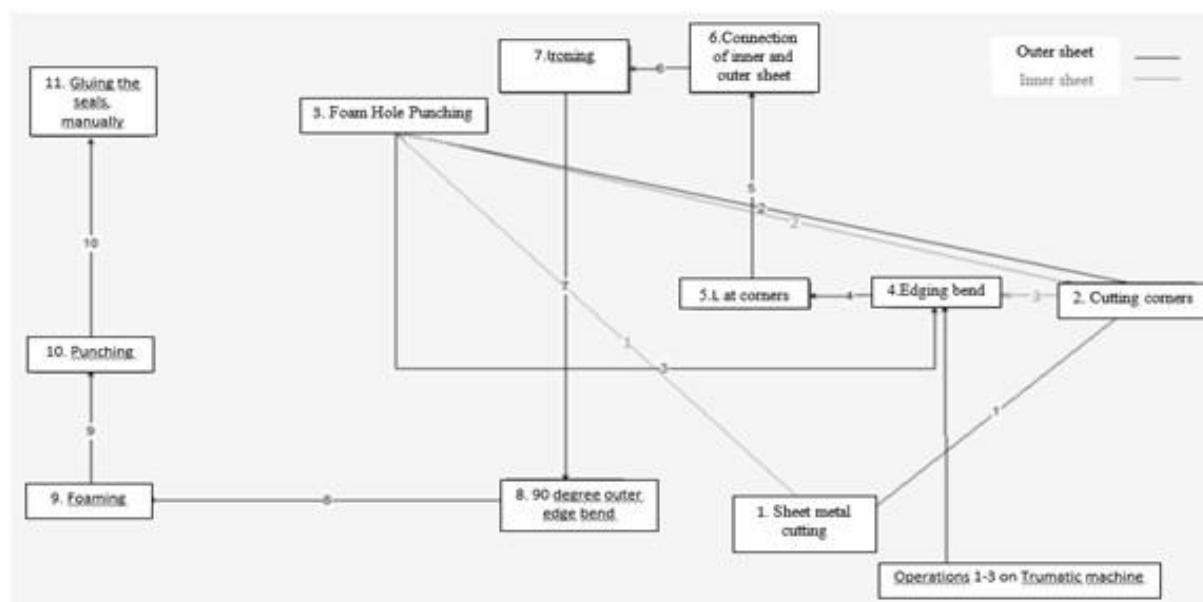


Figure 1 – Scheme of the production process (Poništ, 2018)

The production of AHU panels consists of several sequential operations, including sheet metal cutting, corner cutting, punching, bending, assembly of inner and outer sheets, pressing, foaming, and final finishing operations. Among these, the foaming process represents the most time-consuming operation. The production process involves both inner and outer panels, which follow slightly different processing sequences to ensure continuity and avoid bottlenecks. After separate processing, both parts are assembled and completed through subsequent operations. The conceptual model captures the flow of panels through individual workstations, including queues, processing times, and resource constraints. Special attention is given to machine utilization, worker allocation, and the identification of bottlenecks. Additionally, the model considers the potential integration of a Trumatic machine, which is currently not utilized in the analysed production line but represents a possible improvement scenario to increase production efficiency (Poništ, 2018).

2.2 Collection and analysis of input/output data

Input data for the simulation model were collected (Poništ, 2018) through direct observation of the production process over a three-month period. The collected dataset includes production volumes, processing times of individual operations, and material handling times. Table 1 summarizes production volumes and averages. Tables 2–5 summarize processing times, transport times, and workforce allocation based on company's data.

Table 1 – Production and average production

panel type	month1	month2	month3	avg. week	avg. day
small	365	369	392	90	18
middle	156	148	151	35	7
large	118	125	133	30	6
SUM	639	642	676	155	31

Table 2 – Time to finish the operation (in minutes)

time:	Operation											Trumatic
	1	2	3	4	5	6	7	8	9	10	11	
maximum	0.62	0.9	0.78	0.8	0.68	1.28	1.25	0.72	21.52	0.43	3.62	0.48
minimum	0.5	0.85	0.62	0.73	0.61	1.5	1.17	0.62	20.72	0.38	3.52	0.41
average	0.55	0.88	0.66	0.77	0.65	1.54	1.2	0.66	21.21	0.41	3.57	0.46

Table 3 – Time of material transfer to machines

time:	time for moving material from machine x to machine y											
	1-2	1-3	2-3	2-4	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11
maximum	2.9	3	3.2	1.2	2.8	1.4	1.2	1.1	1.8	2	1.3	2
minimum	2.3	2.1	2.5	0.5	1.7	0.7	0.6	0.7	1.1	1.3	0.8	1.3
average	2.58	2.64	2.91	0.9	2.1	1.16	0.93	0.88	1.5	1.68	1.09	1.71

Table 4 - Time of transport of product pallets in minutes

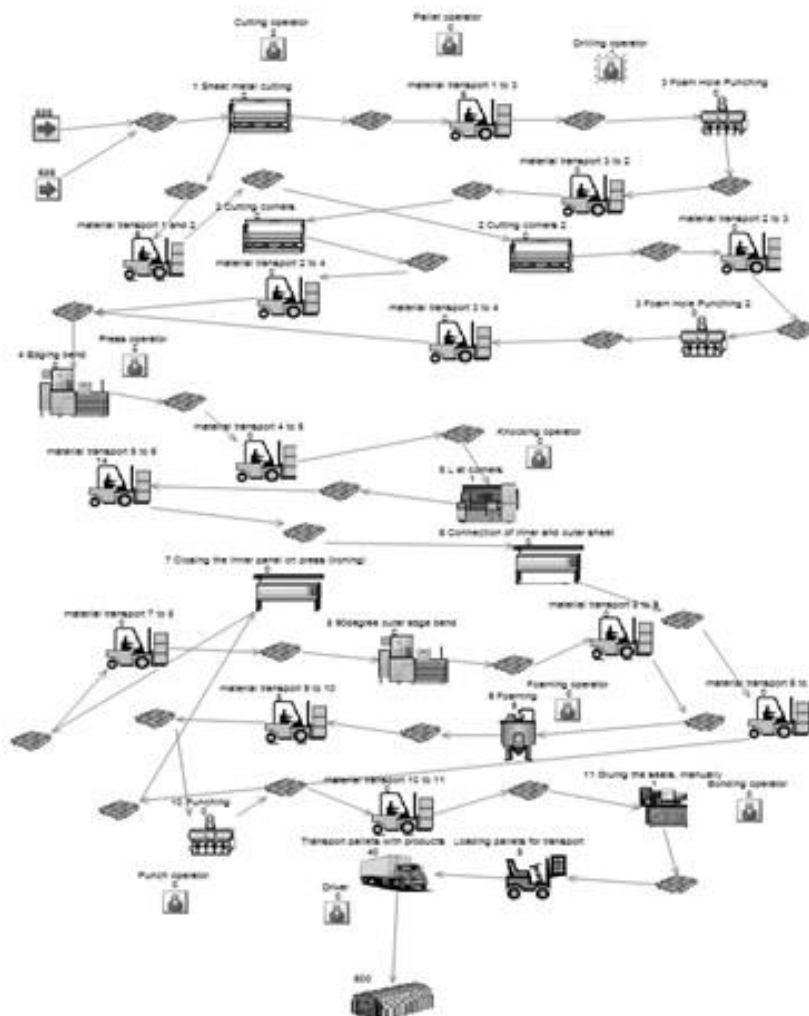
time	Transport of product pallets	Load of material
maximum	3.5	3.6
minimum	2.5	2.7
average	2.92	3.04

2.3 Simulation model

The simulation model was developed in SIMUL8 based on the collected input data. The model (see Figure 2) represents the production flow of AHU panels, where entities correspond to inner and outer sheets moving through a sequence of operations and transport activities.

Table 5 – Number of workers per activity

Activity:	Number of workers
Sheet metal cutting	2
Cutting corners	1
Foam Hole Punching	2
Edging bend	2
L at corners	1
Connection of inner and outer sheet	1
Closing the inner panel on the press (ironing)	1
90 degree outer edge bend	1
Foaming	2
Punching	1
Gluing the seals, manually	3
Trumatic	1
Material transport	5

**Figure 2** – Simulation model of the AHU production

The arrival of material into the system is modelled using a normal distribution with a mean inter-arrival time of 14 minutes and a standard deviation equal to 25% of the mean. After processing, finished panels are grouped into batches of 20 units and transported to the subsequent workplace. The model incorporates both human and machine resources. The

assumed worker productivity (60%) and machine efficiency (70%) were based on expert estimates provided by the company and reflect typical operating conditions of the system. Their impact is partially reflected in the scenario-based analysis presented in the experiments. Resources are assigned to individual operations according to the production requirements.

The production process differentiates between inner and outer sheets after the initial operation. While both entities share the same starting point, their subsequent routing differs based on predefined priority rules, ensuring continuity of the production flow. Processing and transport times were modelled using normal distributions derived from observed averages. Due to the limited size of the dataset and the practical focus of the study, formal goodness-of-fit tests (e.g., Kolmogorov–Smirnov or Anderson–Darling) were not applied. Instead, variability was approximated using standard deviations between 10% and 30% of the mean, which is a commonly used approach in applied simulation studies. This simplification represents a limitation of the model and may influence the accuracy of the results. The model structure consists of interconnected activities representing individual operations and transport steps.

2.4 Verification and validation

After the model development, verification was performed to ensure correct model functionality and logical consistency. Subsequently, the model was validated by comparing simulation outputs with real production data and observed system behaviour. The simulation was configured for an 8-hour working day with a one-month observation period. A total of 100 simulation runs were conducted to ensure statistical reliability of the results. The average number of generated inner sheets was 685.7, while the average number of finished panels reached 603.5, which closely corresponds to real production values. In addition to comparing average output values, the variability of simulation results across replications was analysed to ensure consistency with observed production performance. Although no formal hypothesis testing was conducted, the results fall within a comparable range of real system outputs. This confirms model validity. The simulation results identified a major bottleneck at the final operation (manual gluing of seals), where queues were formed. The average queue length reached 4.9 panels, with a maximum of 25 panels waiting for processing. Another critical issue was the high utilization of pallet operators, reaching approximately 99% with four operators, indicating limited transport capacity. In contrast, other workers showed lower utilization levels, partly due to uneven workload distribution and lower production demand. Material handling was identified as one of the most intensively used processes in the system. Although transport times are relatively short, occasional blocking occurs due to limited availability of resources.

2.5 Experiments with the model

The baseline simulation model revealed several issues related to workforce allocation, transport capacity, and machine performance. Therefore, a set of simulation experiments was designed to evaluate potential improvements in the production system. The experiments focused primarily on two areas: workforce optimization and technological upgrade of the production process. In addition, a scenario reflecting increased production demand was considered.

The following changes were implemented in the simulation model:

- a) The number of operators at the final operation (manual gluing of seals) was increased from 3 to 6.
- b) The number of pallet operators (for material handling) was increased from 4 to 8.
- c) A new, more efficient machine (Trumatic) was introduced into the production process, replacing three existing machines.

- d) The inter-arrival time of inner and outer sheets was reduced from 14 minutes to 8 minutes to simulate increased production demand associated with a new contract.

These scenarios were tested individually and in combination to evaluate their impact on system performance, particularly in terms of throughput, queue lengths, and resource utilization.

3 RESULTS AND DISCUSSION

A series of simulation experiments was conducted to evaluate the impact of workforce adjustments, machine upgrades, and increased production demand on system performance (see Table 6 and Figure 3 and 4). The baseline scenario (operator performance 60%) resulted in an average monthly production of approximately 604 AHU panels, with significant queues forming due to limited capacity in material handling and the final bonding operation. Increasing operator performance to 80% raised production to approximately 650 panels per month, representing the maximum output without structural changes.

Table 6 – Experiments settings and results

	No. of operators		Settings			Results		
	bonding	pallets operators	operators performance	Trumatic	new contract	Avg. number of pieces produced monthly	Avg. % bonding operators utilization	Avg. % pallet operators utilization
basic model	3	4	60%	0	no	604	73.92	99.04
experiment 1	3	4	60%	0	yes	833	85.32	99.56
experiment 2	3	4	60%	1	no	600	73.53	96.62
experiment 3	3	4	60%	1	yes	820	84.86	99.22
experiment 4	3	4	80%	0	no	650	55.02	96.12
experiment 5	3	4	80%	0	yes	1100	79.5	99.42
experiment 6	3	8	80%	0	no	650	54.92	61.7
experiment 7	3	8	80%	0	yes	1117	81.01	75.28
experiment 8	6	8	60%	0	no	650	57.63	71.22
experiment 9	6	8	80%	1	no	650	57.99	32.29
experiment 10	6	8	80%	0	yes	1147	44.01	70.49
experiment 11	6	8	80%	1	yes	1150	71.21	43.73
experiment 12	4	8	80%	0	yes	1150	66.18	72.51
experiment 13	4	8	80%	1	yes	1118	65.66	62.34

The introduction of the Trumatic machine, replacing three existing machines, did not significantly increase production output on its own. However, it enabled a reduction in the number of operators required in upstream operations, allowing their reallocation to bottleneck activities such as material transport and bonding. Increasing the number of pallet operators significantly improved system performance. With higher operator capacity and performance, production increased by approximately 8%, while resource utilization decreased to more sustainable levels. Under increased demand conditions, production reached approximately 1,115 panels per month; however, this resulted in higher utilization of bonding operators and longer queues. Further improvements were achieved by increasing the number of bonding operators. The highest production level (approximately 1,150 panels per month) was achieved with four bonding operators and eight pallet operators, while maintaining acceptable resource utilization. In scenarios with increased demand, improving operator performance proved to be a key factor. Without changes in workforce allocation, production growth remained limited even with the introduction of the Trumatic machine. However, when combined with higher operator performance and optimized workforce distribution, the system achieved significant

production increases. From a resource perspective, the use of the Trumatic machine enabled maintaining the original workforce size while increasing production capacity. In contrast, scenarios without this machine required additional operators to achieve similar production levels. Overall, the results demonstrate that a combination of workforce optimization and technological upgrade leads to the most effective improvement of the production system.

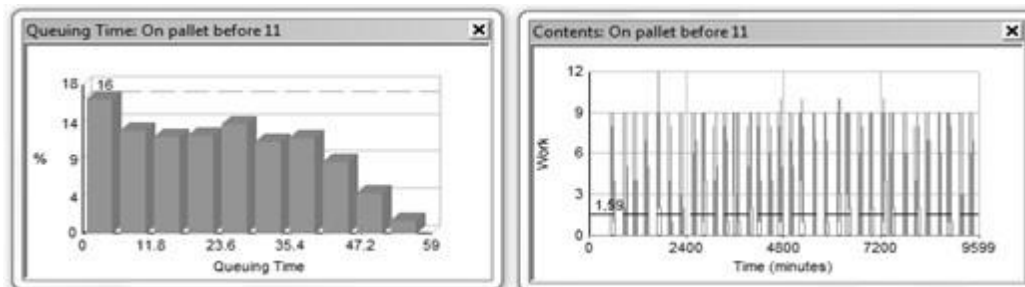


Figure 3 – Queuing time and the queue length before the start of activity 11 – experiment 1

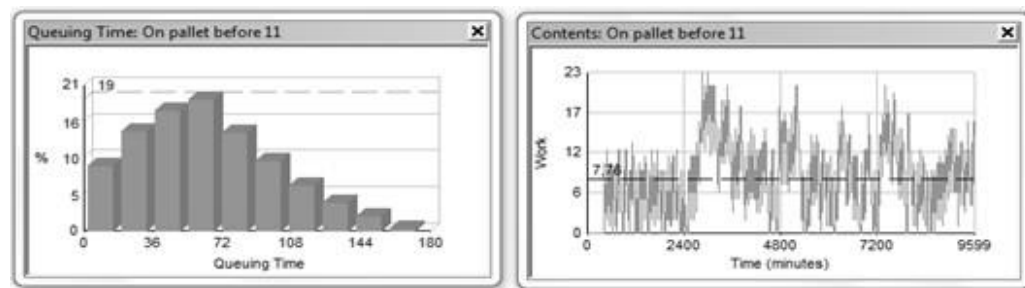


Figure 4 – Queuing time and the queue length before the start of activity 11 – experiment 7

4 CONCLUSIONS

Simulation modelling is an effective tool for production system analysis. This study demonstrated its applicability through a case study of AHU panel production in JANKA Engineering. A DES model was developed based on real data and used to evaluate multiple improvement scenarios. The results showed that the current production system is limited primarily by workforce allocation and bottlenecks in material handling and final assembly operations. The experiments confirmed that increasing operator performance and optimizing workforce distribution can significantly improve production output. In addition, the implementation of a more advanced machine (Trumatic) enables more efficient use of resources and reduces the need for additional workforce. Overall, the combined effect of these measures can increase production capacity by approximately 50–100% without necessarily increasing the total number of employees. These findings support managerial decision-making in production planning. Although the study does not represent a formal optimization approach, the scenario-based analysis provides actionable managerial insights and supports decision-making in real production conditions. A formal sensitivity analysis of key parameters was not conducted, which represents a limitation of the study. However, variability was partially addressed through stochastic modelling of processing and transport times. The model includes several simplifying assumptions. It does not consider shift scheduling, machine breakdowns, rework, or scrap rates. These factors may influence system performance and limit the generalisability of the results. Future research may address these aspects to increase model realism.

Acknowledgements: The research was supported by the grant No. F4/18/2024 of the Faculty of Informatics and Statistics, Prague University of Economics and Business.

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FUEL PRICES IN EUROPEAN UNION COUNTRIES FOLLOWING THE SHOCK CLOSURE OF THE STRAIT OF HORMUZ

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Abstract

In this paper, we compare price bias results from fuel market asymmetry using the static linear-exponential adjustment cost function, with dynamic oil shock effects from models such as the asymmetric error correction, threshold autoregressive cointegration, momentum threshold autoregressive cointegration, and the nonlinear autoregressive distributed lag. These asymmetric models allow the estimation of the half-life of oil price shocks, thereby supporting comparisons of countries' responses. Our goal is to use dynamic methods to examine how long a significant adverse oil price shock resulting from a blockade of the Strait of Hormuz will persist and how it will affect European economies. In particular, we aim to determine whether the results on price bias are related to those on the speed of adjustment.

Keywords: *linear exponential adjustment cost function, asymmetric error correction model, threshold autoregressive cointegration, nonlinear autoregressive distributed lag model*

JEL Classification: C26, C51, Q41

AMS Classification: 62P20, 91B84

1 INTRODUCTION

In our research, we have repeatedly examined the asymmetric responses of retail gasoline and diesel prices to oil price shocks (Szomolányi, Lukáčik and Lukáčiková, 2020; 2024; Lukáčik, Lukáčiková and Szomolányi, 2025). Using several methods, we examined markets in Slovakia and Croatia. The first group of methods consisted of simple asymmetry models that utilized variables to capture price increases and decreases, which we gradually expanded to include dynamic components. The second group consisted of models with an error-correction term, which allows for the analysis of non-stationary data and distinguishes between short-term and long-term asymmetric responses. The third group included threshold autoregressive cointegration models, which addressed the shortcomings of classical cointegration tests in the presence of asymmetry and differed in the methods used to determine the threshold value. Finally, we proposed using a linear-exponential function of adjustment costs (LINEX), which allows us to capture asymmetric adjustment costs and derive the value of the bias.

Unlike other methods, which do not always capture asymmetry in weekly data, the LINEX approach we used consistently yielded a specific value for the bias. Therefore, we used it to compare fuel markets across all European Union countries (Lukáčik, Lukáčiková and Szomolányi, 2025), yielding the results presented in Figure 1. Using this approach, we have identified differences across countries, but we cannot determine the average speed and duration of changes in fuel prices following an oil price shock, as these are defined by a static model (1). From this model, we can derive the average direction and bias. However, since the focus of this paper is on estimating the duration of the effects of an extremely severe and adverse shock to oil prices, which occurred following Iran's closure of the Strait of Hormuz in March 2026, we will return to the other dynamic models used. Another issue with the resulting comparison is the inconsistent and ad hoc market interventions in some countries, which will be discussed in the conclusion.

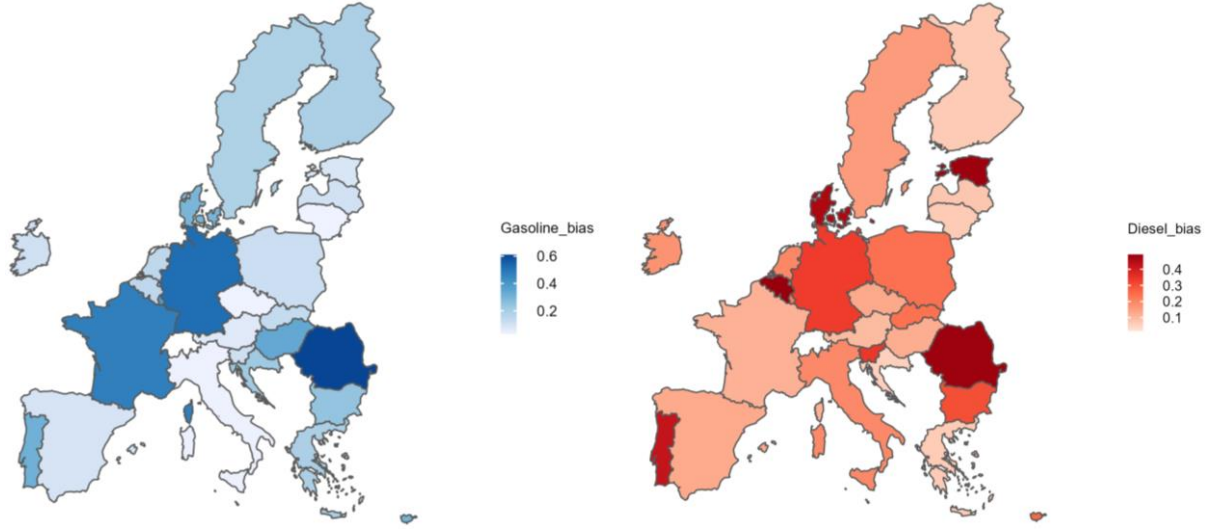


Figure 1. EU Country Maps of Retail Prices of Gasoline (left) and Diesel (right) Biases

2 METHODS

Consider the linear exponential adjustment cost function in the form

$$F[p_t, E_{t-1}(c_t)] = \frac{-\gamma[p_t - kE_{t-1}(c_t)] + \exp\{\gamma[p_t - kE_{t-1}(c_t)]\} - 1}{\gamma^2} \quad (1)$$

where p_t is the retail gasoline or diesel price, c_t is the crude oil price, E_{t-1} denotes the expectation conditional upon the information available at the time $t-1$, k is the technology coefficient. If the asymmetry coefficient γ is negative, the negative difference $p_t - kE_{t-1}(c_t)$ is costlier for the price-making firm than the positive.

From the first-order condition of the firm choosing the output price to minimize the cost function (1) the econometric specification of the firm's price reaction function is obtained:

$$\Delta p_t = k\Delta c_t - \frac{1}{2}\gamma\Delta[(p_t - kc_t)^2] + u_t \quad (2)$$

where Δ denotes the first difference operator, which calculates the change between consecutive values in a sequence; u_t is a stochastic term containing the first differences of terms of third or higher order in the expansion, capturing the changes in those higher-order terms.

The orthogonality conditions implied by the rational expectation hypothesis make the GMM a natural candidate to estimate the average price biases:

$$E(\Delta p_t) = -\frac{k^2\gamma}{2}\sigma^2 \quad (3)$$

We assume that the changes in the input prices Δc_t follow a normally distribution. The process has zero mean and variance σ^2 .

The dynamic asymmetric model with an error-correction term (A-ECM) involving multiple lags is based on the principle of cointegration (Engle and Granger, 1987) and asymmetric price transmission. The long-run relationship is linear, but short-run adjustment is asymmetric depending on the sign of the oil price shock and the deviation from equilibrium. Multiple lags capture the dynamics of shock transmission over time (Borenstein, Cameron and Gilbert, 1997).

The long-run relationship (with ECT_t as stationary stochastic term) is given by:

$$p_t = \theta c_t + ECT_t \quad (4)$$

and dynamic asymmetric model with a lagged error correction term ECT_{t-1} :

$$\Delta p_t = \alpha + \sum_{i=1}^r \beta_i \Delta p_{t-i} + \sum_{i=0}^{s+} \gamma_i^+ \Delta c_{t-i}^+ + \sum_{i=0}^{s-} \gamma_i^- \Delta c_{t-i}^- + \lambda^+ ECT_{t-1}^+ + \lambda^- ECT_{t-1}^- + u_t \quad (5)$$

where the increase in the oil price is $\Delta c_t^+ = \max(\Delta c_t, 0)$, and a decrease is $\Delta c_t^- = \min(\Delta c_t, 0)$. The positive and negative deviations from the long-run equilibrium relationship are included in the terms: $ECT_{t-1}^+ = \max(ECT_{t-1}, 0)$, $ECT_{t-1}^- = \min(ECT_{t-1}, 0)$. Using the Wald test, we can test the symmetry of the short-run transmission under the hypothesis $H_0: \sum \gamma_i^+ = \sum \gamma_i^-$ and the symmetry of the speed of adjustment to equilibrium under the hypothesis $H_0: \lambda^+ = \lambda^-$.

The Threshold Error Correction Model (TAR–ECM) with lags extends the asymmetric ECM by incorporating nonlinearity in the adjustment rate, depending on the magnitude of the deviation from the lagged long-run equilibrium ECT_{t-1} . If the deviation is small, the adjustment may be slower; if the deviation is large, it is faster (Enders and Siklos, 2001). The model allows for two regimes based on the threshold τ at ECT_{t-1} . The short run model:

$$\Delta p_t = \begin{cases} \alpha_1 + \sum_i \beta_{1i} \Delta p_{t-i} + \sum_i \gamma_{1i}^+ \Delta c_{t-i}^+ + \sum_i \gamma_{1i}^- \Delta c_{t-i}^- + \lambda_1 ECT_{t-1} + u_{1t}, & \text{if } ECT_{t-1} \leq \tau \\ \alpha_2 + \sum_i \beta_{2i} \Delta p_{t-i} + \sum_i \gamma_{2i}^+ \Delta c_{t-i}^+ + \sum_i \gamma_{2i}^- \Delta c_{t-i}^- + \lambda_2 ECT_{t-1} + u_{2t}, & \text{if } ECT_{t-1} > \tau \end{cases} \quad (6)$$

The Momentum-Threshold Error Correction Model (MTAR–ECM) is applied to the change of the magnitude of the deviation from the long-run equilibrium ΔECT_{t-1} (Enders and Granger, 1998; Enders and Siklos, 2001). The model allows for two regimes based on the threshold τ at ΔECT_{t-1} . The identified regimes classify the data based on whether the deviation is increasing (positive change) or decreasing (negative change). The short run model:

$$\Delta p_t = \begin{cases} \alpha_1 + \sum_i \beta_{1i} \Delta p_{t-i} + \sum_i \gamma_{1i}^+ \Delta c_{t-i}^+ + \sum_i \gamma_{1i}^- \Delta c_{t-i}^- + \lambda_1 ECT_{t-1} + u_{1t}, & \text{if } \Delta ECT_{t-1} \leq \tau \\ \alpha_2 + \sum_i \beta_{2i} \Delta p_{t-i} + \sum_i \gamma_{2i}^+ \Delta c_{t-i}^+ + \sum_i \gamma_{2i}^- \Delta c_{t-i}^- + \lambda_2 ECT_{t-1} + u_{2t}, & \text{if } \Delta ECT_{t-1} > \tau \end{cases} \quad (7)$$

The existence of the threshold in models is tested using the sup-Wald statistic (Hansen, 1996):

$$\sup_{\tau} F(\tau) = \sup_{\tau} \frac{(SSR_r - SSR_u(\tau))/k}{SSR_u(\tau)/(T-2k)} \quad (8)$$

where SSR_r is the residual sum of squares of the linear model (without a threshold), $SSR_u(\tau)$ is the sum of the two modes, k is the number of parameters, and T is the number of observations.

The Autoregressive Distributed Lag (ARDL) model allows for modelling relationships between variables of different integration orders (I(0)/I(1) – stationary/nonstationary) in a single step and testing for cointegration using a bounds test (Pesaran, Shin and Smith, 2001). $ARDL(p, q)$:

$$\Delta y_t = c + \sum_{i=1}^{p-1} \phi_i \Delta y_{t-i} + \sum_{j=0}^{q-1} \gamma_j \Delta x_{t-j} + \rho y_{t-1} + \theta x_{t-1} + u_t \quad (9)$$

The Nonlinear ARDL (NARDL) model extends this framework to account for asymmetry by decomposing the explanatory variable into positive and negative partial sums (Shin, Yu and Greenwood-Nimmo, 2014). $NARDL(p, q)$:

$$\Delta y_t = c + \sum_{i=1}^{p-1} \phi_i \Delta y_{t-i} + \sum_{j=0}^{q-1} \gamma_j^+ \Delta x_{t-j}^+ + \sum_{j=0}^{q-1} \gamma_j^- \Delta x_{t-j}^- + \rho y_{t-1} + \theta^+ x_{t-1}^+ + \theta^- x_{t-1}^- + u_t \quad (10)$$

where $x_t^+ = \sum_{k=1}^t \max(\Delta x_k, 0)$; $x_t^- = \sum_{k=1}^t \min(\Delta x_k, 0)$. The bounds test of cointegration compares the F -statistic with critical values based on the number of predictors and the case (I(0) vs. I(1)) under the hypothesis $H_0: \rho = \theta = 0$ in the ARDL, or $H_0: \rho = \theta^+ = \theta^- = 0$ in the NARDL model. Using the Wald test, we can test the short-run symmetry under the hypothesis $H_0: \sum \gamma_j^+ = \sum \gamma_j^-$ and the long-run symmetry under the hypothesis $H_0: \theta^+ = \theta^-$.

3 DATA

The analysis uses weekly retail gasoline and diesel price data for European countries from the European Commission Weekly Oil Bulletin and daily Spot Brent crude oil prices from the US Energy Information Administration website. Crude oil prices were converted to euros per 1000 liters using the daily exchange rate series from the European Central Bank database. The daily data series was aggregated to the weekly by averaging. In our analysis, we use all available data from January 2005 through the end of March 2026.

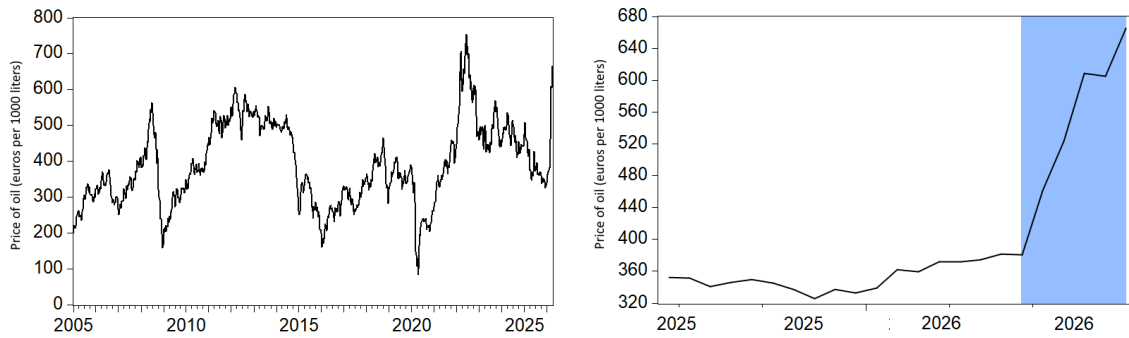


Figure 2. Spot Brent Crude Oil Prices – the Entire Time Series and the Last Six Months

Figure 2 shows the weekly Spot Brent crude oil prices during the same period as the analyzed fuel prices. All used time series show a very similar trend, with a significant increase following the closure of the Strait of Hormuz in early March 2026. The last six months, with a focus on the past month – the period when the Strait of Hormuz was blocked – are highlighted on the right. We saw the opposite trend during the pandemic. Nevertheless, the decline in the third and fourth quarters of 2014, which continued slowly until 2016, was also interesting. The slowdown in global economic activity and oil-market shocks caused it. Fuel prices in all countries also copied this development.

The time-series pattern suggests that the data-generating processes are nonstationary. We tested non-stationarity using the augmented Dickey-Fuller (ADF) unit root test (Dickey and Fuller, 1981). We did not reject the null hypothesis of non-stationarity for the original variables. This indicates all prices are generated by non-stationary processes of order one. Thus, they are suitable for tests of co-integration.

4 RESULTS

The average rate of change in fuel prices following an oil price shock is obtained directly as the absolute value of the parameters in the error-correction term; specifically, in equation (5), these are the parameters λ^+ and λ^- , and in equations (6) and (7) they are the parameters λ_1 and λ_2 . The average rate of change in fuel prices following an oil price shock is obtained directly as the absolute value of the parameter associated with the lagged initial level of the dependent variable in equation (10), which is the parameter ρ . If any of the specified parameters is equal to, say, 0.25, then 25% of the deviation will return to equilibrium within a week.

There is another way to estimate the duration of the reaction (decrease/increase), namely, using the half-life formula as a function of the speed of adjustment:

$$\text{half-life (weeks)} = \frac{\ln(0.5)}{\ln(1+\lambda)} \quad \text{or} \quad \text{half-life (weeks)} = \frac{\ln(0.5)}{\ln(1+\rho)} \quad (11)$$

To facilitate a clearer comparison, we present the results using the reaction's half-life to a rise in oil prices as a shock. A darker shade of the country indicates a longer duration of the shock. A white color indicates the country's absence from the EU or that the asymmetry could not be

confirmed using this method, in which case the reaction's half-life could not be calculated. Our results confirm the conclusion of Deltas and Polemis (2020) that the outcome of traditional methods for examining asymmetric pricing depends on the method used.

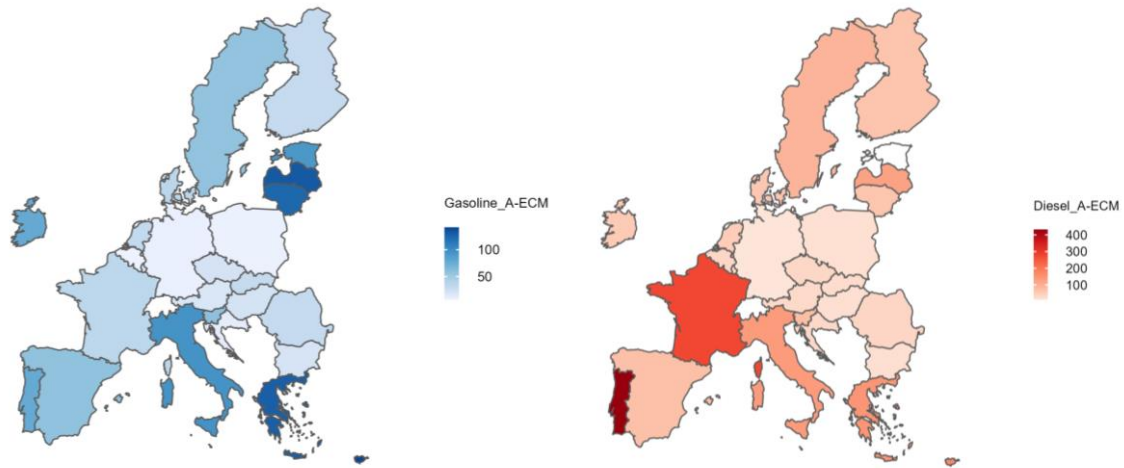


Figure 3. EU Country Maps of Reaction's Half-Life using A-ECM – Equation (5)

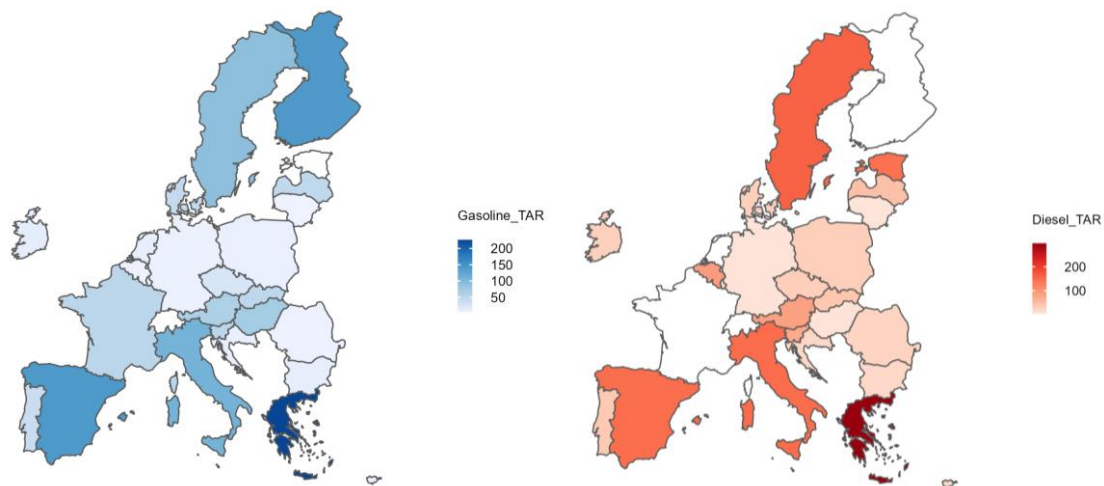


Figure 4. EU Country Maps of Reaction's Half-Life using TAR-ECM – Equation (6)

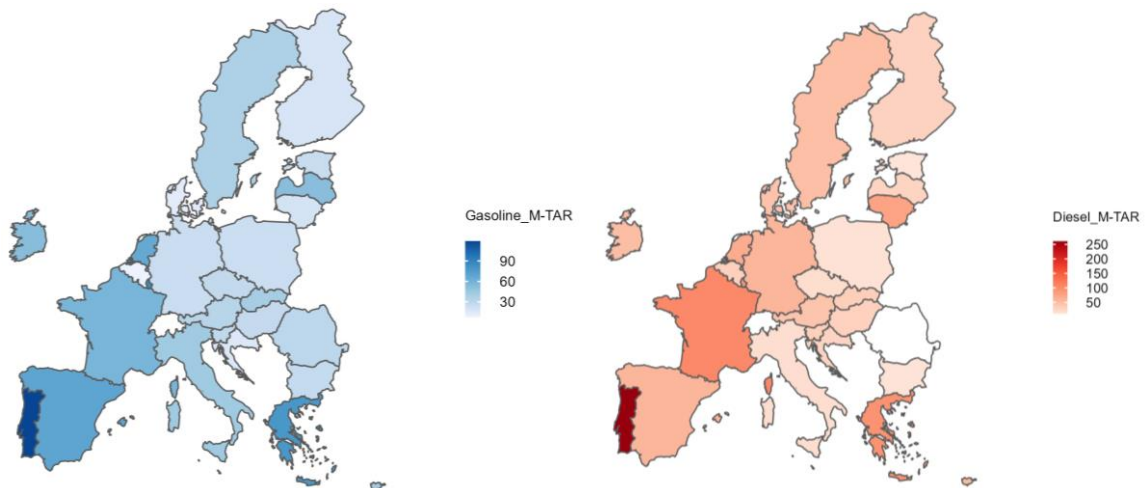


Figure 5. EU Country Maps of Reaction's Half-Life using MTAR-ECM – Equation (7)

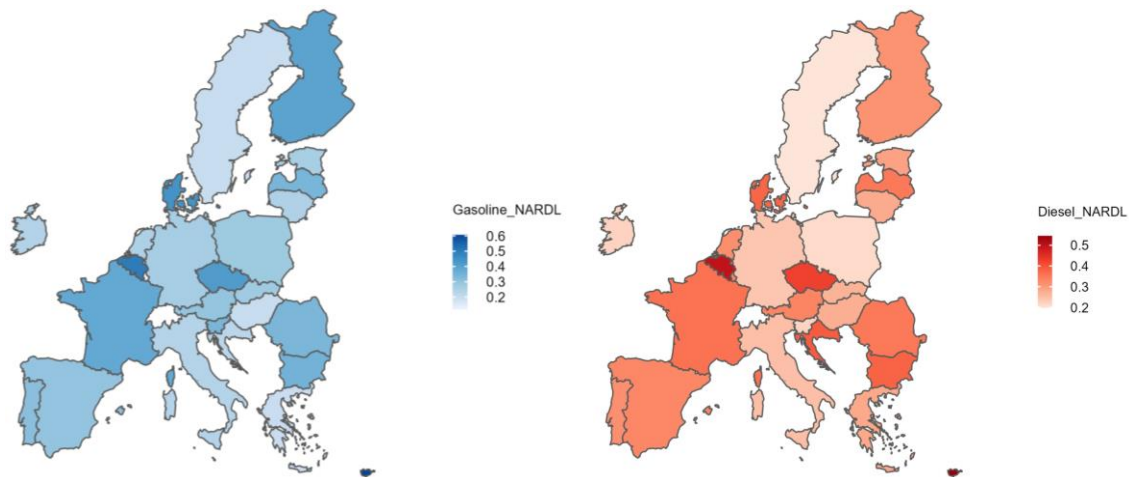


Figure 6. EU Country Maps of Reaction's Half-Life using NARDL – Equation (10)

5 CONCLUSIONS

During the first month of the Strait of Hormuz blockade (March 2026), the price of Brent crude oil rose by more than 70%. Table 1 shows how this effect was reflected in retail gasoline prices, and Table 2 shows how this effect was reflected in retail diesel prices in European countries. Retail fuel prices in Malta change much less frequently (or not at all, as is currently the case), as shown in Table 1 and Table 2. For this reason, we decided not to include fuel prices in Malta in the analysis. We made the same decision in our analysis shown in Figure 1.

Table 1: Retail Gasoline Prices in EU Countries at the End of February and March (€/1000 l)

Country	Price02	Price03	Change	Country	Price02	Price03	Change	Country	Price02	Price03	Change
Austria	1509.0	1879.0	24.52%	Finland	1712.5	2049.0	19.65%	Latvia	1531.7	1869.2	22.04%
Belgium	1503.7	1848.3	22.92%	France	1710.3	2009.2	17.48%	Malta	1340.0	1340.0	0.00%
Bulgaria	1218.0	1444.3	18.58%	Greece	1738.0	2046.0	17.72%	Netherlands	2061.5	2334.1	13.22%
Cyprus	1312.0	1578.8	20.33%	Croatia	1471.0	1680.0	14.21%	Poland	1376.7	1680.6	22.07%
Czechia	1371.8	1685.0	22.83%	Hungary	1461.7	1537.7	5.20%	Portugal	1686.0	1917.0	13.70%
Germany	1824.0	2134.0	17.00%	Ireland	1738.6	1958.1	12.63%	Romania	1546.0	1837.3	18.84%
Denmark	1938.2	2232.1	15.16%	Italy	1654.6	1733.4	4.76%	Sweden	1433.5	1711.5	19.40%
Estonia	1453.0	1730.0	19.06%	Lithuania	1457.5	1738.7	19.29%	Slovenia	1419.7	1507.0	6.15%
Spain	1471.2	1557.6	5.87%	Luxembourg	1457.0	1700.0	16.68%	Slovakia	1474.0	1577.0	6.99%

Source: European Commission and own computations

Table 2: Retail Diesel Prices in EU Countries at the End of February and March (€/1000 l)

Country	Price02	Price03	Change	Country	Price02	Price03	Change	Country	Price02	Price03	Change
Austria	1546.0	2204.0	42.56%	Finland	1737.9	2270.4	30.64%	Latvia	1528.0	2048.0	34.03%
Belgium	1627.2	2225.5	36.77%	France	1649.7	2188.8	32.68%	Malta	1210.0	1210.0	0.00%
Bulgaria	1227.6	1619.9	31.96%	Greece	1548.0	2118.0	36.82%	Netherlands	1873.0	2463.3	31.51%
Cyprus	1406.4	1842.7	31.02%	Croatia	1494.0	1881.0	25.90%	Poland	1423.7	2017.4	41.70%
Czechia	1362.5	1966.6	44.34%	Hungary	1520.7	1619.9	6.52%	Portugal	1600.0	2072.0	29.50%
Germany	1733.0	2286.0	31.91%	Ireland	1737.0	2178.0	25.39%	Romania	1607.5	2032.0	26.41%
Denmark	1780.3	2355.2	32.30%	Italy	1701.8	2032.7	19.44%	Sweden	1571.2	2148.9	36.77%
Estonia	1391.0	2012.0	44.64%	Lithuania	1614.6	2124.7	31.59%	Slovenia	1459.7	1619.9	10.97%
Spain	1423.2	1777.1	24.87%	Luxembourg	1465.0	2005.0	36.86%	Slovakia	1467.0	1687.0	15.00%

Source: European Commission and own computations

In some countries, interventions in the fuel market – particularly for diesel – are underway amid a significant rise in inflation driven by rapidly rising transportation costs. Hungary was the first to implement this policy, introducing a fuel price cap (gasoline and diesel, for domestic vehicles) as early as March 9, 2026. On March 10, 2026, Slovenia and Croatia intervened in fuel prices. The former reduced excise taxes, capped prices, and became the first EU country to

implement rationing (private individuals limited to 50 liters per day). The latter capped retail prices for both gasoline and diesel. On March 23, 2026, Romania and Slovakia followed suit, with the former introducing a cap on fuel markups and export restrictions (emergency decree) and the latter implementing dual pricing (a higher regulated diesel price for foreign vehicles) and sales limits. On April 8, 2026, the Czech Republic announced the introduction of a fuel margin cap and a cut to the diesel excise tax. However, many other countries are also using indirect tools; for example, Poland is putting pressure on the margins of state-owned companies. Germany and Austria are introducing limits on the frequency of fuel price changes. Other countries are monitoring the situation with a view to a possible future response.

Twelve countries directly intervened in prices, with the most active being those in the south and east of the EU: Spain, Italy, Portugal, Hungary, Croatia, Slovenia, Poland, Slovakia, etc. Prices were not affected or are only expected to be affected in fifteen countries; countries in the west and north of the EU are generally more cautious. Thus, the actual price differences are not solely due to volatility, market-specific reactions, and taxes, but also to specific government interventions. Given its magnitude, the persistence of the fuel price shock caused by the closure of the Strait of Hormuz will fade over a very long period. Unfortunately, these shocks occur at the very end of the data series, so it makes no sense to include dummy intervention variables or regime changes in the models to distinguish market adjustments from policy effects.

Table 3: Correlations of Results of Different Methods: Gasoline Prices Asymmetries

Correlation (Probability)	GASOLINE_BIAS	GASOLINE_A-ECM	GASOLINE_TAR	GASOLINE_M-TAR	GASOLINE_NARDL
GASOLINE_BIAS	1.000000 ----				
GASOLINE_A-ECM	-0.262370 (0.2051)	1.000000 ----			
GASOLINE_TAR	-0.218615 (0.2938)	0.315157 (0.1249)	1.000000 ----		
GASOLINE_M-TAR	0.103140 (0.6237)	0.351439 (0.0849)	0.258728 (0.2117)	1.000000 ----	
GASOLINE_NARDL	0.043726 (0.8356)	0.095183 (0.6509)	-0.208858 (0.3164)	-0.306024 (0.1368)	1.000000 ----

Source: Own computations

Table 4: Correlations of Results of Different Methods: Diesel Prices Asymmetries

Correlation (Probability)	DIESEL_BIAS	DIESEL_A-ECM	DIESEL_TAR	DIESEL_M-TAR	DIESEL_NARDL
DIESEL_BIAS	1.000000 --				
DIESEL_A-ECM	0.214210 (0.3511)	1.000000 --			
DIESEL_TAR	-0.235144 (0.3049)	0.191356 (0.4060)	1.000000 --		
DIESEL_M-TAR	0.237615 (0.2997)	0.836003 (0.0000)	0.063641 (0.7840)	1.000000 --	
DIESEL_NARDL	0.218184 (0.3420)	0.068338 (0.7685)	-0.191048 (0.4068)	-0.050833 (0.8268)	1.000000 --

Source: Own computations

Correlations between results obtained from various dynamic methods and price bias did not confirm a link between these methods. Nor did correlations among the dynamic methods themselves exist, except for the MTAR-ECM and A-ECM results for diesel prices and, to some extent, gasoline prices.

Acknowledgements: This work was supported by The Grant Agency of Slovak Republic, grant no. 1/0052/24, "Estimation of Key Structural Parameters of Current Macroeconomic Models" and grant no. 1/0435/26, "Investment Effectiveness and Spatial Spillover Effects in the EU Green Transformation: A Spatial Econometric and Machine Learning Approach".

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INVESTMENT ADVISORY WITH SRI PORTFOLIO OPTIMIZATION UNDER UCITS REGULATION

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Abstract

Investment advisory for European investors is constrained by the MiFID II directive, where the investors are assigned to risk groups and can invest only in funds consistent with their risk profile. This may limit access to desired, riskier investments. This paper proposes a two-step model that expands the set of available investment solutions. First step is to construct the efficient frontier of equity portfolios using a minimization Conditional Value-at-Risk (CVaR) model from referential equity index Dow Jones (DJIA). In the model there are multiple European regulatory constraints that influence the weights of holdings in the portfolio. Then we classify portfolios on the efficient frontier into categories based on the PRIIPs methodology, where each portfolio has Summary Risk Indicator (SRI) that sets the fund riskiness. Our approach extends a SRI assigned to a fund to all seven SRI categories, thereby making investment accessible to investors across all risk groups. Equities portfolios from efficient frontier are not sufficient for low-risk investors. For these investors we add the risk-free asset which reduces the riskiness of the portfolio, also with reducing its expected returns. Our min-CVaR model achieves portfolio with the same SRI as the reference index, while delivering more than 30% higher returns and approximately 20% lower risk across both five-year periods.

Keywords: UCITS, SRI, portfolio optimization

JEL Classification: G11, G28, G32

AMS Classification: 91G10, 90C15, 91G70

1 INTRODUCTION

Selecting the appropriate investment instrument is a complex process, particularly in the context of investment advisory. Although investors enter the process with certain ideas about the structure of their portfolio, it is not always possible to fully realize those ideas. The outcome of investment advice is recommendations for the investor on which assets to allocate their funds to. These recommendations are based on an investment questionnaire, which is a mandatory component of advisory services under the Markets in Financial Instruments Directive II (MiFID II). In the early days of advisory services, advisors could simply advise or influence the investor's responses (Pallavi, 2024). Currently, however, human advisors are increasingly being replaced by digital advisors, or robo-advisors, where this aspect of influencing responses is eliminated (Jung, 2018). After completing the questionnaire, the investor is typically classified into one of several investment categories (most commonly conservative, balanced, or aggressive). Based on this classification, the investor is then recommended funds that match their risk profile (Wagner, 2024). If an investor expresses interest in investing, for example, in the Dow Jones Industrial Average stock index but does not meet the criteria for a dynamic profile, such an investment cannot be arranged for them under standard procedures. A common solution is to combine an equity fund with a conservative asset (e.g., a bond fund), thereby reducing the overall risk of the portfolio; however, the ratios of these components are not defined anywhere. Furthermore, European legislation does not explicitly define uniform risk categories for investors, but only risk categories for funds, of which there are seven (Graf, 2019). We consider it appropriate for the results of the investment questionnaire - risk profiles - to reflect

this seven-point scale as well. This classification of funds is introduced by the PRIIPs Regulation through the Summary Risk Indicator (SRI), which quantifies the riskiness of an investment product and must be included in the Key Information Document (KID) (Amzallag, 2023). The proposed model is based precisely on these seven SRI levels and applies them to portfolios created from a single selected stock index. In the first step, an efficient frontier of portfolios is constructed while respecting the restrictions arising from the Undertakings for Collective Investment in Transferable Securities (UCITS), which regulates the composition of funds available to European investors. The portfolios are optimized by minimizing risk measured by the Conditional Value at Risk (CVaR). Subsequently, SRI values are calculated for the all-equity portfolios. Missing risk levels are supplemented by combining the equity component with a risk-free asset, represented by money market funds or a fund tracking the euro short-term rate. If we were to consider a risk-free asset in the true sense of the word, we would need to invest in government bonds. However, their face value is typically in the thousands of euros, which, combined with the equity component, makes them practically inaccessible to retail investors. In our work, we therefore define as a risk-free asset a fund that tracks the euro short term rate (ESTR). The ESTR is a reference interest rate for the euro area, published daily by the European Central Bank (ECB). It reflects the cost of banks for unsecured overnight loans and is calculated based on actual transactions from the previous day. Our approach Portfolios constructed in this way make it possible to cover the entire spectrum of investors—from conservative to dynamic—while maintaining exposure to the same underlying index. The benefit lies in the possibility of investing in a preferred equity asset for all groups of investors. In this paper, we will introduce the European UCITS and PRIIPs regulations, which must be included in every portfolio (fund) offered to European investors. These regulations affect the composition of the portfolio. We will explain the model in which we minimize risk using CVaR and subsequently seek maximum return. The third chapter provides an overview of the results, where we compare the parameter on a seven-point scale and the DJIA benchmark index, while also showing the structure of a portfolio whose SRI agrees with the value of the benchmark index.

2 METHODOLOGY

In this section, we present methods, regulatory frameworks, and investment model with which we optimize portfolios and create an efficient frontier in this work.

2.1 UCITS

UCITS is a European regulation that sets out rules for funds offered to European investors. The basic rules for the distribution of funds are the liquidity of the fund, the need for a fund depositary and two conditions setting maximum limits for the share of assets in the fund (2014/91/EU). The first rule sets the maximum share of a single asset in the fund 10% of the fund's value.

$$w_i \leq 0.10, \quad \forall i \in M \quad (1)$$

Where w_i represents the weight of asset i and M represents all assets in the fund. The second rule is that the sum of stocks whose weight is greater than 5% must be less than or equal to 40% of the fund.

$$\sum_{i \in N: w_i > 0.05} w_i \leq 0.40, \quad \forall i \in M \quad (2)$$

Thanks to these two rules, the minimum number of assets in the portfolio is always at least 17.

2.2 PRIIPs

PRIIPs is a European regulation effective since 2018 and is used to protect retail investors. The basis of PRIIPs for funds is the requirement for each fund to provide a Key Investor Document (KID) in the language of the country where the fund is distributed. Each KID must be easily readable by retail investors. This three-page document also includes a Summary Risk Indicator (SRI). The value of this indicator represents the riskiness of the given fund on a scale from 1 to 7. This value serves as an indicator of the riskiness of the fund, according to which it is possible to quickly identify which investor risk profile the fund may suit. SRI is composed of the Market Risk Measure (MRM) and Credit Risk Measure (CRM). Since companies from the Dow Jones index are used in this work, we will not consider CRM due to their low default probability, and the SRI value will be proportional to the MRM value. The intervals for calculating MRM - and so the SRI - are shown in Table 1:

Table 1 MRM (SRI) intervals

MRM (SRI)		1	2	3	4	5	6	7
Volatility intervals (VEV)	Equal or more	0%	0.5%	5%	12%	20%	30%	80%
	Less	0.5%	5%	12%	20%	30%	80%	

VEV value is calculated following this formula:

$$VEV = \frac{\sqrt{(3.842 - 2 * VaR_{SPACERETURN}) - 1.96}}{\sqrt{T}} \quad (3)$$

Where the $VaR_{SPACERETURN}$ is defined as follows:

$$VaR_{SPACERETURN} = \sigma \sqrt{P} * \left(-1.96 + 0.474 * \frac{\mu_1}{\sqrt{P}} - 0.0687 * \frac{\mu_2}{\sqrt{P}} + 0.146 \frac{\mu_1^2}{\sqrt{P}} \right) - 0.5 \sigma^2 P \quad (4)$$

P stands for trading periods in the holding period, σ is volatility, T represents period for holding the asset (years), μ_1 represents skewness and μ_2 is the excess kurtosis.

2.3 Min-CVaR model

The first is to calculate the efficient frontier based on historical returns over a five-year period and risk represented by a CVaR of 95 %. Coherent risk measure CVaR measures the average loss in worst-case scenarios, not just the limit like VaR. Unlike VaR, it takes into account the

“tail” of the distribution, i.e. extreme losses, which gives a more realistic view of risk. It is more practical and reliable risk management tool than VaR. The objective function of the model minimizes the CVaR. After finding the portfolio with the minimum CVaR, we increase the annual return by 0.01% and minimize the CVaR. In this way, we capture the efficient frontier of portfolios that are modeled from the DJIA benchmark index. We then add the UCITS constraints to the efficient frontier. The notation of the minimizing objective function is as follows:

$$\min_{w,z,VaR} VaR + \frac{1}{\alpha T} \sum_{t=1}^T z_t \quad (5)$$

Where z_t represents an CVAR auxiliary variable, $t \in T = \{1, \dots, n_T\}$, T is the set of weekly periods $T = \{1, \dots, n_T\}$, N stands for the set of all investment assets, confidence level α for CVaR is from the interval (0,1), confidence level is in our model 0.05. Then the z_t represents scenario constraints:

$$z_t + \sum_{j=1}^n w_i r_{t,i} + VaR \geq 0, \quad \forall t = 1, \dots, n_T \quad (6)$$

Where w_i is the weight in asset i . Assets weights must not be negative. Shorting is not allowed:

$$w_i \geq 0, \quad \forall i \in N \quad (7)$$

Only losses (negative deviations) enter the CVaR:

$$\begin{aligned} z_t &\geq -(E_p^{(w)} + VaR) \\ z_t &\geq 0, \quad \forall t = 1, \dots, n_T \end{aligned} \quad (8)$$

Where $E_p^{(w)}$ is weekly expected return of the portfolio and the sum of the assets must equal 1. Once the efficient frontier has been derived, the portfolios are categorized according to their SRI values. From each category, a representative portfolio is selected based on the minimization of CVaR, resulting in one equity portfolio for every available SRI group. Portfolios for which no direct SRI is available are then incorporated by adjusting their risk exposure relative to the least risky portfolio in terms of SRI. This is achieved by determining an appropriate proportion between the pure equity component of the reference portfolio and the risk-free component, with the allocation guided by the VEV metric. The determination of the equity and bond weights can be expressed mathematically as follows:

$$\begin{aligned} ES &= \frac{VEVx}{VEVy} \\ RFS &= 1 - ES \end{aligned} \quad (9)$$

Where ES is the equity share and the RFS is the risk free share. $VEVx$ is the median of the value range defined in PRIIPs, $VEVy$ represents a lowest possible SRI fully equity portfolio.

3 RESULTS

In this section, we present the results of a min-CVaR model that ranks optimal portfolios based on SRI values. For each SRI category, exactly one portfolio is selected that has the smallest CVaR value among all portfolios in the given SRI category. Two five-year periods are examined. The first five-year period, starting in 2016, is shown in Table 2.

Table 2 Optimized min-CVaR model during period 01/2016-12/2020

Metric	SRI 1	SRI 2	SRI 3	SRI 4	SRI 5	SRI 6	SRI 7 (SRI 6)	DJIA index
Annual return	2.33%	4.65%	10.18%	20.83%	22.09%	34.43%	37.95%	15.21%
CVaR	0.0006	0.0066	0.0195	0.0468	0.0471	0.0575	0.0695	0.0683
VaR	0.0003	0.0028	0.0087	0.0192	0.189	0.0229	0.0271	0.0327
Standard dev.	0.0018	0.0196	0.0605	0.1337	0.1361	0.171	0.2046	0.1885
Sharpe ratio	1.2876	1.3019	1.3355	1.4013	1.4686	1.89	1.7518	0.7008
VEV	1.3	3.66	9.08	18.82	20.15	30.06	39.31	27.37
Holdings	19	19	19	19	17	18	17	30
Equities	1.33%	14.62%	45.18%	100%	100%	100%	100%	100%
Money Market	98.67%	85.38%	54.82%	0%	0%	0%	0%	0%

Two five-year periods are chosen to avoid random results from only one five-year period, as one chosen period could affect the results and credibility of the model. Five years were set because it is the minimum recommended horizon by investment companies, including Erste Asset Management. From Table 2, we see that the DJIA benchmark index has a VEV value of 27.37, which equals SRI 5 risk profile. Comparing the index and the SRI 5 portfolio, it is seen that the optimized portfolio return is almost 7% higher per year. The risk measured based on CVaR is one third lower for the optimized portfolio. The Sharpe ratio is two times in favor of the optimized full equity portfolio with 17 companies. Last SRI 7 portfolio couldn't be obtained since there is no VEV value above 80, so the last portfolio on the effective border with SRI 6 represents the SRI 7 portfolio. We show optimized portfolios and the benchmark in Figure 1.

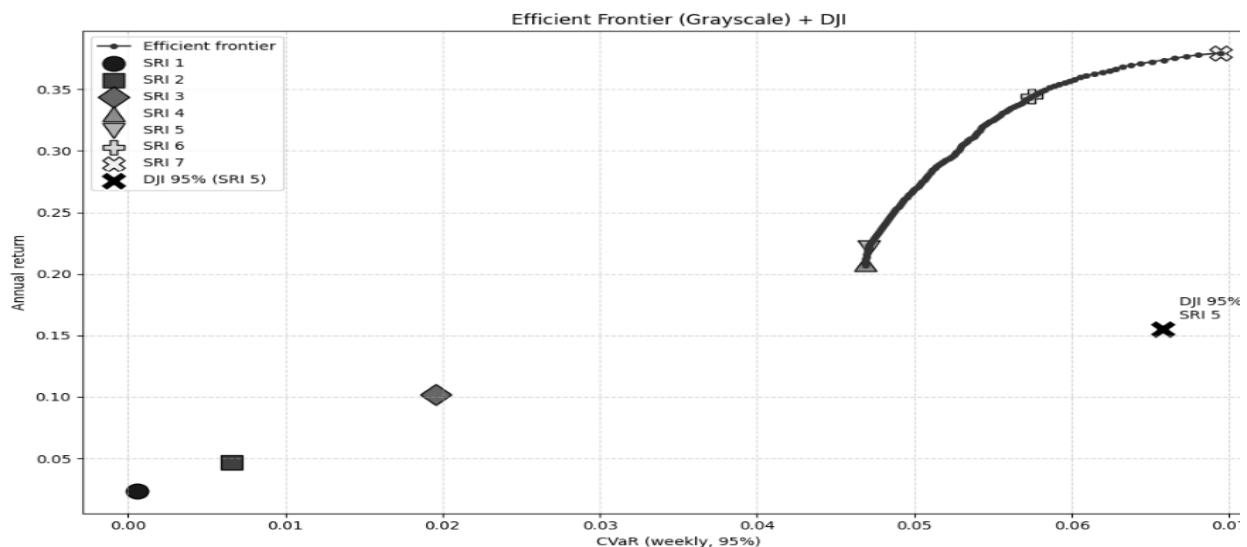


Figure 1 Efficient frontier during period 01/2016-12/2020

Source: Author's own elaboration

We applied the same approach to a five-year period, starting in 2021 and ending in 2025. The VEV value of DJIA is in the range corresponding to SRI 4. However, if we look at the results from Table 3, we see that the given portfolio could be benchmarked against SRI 5 in addition to SRI 4, since this portfolio is closer to the reference index in terms of VEV. The SRI 7 again couldn't be obtained so the same approach applies as before.

Table 3 Optimized min-CVaR model during period 01/2021-12/2025

Metric	SRI 1	SRI 2	SRI 3	SRI 4	SRI 5	SRI 6	SRI 7 (SRI 6)	DJIA index
Annual return	2.24%	3.69%	7.09%	13.85%	15.15%	28.52%	28.65%	10.16%
CVaR	0.0004	0.0039	0.0122	0.0283	0.0284	0.0434	0.0449	0.0393
VaR	0.0003	0.0031	0.0095	0.0217	0.0223	0.0313	0.0303	0.0286
Standard dev.	0.0015	0.016	0.0496	0.1129	0.1129	0.1609	0.1632	0.1368
Sharpe ratio	0.9842	0.991	1.0069	1.04	1.156	1.6421	1.627	0.5964
VEV	1.3	3.66	9.1	19.41	20.02	30.42	30.73	19.8
Holdings	19	20	20	20	19	17	17	30
Equities	1.58%	14.17%	53.80%	100%	100%	100%	100%	100%
Money Market	98.42%	85.83%	56.20%	0%	0%	0%	0%	0%

In Table below we demonstrate the composition of SRI 4 portfolio.

Table 4 Composition of SRI 4 portfolio

AAPL	AMGN	AMZN	CVX	HD	HON	JNJ	KO	MCD	MMM
4.99%	4.99%	4.99%	0.35%	4.99%	0.18%	4.99%	4.99%	10%	4.99%
MRK	SMFT	NKE	PG	SHW	TRV	UNH	VZ	WMT	
4.99%	4.99%	3.84%	10%	4.99%	4.99%	0.74%	10%	10%	

In SRI 4 and SRI 5 portfolios have higher returns and lower risk than the DJIA. Since we applied the conditions in the optimization, the achieved frontier is only “sub-efficient” from a theoretical point of view. In other words, without these restrictions it would be possible to achieve the real efficient frontier, but the UCITS requirements do not allow this. This fact is also reflected in the wavy curvature of the efficient frontier, specifically at a CVaR value of 0.052 and a return of approximately 0.30 (Figure 1). From both methods, whether graphically or by reading from a table, it is possible to see the benefit of this optimization approach. Apart from improving the portfolio parameters, the possibility of assigning a portfolio to each investor based on his risk profile is also a benefit. One stock index can thus represent up to seven portfolios, which can cover the entire range of investment (risk) profiles of investors. It is not necessary to be tied to only one pre-established fund or risk group. The analysis was performed in programming language Python based on weekly closing data for companies from DJIA and the DJIA itself.

4 CONCLUSIONS

The aim of this paper was to introduce an optimization model, which expands the investment options using the same underlying asset during investment advisory for all risk profiles of investors. From an SRI perspective, we created 7 portfolios consisting of an equity component and a risk-free security component. Based on the ratio of these two components, we enabled

investors with high risk aversion to partially participate in the returns of the underlying equity index. The optimization of the portfolios from the reference index was carried out under UCITS conditions, which set requirements for maximum weights of stocks in the portfolio. We analyzed the resulting portfolios over two consecutive five-year periods. For portfolios optimized by minimizing CVaR, we recorded positive results compared to the DJIA reference index. The number of equity securities was reduced from the initial 30 to the span between 17 to 20, depending on the specific portfolio. The risk-free component with an average return of 2% is represented by the Money Market ETF, which tracks the euro short-term rate. This interbank overnight rate has averaged around 2% over the long term. The optimized portfolio with the same SRI value as the benchmark index achieved a return of 13.85% compared to 10.16% for the benchmark index. From a risk perspective, measured by the CVaR indicator, we recorded a decrease from the index value of 0.0393 to 0.0283 for the portfolio with SRI 4. These results were achieved over a five-year horizon from 2021 to 2025. We also recorded a similar improvement in the previous five-year period from 2016 to 2020. Thanks to the proposed model, each investor can enhance their assets while minimizing risk through a stock index or a group of assets to which they prefer to allocate their equity portion of their investment. This removes the limitation that a conservative investor does not have access to riskier assets, while the SRI scale allows for a more precise assignment of a suitable portfolio to a specific investor.

Acknowledgements: This work was supported by the Grant Agency of Slovak Republic: VEGA 1/0285/24 The Impact of Inflation on Poverty and Social Exclusion in Slovakia and the EU

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THE EUROPEAN GREEN DEAL'S 'FIT FOR 55' PACKAGE: A CATALYST FOR LABOR MIGRATION?

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Abstract

This paper utilizes a dynamic computable general equilibrium model, GTAP-E, to analyze the global economic and sectoral implications of the European Green Deal's "Fit for 55" package. The simulation models a 55% reduction in emissions quotas in European countries, serving as a high-fidelity proxy for the policy's shock. Our findings reveal that the policy is a powerful decarbonization tool, leading to the dramatic contraction of fossil fuel-intensive sectors and a notable expansion in electricity value-added, signaling a successful energy transition. Counter to some concerns, the analysis indicates a net positive macroeconomic outcome for the implementing regions, driven by resource reallocation and an improved economic structure. Positive spillover effects, mediated through trade diversion and shifts in global energy prices, benefit other regions. The study also highlights critical policy challenges, such as potential carbon leakage and the social costs of industrial restructuring. Furthermore, the analysis reveals that the policy creates significant changes in regional labor endowments, suggesting a potential for new global labor migration patterns, underscoring the necessity of complementary measures like the Carbon Border Adjustment Mechanism for the policy's long-term success.

Keywords: *computable general equilibrium, green deal, energy, GHG emissions, migration*

JEL Classification: C68, O44

AMS Classification: 91B50

1 INTRODUCTION

The European Green Deal (EGD), formally introduced in 2019, represents a transformative growth strategy for the European Union (EU) with the overarching goal of achieving climate-neutrality by 2050 [7]. The EGD is not merely an environmental policy; it is a comprehensive economic framework designed to de-couple economic growth from resource consumption and environmental degradation. The central and most ambitious legislative measure of this strategy is the "Fit for 55" package, a series of proposals intended to deliver a 55% net reduction in greenhouse gas (GHG) emissions by 2030, compared to 1990 levels. This target is a critical intermediate milestone on the path to full decarbonization and provides the impetus for a profound economic transition.

Scientific literature examining the European Green Deal's "Fit for 55" package shows that while direct analyses of labor migration are scarce, several research outputs highlight mechanisms through which the package may indirectly influence cross-border worker mobility. OECD empirical modelling indicates that "Fit for 55" driven structural changes, particularly declining employment in carbon-intensive sectors and rising demand in high-skill green industries, reshape European labor markets, potentially generating regional labor imbalances and thus incentives for mobility [2], [4]. Eurofound's research likewise documents projected employment contractions in energy-intensive regions and expansions in renewable-energy and technology-driven sectors, implying possible displacement effects that could translate into intra-EU labor migration flows [6]. Complementing these international findings, Slovak analytical studies conducted by the Institute for Environmental Policy (IEP) emphasize that decarbonization requirements under "Fit for 55" will substantially restructure Slovakia's industrial base, potentially necessitating labor reallocation both within the country and across

EU borders, especially in metallurgy, energy, and transport [5]. ESG-CVaR portfolio modeling offers an additional way to study environmental policy effects, showing how environmental criteria restrict feasible investments and shift capital toward low-carbon assets [9]. Collectively, these studies suggest that although explicit academic work linking “Fit for 55” to labor migration is limited, the documented sectoral shifts, skill transformations, and regional disparities provide a credible foundation for hypothesizing that “Fit for 55” could act as a catalyst for increased labor mobility within Europe.

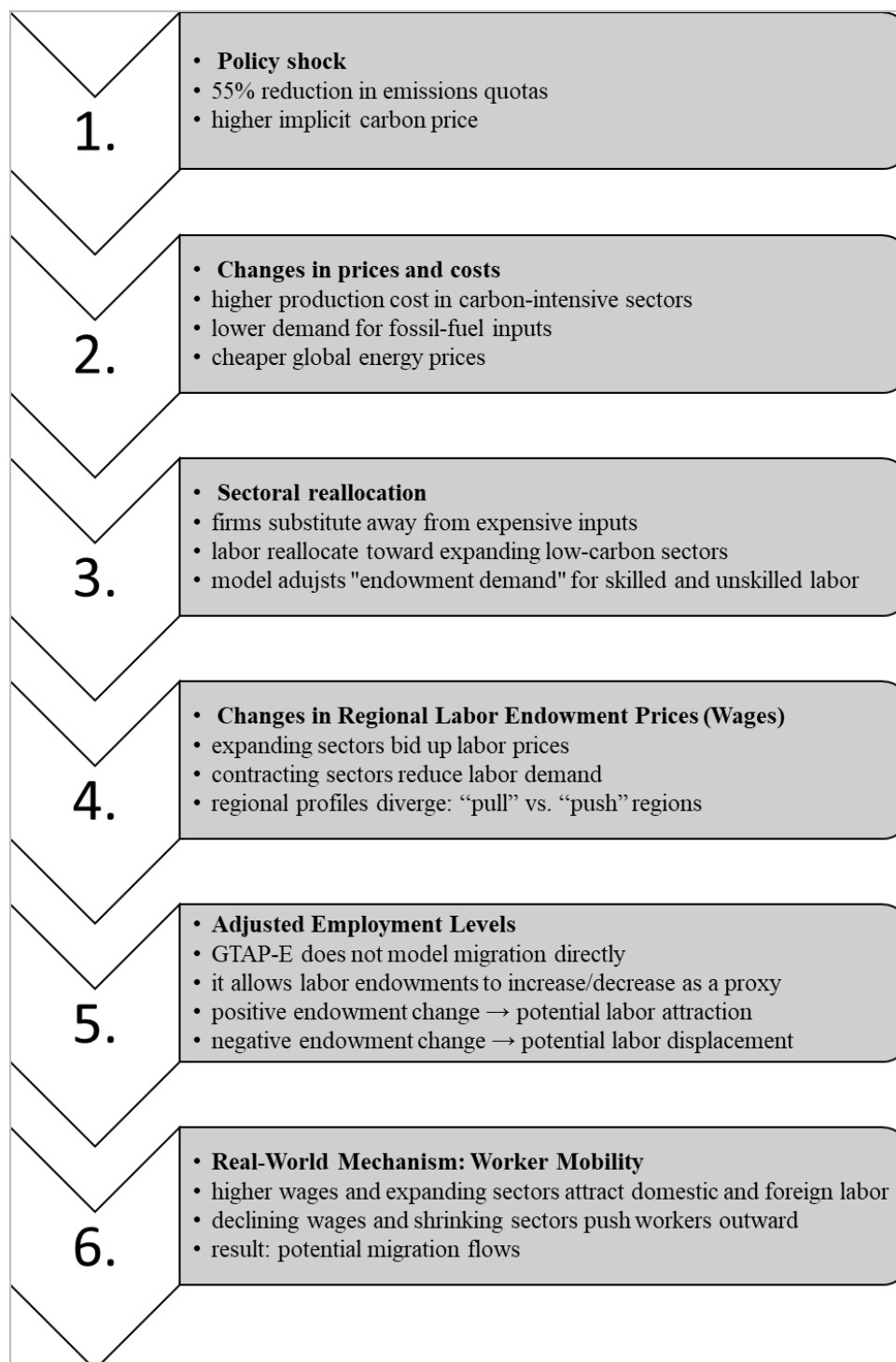


Figure 1: Conceptual Diagram of the Labor Reallocation → Worker Mobility Mechanism in GTAP-E, AI generated scheme

The simulation analyzed here, which models a 55% emissions quota reduction in Slovakia and Western Europe, serves as a direct and high-fidelity proxy for the economic and environmental impacts of the "Fit for 55" package. By utilizing the GTAP-E (Global Trade Analysis Project-Energy) model, we can move beyond a simple report of outcomes to explore the complex interdependencies between macroeconomic variables and sectoral transformations. This analytical framework is crucial because it accounts for how a policy shock in one region propagates through global value chains, affecting production, trade, and consumption patterns worldwide. This rigorous, evidence-based examination of these effects on a global scale will focus on the policy's intended consequences, as well as its unintended ripple effects, particularly on labor markets and migration.

2 DATA

The GTAP-E 11 database [1] includes 160 countries, accounting for 99.1% of the world's GDP and 96.4% of the world's population, and is categorized into 11 aggregated regions for this study. The 65 original sectors are divided into eight new sectors, corresponding to the aim of capturing energy demand. The primary factors use the five main endowments categories, as is shown in Table 1.

Table 1: Mapping in GTAP-E

Regions	Sectors	Primary Factors
Slovakia	Primary Agriculture	Land
Oceania	Coal mining	Unskilled Labor
East Asia	Crude Oil	Skilled Labor
South-East Asia	Natural Gas Extraction	Capital
South Asia	Refined Oil Products	Natural Resources
North America	Electricity	
Latin America	Energy Intensive Industries	
West Europe	Other industries and services	
Middle East and North Africa		
Sub/Saharan Arica		
Rest of World		

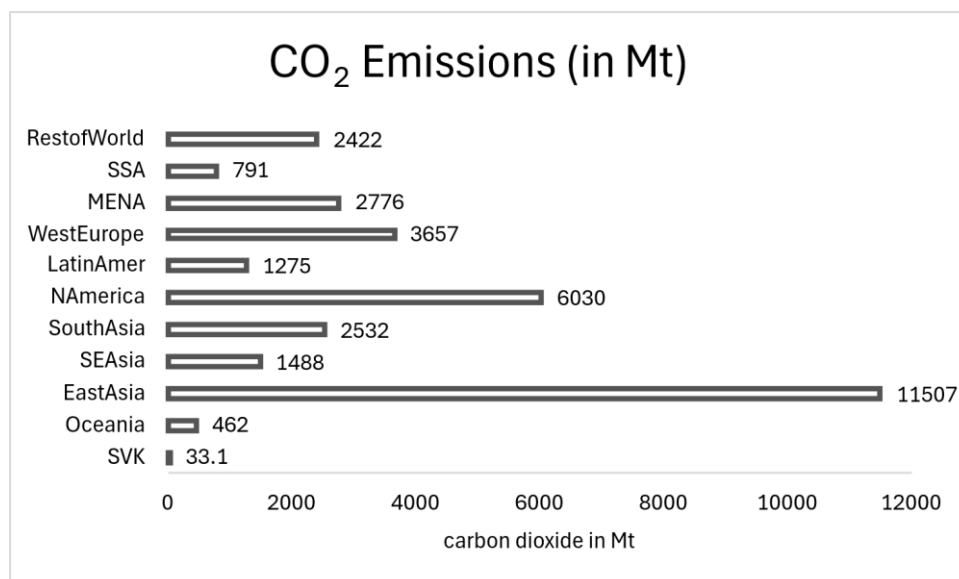


Figure 2: Carbon Dioxide emission levels in 2017 based on [1]

The chart in Figure 2 illustrates regional carbon dioxide emissions measured in million tons, revealing substantial disparities across global areas. East Asia is the dominant emitter with 11,507 Mt, followed by North America at 6,030 Mt and Western Europe at 3,657 Mt. In contrast, smaller regions such as Oceania (462 Mt) and Slovakia (33.1 Mt) contribute only marginal amounts. Overall, the data highlights the concentration of global emissions in a few highly industrialized regions.

3 METHODOLOGY

In the GTAP-E model used in this research [3], production is represented through a sequence of nested constant elasticity of substitution (CES) functions that explicitly incorporate energy use into firms' technological choices. At the top level, sectoral output is produced by combining an aggregate of intermediate inputs with a value added energy composite (see equations 1 - 3), while lower nests govern substitution between primary factors and aggregate energy input (equations 4 - 6). The energy block further differentiates between electricity and non-electric fossil fuels, allowing for substitution among coal, oil, and gas based on relative prices (equations 7 - 11). Household behavior is modelled via a representative regional household that maximizes utility subject to an income constraint, with preferences specified by a Constant Difference of Elasticities (CDE) demand system, which permits non-homothetic consumption patterns and income dependent substitution effects (equations 23 - 27). General equilibrium is ensured through a system of market-clearing and zero-profit conditions requiring equality between supply and demand for all commodities and primary factors, as well as consistency between regional savings and investment under the adopted macroeconomic closure (equations 30 - 35). Carbon emissions are modelled as proportional to the physical consumption of fossil fuels using fixed emission coefficients (equations 36 - 38), and an emissions quota shock is introduced by imposing an exogenous constraint on total allowable CO₂ emissions. This constraint generates an endogenous shadow price of carbon, which enters directly into effective energy prices faced by producers (equations 39 - 41), thereby inducing endogenous adjustments in input substitution, production levels, and trade flows throughout the economy.

4 MACROECONOMIC IMPACTS: THE EGD'S ECONOMIC RATIONALE AND GLOBAL REPERCUSSIONS

The EGD's core economic premise is that aggressive climate action can be a catalyst for new growth rather than a drag on the economy. The macroeconomic results of the GTAP-E simulation provide compelling evidence in support of this hypothesis. The policy-implementing regions, Slovakia and Western Europe, experience a net positive macroeconomic outcome, validating the EGD's "green growth" narrative.

Slovakia demonstrates a particularly robust and positive response, with household income increasing by 1.52% (see Figure 3), government consumption by 1.55%, and private consumption by 1.5%. This outcome is striking given the severe contraction of its energy sectors and underscores the power of resource reallocation. The policy mandates a steep decline in carbon-intensive production, freeing up significant amounts of capital and labor. The GTAP-E model suggests that this freed-up productive capacity is successfully re-directed toward other, more efficient sectors not subject to the same emissions quotas. These could include high-tech manufacturing, energy-efficient services, or the burgeoning renewable energy sector. The result is a more productive and dynamic economy at a new, low-carbon equilibrium, leading to a net gain in overall economic welfare.

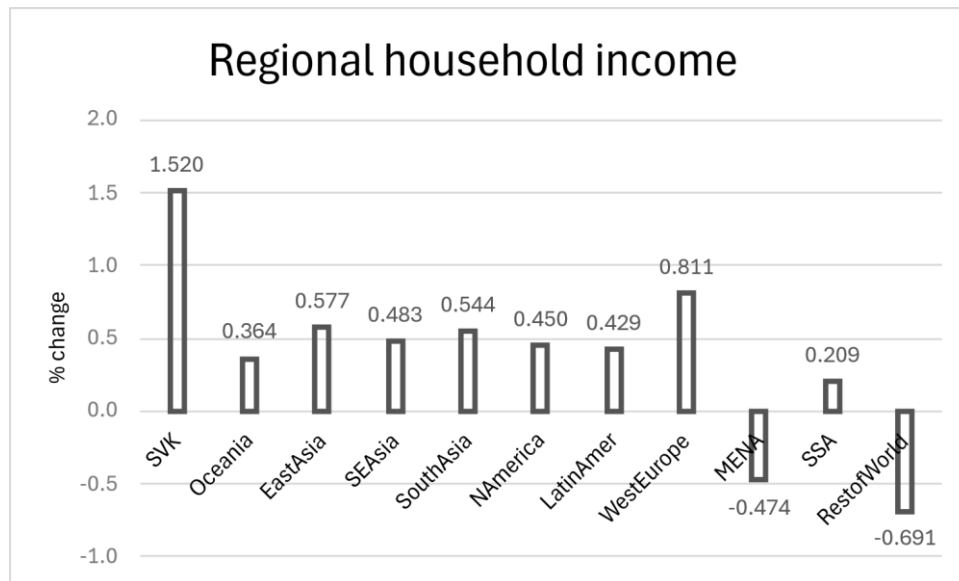


Figure 3: Regional household income change, own calculations

Western Europe, as a larger and more diversified economy, experiences a more moderate but still positive change, with all regional household income increasing by 0.45%. This result suggests that while the scale of the required transformation is immense, the region's economic structure is resilient and capable of managing a substantial transition. The more muted effect is likely due to the size and diversity of the economy, which can absorb shocks more smoothly than a smaller, more concentrated economy like Slovakia. This supports the EGD's underlying assumption that a well-managed transition can lead to a stable and sustainable economic trajectory.

The spillover effects on other regions, while smaller in magnitude, are also predominantly positive, suggesting a net beneficial impact on global welfare. The interconnectedness of the global economy means that a major policy shift in Europe cannot happen in isolation. These positive spillovers are primarily mediated through two key channels: changes in global energy prices and trade diversion. As Europe drastically reduces its demand for fossil fuels, the global price of these commodities is likely to fall. Regions that are net importers of energy, such as East Asia and South Asia, benefit from this price decrease (increase by 0.58% and 0.54% respectively). A lower cost of energy for these regions' industries acts as an economic stimulus, boosting their competitiveness and leading to the observed increases in household income and consumption.

Furthermore, trade diversion creates new opportunities. As European industries face higher production costs due to emissions quotas, some production may shift to other regions. This allows non-EU economies to increase their export-oriented production, particularly in goods that are now more expensive to produce in Europe. For instance, the data for Latin America (increase by 0.47%) and North America (by 0.45%) suggests that they are also experiencing a net beneficial trade adjustment. This dynamic portrays the "Fit for 55" policy as a global economic catalyst that, while originating in Europe, generates positive externalities for many of its trading partners through market mechanisms.

5 THE EGD'S POTENTIAL TO RESHAPE LABOR MARKETS AND MIGRATION

Beyond the macroeconomic and sectoral shifts, a critical and often-overlooked consequence of the EGD is its potential to drive changes in global labor endowments and, consequently, new patterns of immigration. The data on the percentage change in labor endowments purchases (see Table 2) highlights an asymmetric response across regions.

The simulation shows that in Slovakia, both the unskilled labor force increase by 0.58% as for the skilled labor force increases by 0.28%. This positive change suggests a robust "pull" factor, where the policy's economic restructuring makes the country more attractive for both domestic and international workers. This is consistent with the positive macroeconomic outcomes observed in Slovakia, indicating that as new, less carbon-intensive sectors expand, they create a demand for labor that can be met through increased internal mobility or immigration. The increase in capital and natural resources further supports this, suggesting that the policy is attracting and retaining key endowments.

Table 2: Labor endowment purchases change, own calculations

EVFB change in %	Unskilled labor	Skilled labor
SVK	0.58%	0.28%
Oceania	-0.45%	-0.44%
EastAsia	-0.54%	-0.53%
SEAsia	-0.50%	-0.46%
SouthAsia	-0.52%	-0.48%
NAmerica	-0.47%	-0.46%
LatinAmer	-0.48%	-0.48%
WestEurope	0.06%	-0.01%
MENA	-0.12%	0.02%
SSA	-0.45%	-0.39%
RestofWorld	0.05%	0.16%

In stark contrast, a number of other regions experience a contraction in their labor forces. Oceania, for example, sees a -0.45% change of unskilled labor and a -0.44% change in skilled labor. Similarly, East Asia, Southeast Asia, South Asia, and North America all show a decline in both labor categories. This suggests that the policy, through its global ripple effects on trade and energy prices, is creating a "push" factor in these regions. As certain industries contract or become less competitive due to global market shifts, they shed labor, which may then seek opportunities in more dynamic economies. The most significant decrease is seen in the Middle East and North Africa (MENA) for unskilled labor, which decreases by -0.12%.

Western Europe, while implementing the policy, shows a much more muted change, with a slight 0.06% increase in unskilled labor and a negligible -0.01% decrease in skilled labor. This indicates a more stable labor market compared to the dynamic shifts seen in Slovakia. The broader global picture, therefore, is one of labor reallocation, where the EGD's push for a green economy in Europe could lead to a net flow of human capital from regions experiencing negative impacts to those that are adapting successfully. This is not a simple "brain drain" from one country to another, but a more complex, global-scale rebalancing of labor in response to a major climate policy shock. Policymakers will need to consider these potential migratory flows as a direct consequence of climate action and develop complementary policies to manage both the economic and social impacts of this new form of climate-driven migration.

6 SECTORAL TRANSFORMATIONS: A DIRECT CONSEQUENCE OF EGD'S DECARBONIZATION GOALS

The most profound and direct effects of the EGD are evident in the transformation of sectoral value-added composition as shows Table 3. The policy's success is measured by its ability to drive a fundamental shift away from carbon-intensive industries and toward a more sustainable economic structure.

6.1 The Targeted De-carbonization of Fossil Fuels

The simulation results provide unambiguous evidence of the policy's efficacy in achieving its primary decarbonization goals. The value-added of the Coal sector in Slovakia contracts by an unprecedented 330%. Similarly, Western Europe's coal sector experiences a severe decline of 158%. These figures, which represent a decrease in value-added, far exceeding 100%, indicate that the industries are effectively ceasing to exist in their previous form, with production plummeting to an almost negligible level. The Gas sector also faces significant contractions, with Western Europe seeing a decline of 47.6% and Slovakia a drop of 18.8%. While the Oil sector is slightly more resilient, it too experiences notable reductions in value-added.

Table 3: Value -added change in selected sectors, own calculations

	SVK	EastAsia	SouthAsia	WestEurope	MENA	RestofWorld
Agr	0.25	0.00	0.02	-0.44	0.31	0.47
Coal	-330.00	0.03	-0.01	-158.00	0.52	-5.57
Oil	-9.43	-1.78	-1.77	-3.25	-1.73	-2.79
Gas	-18.80	-2.30	-2.99	-47.40	-3.99	-4.38
Oil_pcts	-40.30	1.33	0.99	-18.80	1.52	2.05
Electricity	6.56	0.06	0.08	-19.80	1.32	2.15
En_Int_ind	-18.50	0.59	1.05	-5.30	3.29	4.51
Oth_ind_ser	0.06	-0.04	-0.02	0.17	0.23	0.17
Total	-410.16	-2.12	-2.65	-252.82	1.47	-3.39

This dramatic restructuring is a direct and intended consequence of the "Fit for 55" emissions reduction target. It signifies a successful, albeit disruptive, transition away from fossil fuels, a core objective of the Green Deal. However, it is also a powerful illustration of the significant social and economic challenges that accompany this policy. The collapse of these industries implies a massive loss of jobs, the stranding of capital assets, and the decimation of communities that have historically depended on fossil fuel extraction and processing. The EGD acknowledges this through the concept of a "just transition," which seeks to support workers and regions impacted by decarbonization through re-skilling programs, financial aid, and strategic investments. While a CGE model can show the macroeconomic benefits of this reallocation, it cannot capture the human cost, underscoring the need for strong social policies to complement climate action.

6.2 Carbon Leakage: A Critical Policy Challenge and CBAM as a Response

One of the most significant challenges and potential failure points of the EGD is carbon leakage. This phenomenon occurs when a country's stringent climate policies cause carbon-intensive production to shift to regions with less strict environmental regulations, potentially leading to a net increase in global emissions [8].

While the majority of regions see contractions in their fossil fuel sectors, the Middle East and North Africa (Coal: +0.52%) and East Asia (Coal: +0.03%) show slight increases in coal value-added. This could be interpreted as a form of carbon leakage, where these regions increase their production to meet global demand that is no longer being met by European producers operating under strict quotas. This is a critical issue that threatens to undermine the global climate benefits of the EU's policy. The EGD attempts to mitigate this risk through policies such as the Carbon Border Adjustment Mechanism (CBAM). CBAM is a policy designed to level the playing field by placing a carbon price on imports from non-EU countries, forcing them to pay a carbon cost equivalent to what European producers face under the ETS. The existence of CBAM as a policy response highlights the critical nature of this issue and demonstrates the EU's strategic use of trade policy to export its climate ambition. While this simulation does not explicitly model CBAM, the data on regional shifts provides a strong rationale for its existence.

6.3 The Agricultural and Energy Nexus: Shifting Value Chains

The agricultural sector presents a mixed and complex picture, highlighting the interconnectedness of global value chains. Western Europe's agricultural sector experiences a contraction of -0.44%. This is likely a result of several factors, including rising input costs (e.g., energy, fertilizer) due to the emissions quotas and a reallocation of land and labor to more profitable, less carbon-intensive sectors. In contrast, regions like the Middle East and North Africa (+0.31%), see expansion. This may indicate a significant shift in global agricultural trade, where Europe, with its higher production costs, becomes a net importer, creating new market opportunities for other regions. This demonstrates how the EGD, while focused on decarbonization, also has far-reaching effects on the global food system and agricultural trade flows.

The Electricity sector's response is particularly insightful, serving as a powerful indicator of the structural transition. While Slovakia sees a minor decline (-1.21%), Western Europe experiences a slight increase (+0.11%), and several other regions, most notably East Asia (+0.47%) and South Asia (+0.41%), show significant growth. This highlights a fundamental structural shift: as fossil fuels decline, the demand for and supply of electricity must increase, presumably from renewable and low-carbon sources, to power the new economy. This growth signals a move towards the electrification of various sectors and a rapid decarbonization of the energy mix, which is another key objective of the EGD. The decline in the gas sector, for example, is likely directly linked to the need for a more electricity-intensive infrastructure, driving investments in new generation capacity.

7 CONCLUSION AND FUTURE DIRECTIONS

The simulation of a 55% emissions quota reduction provides a detailed and scientifically grounded look into the potential consequences of the European Green Deal's "Fit for 55" package. The results affirm the policy's efficacy in achieving its primary goal of decarbonization, as evidenced by the dramatic contraction of fossil fuel-intensive sectors in Europe. Counter to some initial concerns, the simulation suggests that this ambitious climate policy can be a catalyst for net positive macroeconomic growth in the implementing regions and their trading partners, driven by a dynamic reallocation of resources and favorable shifts in global trade.

However, the analysis also underscores the significant challenges of the EGD. The near-total collapse of the coal industry, while necessary for decarbonization, represents a major social and economic upheaval that requires careful policy management. Furthermore, the evidence of potential carbon leakage in certain regions highlights the necessity of complementary policies,

such as CBAM, to ensure that the EU's climate efforts are not undermined by shifts in global production. The simulation shows that the EGD's success is not just about domestic policy but also about its ability to reshape global trade in a way that encourages, rather than penalizes, climate action. The new insights into labor endowments also reveal that climate policy can be a driver of global immigration, a consequence that will require proactive policy responses to manage its social and economic effects. Future research is needed to more deeply explore the long-term dynamics of these changes. A more detailed examination of the mechanisms driving macroeconomic growth, the specific impacts on labor markets in transitional regions, and the effectiveness of policies like CBAM in mitigating carbon leakage would all be valuable contributions to a more complete understanding of the EGD's full effects.

Acknowledgements: This work was supported by two projects: APVV-22-0526 “The Economic and Fiscal Effects of Migration in Slovakia: An Integrated Modeling Approach” and VV-MVP-24-0395 “How Climate Policy and Strategic Behavior Can Ensure Financial Stability”.

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SHARPE RATIO-BASED PORTFOLIO OPTIMIZATION OVER BUSINESS CYCLES: AN EMPIRICAL ANALYSIS OF THE GERMAN DAX STOCK MARKET

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Abstract

This paper analyzes the issue of selecting the optimal portfolio on the German stock market represented by the DAX index during crisis and non-crisis periods. The goal of the paper is to identify the optimal stock portfolio using a portfolio selection model based on the Sharpe performance measure, considering the risk-free interest rate. The analysis is conducted on weekly data from 2000 to 2026, taking into account the survivorship bias, as only companies with available data for the entire period are included. The periods are identified through a combination of macroeconomic and market indicators and are divided into crisis and non-crisis phases, with separate calculations performed for each period to determine the optimal portfolio. The findings indicate a significant dependence of the optimal asset allocation on the phase of the economic cycle, with a shift towards defensive sectors and a decrease in risk-adjusted performance during crisis periods. The results allow for an assessment of how much investor recommendations differ between crisis and non-crisis periods on the German stock market.

Keywords: *Sharpe ratio, portfolio optimization, DAX index, financial crises, asset allocation*

JEL Classification: G11, G01, C61

AMS Classification: 91G10, 90C15

1 INTRODUCTION

Portfolio optimization is one of the key issues in modern financial theory. Investors strive to assemble a portfolio of assets to achieve the highest possible return at an acceptable level of risk. Risk-adjusted performance measures, including the Sharpe ratio (Sharpe, 1966), are often used to evaluate portfolio performance. This indicator allows for assessing investment strategies in terms of the ratio of achieved return to the risk undertaken.

However, the development of financial markets is significantly influenced by economic cycles, during which periods of economic expansion and crises alternate. These changes are reflected in investor behaviour, the volatility of financial assets, and the performance of investment portfolios. During crises, there is often increased volatility and a decline in stock prices, while during periods of economic growth, stock markets tend to achieve higher returns. For this reason, it is important to analyze how return indicators and performance measures change depending on the phase of the economic cycle.

The aim of this contribution is to analyze the differences in optimal stock portfolios of the DAX index during crisis and non-crisis periods and compare their performance using the Sharpe ratio (S_p). Particular attention is paid to evaluating the performance of portfolios in different phases of financial market development to identify implications for dynamic portfolio management. The contribution is structured as follows. The first part formulates a portfolio selection model based on maximizing S_p and presents the optimization problem for portfolio selection. The second part is dedicated to analyzing the German stock market DAX, with the observed period from 2000 to 2026, during which crisis and non-crisis phases were identified as a combination of macroeconomic and market indicators. This section also characterizes the weekly data used

and analyzes the companies included in the DAX index. The third part presents the results of portfolio optimization for individual periods and compares their performance. The final section summarizes the main findings of the analysis and discusses their implications for investors.

2 METHODOLOGY

2.1 Sharpe ratio

When searching for the optimal capital allocation relative to performance rates, it is interesting not only to consider the increase in capital but also the risk occurring during the period in which the capital increase took place. The Sharpe ratio (Sharpe, 1966, 1998) is a performance measure adjusted for absolute risk that helps investors compare the profitability of investments and their risk. This ratio indicates how much the return of a particular investment differs compared to a risk-free investment (for example, in the case of safe bonds, the value of S_p would be zero) and is calculated as the difference between the investment's returns and the risk-free return, divided by the standard deviation of the investment's returns. A higher value signifies better relative performance of the fund in relation to its risk.

Sharpe ratio only considers the first and second moments of return distribution, meaning that if the return distribution is not normal, it cannot capture all the information. Therefore, it would be appropriate to supplement the analysis with additional indicators (Sortino ratio, performance based on CVaR). S_p can be calculated as the ratio of the asset's return minus the risk-free rate (r_F) to the standard deviation of returns. (Pekár, Brezina, Reiff, 2022):

$$S_p(\mathbf{r}) = \frac{E(\mathbf{r}) - r_F}{\sigma(\mathbf{r})} = \frac{\sum_{t=1}^T p_t r_t - r_F}{\sqrt{\sum_{t=1}^T p_t (r_t - E(\mathbf{r}))^2}}$$

where $E(\mathbf{r})$ denotes the expected portfolio return,
 $\mathbf{r}$ represents T -dimensional vector $(r_1, r_2, \dots, r_T)$ representing a discrete random variable expressing the rate of return of a portfolio over a certain period,
 p_t the probability density of a discrete random variable in the t -th state, $t = 1, 2, \dots, T$,
 r_F denotes the return of a risk-free asset,
 $\sigma(\mathbf{r})$ denotes the standard deviation of the portfolio returns.

2.2 Portfolio selection model based on the Sharpe ratio

The task of selecting the optimal portfolio by maximizing the Sharpe ratio can be formulated as a mathematical programming problem (Pekár, Brezina, Reiff, 2022).

Let r_{jt} be the t -th component, $t = 1, 2, \dots, T$, of the discrete random variable X_j represented by the vector of returns $\mathbf{r}_j$, $j = 1, 2, \dots, n$, whose values are the historical returns of the j -th asset. Let w_j be the proportion of the j -th asset in the total investment, $j = 1, 2, \dots, n$. Let r_F denote the risk-free interest rate. The optimization problem can be written as the maximization of S_p as follows:

$$\begin{aligned} \max \quad S_p(w_1, w_2, \dots, w_n) &= \frac{\sum_{j=1}^n w_j E[\mathbf{r}_j] - r_F}{\sqrt{\sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} w_i w_j}} \\ \sum_{i=1}^n w_i &= 1 \\ w_1, w_2, \dots, w_n &\geq 0 \end{aligned} \quad (1)$$

The optimization problem of maximizing the Sharpe ratio is solved using differential evolution in the R language.

3 ANALYSIS OF THE GERMAN STOCK MARKET

The German stock market DAX (Deutscher Aktienindex) is the leading German stock index comprising the 40 largest and most liquid companies traded on the Frankfurt Stock Exchange. The original index included 30 companies (DAX30), but in 2021 it was expanded to 40 companies (DAX40).

In the first part of the work, by combining the decline in GDP, increased volatility, and the decrease in the DAX index, we divide the period from 2000 to 2026 into crisis and non-crisis periods, while also identifying the main causes of the recognized crises. The division of individual periods into crisis and non-crisis periods was based on quarterly data on gross domestic product published by the statistical office Destatis (Federal Statistical Office of Germany), with the identification of recession periods based on the standard definition of GDP decline in two consecutive quarters and from literature on economic cycles in Germany e.g (Gehring and Mayer, 2021; Grömling, 2025). Based on these data, the period from 2000 can be divided as follows:

Period	Character
2000 Q1 – 2001 Q1	Non-crisis
2001 Q2 – 2003 Q1	Crisis / stagnation after the dot-com bubble
2003 Q2 – 2008 Q1	Non-crisis (expansion)
2008 Q2 – 2009 Q2	Global financial crisis
2009 Q3 – 2019 Q3	Non-crisis (long growth)
2019 Q4 – 2020 Q2	Crisis (COVID-19)
2020 Q3 – 2022 Q2	Recovery
2022 Q3 – 2026 Q1	Energy and economic crisis

During the analysis, stocks of companies included in the DAX stock index were selected. Companies were included if historical stock prices were available from the year 2000 and if they are among the 30 largest companies making up the DAX index. For the purposes of the analysis, historical weekly stock price data available from 2000 were used, which were obtained from the Yahoo Finance database. The analysis is affected by the survivorship bias problem, meaning only companies whose price information was known throughout the entire observation period (companies with data from 2000) were considered. This approach may lead to an overestimation of the returns of the analyzed portfolios, but for the purpose of demonstrating the methodology used, this approach is sufficient.

Specifically, the following companies were analyzed:

Abbreviation	Company Name	Sector
ADS.DE	Adidas AG	Consumer goods (clothing)
AIR.DE	Airbus SE	Industry (aviation)
ALV.DE	Allianz SE	Finance (insurance)
BAS.DE	BASF SE	Chemical industry
BAYN.DE	Bayer AG	Pharmacy / agrochemicals
BEI.DE	Beiersdorf AG	Consumer goods (cosmetics)
BMW.DE	BMW AG	Automotive industry
CON.DE	Continental AG	Automotive / Technology
DBK.DE	Deutsche Bank AG	Banking
DTE.DE	Deutsche Telekom AG	Telecommunications

Abbreviation	Company Name	Sector
FME.DE	Fresenius Medical Care AG	Healthcare
FRE.DE	Fresenius SE & Co. KGaA	Healthcare
HEI.DE	Heidelberg Materials AG	Building materials
MRK.DE	Merck KGaA	Pharmaceuticals / Chemicals
RWE.DE	RWE AG	Utilities
SIE.DE	Siemens AG	Industry / Technology
VOW3.DE	Volkswagen AG (pref.)	Automotive

We have set $r_F = 0\%$ for the risk-free interest rate, which is a strong simplification acceptable for the purposes of applying the methodology in this paper.

4 COMPARISON OF OPTIMAL PORTFOLIO INDICATORS IN DIFFERENT PERIODS

Based on the identified crisis and non-crisis periods, portfolio optimization was carried out for each period on selected stocks from the DAX index data for the respective period. The goal of the optimization was to maximize the portfolio's Sharpe ratio, relation (1). In addition to the optimal weights and S_p , the average weekly return of the optimal portfolio was also calculated for each period, which characterizes the average growth of the portfolio during that period. The results of the optimization show that the performance of the portfolio varies significantly between different periods (Table 1).

Table 1 Parameters of optimal portfolios.

Period	Character of the period	Average weekly return of the optimal portfolio	Sharpe Ratio (S_p)
2000 Q1 – 2001 Q1	Non-crisis	0.449 %	0.11097
2001 Q2 – 2003 Q1	Crisis / stagnation after the dot-com bubble	-0.180 %	-0.02191
2003 Q2 – 2008 Q1	Non-crisis (expansion)	0.546 %	0.27263
2008 Q2 – 2009 Q2	Global financial crisis	-0.254 %	-0.05553
2009 Q3 – 2019 Q3	Non-crisis (long growth)	0.353 %	0.16171
2019 Q4 – 2020 Q2	Crisis (COVID-19)	0.204 %	0.05507
2020 Q3 – 2022 Q2	Recovery	0.239 %	0.10292
2022 Q3 – 2026 Q1	Energy and economic crisis	0.417 %	0.19256

The results of portfolio optimization show significant differences between the various phases of the economic cycle. During non-crisis periods, the optimized portfolio generally achieved higher performance and more favorable returns. The highest average return was recorded in the period from Q2 2003 to Q1 2008, which represented a phase of strong economic expansion before the global financial crisis (see Figure 1). During this period, the portfolio also achieved the best value of the optimization function, meaning the highest Sharpe ratio.

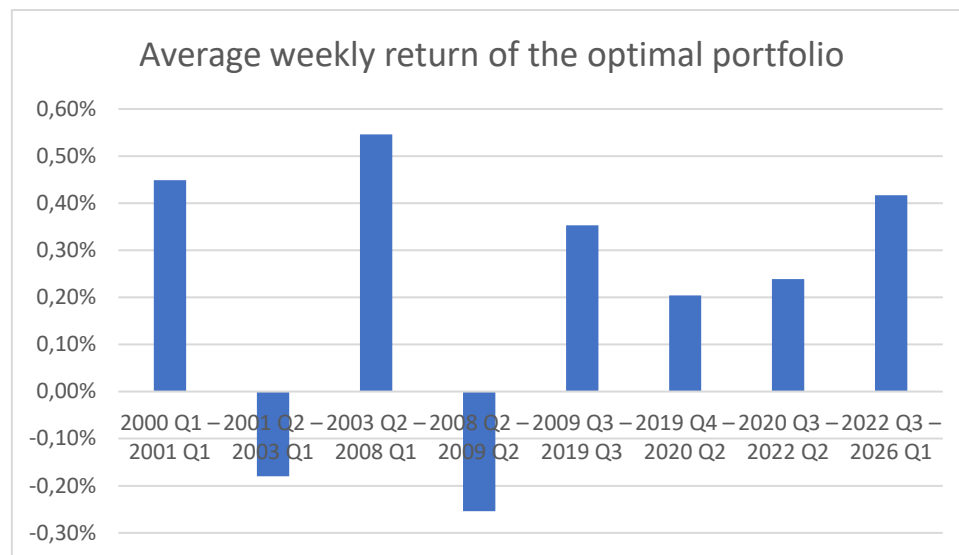


Figure 1 Return development for optimal portfolios.

On the contrary, during periods of crisis, the performance of the portfolio significantly worsened. The lowest average return was recorded during the global financial crisis from Q2 2008 to Q2 2009, when the optimal portfolio calculated based on data from that period achieved a negative average return. Similarly, a negative return was observed during the dot-com bubble period from Q2 2001 to Q1 2003. These results reflect an unfavorable situation in the financial markets, characterized by falling stock prices and increased volatility.

An interesting finding is that during the COVID-19 crisis, the optimized portfolio achieved a slightly positive average return. This result may be due to the relatively quick recovery of financial markets after the initial decline in 2020, as well as the appropriate diversification of the portfolio.

5 CONCLUSIONS

During non-crisis periods, such as from Q2 2003 to Q1 2008 or from Q3 2009 to Q3 2019, the optimized portfolio achieved a positive average return and favorable Sharpe ratio values. Conversely, during crisis periods, such as the dot-com bubble (Q2 2001 – Q1 2003) or the global financial crisis (Q2 2008 – Q2 2009), the average return of the portfolio was negative, reflecting deteriorated conditions in the financial markets.

The results also showed that the optimal weights of individual stocks vary significantly over different periods, indicating that the optimal portfolio structure is highly dependent on the phase of the economic cycle.

The results confirm that during non-crisis periods, Sharpe ratio values are higher, while crisis periods are characterized by a decrease in returns and an increase in volatility. The differences in the calculated Sharpe ratio values between crisis and non-crisis periods are relatively significant.

The results of the presented analysis suggest that static portfolio allocation is not optimal. Dynamic adjustment of the portfolio according to the phase of the cycle leads to higher risk-adjusted performance. The Sharpe ratio used for the analysis has its limitations, especially if the return distribution is not normal. Therefore, in the future, it would be appropriate to supplement the analysis with additional performance measures.

The paper demonstrates that the optimal portfolio is significantly dependent on the phase of the economic cycle. Crisis periods require more conservative strategies and higher diversification. The results have significant implications for investors and portfolio managers.

Acknowledgements: This work was supported by VEGA 1/0435/26 Investment Effectiveness and Spatial Spillover Effects in the EU Green Transformation: A Spatial Econometric and Machine Learning Approach.

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ISBN: 978-80-225-5329-2